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Vendor managed inventory with consignment stock agreement for a supply chain with defective items

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ABSTRACT

The pursuit of better coordination schemes is crucial for contemporary supply chains to survive in a highly competitive environment. A supply chain may employ revenue sharing, information sharing and quantity discounts etc. to gain that viable edge. Another recent trend is to store vendor's inventory at the buyer's warehouse, known as vendor-managed-inventory (VMI) with consignment stock (CS) agreement. This paper brings the present literature in VMI with consignment stock closer to real life by introducing the notion of screening defective items. A single-vendor single-buyer supply chain is considered where the vendor ships every production batch in a number of lots to the buyer's warehouse. The buyer withdraws and screens these products while fulfilling the market demand. An analytical model is developed to depict this scenario. The impact of different fractions of defective items, storage costs as well as disposal schemes is also studied. The results indicate that the prominence of the proposed storage scheme over the conventional one remains proportional to the size and number of shipments in a cycle. Besides, the non-financial component of storage costs has a critical impact on the cost of the supply chain.

1. Introduction

The industry today is striving hard to achieve more and more coordination in their joint businesses. Information technology has had a substantial impact in achieving this goal in contemporary supply chains. Scanners collect sales data at the point-of-sale and electronic data interchange (EDI) allows these data to be shared immediately with all stages of the supply chain [1]. The application of these technologies, especially in the grocery industry, has substantially lowered the time and cost to process an order, leading to impressive improvements in supply chain performance [2].

Recently, there has been a trend to adopt a new strategy for storing inventory in a supply chain, named as consignment stock (e.g. Braglia and Zavanella [3] and Persona et al. [4]). Vendor Managed Inventory (VMI) with consignment stocks (CS) agreement can be defined as 'stocks owned by the vendor' but on the buyer's premises and managed by the buyer [3]. The importance of this stocking policy comes from the fact that it (i) saves the buyer the investment in inventory value, (ii) assures the supplier/vendor of an almost captive buyer, and (iii) reassures the buyer that the supply is conveniently available. This policy ensures quantity and timing of the transfer of inventory from vendor to buyer. This new scheme for

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coordination between vendor and buyer is widely used by a number of well-known companies ranging from Walmart to Intel and Shell [5].

On the other hand, it is evident that traditional inventory models have a tendency to obtain an economic (optimal) order quantity (EOQ) or economic production quantity (EPQ) through the use of storage and ordering/setup costs. These models are usually based on a number of assumptions to get a closed form solution for the optimal batch size to stock/order or to produce. One of these assumptions is that the items produced by the vendor are all of a perfect quality. This assumption is very far from a real world supply chain. A vendor's lot may contain a significant number of defective items. These defective items may be a result of weak process control, deficient planned maintenance, inadequate work instructions and/or damage in transit [6].

The motivation for this paper comes from the popularity of the VMI scheme in that it diminishes the uncertainty of demand and makes the items readily available to a buyer whenever needed. It would be interesting to see how the complex supply chains of Walmart, Proctor & Gamble or Johnson & Johnson etc. can significantly benefit from screening out and disposing the defectives they receive from their suppliers. This paper develops a two level supply chain model for VMI with CS where the defective items from vendor are screened out by the buyer before fulfilling market demand. The rationale for this model is that (i) in many real-life situations vendors' lots are defective that result in heavy warranty and repair costs, (ii) VMI is a well-known practice in today's retailer–supplier relationships, (iii) a buyer may choose different schemes to dispose defective items. This choice of disposal schemes is even more significant in case the supplier is located at a remote distance. A sensitivity of the financial and nonfinancial components of holding costs along with a comparison with earlier works in CS will further enhance the contribution of this work. The rest of the paper is organized as follows. Section 2 provides a review of related literature. Section 3 describes the mathematical model. Section 4 presents numerical examples and discusses the results. Section 5 presents summary and conclusions.

2. Literature review

The notion of coordination is not new. Goyal [7] was one of the first few models to address the problem of a joint economic lot size for a system consisting of a vendor and a buyer. He assumed infinite production rate and a lot-for-lot policy for the shipment from the vendor to the buyer. Banerjee [8] relaxed the assumption of infinite production rate and assumed that the whole production lot is ready before the shipment. Goyal [9] followed the same one-vendor and one-buyer scenario to propose a number of equal-sized shipments of the production lots, after the whole lot is produced. Lu [10] extended upon this approach to allow shipments to take place during production. Goyal [11] proposed a different approach for the shipment of lots in a similar scenario. He suggested that the successive shipments within a production lot should increase by a factor equal to the ratio of production rate and demand rate. Hill [12] generalized the approach in Goyal [11] by assuming a successive growth factor as a decision variable. He concluded that the ratio of the unit holding cost of the vendor and the buyer may decide whether a batch should be delivered in lots of equal size.

The concept of economic ordering/production size has still not lost its significance and researchers are exploring new frontiers in this field. Zhang et al. [13] have recently derived an optimal inventory policy for the case where payments are made before delivery. They showed that the length of the advance payment does not influence this inventory policy. However, the discount associated with advance payment makes the replenishment cycle larger. Zhou et al. [14] developed EPQ (economic production quantity) models to evaluate the optimal make-or-buy decisions when a buyer faces a one-time-only discount offered by a vendor. They showed that their decision depends on the discounted price as well as its timing. Shu et al. [15] assumed that transportation lead time in a vendor–buyer supply chain is exponentially distributed and the transportation cost is a function of lot size. They showed that these assumptions lead to noticeable cost reductions when compared with the traditional coordination models.

