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Corrigendum to "Eulerian rates of elastic incompatibilities for crystal plasticity applied to size-dependent hardening in finite bending" [Int. J. Solids Struct. 316 (2025) 113376]

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The authors regret to inform that, after the publication of the paper (Bardella et al., 2025), they found out a few oversights. The main reason for writing this corrigendum is to correct a mistake in an equation related to the size-dependent hardening, which has required revising some of the obtained numerical results. This notwithstanding, the conclusions drawn from this investigation remain unchanged.

Each of the unnumbered equations at the end of page 10 and at the beginning of page 11 contains the same two errors. The correct forms of those equations are

$$
{}_1\mathbf{s} \otimes \mathrm{curl}({}_1\mathbf{n}) + (\partial_1 \mathbf{s}/\partial \mathbf{x})({}_1\mathbf{s}\otimes \mathbf{e}_z - \mathbf{e}_z \otimes {}_1\mathbf{s}) = \left[\left(-\frac{\cos(2\alpha)}{r} + \sin(2\alpha)\frac{\partial\alpha}{\partial r}\right)\mathbf{e}_r + \left(\frac{\sin(2\alpha)}{r}+\cos(2\alpha)\frac{\partial \alpha}{\partial r}\right)\mathbf{e}_\theta\right]\otimes \mathbf{e}_z \,.
$$

$$
{}_2\mathbf{s} \otimes \mathrm{curl}({}_2\mathbf{n}) + (\partial_2 \mathbf{s}/\partial \mathbf{x})({}_2\mathbf{s}\otimes \mathbf{e}_z - \mathbf{e}_z \otimes {}_2\mathbf{s}) = \left[\left(-\frac{\cos(2\alpha)}{r} + \sin(2\alpha)\frac{\partial\alpha}{\partial r}\right)\mathbf{e}_r - \left(\frac{\sin(2\alpha)}{r}+\cos(2\alpha)\frac{\partial \alpha}{\partial r}\right)\mathbf{e}_\theta\right]\otimes \mathbf{e}_z \,.
$$

Because of this, Eq. (80) in (Bardella et al., 2025) must be modified to read

$$
\sum_{I=1}^{2} {}_I\Gamma \,\mathrm{curl}({}_I\mathbf{s} \otimes {}_I\mathbf{n}) = 2\Gamma\left(\frac{\sin(2\alpha)}{r}+\cos(2\alpha)\frac{\partial \alpha}{\partial r}\right)\mathbf{e}_\theta \otimes \mathbf{e}_z \,,
$$

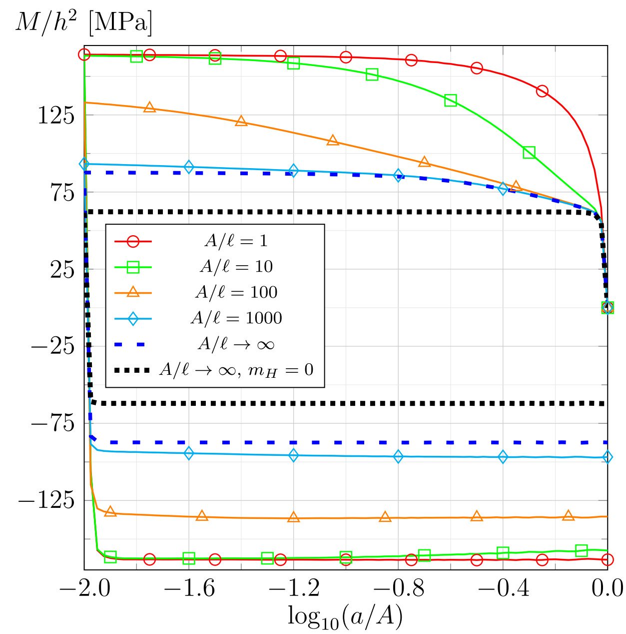
where, specifically, the denominator in the first addend on the right-hand side is r instead of $2r$. Then, other equations must be consistently modified, along with other minor corrections, as detailed below.

