A deposit-refund system for managing the economical circulation of returnable transport items

Abstract

The use of Returnable Transport Items (RTI) is a consolidated practice in supply chains committed to sustainability. Therefore, a mechanism ensuring profitability must be implemented to boost the efficiency of RTI circulation. This paper presents a deposit-refund system (DRS) designed for RTI circulation to maximize supply chain profitability. The profit optimization analysis considered three scenarios: vendor's profit, retailer's profit, and overall system profit. Tailored solution strategies have been developed for each case, considering the retail price components. The optimal deposit amount required to maximize RTI circulation has been derived. Then, it has been examined how the retail price affects the profits of supply chain actors relative to the RTI return rate and observed how changes in return rates influence the profit patterns of supply chain actors. The results are scrutinized via several sensitivity analyses, leading to important implications for the design of deposit-refund systems for RTI. It is noted that the vendor's wholesale price has a significant effect in establishing the decision-maker's optimal policy in designing a DRS for RTI circulation. In other words, the wholesale price is a key component in assessing the economic viability in determining the deposit amount for RTI. Moreover, through a couple of real cases of RTI application with specific peculiar parameter settings, it has been shown that the vendor's position would be significant or insignificant according to the difference between the value of the RTI and that of the goods it holds.

Keywords

Returnable transport items; Deposit-refund system; Price-dependent demand; Return rate; Profit maximization

1. Introduction

Recent studies have shown that using reusable containers instead of single-use ones is more environmentally sustainable [1]. Thus, the economical circulation of Returnable Transport Items (RTI), which can be pallets, trays, boxes, or crates [2], has become a critical concern for companies, striving for enhanced sustainability and closed-loop supply chains, while at the same time keeping economic considerations in mind. Since this effective circulation relies on the collaboration and integration of various supply chain members, agreeable and mutually profitable operational conditions must be designed for supply chain entities. Once this alignment has been achieved, meeting the economic objectives of all partners involved, sustainability can be pushed to the next level. Interested readers are encouraged to look at the study [3], for a comprehensive review and future challenges of inventory models for reverse logistics .

Several studies have analyzed real cases related to RTI: e.g. [4] discussed the design of the return logistic system of returnable collapsible containers in The Netherlands, [5] looked at the impact of control strategies of roll containers for food in Sweden. Moreover, as outlined in [5], a single RTI can range in cost from a few euros to thousands of euros each, and it is not uncommon for the value of the RTI to surpass that of the goods it contains. Therefore, maintaining an RTI fleet, which often requires a substantial initial capital investment, can also incur significant operating costs due to shrinkage. Moreover, it should be noted that RTI can also be equipped for the transportation of refrigerated items (in food or pharmaceutical sectors) with significant environmental benefits as shown in [6].

How to design and structure a deposit-refund system that meets the objectives of all supply chain members is however a non-trivial undertaking. Critical factors to be considered in this vein include the costs of utilizing the RTI, which can be associated with their procurement, delivery, return, inspection, and repair. These factors can impact the retail price, rendering this problem both relevant and important. In this vein, the ways of promoting the willingness to return the used RTI have been of particular concern [7]. As an option for promoting the return performance, it has been said that RTI returns can be incentivized by offering a refund [8]. However, related research that investigates this option is scarce. We, therefore, follow the claim by [2] for research that considers alternate design options for RTI deposit-refund systems.

Specifically, we study in the present work the design of a deposit-refund system and its impact on the retail price. We consider the circulation of RTI between a vendor and a retailer, with the RTI deposit amount being included in the retail price. This is done since the number of RTI needed for supplying the finished products to the customers depends on the pricing mechanism, which considers the cost factors involved for the effective use of RTI among supply chain partners (i.e., the vendor and the retailer in our setting). As the ultimate objective we consider the profit of the vendor, the retailer, and the system overall, and develop solution mechanisms for the different cases based on the composition of the retail price. The results are scrutinized via several sensitivity analyses, leading to important implications for the design of deposit-refund systems for RTI.

