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In this article we use a stochastic model with one representative firm to study business tax policy under default risk. We will show that, for a given tax rate, the government has an incentive to reduce (increase) financial instability and default costs if its objective function is welfare (tax revenue).

**Keywords:** Capital Structure, Default Risk, Business Taxation and Welfare

**JEL Classification:** H25, G33, G38

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# Business Tax Policy under Default Risk

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## Abstract

In this article we use a stochastic model with one representative firm to study business tax policy under default risk. We will show that, for a given tax rate, the government has an incentive to reduce (increase) financial instability and default costs if its objective function is welfare (tax revenue).

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# 1 Introduction

Business taxation and financial stability are intrinsically related.<sup>1</sup> This is mainly due to the fact that, despite recent financial crises, almost all tax systems encourage the use of debt over equity finance. Though this "debt bias" has been reduced by tax devices such as thin cap and earning stripping rules, it still persists (see, e.g., De Mooij and Hebous, 2018, and the articles quoted therein, as well as Sinn, 2010).

In this article, we use a simple stochastic model to analyze the effects of default on a representative firm's value, as well as on tax revenue and welfare (that is equal to the summation between the firm's value and the expected tax revenue).<sup>2</sup> Given this simple framework we show that welfare and tax revenue crucially depend not only on the relevant statutory tax rate but also on default risk and its expected cost.<sup>3</sup> It is worth noting that the cost of default is affected by both market conditions and default rules. Since a change in these rules is feasible, we can say that, to some extent, a policy-maker can affect default costs.<sup>4</sup> Similarly, volatility is affected by both systemic and firm-specific risk. If therefore a policy-maker can affect systemic risk, it is useful to study the effects of volatility on a firm's value as well as on welfare.<sup>5</sup>

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<sup>1</sup>For instance, Kocherlakota (2010) argues that bailouts are inevitable if the default of firms causes systemic failure. For this reason, he proposes a Pigouvian tax, aimed at offsetting negative externalities arising from financial instability.

<sup>2</sup>For simplicity, we assume symmetric information and full interest deductibility. For a detailed analysis on business taxation under asymmetric information, see Cohen et al. (2016) and the articles cited therein. Partial interest deductibility is left for future research.

<sup>3</sup>In this article we focus on the tax rate effects. For a complementary analysis on the effects of corporate tax base changes, see Panteghini and Vergalli (2016).

<sup>4</sup>Adalet McGowan and Dan (2018) provide a comprehensive analysis of insolvency procedures across OECD countries and find that they are quite heterogeneous. In particular, they state that "[a] comparison of the 2010 and 2016 values suggests that recent reform efforts have been largest for prevention and streamlining, with reforms observable in 11 countries, especially European countries (e.g. Portugal). This may reflect the fact that such measures have been recently endorsed by the European Commission and the IMF, in response to the crisis [...]. Barriers to restructuring have also declined in 10 countries, while reform activity affecting the personal costs to failed entrepreneurs has been less ambitious, with only Chile, Greece and Spain undertaking reforms since 2010". Of course, heterogeneous rules can lead to heterogeneous default costs.

<sup>5</sup>Here, we deal with one policy-maker who can implement both monetary and fiscal policies, although we are aware that separate entities deal with them. As pointed out by Sinn (2018) however, the separation of roles sometimes vanishes and a central bank can

As will be shown, welfare and tax revenue crucially depend on tax rates as well as on volatility and the expected cost of default.<sup>6</sup> In particular, welfare is decreasing in the tax rate under default risk. Tax revenue is always increasing in the statutory tax rate and hence no Laffer curve is found. Moreover, an increase in the expected cost of default raises tax revenue and decreases welfare. Similarly, volatility raises (reduces) the expected value of tax revenue (welfare). Thus, a more stable financial system is beneficial from a social point of view, although it leads to lower tax revenue and vice versa. A similar effect is found for the default cost which increases (reduces) tax revenue (welfare). This means that the government faces a policy dilemma. For any given tax rate indeed, the government should reduce (increase) financial instability and default costs if its objective function is welfare (tax revenue).

The structure of this paper is as follows. Section 2 introduces a simple trade-off model and calculates the value function of a representative firm, as well as tax revenue and the value of welfare. Section 3 provides a numerical analysis where the effects of changes in the tax rate, default risk and its expected cost are examined. Section 4 summarizes our findings, discusses their policy implications and discusses possible extensions of our analysis.

## 2 The model

In this section we apply a continuous-time model in line with Goldstein et al. (2001). By assumption a representative firm can borrow from a perfectly competitive credit sector, characterized by a risk-free interest rate  $r$ . Moreover, we let the firm's Earning Before Interest and Taxes (EBIT), defined as  $\Pi$ , be stochastic and follow a geometric Brownian motion:

$$\frac{d\Pi}{\Pi} = \sigma dz, \text{ with } \Pi_0 > 0, \quad (1)$$

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dramatically affect fiscal policy. The reverse may also be true when bank taxes are levied (Keen and de Mooij, 2016).