The impact of quality has also been of great motivation for the researchers in this field. Rahdar and Nookabadi [16] have recently studied the coordination between a single vendor and multiple buyers where inventory levels decrease due to deteriorating items. They formulated buyer's synchronizing delivery days and their coordination with vendor's production cycle. They also suggested offering profit sharing to buyer to encourage them for cooperation. Lee and Kim [17] assumed deterioration to be an endogenous characteristic of products in a vendor–buyer supply chain. They conducted a sensitivity analysis to differentiate between the impacts of deterioration and quality in a two level supply chain.

Salameh and Jaber [18] presented the model that gave this field a new direction of research by introducing the idea of screening at a buyer's facility. Further, they assumed that nonconforming (low grade) items are withdrawn from inventory and as a single batch and sold at a salvage price (below purchase price) in a secondary market. This work has been extended for so many practical aspects. A review of some of those extensions can be found in Khan et al. [19].

Recently, there has been a growing interest in the field of supply chains about exploring the issues related to coordination in vendor managed inventory and consignment stock policy. Valentini and Zavanella [20] was the one who showed how consignment stock can be used in joint-profit maximizing models that use (s, S) policies. They used simulation experiments to demonstrate the rationale behind the implementation of a CS policy. Braglia and Zavanella [3] demonstrated that the most evident difference between consignment stock and the conventional model of Hill [12] lies in the location of the stocks.

The work of Braglia and Zavanella [3] has been extended to study a number of situations with the consignment stock policy. For example, Zanoni and Grubbstrom [21] provided explicit analytical expressions for the optimal lot size and the optimal total number of lots; Persona et al. [4] accounted for obsolescence in a supply chain; Zavanella and Zanoni [22] studied a multi-buyer system; Huang and Chen [23] demonstrated that all financial costs are borne by the vendor until the goods are used or sold. Battini et al. [24] extended the work of Braglia and Zavanella [3], Valentini and Zavanella [20], and Persona et al. [4] to bring in the issues such as safety stocks, stock-out risk and the availability of the storage space. They demonstrated that consignment stock policy is always convenient when compared with the EOQ policy. Braglia et al. [25] extended the work of Braglia and Zavanella [3] by assuming an uninterruptable batch processing at vendor's facilities. They demonstrated a step by step heuristic procedure to compute the number of batches, number of shipments and the size of each shipment in a production run. Bazan et al. [26] extended the model of Braglia and Zavanella [3] by screening defective items from a vendor's facility to either scrap or rework them before dispatching to the buyer. They studied interrupting the vendor's production process to reduce the number of defective items. They also showed that consignment stocking performs better than the classical coordination in a vendor–buyer relationship in the presence of defective items.

Some interesting and practical aspects in vendor managed inventory and consignment stock have been studied very recently. Among them are Jaber et al. [27] who developed a production, manufacturing and waste disposal model for a vendor–buyer supply chain that follows a consignment stock policy. They also analyzed the sensitivity of the collection rate of used items, production cost and the remanufacturing cost. Zanoni et al. [28] studied the consignment stock policy for a vendor–buyer supply chain to investigate the impact of transfer of learning in vendor's production cycle. They showed that learning rate and the interruption between production cycles significantly affect the total cost of the supply chain. Zanoni et al. [29] extended a model in the literature to account for a VMI-CS based coordination in a vendor–buyer supply chain while minimizing the amount of greenhouse gases emissions and the penalties paid for exceeding the annual quota. They showed that consignment stock mechanism may help in reducing the total cost as well as emission level of the supply chain studied. Ben-Daya et al. [30] studied consignment stocking scheme for a single vendor multiple buyers supply chain. They investigated three policies of coordination and found that VMI and CS is beneficial when the vendor has a flexible capacity. Mateen and Chatterjee [31] highlighted the benefits of vendor managed inventory through various models in a single vendor and multiple buyer supply chain. They presented a sensitivity analysis of different parameters to choose the best of the VMI schemes. A survey of different consignment stock models in supply chains can be found in Sarker [32]. They categorized these models on the basis of (a) structural configuration, (b) operational policies, and (c) cost and profit measurement. They pointed out comparative perspectives of different models ranging from practical applications to theoretical foundations. A very recent study by Zahran et al. [33] investigated the model in Braglia and Zavanella [3] for different payment schemes collected from real CS contracts. This is a significant improvement to the work in [3] that implied that payments by the buyer are made to the vendor are on continuous basis.

One of the missing aspects of the above literature is the efforts to reduce the fraction of defective items. Very few researchers have addressed the fact that products from a vendor would not be free of defects. This paper extends the work of Braglia and Zavanella [3] in this direction and incorporates the approach in Salameh and Jaber [18] for the products received at the buyer's warehouse. That is, the buyer would institute an inspection process for each lot to screen out the imperfect products. These imperfect items would be separated from the inventory and disposed after the screening is finished. This paper addresses several limitations of the model in Braglia and Zavanella [3] by (i) introducing a screening process at the buyer's facility to separate the defective fraction of each lot, (ii) introducing financial and non-financial component of holding costs of the two stakeholders, (iii) providing a sensitivity of these components, (iv) studying the difference between different schemes to dispose defective items, and finally by (v) comparing the results of the proposed model with models in the literature. Table 1 highlights the contribution in this paper.

It should be noticed that albeit the little similarity with the models in Bazan et al. [26] and Lee and Kim [17], the work in our paper is different in the following respects: (a) It extends the model in Salameh and Jaber [25] to present a simpler coordination scheme for a vendor–buyer supply chain (b) the screening takes place at the buyer's facility, and (c) it does not account for deterioration in the vendor's production process (d) it highlights the difference between financial and non-financial components of storage costs with the two stakeholders.

