Technical Note

Integration of power energy aspects into the Economic Lot Scheduling Problem

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Abstract
Beck et al. [2019. Integration of energy aspects into the economic lot scheduling problem. International Journal of Production Economics 209, 399-410] extended the Economic Lot Scheduling Problem (ELSP) to account for energy costs as well as tool change and inventory holding costs. In particular, the authors considered the cost arising from the product-dependent energy usage of the production facility during machine startups and shutdowns as well as during tool change, idle, and production phases. This note extends the model proposed by Beck et al. (2019) in two ways: it 1) considers variable production rates and 2) includes power-demand costs, i.e. the energy cost component that depends on the maximum power demand required, thus taking account of a more realistic representation of energy costs in the model.
The resulting problem is solved using the common cycle policy, and a numerical experiment is performed to investigate the behaviour of the proposed model. The experiment illustrates that the modified model leads to significant cost savings as compared to the traditional ELSP or the model proposed by Beck et al. (2019), which illustrates the potential usefulness of the proposed approach in practice for reducing energy costs.

Keywords. ELSP, Economic Lot Scheduling Problem, Energy Scheduling, Variable Production Rates

1. Introduction

The economic rebound after the COVID-19 pandemic and the conflict in Ukraine has led to a surge in energy prices, thus forcing most companies to look at viable ways for reducing energy costs.
Companies have two main approaches to improve energy efficiency:

1. Improve equipment, e.g. by upgrading facilities or by replacing old machines by more energy-efficient ones;
2. Use existing equipment in a more energy-efficient way.

Especially the second objective has attracted some attention in the industrial engineering literature recently, as it is usually not associated with high investment cost (see Biel and Glock, 2016, for an overview).

Beck et al. (2019) propose an extension of the classical Economic Lot Scheduling Problem (ELSP) by considering different machine operating states associated with different energy requirements. The machine operating states considered in their paper are: I) startup, II) tool change, III) processing, IV) idle, V) shutdown, and VI) off. The authors showed that taking account of machine operating states and the consequent energy requirements in the ELSP influences the scheduling of products on the machine and the resulting total costs. Hence, including energy costs explicitly in the ELSP may both lower total production costs and improve the energy efficiency of the company's production processes.
In developing their model, Beck et al. (2019) did not take into account the fact that production rates can often be varied in practice, and that they have frequently been considered as a main determinant of the energy demand during the actual processing of a product (e.g., Gutowski et al., 2006). Moreover, Beck and Glock (2020) publish a literature review about the evolution of the ELSP over time. In particular, after the examination of 242 papers, they underline a strong focus of the research about the development of solution methodologies, but not about aspects such as energy cost and sustainability. Böning et al. (2017) present a memetic algorithm with local search to generate a production plan by considering energy costs that result from the power peak. They show that energy costs affect the optimal solution. Compared to the present work, the size of the jobs is given in their model. Mokhtari and Hasani (2017) define a multi-objective optimization model for a flexible job-shop production system by considering production and maintenance operations together with energy consumption and three objectives (total completion time, total availability of the system, and energy consumption) in the objective function. They determine both the best sequence of operations on the machines as well as a suitable schedule for maintenance operations that minimizes energy consumption. Masmoudi et al. (2019) propose two linear models that minimize the energy cost for a job-shop scheduling problem considering the power peak limitation. These models do not consider lot sizing as in the model proposed in this research, and again assume a given job size. Their results show that the power limit changes the scheduling of the different jobs. Park and Ham (2022) propose an energy-efficient flexible job shop scheduling problem considering time-of-use pricing and scheduled downtime. They develop an integer linear programming model and a constrained model where it is possible to shift production to off-peak periods to minimize energy cost. As in the previous cases, the size of the jobs is given in this paper. Ferretti et al. (2022) propose an analytical method based on queueing theory useful to define the contractual power level and calculate the related service level in a productive system where it is not possible to obtain field data (green field design). With this work they underline the importance to consider the power requirements in the definition of the energy production systems contracts.
In the inventory control literature, research on variable production rates has mainly focused on single-item inventory models, with examples being the works of Khouja (1997), Glock (2010, 2011) and Zanoni et al. (2014). In the context of the ELSP, prior research has shown that lowering production rates may reduce inventory carrying cost, and it has therefore mainly concentrated on identifying those products for which the production rate should be reduced (e.g., Silver, 1990; Moon, 1991; Eynan, 2003). The influence of variable production rates on energy consumption has not yet been discussed in the context of the ELSP.
A second aspect that Beck et al. (2019) did not consider is that energy costs are usually not only linked to the energy consumed in kWh in practice. To correctly account for the cost of energy, it is necessary to add to the cost per kWh the peak power demanded during a given period, i.e., the cost per peak power demand in kW, such that a price premium has to be paid the higher the peak power demand, regardless of how much energy is consumed in

total during the period (e.g., Artigues at al 2013). Utility companies offer such pricing schemes to incentivize consumers to avoid peaks in their energy consumption, which helps to avoid periods of high energy demand on the grid level.

This note extends the work of Beck et al. (2019) to take into account variable production rates and a peak power cost component, and it investigates how varying the production rates and the power-demand cost influence both energy demand and the total cost. The paper is organized as follows: Section 2 defines the problem to be solved and its main assumptions, while Section 3 presents the model formulation and solution procedure. Section 4 offers a numerical example along with numerical results and sensitivity analyses. Finally, Section 5 covers conclusions and suggestions for discussion.

2. Problem definition, notation and assumptions

This research studies the Economic Lot Scheduling Problem (ELSP) where multiple items are produced on a single production facility. The paper assumes that the energy demand during the processing of items is a function of the production rate, and that the production rate can be varied within given limits.