Equation (82) becomes

$$
\mathrm{curl}(\mathbf{L}_p) = \left[2\Gamma\left(\frac{\sin(2\alpha)}{r}+\cos(2\alpha)\frac{\partial \alpha}{\partial r}\right) + \frac{\partial \Gamma}{\partial r}\sin(2\alpha)\right]\mathbf{e}_\theta \otimes \mathbf{e}_z \,.
$$

The third relation in Eq. (83) becomes

$$
R_{23}^{\mathrm{ed}} = -2\Gamma\left(\frac{\sin(2\alpha)}{r}+\cos(2\alpha)\frac{\partial \alpha}{\partial r}\right) m'_{2\theta} m'_{3z} \,,
$$



Replacement Figure 9. For the caption see Bardella et al. (2025).

where also a previously missing minus sign has been added. Consistently, the third relation in Eq. (88) becomes

$$
R_{23}^{\mathrm{ed}} = -2\Gamma\left(\frac{\sin(2\alpha)}{r}+\cos(2\alpha)\frac{\partial \alpha}{\partial r}\right)\frac{1}{m'_{1r}} \,.
$$

Note that also the right-hand sides of the relations for R_{23}^{GND} in Eqs. (83) and (88) must change sign. However, fixing the sign in the expressions

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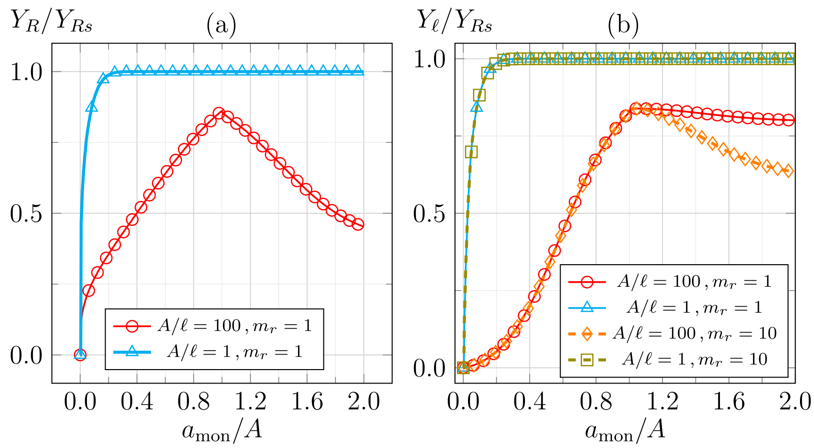
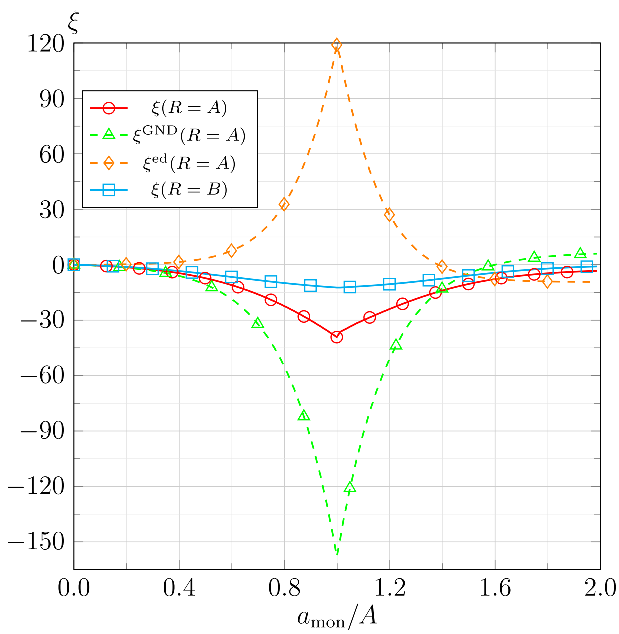


Replacement Figure 10. For the caption see Bardella et al. (2025).