3. Model description

Consider a vendor–buyer supply chain in which the vendor produces in batches. Each batch is transported to the buyer in a number of equal size lots. Following the consignment stock policy the vendor uses the buyer's warehouse to store/stock its products. The buyer withdraws from this inventory based on the market demand. The buyer pays the products to the vendor on the basis of the quantity withdrawn and it is not related to the quantity stored (which is the property of the vendor). It is assumed that each lot withdrawn by the buyer contains a fixed fraction γ of defective items. An inspector screens out the defective items from the lot at a fixed rate. These defective products are disposed off from each lot after finishing the screening process. The behavior of the inventory level is illustrated in Fig. 1, where T is the cycle length. It should be noticed that the number of defective items (γq) may vary from one lot to another. The following nomenclature is used throughout the paper.

D	buyer's demand rate
P	vendor's production rate
Q	quantity of products transported per lot
λ	number of transport operations or lots per batch
Q	vendor's batch size ($Q=\lambda q$)

Table 1
Comparison of the proposed model with literature.

	Two level SCM	Defective items	Screening at buyer's facility	Screening at vendor's facility	Consignment stock	Deterioration at vendor's facility	Financial and non- financial holding cost	Sensitivity of holding cost components	Disposal schemes	Advance payment and discount	Learning in production	Vendor's capacity	Greenhouse gases	Energy usage
Hill [12]	X													
Goyal [7]	X													
Banerjee [8]	X													
Zhang et al. [13]	X									X				
Zhou et al. [14]	X									X				
Zanoni et al. [28]	X				X						X			
Ben-Daya et al. [30]	X				X							X		
Bazan et al. [36]	X												X	X
Salameh and Jaber [18]		X	X											
Braglia and Zavanella [3]	X				X									
Khan et al. [35]	X	X	X											
Lee and Kim [17]	X	X		X		X								
Bazan et al. [26]	X	X		X	X				X					
Jaber et al. [27]	X	X		X	X			X	X					
Model in the paper	**X**	**X**	**X**		**X**		**X**	**X**	**X**					

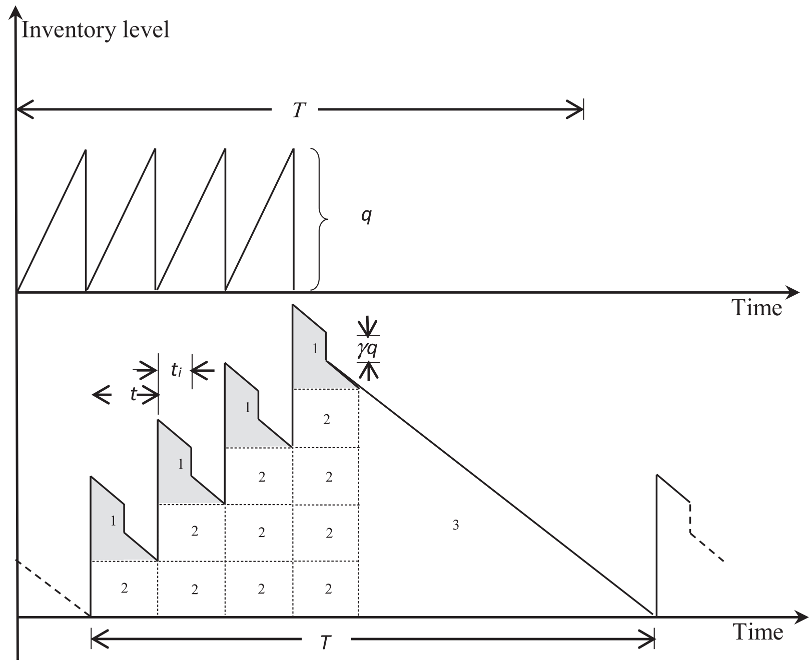

Fig. 1. The behavior of inventory for the vendor and the buyer (with $\lambda = 4$).

A_v	vendor's setup cost for a batch
A_b	buyer's order cost for each lot (λ orders per batch)
γ	fraction of defective items in a lot
X	buyer's screening rate
d	buyer's unit screening cost
h_b	buyer's unit holding cost (financial and non-financial components)
h_b'	buyer's unit holding cost (only financial component)
h_b''	buyer's unit holding cost (only non-financial component)
h_v	vendor's unit holding cost (financial and non-financial components)
h_v'	vendor's unit holding cost (only financial component)
h_v''	vendor's unit holding cost (only non-financial component)
T	cycle length

Using this notation, the objective of the paper is to determine an optimal inventory policy and the number of lots per batch, using the total annual cost of the supply chain. The impact of the fraction of defective items and the vendor and buyer holding costs on the annual cost of the supply would also be studied.

As shown in Fig. 1, the inventory with the buyer contains several profiles similar to those given by Salameh and Jaber [18]. Let us first of all determine the area of buyer's inventory profile in Fig. 1. Taking into account the expected value of the fraction of defectives, the area of the shaded profiles '1' would be:

$$Area\ 1 = \frac{1}{2}\left(\frac{q}{P}\right)\left(\frac{\lambda Dq}{P}\right) + (\lambda\gamma q)\left(\frac{q}{x}\right) = \frac{\lambda Dq^2}{2P^2} + \frac{\lambda\gamma q^2}{x} \tag{1}$$

Now for the rectangles '2', we notice that they increase by one every time the buyer starts his replenishment process. So, their area is given by:

$$Area\ 2 = \frac{\lambda(\lambda+1)}{2}\left(\frac{q}{P}\right)\left[(1-\gamma)q - \frac{Dq}{P}\right] = \frac{\lambda(\lambda+1)q^2}{2P}\left(1-\gamma-\frac{D}{P}\right) \tag{2}$$

The rest of the inventory profile is the big triangle '3'. Its area is:

$$Area\ 3 = \frac{1}{2}\left\{\lambda\left(\frac{(1-\gamma)q}{D} - \frac{q}{P}\right)\right\}\left\{\lambda\left(\left[(1-\gamma)q - \frac{Dq}{P}\right]\right)\right\} \text{ or}$$

$$Area\ 3 = \frac{\lambda^2 q^2}{2}\left(\frac{1-\gamma}{D} - \frac{1}{P}\right)\left(1-\gamma-\frac{D}{P}\right). \tag{3}$$