In developing the proposed model, the following notation is used:

<u>Parameters</u>

i item index

n number of items produced on the facility

r_i demand rate of item i (units/h)

A_i setup cost of item i ($/setup)

s_i setup time of item i (h/setup)

h_i unit inventory holding cost of item i ($/h/unit)

v_i fixed energy component to process one unit of item i (kWh/unit)

f_i Multiplication factor representing the relation of the power required during the tool change of the machine for product i to the idle power of the machine (-)

e unit energy-based cost ($/kWh)

ep unit power demand-based cost that has to be paid for the maximum power demand ($/kW)/h: this corresponds to the ratio between the charge for the maximum rate at which the electricity is consumed during a specified period, and the length of the reference period (i.e. given a monthly charge of 20 $/kW and considering 160 productive hours in the month, $ep = 20/160 = 0.125$($/kW)/h.

g multiplication factor for the power required during the shutdown and startup of the production line (-), with $g>1$;

W idle power demand of the machine (kW)

l^{sdsu} sum of the durations of the shutdown and startup phases (h)

PL power-demand limit (kW)

p_i^{min} minimum production rate (units/h), with $p_i^{min} \geq r_i$

p_i^{max} maximum production rate (units/h)

<u>Decision variables:</u>

p_i production rate (units/h)

T cycle time (h)

<u>Definitions:</u>

SC total setup cost ($/h)

HC total inventory holding cost ($/h)

EC total energy-based cost ($/h)

PC total power demand-based cost ($/h)

TC total cost ($/h)

x non-productive time interval, that can be assigned as *idle* or *off* state

LBE break-even duration of the non-productive time interval to be assigned as *idle* state
z binary variable that is equal to 1 if the non-productive time is assigned to the idle state or equal to 0 if it is assigned to the off state

The objective of the proposed model is to minimize the total costs of producing a given set of items in a cyclic fashion, considering controllable production rates and energy costs. In addition to what has already been stated, this paper makes the following assumptions in developing the proposed model:

- Each product has a deterministic, known and constant demand rate.
- Setup costs and setup times do not depend on the production sequence.
- The production facility's capacity is sufficient to satisfy the entire demand over the planning horizon.
- Energy consumed during processing is a function of the production rate that can be adjusted for each item only once before the start of the production process. This has often been referred to as the "rigid case" in the literature; (see, e.g., Glock, 2011).
- During idle phases, the machine can be switched-off or be kept in the idle mode.

In order to decide on whether the machine should be switched off or be kept in the idle mode, we extend the concept of break-even duration (*LBE*) introduced by Mouzon et al. (2007). In particular, the machine should be shut down and switched on between processing two successive lots if the energy required to shut the machine down and start it up again is lower than the energy required to keep the machine in the idle state. Since in this study, we consider the effect of power on the total cost, it is necessary to introduce the power cost in the definition of the *LBE*. Therefore, the machine should be shut down and switched on if the energy costs and the power costs required to shut the machine down and start it up again is lower than the energy costs and the power costs required to keep the machine in the idle state. Assuming that the maximum production power is $\max_i\{W + v_i p_i\}$ and the power required during start-up and shutdown of the machine equals gW, the break-even duration *LBE* can be calculated by equating the energy costs and power costs of the idle state with the energy costs and power cost of switch-on/off state if $\max_i\{W + v_i p_i\} < g \cdot W$, otherwise *LBE* is calculated only considering the energy costs. The power cost is paid only for the maximum value between *Productive* and *Switched-off* state. In equation (1) we explicit the relation in case of $\max_i\{W + v_i p_i\} < g \cdot W$.

$$LBE \cdot W \cdot e = l^{sdsu} \cdot g \cdot W \cdot e + (g \cdot W - \max_i\{W + v_i p_i\}) \cdot ep \cdot T \quad (1)$$

Following, we integrate the formulation of the *LBE* presented by Beck et al. (2019) adding the power demand cost component.

$$LBE = \begin{cases} l^{sdsu} \cdot g & \max_i\{W + v_i p_i\} \geq g \cdot W \\ l^{sdsu} \cdot g + \dfrac{(g \cdot W - \max_i\{W + v_i p_i\}) \cdot ep \cdot T}{e \cdot W} & \max_i\{W + v_i p_i\} < g \cdot W \end{cases} \quad (2)$$

For each machine, we consider the following operating states associated with a certain power demand (note that some of the states discussed in Beck et al. (2019) have been aggregated in the list below):

- *Idle* state: the machine is turned on, but it does not produce; the energy consumed during this state is denoted *idle energy*;
- *Productive* state: the machine produces; the energy consumed during this state is denoted *processing energy*;
- *Switched-off* state: the machine is turned off (thus, its energy consumption is zero); at the end of a switch-off period, the machine must be turned on, which consumes energy referred to as *start-up energy*.

This work considers the common cycle policy for solving the ELSP (presented by Hanssmann in 1962), where the objective is to produce a set of items in a cyclic fashion, minimizing the total cost in only one cycle (common). Figure 1 illustrates the inventory levels of three items in an example. In the lower part of the figure, the cycle time T is split up into its components for the example with three products, namely setup times s_1, s_2 and s_3, processing times t_1, t_2 and t_3, and idle time t_{idle}. During t_{idle}, the facility can either be kept in an idle state, or it may be switched off and switched on again as soon as the next cycle starts. The decision whether to keep the facility in the idle state during t_{idle} is made based on its impact on the energy- and power demand-based costs.
Moreover, we introduce a limit for the power demand. Artigues et al. (2013) stated that the electricity bill is based on the cost of the energy consumed and on penalties for power overrun, in reference to a subscribed maximum power demand. In the following, the effects of introducing a power-demand limit (PL) are studied. The value of PL influences the production rates, as for each value of p_i, a related power-demand is computed.
In the following chapter we present in detail the mathematical model and the solution procedure.