Replacement Figure 13. For the caption see Bardella et al. (2025).

for $R_{23}^{\rm GND}$ and $R_{23}^{\rm ed}$ affects only the sign of the $\dot{\xi}^{\rm GND}$ and $\dot{\xi}^{\rm ed}$ fields, while it does not influence the elastoplastic response because only the moduli of $\dot{\xi}^{\rm GND}$ and $\dot{\xi}^{\rm ed}$ enter the size-dependent hardening.

Then, moving to the numerical results, Fig. 9 in (Bardella et al., 2025) has to be replaced by Fig. 9 presented here. The sole difference appears at about the end of the curve for $A/\ell = 1$, which, here, does not exhibit any softening. This is due to the corrected expression for $R_{23}^{\rm ed}$, which, for the selected material parameters, leads to a ξ field that does not sufficiently decrease to significantly reduce the size-dependent hardening relative to its saturated value Y_R. Further details are provided below in the comments on Fig. 13.

Figure 10 in (Bardella et al., 2025) has to be replaced by Fig. 10 presented here. Differences are again due to the corrected equation for $R_{23}^{\rm ed}$ that affects the evolution of the ξ field and, in turn, of the hardening variables. For the case $A/\ell = 100$, both Y_R and Y_ℓ reach a lower maximum value, at about the end of the loading (i.e., $a_{\rm mon}/A = 0.99$). For the case $A/\ell = 1$, similarly to what commented about Fig. 9, the ξ field does not sufficiently decrease during unloading to observe any reduction in both Y_R and Y_ℓ.

Figure 13 in (Bardella et al., 2025) has to be replaced by Fig. 13 presented here. This figure shows the largest difference due to the fixed error in the expression for $R_{23}^{\rm ed} = \dot{\xi}^{\rm ed}/\ell$, which affects the $\xi^{\rm ed}$ and ξ fields. First, it is recalled that, in the elastoplastic bending problem under investigation, the contribution to $R_{23}^{\rm ed}$ of the term depending on $\partial\alpha/\partial r$ is negligible. Therefore the correct $R_{23}^{\rm ed}$ (and, then, $\xi^{\rm ed}$) is about double (in magnitude) than that in (Bardella et al., 2025). This makes the contributions $\xi^{\rm GND}$ and $\xi^{\rm ed}$ to ξ to be even closer in magnitude, thus strengthening the main result of this investigation. Additionally, the value of $\xi(R = A)$ at the end of the analysis (i.e., at $a_{\rm mon}/A = 1.98$) now assumes a value ≈ 3.25, which is no longer small enough to observe a significant decrease in the size-dependent hardening variables, as commented about Figs. 9 and 10. Finally, the curves in Fig. 13 have opposite sign with respect to those in Bardella et al. (2025) because of the sign correction in the expressions of $R_{23}^{\rm GND}$ and $R_{23}^{\rm ed}$.

Table 1 and Figs. 12, 14, 15 in (Bardella et al., 2025) are also affected by the foregoing corrections. However, there are no significant differences to discuss along their corrected versions, such as those figures are not reported here for the sake of brevity.

Moreover, because of mistakes analogous to those affecting the computation of the curl in the main benchmark problem, in Appendix B of (Bardella et al., 2025), Eq. (B.4) becomes

$$
\begin{aligned}
{}_1\Gamma\,\mathrm{curl}({}_1\mathbf{s}\otimes{}_1\mathbf{n}) &= {}_1\Gamma[{}_1\mathbf{s}\otimes\mathrm{curl}({}_1\mathbf{n}) + (\partial_1\mathbf{s}/\partial\mathbf{x})({}_1\mathbf{s}\otimes\mathbf{e}_z - \mathbf{e}_z\otimes{}_1\mathbf{s})] \\
&= {}_1\Gamma\left[\left(-\frac{\cos(2\alpha)}{r} + \sin(2\alpha)\frac{\partial\alpha}{\partial r}\right)\mathbf{e}_r + \left(\frac{\sin(2\alpha)}{r} + \cos(2\alpha)\frac{\partial\alpha}{\partial r}\right)\mathbf{e}_\theta\right]\otimes\mathbf{e}_z\,,
\end{aligned}
$$