Therefore, the buyer's expected holding cost in vendor's one cycle of production is computed using the areas in (1), (2) and (3), as:

$$HC_b = h_b''\lambda q^2\left(\frac{D}{2P^2}+\frac{\gamma}{x}\right)+\frac{h_b''q^2}{2}\left(1-\gamma-\frac{D}{P}\right)\left\{\frac{\lambda(\lambda+1)}{P}+\lambda^2\left(\frac{1-\gamma}{D}-\frac{1}{P}\right)\right\} \tag{4}$$

The reader may refer to Hill [12] to compare this policy to a conventional stocking scheme. It should also be noticed that the above holding cost is different from that in Salameh and Jaber [18] in that it accounts for only the non-financial component, a finite production rate and a number of shipments from vendor: i.e. if $\lambda=1$, $h_b''=h$ and $P=\infty$,the above holding cost turns to Eq. (5) of [18].

Having computed the above holding cost, the buyer's total expected cost in a cycle, which is composed of ordering, holding, inspecting will be given by:

$$AC_b = \lambda A_b + d\lambda q + h_b''\lambda q^2\left(\frac{D}{2P^2}+\frac{\gamma}{x}\right)+\frac{h_b''q^2}{2}\left(1-\gamma-\frac{D}{P}\right)\left\{\frac{\lambda(\lambda+1)}{P}+\lambda^2\left(\frac{1-\gamma}{D}-\frac{1}{P}\right)\right\} \tag{5}$$

Similarly, the vendor's total expected cost in a cycle, would be:

$$AC_v = A_v + \frac{\lambda h_v q^2}{2P} + h_v'\left[\lambda q^2\left(\frac{D}{2P^2}+\frac{\gamma}{x}\right)+\frac{q^2}{2}\left(1-\gamma-\frac{D}{P}\right)\left\{\frac{\lambda(\lambda+1)}{P}+\lambda^2\left(\frac{1-\gamma}{D}-\frac{1}{P}\right)\right\}\right] \tag{6}$$

There are two elements of holding cost in this equation. That is the sum of financial and non-financial components for production while only financial component for the storage of inventory in Eqs. (1)–(3).

It should be noted from Eqs. (5) and (6) that the holding costs for stocking items at the buyer's warehouse, according to the VMI with consignment stock agreement, are carried by the buyer for the non-financial component while the financial component are carried by the vendor. One may need to understand that the financial component includes the investment in the space while the non-financial components include the cost due to physical storage, movement and insurance of the products. This point was not explicitly considered in the analytical model of Braglia and Zavanella [3] and the readers might end up underestimating the total costs, while it is clearly addressed in Valentini and Zavanella [20]. Thus, the total expected cost of the supply chain, after simplification is:

$$TC = A_v + \lambda A_b + d\lambda q + \frac{\lambda h_v q^2}{2P} + \left(h_b''+h_v'\right)q^2\left[\lambda\left(\frac{D}{2P^2}+\frac{\gamma}{x}\right)+\frac{1}{2}\left(1-\gamma-\frac{D}{P}\right)\left\{\frac{\lambda(\lambda+1)}{P}+\lambda^2\left(\frac{1-\gamma}{D}-\frac{1}{P}\right)\right\}\right]$$

Using a uniform distribution for the probability of defective items, this expected total cost can be written as:

$$\begin{aligned}\mathrm{E}[TC] &= A_v + \lambda A_b + d\lambda q + \frac{\lambda h_v q^2}{2P} + \left(h_b''+h_v'\right)q^2\\ &\times\left[\lambda\left(\frac{D}{2P^2}+\frac{\mathrm{E}[\gamma]}{x}\right)+\frac{1}{2}\left(1-\mathrm{E}[\gamma]-\frac{D}{P}\right)\left\{\frac{\lambda(\lambda+1)}{P}+\lambda^2\left(\frac{1-\mathrm{E}[\gamma]}{D}-\frac{1}{P}\right)\right\}\right]\end{aligned} \tag{7}$$

The expected length of a cycle is given by:

$$\mathrm{E}[T] = \frac{\lambda(1-\mathrm{E}[\gamma])q}{D} \tag{8}$$

The annual cost of the supply chain, using renewal reward theorem [34], would be written as:

$$\mathrm{E}[ATC] = \frac{\mathrm{E}[TC]}{\mathrm{E}[T]},$$

or

$$\begin{aligned}\mathrm{E}[ATC] &= \frac{M_2}{q}\left\{\frac{A_v}{\lambda}+A_b\right\}+M_2dD+\frac{h_vDM_2q}{2P}+\left(h_b''+h_v'\right)M_2qD\\ &\times\left[\left(\frac{D}{2P^2}+\frac{M_1}{x}\right)+\frac{1}{2}\left(1-M_1-\frac{D}{P}\right)\left\{\frac{(\lambda+1)}{P}+\lambda\left(\frac{1-M_1}{D}-\frac{1}{P}\right)\right\}\right],\end{aligned} \tag{9}$$

where $M_1 = \mathrm{E}[\gamma]$ and $M_2 = \frac{D}{1-\mathrm{E}[\gamma]}$.