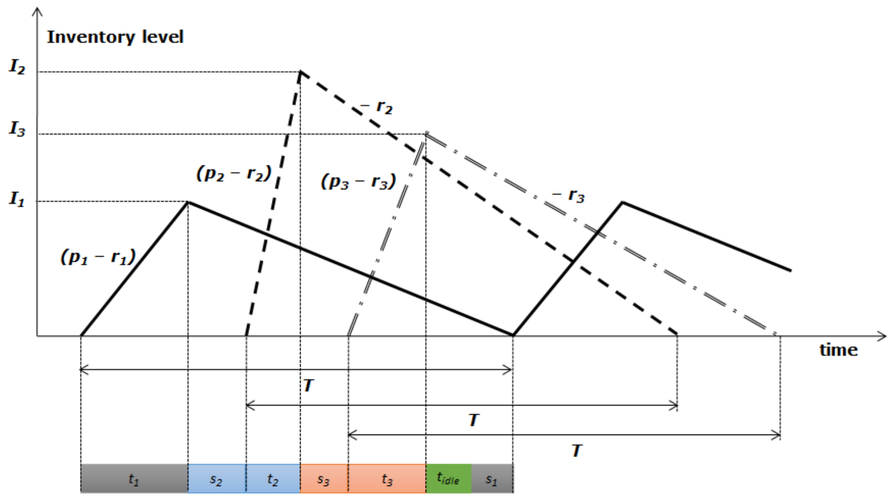

Figure 1. Inventory levels for three items scheduled according to the Common Cycle policy.

3. Model formulation and solution procedure

This section extends the Common Cycle model for solving the ELSP proposed in Beck et al. (2019) to explicitly take account of electricity costs depending on the items' production rates and the power peak cost and constraint.

The classical ELSP considers two types of costs, namely setup and inventory carrying costs:

$$SC = \sum_i \frac{A_i}{T} \quad (3)$$

$$HC = \sum_i \frac{h_i}{2} r_i \left(1 - \frac{r_i}{p_i}\right) T \quad (4)$$

In addition to setup and inventory carrying costs, energy-based costs are considered in the work at hand. Eq. (5) takes account of four energy-related costs, namely processing, setup, idle and switched-off/on energy cost:

$$EC = \sum_i (W + v_i p_i) \frac{r_i}{p_i} \cdot e + \sum_i \frac{f_i s_i}{T} \cdot W \cdot e + z \frac{x \cdot W \cdot e}{T} + (1 - z) \frac{g \cdot l^{sdsu} \cdot W \cdot e}{T} \quad (5)$$

where $z = \begin{cases} 1 & if \text{ x } < LBE \\ 0 & otherwise \end{cases}$

The first term (processing energy) is adapted from Gutowski et al. (2006) and Li and Kara (2011). The authors decompose processing energy-based cost into two components:

- when the machine is *idle*, a fixed amount of power is required by the auxiliary components to ensure that the machine is ready for operation when required. The power demand during this phase does not depend on the production rate, and it is therefore represented by a constant coefficient W in Eq. (5);
- when the machine produces, the power demand is a function of the production rate, and it is represented by the term $W + v_i p_i$. in Eq. (5).

During setups, the machine has a fixed, item-specific power demand that is independent of the production rate. The energy costs from setups are computed according to the second term in Eq. (5).

The third and fourth terms of Eq. (5) finally define the energy costs resulting from idle and switched-off/on states. The duration of these states is x units of time. If the machine is in the idle mode (i.e., if x < $g \cdot l^{sdsu}$), it requires W power-demand level during this time span. In contrast, if the machine is switched off and switched on again, it consumes energy during these operations, which takes l^{sdsu} units of time, and the power-demand level during this time span would be $g \cdot W$.

Finally, a cost component depending on the maximum power demand reached during the cycle is considered (see, e.g., Artigues et al., 2013):

$$PC = max\left\{\max_i\{W + v_i p_i\}; (1 - z) \cdot g \cdot W\right\} ep \quad (6)$$

The total cost can now be expressed as the sum of the cost components in Eqs. (3), (4) (5) and (6):

$$TC = SC + HC + EC + PC \quad (7)$$

The decision variables are the cycle time T and production rates p_i, $i = \{1, \dots, n\}$. Eq. (7) differs from the objective function proposed in Beck et al. (2019) by considering the power cost PC and by treating p_i as a decision variable.

To further highlight the novelty of the present work, Table 1 summarizes the different cost components of the objective functions of the Traditional model (Bomberger, 1996), the Energy model (Beck et al., 2019) and the Power model (present study).

Table 1. Cost components of the objective functions of the Traditional model, Energy model and Power model.

Model	Traditional costs		

	Setup costs SC	***Inventory carrying Costs HC***	**Energy costs EC**	**Power costs *PC***
Traditional (Bomberger, 1996)	X	X		
Energy (Beck et al., 2019)	X	X	X	
Power (present study)	X	X	X	X

The aim of the proposed "Power" model is to minimize the *TC* and find the optimal values of *T*, p_i and *z*.

$$min\, TC(T, p_i, z) \qquad (8)$$

Subject to:

$$r_i - p_i < 0 \qquad \forall\, i = 1, \dots, n \qquad (9)$$

$$p_i \le \frac{PL - W}{v_i} \qquad \forall\, i = 1, \dots, n \qquad (11)$$

$$x = T - \left(\sum_i s_i + \sum_i \frac{r_i}{p_i} \cdot T\right) \ge 0 \qquad (12)$$

In constraints (11), we consider that the power demand during production has to be less than the power limit *PL* (it should be noted that the power limit *PL* must be greater than the power used during the switch-on phase *gW*). Constraint (12) defines that the non-productive time interval *x*, that can be assigned as *idle* or *off* state, has to be greater than or equal to zero; this constraint also guarantees that the production facility's capacity is sufficient to satisfy the demand.

In the solution of the non-linear problem, we determine the energy costs resulting from idle or switched-off/on states and the cost related to the power demand based on the break-even duration. In order to determine if the machine is in the idle state or in switched-off/on states, we have to calculate *x*.