<sup>6</sup>Welfare analysis of business taxation has already been studied by other authors. See, e.g., Sørensen (2017) who shows that the socially optimal debt-asset ratio is 2-3% below the current debt level. As a consequence, a reduction in leverage due to limitations on interest deductibility would lead to a welfare gain of about 5% of corporate tax revenue. See also Gordon (2010) and Weichenrieder and Klautke (2008) who calculate the efficiency cost of a tax distortion in the debt-equity decision. However, these articles are based on a deterministic framework and therefore disregard the effects of default risk (and financial (in)stability).

where  $\Pi_0$  is the initial EBIT,  $\sigma$  is the instantaneous standard deviation of  $\frac{d\Pi}{\Pi}$ , and  $dz$  is the increment of a Wiener process.<sup>7</sup> Moreover, we introduce the following:

**Assumption 1** *At time 0, the firm borrows some resources and pays a coupon  $C$ , which cannot be renegotiated.*

**Assumption 2** *If the firm does not meet its debt obligations, default occurs and hence, the firm is expropriated by the lender.*

**Assumption 3** *The cost of default is equal to a percentage  $\alpha \in [0, 1)$  of the defaulted firm's value.*

Assumption 1 means that the firm sets a coupon and then computes the debt market value. Without arbitrage, this is equivalent to first setting the debt value and then calculating the effective interest rate. For simplicity, we also assume that debt cannot be renegotiated: this means that we apply a *static* trade-off approach where the firm's financial policy cannot be reviewed later.<sup>8</sup>

Assumptions 2 and 3 introduce default risk and its cost, respectively. Given (1), if the firm's EBIT falls to a given threshold value, denoted  $\bar{\Pi}$ , the firm is expropriated by the lender (assumption 2), who becomes equityholder. Default causes a sunk cost borne by the lender. By assumption, this cost is proportional to the value of the firm (assumption 3).

Following Goldstein et al. (2001) we also introduce the following:

**Assumption 4** *The threshold level  $\bar{\Pi}$  is chosen by equityholders at time 0.*

Assumption 4 implies that equityholders behave as if they owned a put option, whose exercise leads to default.<sup>9</sup>

Given these assumptions, the firm's net profit function is equal to  $\Pi^N = (1 - \tau)(\Pi - C)$ , where  $\tau$  is the relevant tax rate.

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<sup>7</sup>The general form of the geometric Brownian motion is  $d\Pi = \mu\Pi dt + \sigma\Pi dz$  where  $\mu$  is the expected rate of growth. With no loss of generality, here we set  $\mu = 0$ .

<sup>8</sup>The analysis of a dynamic trade-off model, where firms can subsequently adjust their capital structure is left for future research.

<sup>9</sup>For further details on the characteristics of default conditions see, e.g., Leland (1994) and Panteghini (2007a).

It is worth noting that a tax saving due to debt-finance arises as long as the business tax rate is higher than the lender's rate (see, e.g., Panteghini, 2007b). For simplicity, here we assume that the lender's pre-default tax rate is nil. When, however, default takes place, the lender becomes equityholder and is therefore subject to corporate taxation.

Finally, the firm's value function is equal to the summation between debt,  $D(\Pi)$ , and equity,  $E(\Pi)$ :

$$V(\Pi) = D(\Pi) + E(\Pi). \quad (2)$$

By maximizing  $V(\Pi)$ , the optimal coupon  $C$  will be found.

## 2.1 The value of debt and equity

Let us start with debt. According to assumption 3, the (sunk) default cost is a percentage  $\alpha$  of the defaulted firm. Hence, the lender will own  $(1 - \alpha)$  of the defaulted firm.<sup>10</sup> Using dynamic programming, we can therefore write debt as follows:

$$D(\Pi) = \begin{cases} (1 - \alpha)(1 - \tau)\Pi dt + e^{-r dt} \xi [D(\Pi + d\Pi)] & \text{after default,} \\ C dt + e^{-r dt} \xi [D(\Pi + d\Pi)] & \text{before default.} \end{cases} \quad (3)$$

As shown in Appendix (A.1), (3) can be rewritten as:

$$D(\Pi) = \begin{cases} \frac{(1-\alpha)(1-\tau)\bar{\Pi}}{r} & \text{after default,} \\ \frac{C}{r} + \left[ \frac{(1-\alpha)(1-\tau)\bar{\Pi} - C}{r} \right] \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_2} & \text{before default,} \end{cases} \quad (4)$$

where  $\beta_2 = \frac{1}{2} - \sqrt{\left(\frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0$ . As shown in (4), before default the debt value consists of two terms. The first one,  $\frac{C}{r}$ , is a perpetual rent which measures the debt value without default, while the second term accounts for the default effects. In particular, term  $\left(\frac{\Pi}{\bar{\Pi}}\right)^{\beta_2}$  measures the present value of 1 Euro contingent on the default event. After default, the lender becomes equityholder and the value of his/her claim is equal to  $\frac{(1-\alpha)(1-\tau)\bar{\Pi}}{r}$ .