For a fixed number of shipments for a batch, the quantity in each shipment would be:

$$q = \sqrt{\frac{\frac{A_v}{\lambda}+A_b}{\frac{h_v}{2P}+\left(h_b''+h_v'\right)\left[\left(\frac{D}{2P^2}+\frac{M_1}{x}\right)+\frac{1}{2}\left(1-M_1-\frac{D}{P}\right)\left\{\frac{(\lambda+1)}{P}+\lambda\left(\frac{1-M_1}{D}-\frac{1}{P}\right)\right\}\right]}}. \tag{10}$$

The above expression would be iterated to find an optimal number of lots per batch (λ) using the following algorithm:

1. Assume $\lambda=1$ and $Cost=\infty$
2. Compute the lot size using Eq. (10).

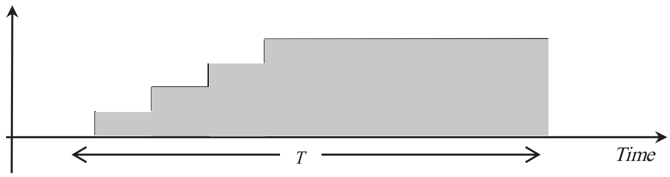

Fig. 2. Defectives screened out by buyer in a cycle.

3. Compute the annual cost of the supply chain using Eq. (9).
4. If $E[ATC] < Cost$, set $\lambda = \lambda + 1$ and repeat steps 1 through 4. Else Stop.

To bring in the notion of sustainability, it is assumed that defectives are salvaged at the end of the whole cycle, i.e. after they are accumulated as in Fig. 2, the annual cost in Eq. (9) will be written as:

$$\begin{aligned} E[TCU(q,\lambda)] &= \frac{DM_2}{q}\left(\frac{A_v}{\lambda} + A_b\right) + dDM_2 \\ &+ \frac{qM_2}{2}\left[\frac{h_v D}{P} + \left(h_b + h_v'\right)\left\{(1-M_1)\left(\lambda + \frac{D}{P}\right) + D\left(\frac{2M_1}{x} - \frac{\lambda}{P}\right)\right\}\right] \\ &+ \frac{\left(s_b + h_b + h_v'\right)}{2} M_1 \left\{2\lambda q - \frac{qDM_2}{P}(\lambda - 1) - \frac{2D^2M_2}{x}\right\}. \end{aligned} \tag{11}$$

The holding cost of defective items is derived in Appendix 2. Next, it is assumed that all the defectives screened in each shipment are repaired and utilized to fulfill demand. It should be noted that the repair process starts after the defectives from the first shipment are screened out. Thus, it would require a longer cycle time to utilize all the items in a cycle. Adding the cost of repair and replacing $M_2 = 1$, in Eq. (9):

$$\begin{aligned} E[TCU(q,\lambda)] &= \frac{D}{q}\left(\frac{A_v}{\lambda} + A_b\right) + dD \\ &+ \frac{q}{2}\left[\frac{h_v D}{P} + \left(h_b + h_v'\right)\left\{(1-M_1)\left(\lambda + \frac{D}{P}\right) + D\left(\frac{2M_1}{x} - \frac{\lambda}{P}\right)\right\}\right] \\ &+ \frac{\left(r_b + h_b + h_v'\right)}{2} M_1 \left\{2\lambda q - \frac{qD}{P}(\lambda - 1) - \frac{2D^2}{x}\right\}. \end{aligned} \tag{12}$$

4. Numerical analysis

Consider the two-level vendor–buyer supply chain model in Section 3. Most of the data is obtained from Salameh and Jaber [18]. It should be noticed that the non-financial component of storage cost at the buyer's end is taken to be higher than that at the vendor's end. The fraction of defectives is assumed to be uniformly distributed as given by:

$$f(\gamma) = \begin{cases} 25, & 0 \le \gamma \le 0.04, \\ 0, & otherwise. \end{cases}$$

The results indicate that following Salameh and Jaber [18] approach with the consignment stock policy (CS) addressed by Braglia and Zavanella [3], turns up decreasing the annual cost of the supply chain. The major contributors of this difference are the costs of inspection and that of storing the defective items. Fig. 3 shows the convex behavior of the model in this paper and that of Khan et al. [35]. It should be noticed that the practical methodology introduced in this paper would enforce the reduction not only in the fraction of defective products but the total cost of the supply chain as well. Besides, the shift in the cost is mostly governed by the difference in storage cost of the two stakeholders.

Fig. 4 shows the sensitivity of our model to the fraction of defective products. It was noticed that an increasing fraction of defective products goes on enhancing all the cost elements for the two stakeholders while buyer's screening costs and storage costs are affected the most. It should be noticed that Khan et al. [35] model would have a similar behavior against the fraction of defectives at a higher level of costs.

Next, Fig. 5 shows how the annual cost of the supply chain under two disposal schemes (salvage and repair) for different levels of fraction of defectives. For the given parameters of the model, it can be seen that 4 % defectives acts as a breakeven point for the two schemes. That is, it is advisable to salvage the defectives if the fraction of defectives is lesser, and vice versa.

The impact of different measures of storage cost of the two stakeholders was studied through simulation. Fig. 6 shows the behavior of annual cost in the proposed scheme with a ratio of financial and non-financial components of the storage cost of the two stakeholders (i.e. hb'/hv' and hb"/hv"). It can be seen that non-financial component of the storage cost is

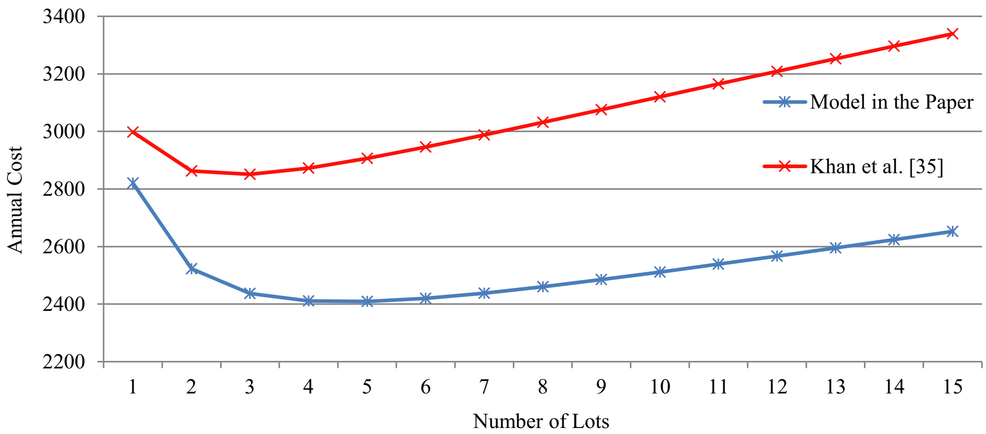

Fig. 3. Variation in the annual cost of the two models.