As shown in Figure 2, we specify that if:

- *x* > *LBE*, then the machine should be shut down and switched on, so the additional energy cost will be $\frac{g \cdot l^{sdsu} \cdot W \cdot e}{T}$, while the cost related to the power demand will be corrected to $max\left\{\max_i\{W + v_i \cdot p_i^*\}\,; g \cdot W\right\} ep - \max_i\{W + v_i \cdot p_i^*\}\, ep.$
- *x* <= *LBE*, then the machine is in idle state, the additional energy cost will be $\frac{x \cdot W \cdot e}{T}$, while the cost related to the power demand does not change because the power demand during the idle mode is less than the power demand during production.

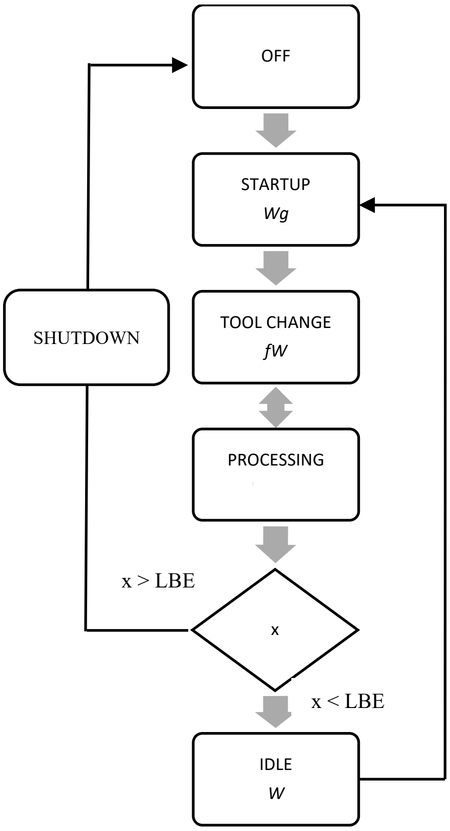

Figure 2. Machine operating states and associated power demands.

As reported in Khouja (1997), no proof of convexity of the objective function where p_i are variables in a specific range can be provided. So, it is not possible to determine a closed form that permit to obtain the optimal value of T and p_i. For this reason, we solve the non-linear optimization problem presented above by using Matlab (R2022b).

4. Numerical analysis

This section develops a numerical analysis to investigate the influence of variable production rates and the peak power demand penalty on the ELSP using the dataset introduced by Bomberger (1966) and Beck et al. (2019).

4.1 Base case solution

Table 2 introduces the dataset of Bomberger (1966) that was extended to take account of the energy consumption coefficient v and the multiplication factor f (Beck et al., 2019).

Table 2. Dataset used for numerical experimentation (Bomberger, 1966; Beck et al., 2019)

Product (*i*).	Demand rate	Unit holding cost	Setup cost	Setup time	Multiplicator factor	Energy cons. coeff.
	r_i	h_i	A_i	s_i	f_i	v_i
	items/h	$/(item x h)	$/setup	h/setup	-	kWh/item
1	50	3.39E-07	15	1	1.25	0.0050
2	50	9.24E-06	20	1	1.5	0.0400
3	100	6.64E-06	30	2	2.5	0.0100
4	200	5.21E-06	10	1	1.25	0.0250
5	10	1.45E-04	110	4	3	0.0650
6	10	1.39E-05	50	2	2.5	0.0300
7	3	7.81E-05	310	8	1.5	0.0200
8	42.5	3.07E-04	130	4	2.75	0.0500
9	42.5	4.69E-05	200	6	1.75	0.0700
10	50	2.0z8E-06	5	1	2	0.0150

Table 3 presents data related to energy-based and power demand-based cost components taken from the work of Beck et al. (2019). We introduce in this table a new data *ep*, unit power demand-based cost, not considered in the work of Beck et al. (2019).

Table 3. Energy data used for numerical experimentation (Beck et al., 2019).

Idle power-demand	***W***	kW	50
Unit energy-based cost	***e***	$/kWh	0.15
Unit power demand-based cost	***ep***	$/kW	0.1
Multiplication factor for power during shutdown	***g***	-	4
Shutdown time limit	l^{SDSU}	h	2
Power Limit	***PL***	kW	200

The goal of this work is to introduce a new model that considers variable production rates and a peak power cost component in order to investigate how varying these components influences both energy demand and the total cost of production. In this section, we compare the behaviour of three different models: the *Traditional model*, i.e., the model presented by Bomberger (1966) that solves the ELSP using the Common Cycle method, the *Energy model*, i.e., the model presented by Beck et al. (2019) that solves the ELSP by considering the energy cost using the common cycle method, and the *Power model*, i.e., the model introduced in this work that solves the ELSP by considering the energy cost and the power cost using the common cycle method. In order to compare the different models in terms of the total cost, we calculate the energy cost and power cost for the *Traditional model* and the power cost for the *Energy model*. This is necessary to ensure that all models consider the same cost components in the comparison. To calculate optimal solutions, the different models were coded in Matlab R2022b (see Appendix A) and results were obtained on a computer equipped with an Intel Core i7 processor in less than 10 seconds for each instance. Table 4 presents the results for each cost component.

Table 4. Results obtained for the optimal solutions of three models.

	Traditional model (Bomberger, 1966)	***Energy model*** (Beck et al., 2018)	***Power model*** (present study)
Total cost (*TC*) [$]	**24.60**	**24.59**	**23.26**
Setup cost (*SC*) [$]	2.57	2.08	2.34
Inventory holding cost (*HC*) [$]	2.57	3.18	2.99
Traditional cost (*SC* + *HC*) [$]	**5.15**	**5.26**	**5.33**
Energy-based cost (*EC*) [$]	10.46	**10.33**	10.39
Power demand-based cost (*PC*) [$]	9.00	9.00	**7.54**
Electricity cost (*EC*+*PC*) [$]	**19.46**	**19.33**	**17.94**

The results show that the cost components that are traditionally considered in the ELSP (*SC*+*HC*) are lowest in the *Traditional model*, while *EC* is lowest in the *Energy model*. *PC* adopts its lowest value in the *Power model*. The solution obtained in the *Power model* leads to an increase of 0.73% in traditional cost compared to the *Traditional model* and to an increase of 0.67% in energy cost compared to the *Energy model*, while we have a reduction of the power cost of 5.38% in the *Energy model*. The total costs obtained in the *Power model* are the lowest, as expected.