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<sup>10</sup>As pointed out by Estrin et al. (2017), economic agents are sensitive to different elements of the default codes. Moreover, the authors show that some countries are more debt-friendly than others. All of these features are here summarized by our parameter cost  $\alpha$ . If therefore countries are debtor-friendly (-unfriendly),  $\alpha$  is expected to be lower (higher). For a detailed (and economic) analysis of default procedures see also Claesens et al. (2001).

Applying dynamic programming we can write the value of equity as follows:

$$E(\Pi) = \begin{cases} 0 & \text{after default,} \\ (1-\tau)(\Pi-C) dt + e^{-rdt} \xi [E(\Pi + d\Pi)] & \text{before default.} \end{cases} \quad (5)$$

As can be seen, after default the equity value is nil (according to assumption 2). As shown in Appendix (A.2), (5) can be written as:

$$E(\Pi) = \begin{cases} 0 & \text{after default} \\ \frac{(1-\tau)(\Pi-C)}{r} - \frac{(1-\tau)(\bar{\Pi}-C)}{r} \left(\frac{\Pi}{\bar{\Pi}}\right)^{\beta_2} & \text{before default} \end{cases} \quad (6)$$

As shown in (6), before default the equity value is given by the summation between the perpetual rent  $\frac{(1-\tau)(\Pi-C)}{r}$  and the loss contingent on the event of default,  $-\frac{(1-\tau)(\bar{\Pi}-C)}{r} \left(\frac{\Pi}{\bar{\Pi}}\right)^{\beta_2}$ .

## 2.2 Optimal default

Given (6), we can now calculate the default threshold point under debt financing. Following Leland (1994) and Goldstein et al. (2001), we solve the following problem:

$$\max_{\bar{\Pi}} E(\Pi). \quad (7)$$

Using (6) and rearranging the F.O.C. of (7) gives

$$\bar{\Pi} = \frac{\beta_2}{\beta_2 - 1} C < C. \quad (8)$$

This means that if the firm's net cash flow is negative, equityholders can decide whether to inject further resources to meet the firm's debt obligations or to default. As long as they pay the coupon they can exploit future recoveries in the value of their claim.

## 2.3 Optimal coupon

Substituting (4) and (6) into (2) gives the pre-default value of the firm: <sup>11</sup>

$$V(\Pi) = \frac{(1-\tau)\Pi}{r} + \tau \frac{C}{r} - \left[ (1-\tau) \alpha \frac{\bar{\Pi}}{r} + \tau \frac{C}{r} \right] \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_2}. \quad (9)$$

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<sup>11</sup>Recall that the after-default value of the firm is simply  $\frac{(1-\alpha)(1-\tau)\bar{\Pi}}{r}$ .

To find the optimal coupon we maximize (9) with respect to  $C$ . Differentiating the value function (9) with respect to  $C$  and rearranging gives:

$$C = \frac{\beta_2 - 1}{\beta_2} \left\{ \frac{\tau}{(1 - \beta_2) \left[ (1 - \tau) \alpha \frac{\beta_2}{\beta_2 - 1} + \tau \right]} \right\}^{-\frac{1}{\beta_2}} \Pi. \quad (10)$$

Given (10), it is straightforward to see that  $\frac{\partial C}{\partial \alpha} < 0$ . Moreover it is easy to prove that  $\frac{\partial C}{\partial \tau} > 0$ .<sup>12</sup> Finally,  $\frac{\partial C}{\partial \sigma^2}$  is expected to be negative for realistic parameter values. The intuition behind this result is straightforward: *coeteris paribus*, an increase in volatility (i.e.,  $\sigma^2$ ) is expected to anticipate default and its sunk cost. This induces firms to borrow less (and hence, choose a lower coupon).

## 2.4 Tax revenue and welfare

Using the value function (9), we can easily obtain the present value of tax revenue:

$$T(\Pi) = \frac{\tau}{r} \left[ \Pi - C + (C - \alpha \bar{\Pi}) \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_2} \right]. \quad (11)$$

Of course, tax revenue is increasing in  $\Pi$  and decreasing in  $C$ . Moreover, given the inequality in (8), the term  $\frac{\tau}{r} (C - \alpha \bar{\Pi}) \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_2}$ , which measures the present value of tax revenues contingent on default, is positive. So, we can say that default leads to a twofold effect: on the one hand, it causes a sunk cost; on the other hand, it causes an increase in tax revenue, equal to  $\tau (C - \alpha \bar{\Pi})$  for any short period  $dt$ .