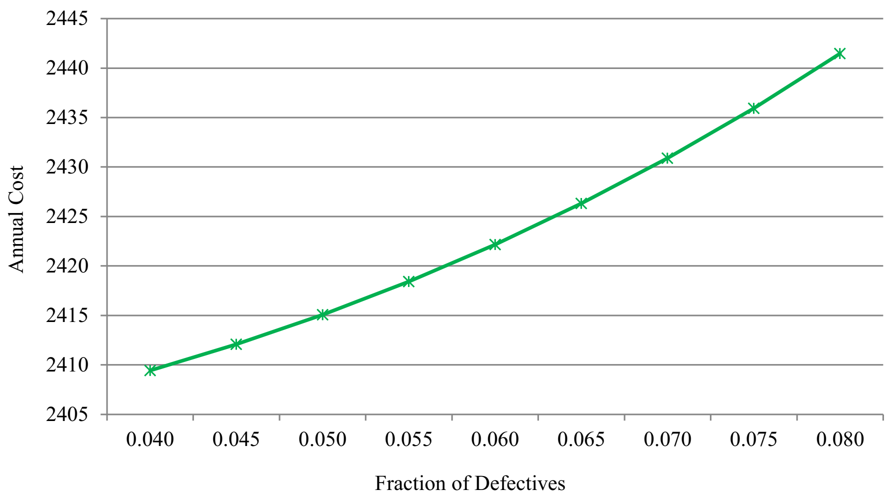

Fig. 4. Variation of the annual cost with fraction of defectives.

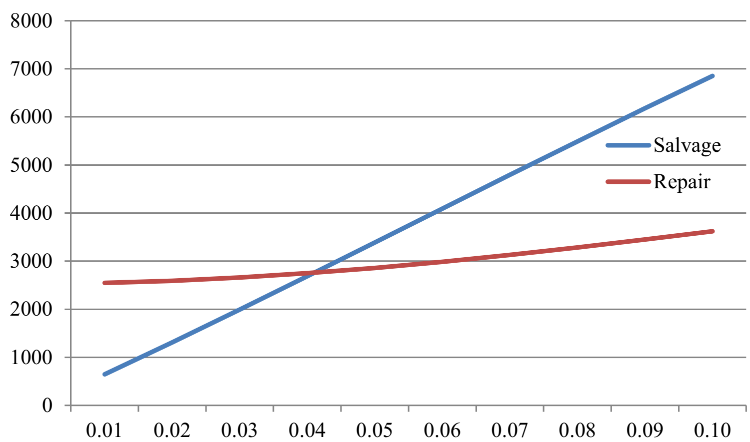

Fig. 5. Comparison of the two disposal schemes with different fraction of defectives.

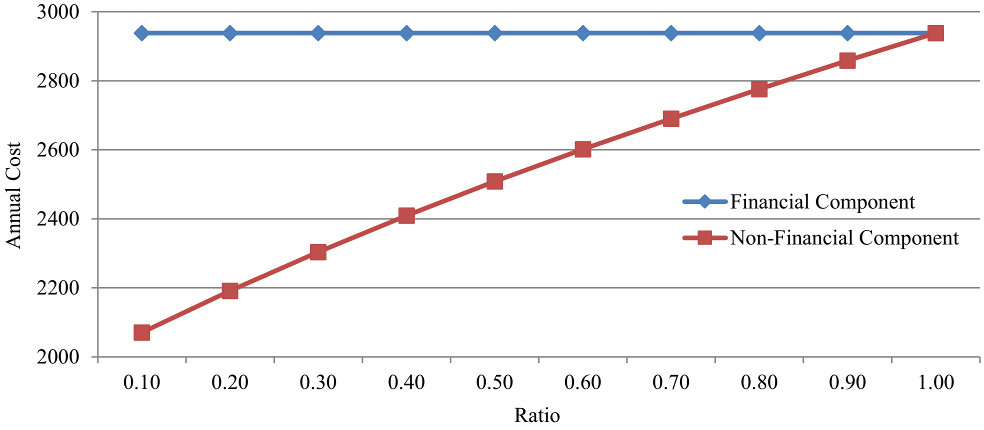

Fig. 6. Annual cost with the ratio of financial and non-financial components.

Table 2
Input data and results of the numerical example.

D	P	A_v	A_b	h'_v	h''_v	h'_b	h''_b	D	x
1000 units/yr	3200 units/yr	400 $/cycle	25 $/cycle	2 $/unit/yr	5 $/unit/yr	3 $/unit/yr	2 $/unit/yr	0.5 $/unit	175,200 unit/yr
				# of lots per cycle		Quantity per lot		Expected annual cost	
	Model in the paper			5		113		2409	
	Khan et al. [35]			3		138		2851	

critical as compared to the financial component. This demonstrates the significance of the proposed model especially in the presence of defective items, as it provides a tool for operations managers to choose an optimal level of investment in their storage cost.

To highlight the significance of consignment stocking scheme used in this paper, an analytical comparison is made with the conventional scheme, in Appendix 1. Using the data in Table 2, the difference in the two annual costs is plotted in Figs. A1 and A2. It can be seen that this difference goes higher and higher with the number as well as the size of shipments in a cycle. This shows the prominence of the proposed scheme in a two level supply chain.