Figure 3 presents the power-demand profile for the three models. Comparing the *Traditional model* and the *Energy model*, we can see that the energy demand decreases (represented by the area under the lines of power-demand profile), while peak-power demand increases (higher value of power-demand profile). The *Power model* exhibits both lower energy demand and lower peak-power demand. In particular, the *Traditional model* schedule requires a maximum power-demand of 90 kW, the *Energy model* schedule requires a maximum power-demand of 90 kW, and the *Power model* schedule, according to the power limit fixed, requires a maximum power-demand of 75,44 kW (table 5).

Table 5. Results obtained for the three models.

	Traditional model (Bomberger, 1966)		***Energy model*** (Beck et al., 2018)		***Power model*** (present study)	
OptimalCycle time (*T*) *[h]*	**342**		**422**		**376**	
Product (*i*)	**Production Rate (p_i) [items/h]**	**Power [kW]**	**Production Rate (p_i)[items/h]**	**Power [kW]**	**Optimal Production Rate (p_i)[items/h]**	**Power [kW]**
1	3750	68.75	3750	68.75	3540	67.7
2	1000	90	1000	90	452	68.08
3	1187.5	61.875	1187.5	61.875	1808	68.08
4	937.5	73.4375	937.5	73.4375	723	68.075
5	250	66.25	250	66.25	278	68.07
6	750	72.5	750	72.5	603	68.09
7	300	56	300	56	904	68.08
8	162.5	58.125	162.5	58.125	220	61
9	250	67.5	250	67.5	258	68.06
10	1875	78.125	1875	78.125	1696	75.44

As shown in Figure 3, in the *Power model*, the change of the production rates, with respect to the *Traditional model* and *Energy Model*, permits minimizing the power demand and related costs. By considering this scenario data the switch-off/on state is avoided by all three models.

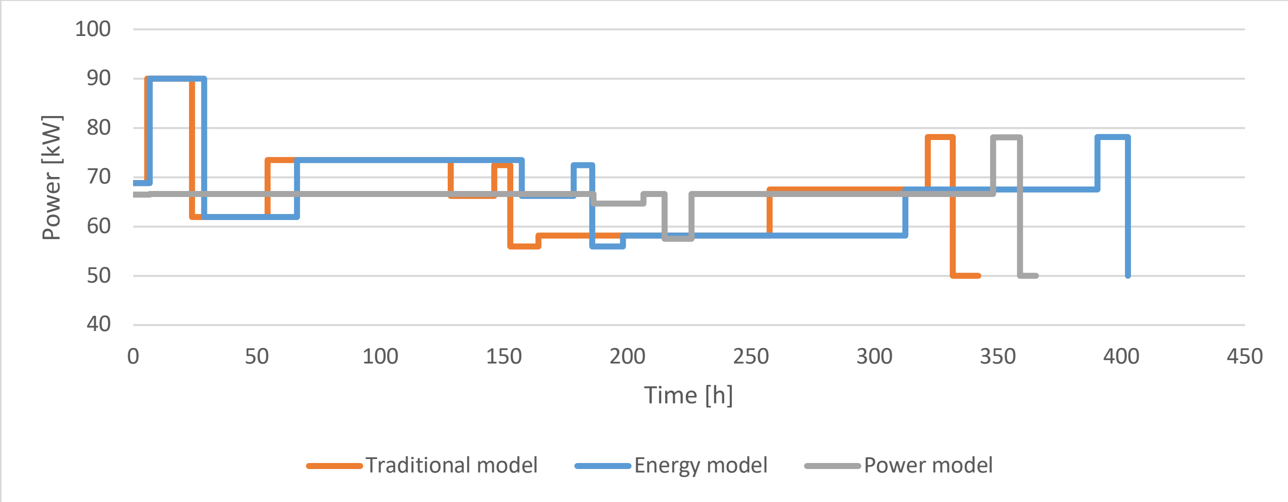

Figure 1. Power-demand profiles for the three models.

4.2 Sensitivity analysis

Considering different values of unit electricity-based and unit power demand-based costs, the following graphs are obtained. In each graph, the x-axis displays the value of *k*, a coefficient that multiplies both unit electricity-based and unit power demand-based costs, starting from the values in the base case:

- Unit energy-based cost *e*: 0.5 $/kWh.
- Unit power demand-based cost *ep*: 0.2 ($/kW)/h.
- Multiplier of electricity-based and power demand-based costs *k*: 0.5, 1, 2.

Note that our sensitivity analysis considers quite high electricity costs compared to the values in Beck et al. (2019) to reflect the current surge in energy prices with the expectation that they will remain higher for longer.