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<sup>12</sup>Taking the log of (10) gives:

$$\log C = \log \left( \frac{\beta_2 - 1}{\beta_2} \right) - \frac{1}{\beta_2} \log \tau + \frac{1}{\beta_2} \log(1 - \beta_2) + \frac{1}{\beta_2} \log \left[ (1 - \tau) \alpha \frac{\beta_2}{\beta_2 - 1} + \tau \right] + \log \Pi.$$

Differentiating it with respect to  $\tau$  gives:

$$\frac{\partial \log C}{\partial \tau} = -\frac{1}{\beta_2} \left[ \frac{1}{\tau} - \frac{1 - \alpha \frac{\beta_2}{\beta_2 - 1}}{(1 - \tau) \alpha \frac{\beta_2}{\beta_2 - 1} + \tau} \right] = -\frac{1}{\beta_2} \frac{\alpha \frac{\beta_2}{\beta_2 - 1}}{\tau \left( (1 - \tau) \alpha \frac{\beta_2}{\beta_2 - 1} + \tau \right)} > 0.$$

As pointed out, the social welfare function is simply given by the summation between  $V(\Pi)$  and  $T(\Pi)$ , i.e.:

$$W(\Pi) = V(\Pi) + T(\Pi) = \frac{\Pi}{r} - \alpha \frac{\bar{\Pi}}{r} \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_2}. \quad (12)$$

As can be seen, without default  $W(\Pi)$  is equal to the perpetual rent  $\frac{\Pi}{r}$ . With default however, it is lower.

### 3 A numerical analysis

In this section we use a numerical simulation to study the effects of default on our representative firm's value as well as on tax revenue and welfare. In line with Bilicka and Devereux (2012) we set  $r = 5\%$ . Also, we let the initial value of EBIT be equal to 5. This allows us to normalize all the effects, since the perpetual rent  $\Pi/r$  is equal to 100. Parameter  $\tau$  is assumed to range from 0 to 50%: most statutory tax rates range between 20% and 30% (e.g., Sørensen, 2017, uses an average rate of about 27%), although lower rates are applied in many countries.

In line with Dixit and Pindyck (1994), we let the benchmark value of standard deviation  $\sigma$  be 20%. Moreover,  $\sigma$  is assumed to range from 10% to 40% in order to run our sensitivity analysis.

As regards the value of  $\alpha$ , the empirical evidence shows quite heterogeneous results. For instance, Andrade and Kaplan (1998) estimate distress costs of 10–23% of firm value for a sample of 31 highly leveraged transactions. Branch (2002) finds a total default-related cost that ranges between 12.7% and 20.5%. Davydenko et al. (2012) estimate the cost of default for an average defaulting firm to be 21.7% of the market value of assets. These costs are shown to range from 14.7% for bond renegotiations to 30.5% for default. Interestingly, Glover (2016) finds that the average firm expects a default cost equal to 45% of its value under default. However, this cost is estimated to be less (25%) among defaulted firms. Given these findings, we therefore let  $\alpha$  range from 10% to 50%.

It is worth noting that the cost of default depends not only on market conditions but also on default rules. This means that, to some extent, the government can affect the value of  $\alpha$  by changing the insolvency regulation.<sup>13</sup>

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<sup>13</sup>For instance, time-consuming default procedures are expected to increase  $\alpha$  and vice versa.

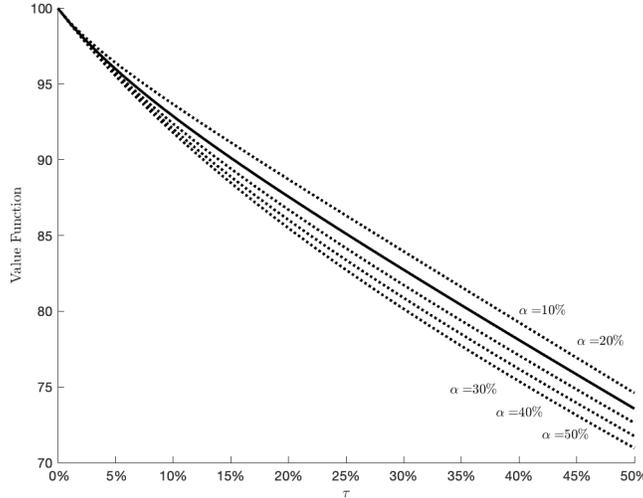


Figure 1: Sensitivity analysis of  $\alpha$  on the value function with respect to  $\tau$ .