The buyer in this paper uses the approach given in the model of Salameh and Jaber [18]. An interesting aspect would be to make sure that he would never incur shortages. A simulation test was carried out with three thousand examples, using the model of Salameh and Jaber [18]. The results showed that we can safely assume the non-shortages policy.

5. Conclusions

A single vendor, single buyer supply chain is studied in this paper. The vendor is supposed to make a single product and it is believed that a known fraction of their lots is defective. The buyer institutes a 100% inspection process to separate these defective products. They follow a consignment stock policy according to which the vendor keeps on supplying its inventory to the buyer's warehouse with regular intervals. The buyer withdraws from this warehouse according to the market demand. A model depicting this scenario is formulated to find an optimal lot size and the number of shipments per batch for the vendor.

The model shows sharp increase in the cost with respect to the fraction of defective items. The results also showed that the annual costs in our model are better than that in Khan et al. [35] when holding costs go higher than a threshold value.

The paper addresses some interesting issues related to operative practice in supply chains. In particular, the contribution from this study can be summarized as:

1. It extends the model of Braglia and Zavanella [3] to address an important practical aspect, i.e. the impact of defective items from a vendor.
2. It sheds light to some practical implications of the work by providing sensitivity analysis of certain parameters such as fraction of defective items, financial and non-financial components of the storage costs of the two stakeholders.
3. It highlights how the cost of a single-vendor-single-buyer supply chain in the presence of defective items and VMI compare to the other conventional models in the literature.
4. It compares the proposed storage scheme with the conventional one.
5. The results indicate that
   (a) the model in this paper performs better than the conventional model in the presence of defective items (Khan et al. [35]) – Fig. 3

(b) the percentage of defective items in a batch is critical to the relationship of the two stakeholders – Fig. 4
(c) non-financial component of storage costs has a critical impact on the cost of the supply chain – Fig. 6
(d) the prominence of the proposed scheme with respect to the conventional model remains proportional to the size and number of shipments in a cycle – Appendix 1

This model is equally beneficial for researchers and practitioners. Some of the implications of this study can be listed as

1. The numerical example reveals the significance (cost savings) for a retailer–buyer supply chain of lots that contain defective items. Besides, the proposed scheme gives both the partners a fair share of the supply chain profit.
2. The above savings offer operations and supply chain managers an added advantage over their competitors.
3. The investigation on fraction of defective items calls for research and investment to keep this fraction as low as possible.
4. The sensitivity analysis helps managers examine and streamline their investment into financial and non-financial parameters of storage cost.
5. The comparison with the conventional CS scheme suggests supply chain planners that the coordination remains more economical when the size and number of their shipments go higher. This would certainly entice retailers to increase their purchase quantity.

An interesting extension of the model in this paper is to account for environmental issues, such as emissions and energy-usage in production and transportation. That is, to extend the work of Bazan et al. [36] to include the concepts of quality and misclassification brought forth in this paper. Another extension would be to investigate if it is beneficial to scrap a fraction of defectives while using the two disposal schemes studied here.

Appendix 1. Comparison of the proposed scheme with Hill [12]

Considering Hill [12], vendor's total cost in a cycle is the sum of setup, storage and production costs:

$$C_v(q,\lambda) = A_v + \frac{h_v \lambda q^2}{2D}\left\{(\lambda-1) - (\lambda-2)\frac{D}{P}\right\}$$

The buyer's total cost in vendor's one cycle is the sum of ordering, storage and screening costs:

$$C_b(q,\lambda) = A_b + \lambda h_b \left\{\frac{q(1-\gamma)T}{2} + \frac{\gamma Q^2}{x}\right\} + \lambda d q$$

The total cost per unit time is:

$$\begin{aligned} TC(q,n) &= \frac{C_v(q,\lambda)}{nT} + \frac{C_b(q,\lambda)}{\lambda T}, \\ TC(q,n) &= \frac{A_v D}{(1-\gamma)\lambda q} + \frac{h_v q}{2(1-\gamma)}\left\{(\lambda-1) - (\lambda-2)\frac{D}{P}\right\} \\ &\quad + \frac{A_b D}{\lambda(1-\gamma) q} + h_b\left\{\frac{q(1-\gamma)}{2} + \frac{\gamma D q}{x(1-\gamma)}\right\} + \frac{dD}{1-\gamma}. \end{aligned} \tag{A1}$$

Using the notion of consignment stock:

$$\begin{aligned} C_v(q,n) &= A_v + \frac{h_v \lambda q^2}{2D}\left\{(\lambda-1) - (\lambda-2)\frac{D}{P}\right\} + \lambda h'_v\left\{\frac{q(1-\gamma)T}{2} + \frac{\gamma q^2}{x}\right\}, \\ C_b(q,n) &= A_b + \lambda h''_b\left\{\frac{q(1-\gamma)T}{2} + \frac{\gamma q^2}{x}\right\} + \lambda d q. \end{aligned}$$

The total annual cost is:

$$TC(q,n) = \frac{C_v(q,\lambda)}{\lambda T} + \frac{C_b(q,\lambda)}{\lambda T}$$

where,

$$\begin{aligned} T &= \frac{(1-\gamma)q}{D}, \\ TC(q,n) &= \frac{A_v D}{(1-\gamma)\lambda q} + \frac{h_v q}{2(1-\gamma)}\left\{(\lambda-1) - (\lambda-2)\frac{D}{P}\right\} + \frac{A_b D}{\lambda(1-\gamma)Q} \\ &\quad + \left(h'_v + h''_b\right)\left\{\frac{q(1-\gamma)}{2} + \frac{\gamma D q}{x(1-\gamma)}\right\} + \frac{dD}{1-\gamma}. \end{aligned}$$

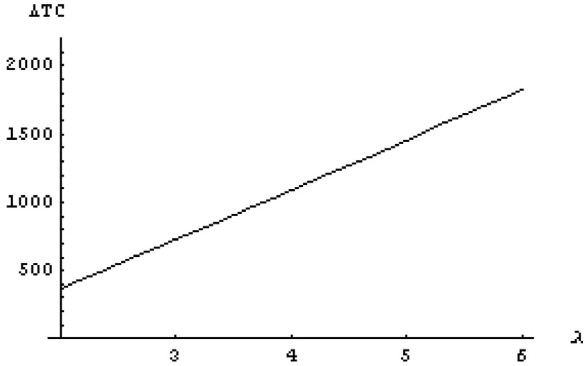

Fig. A1. The difference between two annual costs at $q = 300$.