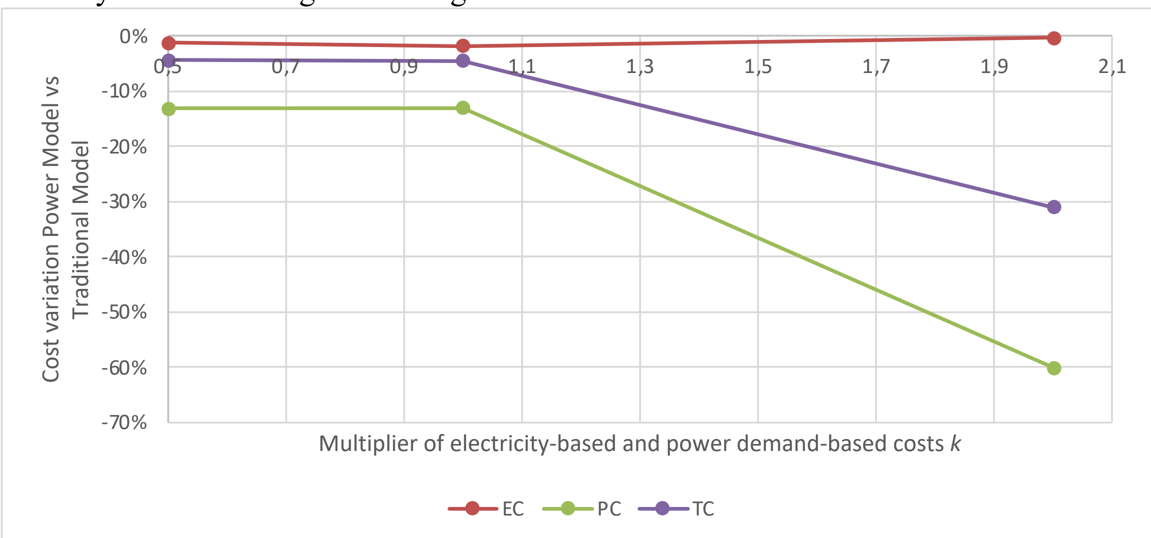

Figure 4. Cost variations while adopting the schedule of the *Power model* instead of the schedule of the *Traditional model*.

Figure 4 illustrates cost savings obtained from the *Power model* compared to the *Traditional model*: by adopting the *Power model*, for all *k* values investigated, the total cost *TC* is reduced between -4.46% and -31.03% compared to the *Traditional model*. This is mainly due to the reduction in the power demand costs PC, while the energy cost EC increases from -1.23% to -0,35%.

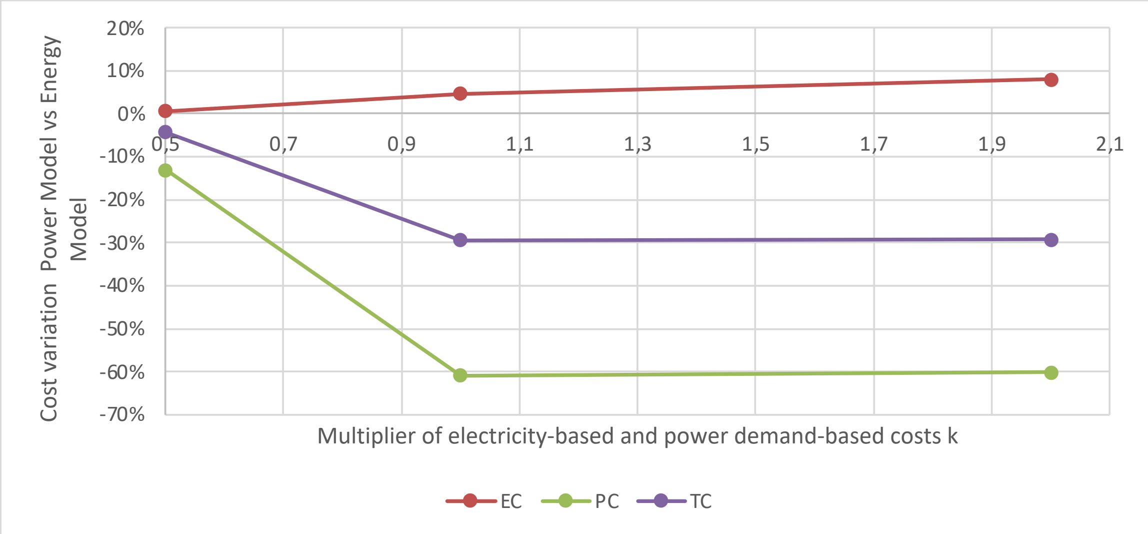

Figure 5. Cost variations while adopting the schedule of the Power model instead of the schedule of the Energy model.

Figure 5 illustrates cost savings obtained from adopting the *Power model* instead of the *Energy model*: by adopting the *Power model*, we observe a reduction in the total cost *TC* for all *k* values investigated (from -4.29% to -29.42%). This is due to the reduction of the power costs PC, while the energy cost EC in the *Power model* increases from +0.56% to +8.01%. Obviously, the *Power model* guarantees the minimization of the total cost *TC*, but not necessarily the minimization of all individual cost components.

In order to evaluate the effect of the production rate p_i variation on the optimal solution, in Table 6 we show the optimal p_i value for the different scenarios and then the mean value, the standard deviation and finally the coefficient of variation (CV).

Table 6. Optimal p_i value for the different scenarios.

k	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
0.75	3374	426	1550	681	262	563	541	298	243	1625
1	3540	452	1808	723	278	603	904	220	258	1696
1.25	3366.6	420.86	1664	673.38	258.98	560	581	328	240	1208
1.5	3269	441	1601	706	271	252	582	353	252	911
1.75	3254	418	1673	669	257	557	535	334	239	1282
2	3241	416	1665	666	256	555	430	333	237	1485
mean	3340.77	428.98	1660.17	686.40	263.83	515.00	595.50	311.00	244.83	1367.83
dev.st	113.22	14.41	86.83	22.98	8.80	130.08	160.99	47.99	8.33	292.97
CV	0.03	0.03	0.05	0.03	0.03	0.25	0.27	0.15	0.03	0.21

As reported in Table 6, the CV is low compared with the variation of *k*, so it is possible to think that the values of p_i obtained are robust with respect the variation of electricity parameters (energy cost *e* and power cost *ep*).

In table 7, it is reported the variation between Bomberger's p_i values and the average value of p_i evaluated in Table 6.

Table 7. Results obtained for the three models.

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Bomberger's p_i value	3750	1000	1187,5	937,5	250	750	300	162,5	250	1875
Optimal p_i mean value	3340.77	428.98	1660.17	686.40	263.83	515.00	595.50	311.00	244.83	1367.83
Percentage Variation	-11%	-57%	40%	-27%	6%	-31%	99%	91%	-2%	-27%

From the results in Table 7 it possible to see that the search for the optimal solution changes significantly the values of the different p_i. This is due to that the model, by respecting the different constraints, changes the p_i values minimizing the sum of the different cost components and penalizing in this case, as shown before, the logistics costs.
Another important consideration is reported in Figure 6, which shows that with the increasing of electricity costs (k) the cycle time increases. This is due to the necessity of the model to maintain low power costs.