Similarly,  $\sigma$  is affected by both systemic and firm-specific risk. If therefore our policy-maker wants to improve financial stability we expect a decrease in  $\sigma$ . For these reasons, we provide a numerical analysis which enables us to analyze the effects of  $\alpha$ ,  $\sigma$  and  $\tau$  on value function (9), tax revenue (11) and welfare (12), respectively. Our benchmark values will be  $\alpha = \sigma = 20\%$  and  $\tau = 25\%$ .

In figures 1-3 and table 1 we show the effects of  $\alpha$  and  $\tau$  on the firm's value, tax revenue and the welfare function, respectively, while in figures 4-6 and table 2, we show the effects of  $\sigma$  and  $\tau$  on the same functions. In all these figures the tax rate  $\tau$  is set on the horizontal axis.

In figure 1 we focus on the value function. The solid line shows the effects of  $\tau$  on the value function, with  $\alpha = \sigma = 20\%$ , while dotted lines show the same effects for different values of  $\alpha$ , with  $\sigma = 20\%$ . As can be seen, the firm's value is decreasing in  $\tau$ , despite the tax benefit arising from the deductibility of  $C$ . Not surprisingly, the greater the (sunk) default cost the lower the value of  $V(\Pi)$ .<sup>14</sup>

As shown in figure 2, tax revenue is increasing in  $\tau$ : this means that no Laffer curve exists. Moreover, if  $\tau$  is low enough (below 38%),  $\alpha$  has a positive

<sup>14</sup>Figures about equity and debt value (as a function of  $\tau$ ) are available upon request.

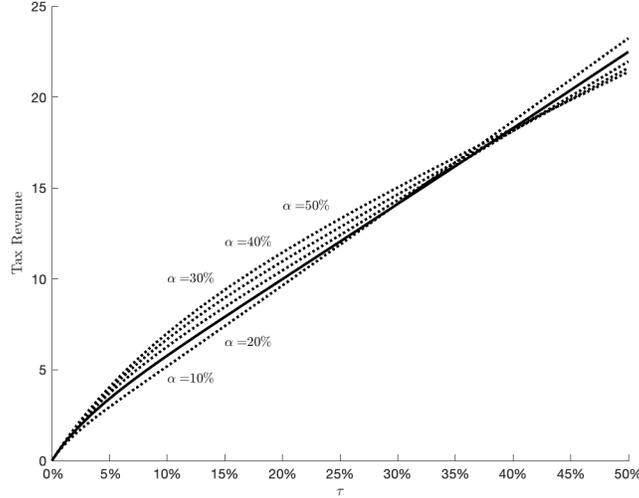


Figure 2: Sensitivity analysis of  $\alpha$  on tax revenue with respect to  $\tau$ .

impact on  $T(\Pi)$ . This is due to the fact that default has a twofold impact on our economic system. On the one hand, it causes a sunk cost; on the other hand, it increases tax revenue, due to the elimination of interest rate deductions (the value of such benefit is equal to  $\tau (C - \alpha \bar{\Pi}) dt$ ). If therefore  $\tau$  is low enough, this latter effect dominates the previous one.

Figure 3 shows the effects of  $\alpha$  and  $\tau$  on welfare (with  $\sigma = 20\%$ ). As can be seen, taxation causes a welfare loss. Moreover,  $\alpha$  has a negative impact on  $W(\Pi)$ : this is not surprising since default causes a default cost. In particular, the welfare loss is about 1.5% irrespective of the value of  $\alpha$ , if  $\tau$  is around 10%. When however the business tax rate reaches higher (and hence more realistic) values, the effects of  $\alpha$  on welfare are much more significant. For instance, if  $\tau$  is 25%, the welfare loss ranges from 1.81 (with  $\alpha = 10\%$ ) to 3.97 (with  $\alpha = 50\%$ ).

Table 1 provides a detailed sensitivity analysis for  $\alpha$ . As can be seen, the higher the parameters  $\tau$  and  $\alpha$ , the greater the welfare loss. To have an idea, assume  $\tau = 25\%$ . As can be seen, the ratio between the welfare loss and tax revenue ranges from 15.2% (with  $\alpha = 10\%$ ) to 29.85% (with  $\alpha = 50\%$ ). This suggests that, if the policy-maker can reduce  $\alpha$ , there may be a substantial welfare gain.