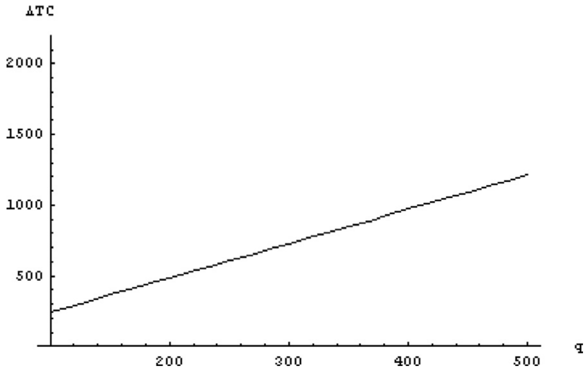

Fig. A2. The difference between two annual costs at $\lambda = 3$.

Rewriting Eq. (A2):

$$TC(q,n) = \frac{(A_v + A_b)D}{(1-\gamma)\lambda q} + \frac{dD}{1-\gamma} + \frac{h_v q}{2(1-\gamma)}\left\{(\lambda-1) - (\lambda-2)\frac{D}{P}\right\} + \left(h'_v + h''_b\right)\left\{\frac{q(1-\gamma)}{2} + \frac{\gamma Dq}{x(1-\gamma)}\right\}$$

Rewriting Eq. (7):

$$ATC = \frac{1}{(1-\gamma)q}\left\{\frac{A_v}{\lambda} + A_b\right\} + \frac{dD}{(1-\gamma)} + \frac{h_v qD}{2P(1-\gamma)}$$
$$+ \frac{\left(h''_b + h'_v\right)qD}{(1-\gamma)}\left[\left(\frac{D}{2P^2} + \frac{\gamma}{x}\right) + \frac{1}{2}\left(1-\gamma-\frac{D}{P}\right)\left\{\frac{(\lambda+1)}{P} + \lambda\left(\frac{1-\gamma}{D} - \frac{1}{P}\right)\right\}\right] \quad \text{(A2)}$$

In order to estimate the difference between expressions A1 and A2, assuming the buyer orders only once in a cycle, the first two terms can be avoided. Taking$(1-\gamma) = \theta$ and $(h'_v + h''_b) = h_c$ the above two expressions can be simplified as:

$$TC1 = \frac{h_v q}{2\theta}\left\{(\lambda-1) - (\lambda-2)\frac{D}{P}\right\} + h_c\left\{\frac{q\theta}{2} + \frac{\gamma Dq}{x\theta}\right\}$$

and,

$$ATC1 = \frac{h_v qD}{2P\theta} + h_c\left[\frac{\gamma qD}{\theta x} + \frac{\lambda\theta q}{2} - \frac{(\lambda-1)qD}{2P}\right]$$

So, the difference between the two expressions, would be:

$$\Delta TC = TC1 - ATC1 = \frac{h_v q}{2\theta}\left\{(\lambda-1) - (\lambda-2)\frac{D}{P}\right\} + h_c\left\{\frac{q\theta}{2} + \frac{\gamma Dq}{x\theta}\right\} - \frac{h_v qD}{2P\theta} - h_c\left[\frac{\gamma qD}{\theta x} + \frac{\lambda\theta q}{2} - \frac{(\lambda-1)qD}{2P}\right]$$
$$\Delta TC = \frac{(\lambda-1)q}{2}\left\{\frac{h_v}{\theta}\left(1-\frac{D}{P}\right) - h_c\left(\theta - \frac{D}{P}\right)\right\},$$

or

$$\Delta TC = \frac{(\lambda-1)q}{2}\left\{\frac{h_v}{(1-\gamma)}\left(1-\frac{D}{P}\right) - \left(h'_v + h''_b\right)\left(1-\gamma-\frac{D}{P}\right)\right\} \quad \text{(A3)}$$

The above expression is plotted in Figs. A1 and A2.

Appendix 2. Holding cost of defective items

Using Fig. 2, for the accumulation of defective items,

$$HC_d = \left(h_b + h'_v\right)\left[yz + 2yz + 3yz + \ldots + ny\left\{T - \frac{(n-1)q}{P} - \frac{D}{x}\right\}\right],$$

where,

$y = \gamma q,$
$n = \lambda,$
$z = \frac{q}{P}.$

The above expression simplifies to:

$$HC_d = \left(h_b + h'_v\right)\left[\frac{\gamma q^2 \lambda}{P}\left(\frac{\lambda - 1}{2}\right) + \lambda\gamma qT - \lambda\gamma q\left\{\frac{(\lambda-1)q}{P} + \frac{D}{x}\right\}\right]$$

Using the cycle length and the terms $M_1$and $M_2$the annual holding cost of these defective items would be:

$$HC_d = \left(h_b + h'_v\right)\left\{\lambda M_1 q - \frac{qDM_1M_2}{P}\left(\frac{\lambda-1}{2}\right) - \frac{D^2M_1M_2}{x}\right\}$$

or

$$HC_d = \frac{M_1\left(h_b + h'_v\right)}{2}\left\{2\lambda q - \frac{qDM_2}{P}(\lambda - 1) - \frac{2D^2M_2}{x}\right\}$$

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