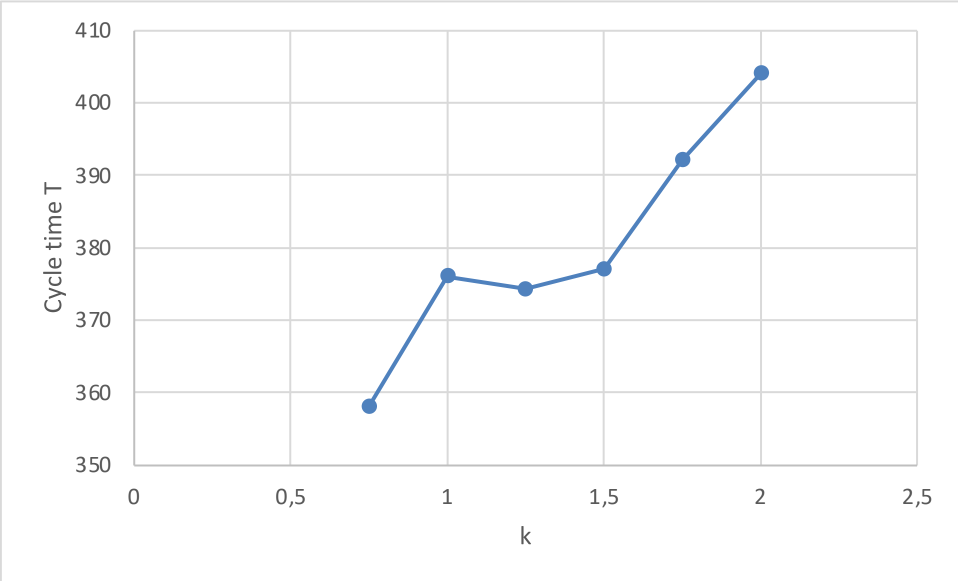

Figure 6. Variation of the cycle time T for the different scenarios k.

5. Conclusions and future research

This note extended the work of Beck et al. (2019) on electricity costs in the Economic Lot Scheduling Problem (ELSP) by considering energy-based and power-demand based cost components. Energy and power demand were assumed to be related to processing and idle-off time intervals of the production line where manufacturing operations are performed. A second extension of the model presented in Beck et al. (2019) is that production rates for the different items produced were considered controllable to minimize total cost. Moreover, a power-demand limit was considered for the definition of production rates, cycle time duration and idle-off assignment to no-productive time intervals of the production line. A numerical analysis was developed that used the common cycle policy to solve the ELSP. The results highlight the influence of energy-based and power-demand based cost components on the optimal decision variables values that minimize the total cost function. The sensitivity analysis shows that by varying the unit energy-based cost e and unit power demand-based cost ep, the *Power model* always leads to the best solution by minimizing the total cost function.
The main managerial insight of the contribution resides in the opportunity of adjusting the production rate for different items produced (which was not included in Beck et al., 2019), which can be successfully implemented by companies while looking for a viable alternative for reducing overall energy consumption and thus lowering energy costs.
Moreover, companies that are interested in reducing energy costs should pay attention not only to energy-based components but also to power-demand cost components that may be responsible for a significant share of the total expenditure. The *Power model* proposed in

this paper, in contrast to the base case, avoids switching the machine off and on if the power demand is particularly large.

In the base case considered in the numerical analysis, a substantial cost reduction could be obtained. Moreover, it should be noted that schedules of the different model variants imply significantly different power-demand, and therefore via a proper adjustment of the schedule, a consistent reduction of the power-demand can be achieved. In particular, with the increasing of the weight of the electricity costs (energy costs and power costs) the optimal solutions present an increasing of the cycle time *T*.

Future developments could consider different scheduling policies, e.g. the basic period policy. Moreover, future work could also investigate the presence of a grid-connected microgrid with renewable energy sources along the line of Smart Energy-Efficient Production-Planning (SEEPP) recently proposed by Golpira et al (2018) and Golpira (2020), where they introduce the problem of the distribution of manufacturing and non-manufacturing loads. Finally, future research could incorporate non-deterministic features, e.g. by considering uncertain or variable data that could lead to the development of stochastic optimization approaches to take into account machine failures, for example.

Data Availability Statement: The authors confirm that the data supporting the findings of this study are available within the article.

Appendix A

This section provides the code related to the mathematical model explained in the section 3 in order to permit to replicate the simulation and the results. In particular we report the code for:

- objective function;
- constraints;

In order to solve the problem, we use the fmincon solver for non-linear problem.