Let us next focus on figures 4 to 6. Again, the solid line shows the effects

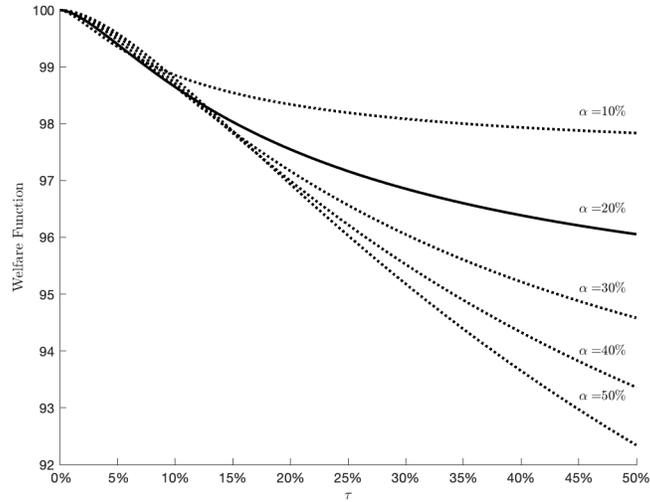


Figure 3: Sensitivity analysis of  $\alpha$  on the welfare function with respect to  $\tau$ .

Sensitivity Analysis on $\alpha$ (with $\sigma = 0.2$ )					
	<i>Value Function</i>				
	$\tau = 10\%$	$\tau = 20\%$	$\tau = 25\%$	$\tau = 30\%$	$\tau = 40\%$
$\alpha = 10\%$	93.66	88.70	86.31	83.95	79.26
$\alpha = 20\%$	92.87	87.56	85.11	82.73	78.10
$\alpha = 30\%$	92.38	86.70	84.16	81.72	77.08
$\alpha = 40\%$	92.03	86.03	83.38	80.87	76.18
$\alpha = 50\%$	91.78	85.49	82.73	80.15	75.37
	<i>Tax Revenue</i>				
	$\tau = 10\%$	$\tau = 20\%$	$\tau = 25\%$	$\tau = 30\%$	$\tau = 40\%$
$\alpha = 10\%$	5.19	9.64	11.88	14.14	18.68
$\alpha = 20\%$	5.78	9.99	12.05	14.12	18.29
$\alpha = 30\%$	6.28	10.47	12.41	14.33	18.14
$\alpha = 40\%$	6.68	10.97	12.85	14.65	18.15
$\alpha = 50\%$	7.01	11.45	13.30	15.03	18.28
	<i>Welfare Function</i>				
	$\tau = 10\%$	$\tau = 20\%$	$\tau = 25\%$	$\tau = 30\%$	$\tau = 40\%$
$\alpha = 10\%$	98.86	98.34	98.19	98.09	97.94
$\alpha = 20\%$	98.65	97.55	97.16	96.86	96.39
$\alpha = 30\%$	98.65	97.17	96.57	96.05	95.22
$\alpha = 40\%$	98.71	96.99	96.22	95.52	94.33
$\alpha = 50\%$	98.79	96.94	96.03	95.18	93.65

Table 1: Numerical results of the sensitivity analysis of  $\alpha$ .

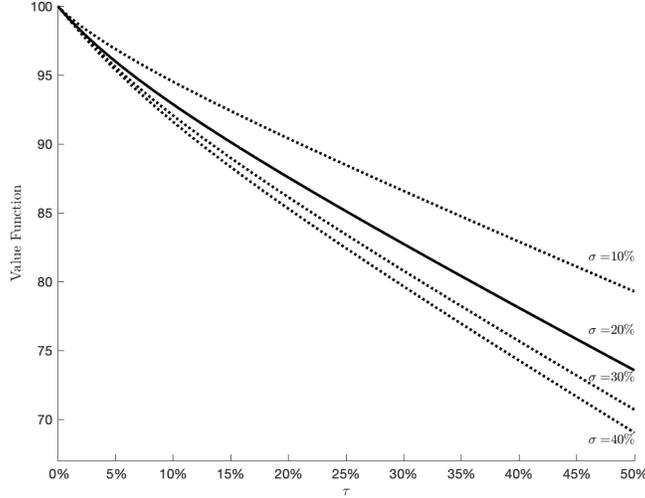


Figure 4: Sensitivity analysis of  $\sigma$  on the value function with respect to  $\tau$ .

of  $\tau$  on the firm's value (with  $\alpha = \sigma = 20\%$ ). Similarly, dotted lines show the effects of different values of  $\sigma$ , given  $\alpha = 20\%$ . If we compare the former three figures with the latter ones, the quality of results is similar. Again, the value function is decreasing in  $\tau$ , while tax revenue is always increasing.

As shown in figure 4, the higher the parameter  $\sigma$ , the lower the firm's value is. This is due to the fact that an increase in volatility (i.e., default risk) rises the contingent value of the default cost thereby reducing  $V(\Pi)$ .