such that Eq. (B.6) must be modified to read

$$
\begin{aligned}
\mathrm{curl}(\mathbf{L}_p) = &\left[{}_1\Gamma\left(-\frac{\cos(2\alpha)}{r} + \sin(2\alpha)\frac{\partial\alpha}{\partial r}\right) + \frac{\partial\,{}_1\Gamma}{\partial r}\sin^2(\alpha)\right]\mathbf{e}_r\otimes\mathbf{e}_z \\
&+ \left[{}_1\Gamma\left(\frac{\sin(2\alpha)}{r} + \cos(2\alpha)\frac{\partial\alpha}{\partial r}\right) + \frac{1}{2}\frac{\partial\,{}_1\Gamma}{\partial r}\sin(2\alpha)\right]\mathbf{e}_\theta\otimes\mathbf{e}_z
\end{aligned}
$$

and the first two unnumbered equations in the second column on page 18 become

$$
\begin{aligned}
R_{13} = &-\left[{}_1\Gamma\left(-\frac{\cos(2\alpha)}{r} + \sin(2\alpha)\frac{\partial\alpha}{\partial r}\right) + \frac{\partial\,{}_1\Gamma}{\partial r}\sin^2(\alpha)\right]m'_{1r}m'_{3z} \\
&- \left[{}_1\Gamma\left(\frac{\sin(2\alpha)}{r} + \cos(2\alpha)\frac{\partial\alpha}{\partial r}\right) + \frac{1}{2}\frac{\partial\,{}_1\Gamma}{\partial r}\sin(2\alpha)\right]m'_{1\theta}m'_{3z}\,, \\
R_{23} = &-\left[{}_1\Gamma\left(-\frac{\cos(2\alpha)}{r} + \sin(2\alpha)\frac{\partial\alpha}{\partial r}\right) + \frac{\partial\,{}_1\Gamma}{\partial r}\sin^2(\alpha)\right]m'_{2r}m'_{3z} \\
&- \left[{}_1\Gamma\left(\frac{\sin(2\alpha)}{r} + \cos(2\alpha)\frac{\partial\alpha}{\partial r}\right) + \frac{1}{2}\frac{\partial\,{}_1\Gamma}{\partial r}\sin(2\alpha)\right]m'_{2\theta}m'_{3z}\,.
\end{aligned}
$$

As further minor corrections, the right-hand side of the unnumbered equation in between pages 3 and 4 holds only if the microstructural vectors remain orthonormal; thus, that equation should be simply understood as a definition for the transverse crystallographic direction, that is ${}_I\mathbf{t} = {}_I\mathbf{s} \times {}_I\mathbf{n}$. Also, in the third relation in Eq. (18) a minus sign is missing in front of its right-hand side; consistently, the third relation in Eq. (33) becomes

$$\dot{\xi}^{\text{ed}} = \ell \sum_{i=1}^{3} \sum_{j=1}^{3} R_{ij}^{\text{ed}} = -\ell \sum_{i=1}^{3} \sum_{j=1}^{3} \left[\sum_{I=1}^{N} {}_I\Gamma \, \text{curl}({}_I\mathbf{s} \otimes {}_I\mathbf{n}) \right] \cdot (\mathbf{m}_i' \otimes \mathbf{m}_j') \, .$$

It is worth noting that these last two oversights do not affect what follows them because the incorrect relations were never used to obtain any of the results presented in (Bardella et al., 2025).

Finally, the authors would like to apologize for any inconvenience caused.

CRediT authorship contribution statement

Lorenzo Bardella: Formal analysis, Writing – original draft, Writing – review & editing, Visualization. **M.B. Rubin:** Writing – review & editing. **Andrea Panteghini:** Software, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

Bardella, L., Rubin, M.B., Panteghini, A., 2025. Eulerian rates of elastic incompatibilities for crystal plasticity applied to size-dependent hardening in finite bending. Int. J. Solids Struct. 316, 113376. http://dx.doi.org/10.1016/j.ijsolstr.2025.113376.