Objective function

```
function f = myfunALLNEW2(x)
%UNTITLED Summary of this function goes here
%   LBE evaluation
e=0.15;
ep=0.1;
g=4;
lsdsu=2;
W=50;
if
max([50+0.005*x(2),50+0.04*x(3),50+0.01*x(4),50+0.065*x(6),50+0.03*x(7)
,50+0.02*x(8),50+0.05*x(9),50+0.07*x(10),50+0.005*x(11),50+0.025*x(5)])
>=g*W
    LBE=lsdsu*g;
else
    LBE=lsdsu*g+(g*W-
max([50+0.005*x(2),50+0.04*x(3),50+0.01*x(4),50+0.065*x(6),50+0.03*x(7)
,50+0.02*x(8),50+0.05*x(9),50+0.07*x(10),50+0.005*x(11),50+0.025*x(5)])
*ep*x(1)/(e*W));
end
if (x(1)-30-
(50/x(2)+50/x(3)+100/x(4)+200/x(5)+10/x(6)+10/x(7)+3/x(8)+42.5/x(9)+42.
5/x(10)+50/x(11))*x(1) < LBE)
    % Idle
    k=1;
else
    % set-up
    k=0;
end
```

```
% objective function
f=50*e*k*(1-
(50/x(2)+50/x(3)+100/x(4)+200/x(5)+10/x(6)+10/x(7)+3/x(8)+42.5/x(9)+42.
5/x(10)+50/x(11))-30/x(1))+(1-
k)*4*2*50*e/x(1)+15/x(1)+0.00000033854*50*x(1)/2*(1-
50/x(2))+((50+0.005*x(2))*50/x(2)+1.25*1*50/x(1))*e+20/x(1)+0.000009244
79*50*x(1)/2*(1-
50/x(3))+((50+0.04*x(3))*50/x(3)+1.5*1*50/x(1))*e+30/x(1)+0.00000664063
*100*x(1)/2*(1-
100/x(4))+((50+0.01*x(4))*100/x(4)+2.5*2*50/x(1))*e+110/x(1)+0.00014505
208*10*x(1)/2*(1-
10/x(6))+((50+0.065*x(6))*10/x(6)+3*4*50/x(1))*e+50/x(1)+0.00001393229*
10*x(1)/2*(1-
10/x(7))+((50+0.03*x(7))*10/x(7)+2.5*2*50/x(1))*e+310/x(1)+0.000078125*
3*x(1)/2*(1-
3/x(8))+((50+0.02*x(8))*3/x(8)+1.5*8*50/x(1))*e+130/x(1)+0.00030729167*
42.5*x(1)/2*(1-
42.5/x(9))+((50+0.05*x(9))*42.5/x(9)+2.75*4*50/x(1))*e+200/x(1)+0.00004
6875*42.5*x(1)/2*(1-
42.5/x(10))+((50+0.07*x(10))*42.5/x(10)+1.75*6*50/x(1))*e+5/x(1)+0.0000
0208333*50*x(1)/2*(1-
50/x(11))+((50+0.015*x(11))*50/x(11)+2*1*50/x(1))*e+10/x(1)+0.000005208
33*200*x(1)/2*(1-
200/x(5))+((50+0.025*x(5))*200/x(5)+1.25*1*50/x(1))*e+max([max([50+0.00
5*x(2),50+0.04*x(3),50+0.01*x(4),50+0.065*x(6),50+0.03*x(7),50+0.02*x(8
),50+0.05*x(9),50+0.07*x(10),50+0.005*x(11),50+0.025*x(5)],(1-
k)*50*4)])*ep;
end
```

```
% objective function
f=50*e*k*(1-
(50/x(2)+50/x(3)+100/x(4)+200/x(5)+10/x(6)+10/x(7)+3/x(8)+42.5/x(9)+42.
5/x(10)+50/x(11))-30/x(1))+(1-
k)*4*2*50*e/x(1)+15/x(1)+0.00000033854*50*x(1)/2*(1-
50/x(2))+((50+0.005*x(2))*50/x(2)+1.25*1*50/x(1))*e+20/x(1)+0.000009244
79*50*x(1)/2*(1-
50/x(3))+((50+0.04*x(3))*50/x(3)+1.5*1*50/x(1))*e+30/x(1)+0.00000664063
*100*x(1)/2*(1-
100/x(4))+((50+0.01*x(4))*100/x(4)+2.5*2*50/x(1))*e+110/x(1)+0.00014505
208*10*x(1)/2*(1-
10/x(6))+((50+0.065*x(6))*10/x(6)+3*4*50/x(1))*e+50/x(1)+0.00001393229*
10*x(1)/2*(1-
10/x(7))+((50+0.03*x(7))*10/x(7)+2.5*2*50/x(1))*e+310/x(1)+0.000078125*
3*x(1)/2*(1-
3/x(8))+((50+0.02*x(8))*3/x(8)+1.5*8*50/x(1))*e+130/x(1)+0.00030729167*
42.5*x(1)/2*(1-
42.5/x(9))+((50+0.05*x(9))*42.5/x(9)+2.75*4*50/x(1))*e+200/x(1)+0.00004
6875*42.5*x(1)/2*(1-
42.5/x(10))+((50+0.07*x(10))*42.5/x(10)+1.75*6*50/x(1))*e+5/x(1)+0.0000
0208333*50*x(1)/2*(1-
50/x(11))+((50+0.015*x(11))*50/x(11)+2*1*50/x(1))*e+10/x(1)+0.000005208
33*200*x(1)/2*(1-
200/x(5))+((50+0.025*x(5))*200/x(5)+1.25*1*50/x(1))*e+max([max([50+0.00
5*x(2),50+0.04*x(3),50+0.01*x(4),50+0.065*x(6),50+0.03*x(7),50+0.02*x(8
),50+0.05*x(9),50+0.07*x(10),50+0.005*x(11),50+0.025*x(5)],(1-
z)*50*4)])*ep;
end
```

Constraints

```
function [c, ceq] = constrTNEW(x)
%UNTITLED Summary of this function goes here
%   Detailed explanation goes here
% j=PL-W (power limit - idel power)
j=150;
c= [-x(1)+30/(1-50/x(2)-50/x(3)-100/x(4)-200/x(5)-10/x(6)-10/x(7)-
3/x(8)-42.5/x(9)-42.5/x(10)-50/x(11))
    -
0.9+50/x(2)+50/x(3)+100/x(4)+200/x(5)+10/x(6)+10/x(7)+3/x(8)+42.5/x(9)+
42.5/x(10)+50/x(11)
    51-x(2)
    51-x(3)
    101-x(4)
    201-x(5)
    11-x(6)
    11-x(7)
    4-x(8)
    43.5-x(9)
    43.5-x(10)
    51-x(11)

    % (PL-W)/v
    x(2)-j/0.005
    x(3)-j/0.04
    x(4)-j/0.01
    x(5)-j/0.025
    x(6)-j/0.065
    x(7)-j/0.03
    x(8)-j/0.02
    x(9)-j/0.05
    x(10)-j/0.07
    x(11)-j/0.015

   1-x(1)
  %x(12)-1
  ];
ceq=[];
end
```

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