Figure 5 shows that, for any  $\tau$ , volatility has a positive effect on  $T(\Pi)$ . As pointed out, an increase in  $\sigma$  makes default more likely. When it takes place,  $C$  is no longer deductible and hence tax revenue rises. On the contrary,  $W(\Pi)$  is decreasing in  $\sigma$  (see figure 6). This is not surprising since a (costly) default has a negative impact on welfare (see function (12)). Overall, our analysis shows that the negative effect of  $\sigma$  on the firm's value dominates the positive one on tax revenue.

Table 2 provides a sensitivity analysis for  $\sigma$ . As can be seen, default risk does have an important impact. If, for instance,  $\tau$  is 25%, the ratio between the welfare loss and tax revenue ranges between 10.73% (with  $\sigma = 10\%$ ) and 24.86% (with  $\sigma = 40\%$ ). Hence, for realistic values of  $\tau$ , a reduction in volatility may lead to a significant welfare gain.

These results have striking policy implications. If welfare is indeed the

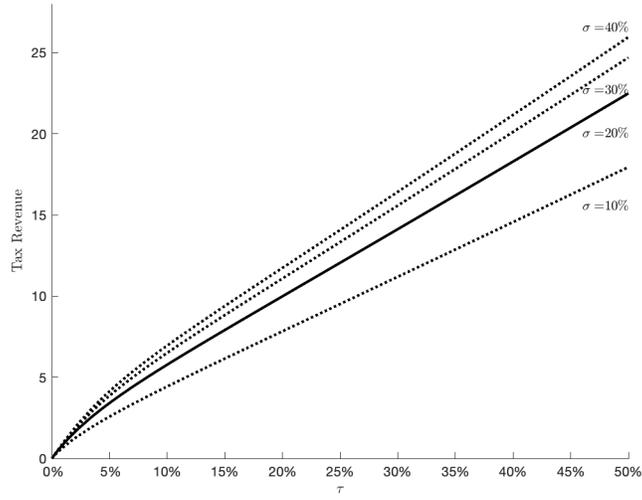


Figure 5: Sensitivity analysis of  $\sigma$  on tax revenue with respect to  $\tau$ .

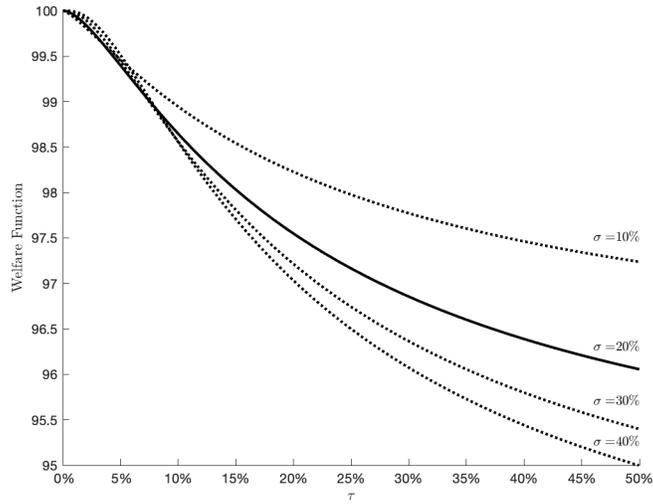


Figure 6: Sensitivity analysis of  $\sigma$  on the welfare function with respect to  $\tau$ .

Sensitivity Analysis on $\sigma$ (with $\alpha = 0.2$ )					
	<i>Value Function</i>				
	$\tau = 10\%$	$\tau = 20\%$	$\tau = 25\%$	$\tau = 30\%$	$\tau = 40\%$
$\sigma = 10\%$	94.52	90.39	88.46	86.58	82.90
$\sigma = 20\%$	92.87	87.56	85.11	82.73	78.10
$\sigma = 30\%$	92.06	86.13	83.41	80.79	75.68
$\sigma = 40\%$	91.60	85.29	82.42	79.65	74.28
	<i>Tax Revenue</i>				
	$\tau = 10\%$	$\tau = 20\%$	$\tau = 25\%$	$\tau = 30\%$	$\tau = 40\%$
$\sigma = 10\%$	4.43	7.84	9.51	11.19	14.56
$\sigma = 20\%$	5.78	9.99	12.05	14.12	18.29
$\sigma = 30\%$	6.50	11.08	13.33	15.58	20.11
$\sigma = 40\%$	6.95	11.74	14.08	16.42	21.16
	<i>Welfare Function</i>				
	$\tau = 10\%$	$\tau = 20\%$	$\tau = 25\%$	$\tau = 30\%$	$\tau = 40\%$
$\sigma = 10\%$	98.95	98.23	97.98	97.77	97.46
$\sigma = 20\%$	98.65	97.55	97.16	96.86	96.39
$\sigma = 30\%$	98.56	97.21	96.74	96.36	95.80
$\sigma = 40\%$	98.56	97.03	96.50	96.07	95.44

Table 2: Numerical results of the sensitivity analysis of  $\sigma$ .

relevant objective function, then the government has an incentive to reduce both  $\alpha$  and  $\sigma$ , for any  $\tau$ . If however, the government's objective function is tax revenue, the converse is true at least for  $\tau < 38\%$ .

## 4 Conclusion

In this article we have shown that, under default risk, taxation is welfare deteriorating. Moreover, we have shown that a government's policy crucially depends on whether its objective function is welfare or tax revenue. In particular, if welfare is the relevant objective function, then the government should improve financial stability (i.e. reduce systemic risk) and cut default costs. If however, the government's objective function is tax revenue, default risk and its costs would be beneficial.

In this article we have used a fairly simplified framework, which can be enriched in the future. For instance, further research could study the welfare effects of business taxation in a dynamic context, i.e., when a firm's capital structure can be subsequently modified and further investment can be made. Moreover, such an analysis could be replicated under asymmetric information

when agency costs arise.

## A Derivations of (4) and (6)

### A.1 The value of debt

Applying Itô's Lemma to (3) gives

$$rD(\Pi) = L + \frac{\sigma^2}{2}\Pi^2 D_{\text{III}}(\Pi), \quad (13)$$

where  $L = (1 - \alpha)(1 - \tau)\Pi, C$ , and  $D_{\text{III}}(\Pi) \equiv \frac{\partial^2 D(\Pi)}{\partial \Pi^2}$ . The general closed-form solution of function (13) is therefore equal to:

$$D^j(\Pi) = \begin{cases} \frac{(1-\alpha)(1-\tau)\bar{\Pi}}{r} + \sum_{i=1}^2 B_i \Pi^{\beta_i} & \text{after default,} \\ \frac{C}{r} + \sum_{i=1}^2 D_i \Pi^{\beta_i} & \text{before default,} \end{cases} \quad (14)$$

where  $\beta_1 = \frac{1}{2} + \sqrt{\left(\frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1$ , and  $\beta_2 = \frac{1}{2} - \sqrt{\left(\frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0$  are the two roots of the characteristic equation  $\Psi(\beta) \equiv \frac{1}{2}\sigma^2\beta(\beta - 1) - r = 0$ . To calculate  $B_i$  and  $D_i$  for  $i = 1, 2$ , we need three boundary conditions. Firstly, we assume that whenever  $\Pi$  goes to zero the lender's claim is nil, namely condition  $D(0) = 0$  holds: this implies that  $B_2 = 0$ . Secondly, we assume that financial bubbles do not exist: this means that  $B_1 = D_1 = 0$ . Thirdly, we must consider that at point  $\Pi = \bar{\Pi}$ , the pre-default value of debt must be equal to the post-default one, net of the default cost. Using the two branches of (14) we thus obtain

$$(1 - \alpha) \frac{(1 - \tau) \bar{\Pi}}{r} = \frac{C}{r} + D_2 \bar{\Pi}^{\beta_2}.$$

Rearranging gives  $D_2 = \left[ \frac{(1-\alpha)(1-\tau)\bar{\Pi}-C}{r} \right] \bar{\Pi}^{-\beta_2}$ . Hence, the value of debt is (4).

### A.2 The value of equity

Using (5) and Itô's Lemma, we obtain the following non-arbitrage condition:

$$rE(\Pi) = (1 - \tau)(\Pi - C) + \frac{\sigma^2}{2}\Pi^2 E_{\text{III}}(\Pi), \quad (15)$$

before default. Since, after default, the general-form solution of (15) is:

$$E(\Pi) = \begin{cases} 0 & \text{after default,} \\ (1 - \tau) \left( \frac{\Pi - C}{r} \right) + \sum_{i=1}^2 A_i \Pi^{\beta_i} & \text{before default.} \end{cases} \quad (16)$$

In the absence of financial bubbles,  $A_1$  is nil. To calculate  $A_2$ , we recall that default occurs when  $\Pi = \bar{\Pi}$ . In this case the value of equity falls to zero, namely,

$$E(\bar{\Pi}) = 0. \quad (17)$$

Substituting (16) into (17), and solving for  $A_2$  gives  $A_2 = -\frac{(1-\tau)(\bar{\Pi}-C)}{r} \bar{\Pi}^{-\beta_2}$ . Hence, the pre-default value of equity is equal to

$$E(\Pi) = \frac{(1 - \tau) (\Pi - C)}{r} - \frac{(1 - \tau) (\bar{\Pi} - C)}{r} \left( \frac{\Pi}{\bar{\Pi}} \right)^{\beta_2} \quad (18)$$

and zero otherwise. These results give (6).

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