The paper is structured as follows: ***Section 2*** presents a brief overview of previous works related to several key topics pertinent to the problem addressed in this study. ***Section 3*** provides the problem description, including basic assumptions and notations used. ***Section 4*** presents the development of a mathematical model that demonstrates the optimal deposit policy based on the retail pricing structure, along with the characteristics of the profit functions for the supply chain actors. ***Section 5*** includes extensive numerical analyses, illustrative examples, and key sensitivity analyses. Finally, ***Section 6*** concludes the paper and suggests possible future research contributions.

2. Literature review

Prior research has focused on optimizing logistical systems responsible for circulating RTI to enable efficient and economical operations of both forward and reverse logistics. While RTI can take on many formats ([2], [4], [9]), a critical concern among all of them has been the economic circulation of used or empty RTI. This is important in setting the foundation for a circular economy since it enables the continuous re-supply of finished goods without interruption or delay. This was also stressed in the literature review [10], which identified the cost of reusable packaging as an influential factor in determining the feasibility and viability of this approach, most notably since it can significantly impact the retail price paid by consumers. As such, the retail price generally has to recoup the cost of the reusable packaging as well as the logistics associated with its return. This makes the study of the deposit-return system an important area of investigation [10].

The broader literature on which we rely in our study can be classified into three literature streams, including (1) RTI inventory management, (2) deposit-refund systems, and (3) retail pricing decisions. Specifically, within the literature on RTI inventory management, [11] proposed a mathematical model to determine the optimal shipment lot size for RTI carrying perishable items to customers, and [12] developed analytical models to investigate the effects of RFID-based RTI management when the rate of used RTI is not deterministic. The authors further considered the time needed to upgrade non-RFID-tagged RTI to RFID-tagged RTI, considering the return rate of used RTI when both tagged and non-tagged RTI are used simultaneously. In addition, [13] proposed safety measures by considering the safety stocks for both the finished goods and the RTI, since a shortage in RTI can be directly responsible for shortages in finished goods (if RTI are not available to transport the finished goods). [14] proposed a decision-making model determining the optimal ratio of owned versus rented RTI to minimize operational costs while at the same time satisfying demand requirements for a single buyer. In addition, [15] developed a model to forecast container returns considering both the return rate and the cycle time, while [16] aiming to quantify the economic and environmental impacts of reusable plastic containers compared a multi-use system to a traditional single-use packaging. The literature on decision support models for managing RTI was reviewed by [2], who called for the future study of deposit-refund systems for RTI in a closed-loop supply chain. We are following their call with the present research.

The second stream of research on deposit-refund systems focuses on the return rate of used RTI as a function of the deposit-refund mechanism. As such, consumers generally pay a deposit that is added to the price of a product, incentivizing them to return the used product by refunding the deposit amount [17]. In this vein, [18] analyzed the beverage packaging deposit-refund system, categorized by material and deposit flows, highlighting challenges related to smart collection, the encouragement of eco-friendly design, and the advancement of circular business models. In addition, factors affecting both the economic and environmental impacts of reusable packaging were reviewed by [19], highlighting that return rates vary between different systems and noting the positive impact of deposit fee approaches. However, few works have studied the consumer's willingness to participate in waste collection programs or the customer's attitude in determining the effectiveness of deposit-refund systems ([20], [21]). In this vein, [10] reviewed the literature on reusable packaging considering their environmental and economic costs, and highlighted the impact of consumers' behavior on the cost of reusable packaging as a promising research avenue. [22] addressed this issue by examining consumers' willingness to pay for reusable food packaging when ordering from food delivery platforms. Respondents expressed a willingness to pay a specific amount for reusable packaging but emphasized the need for convenient return options for used packaging (i.e., the merchant picking up the packaging or the customer returning it to a collection facility). We follow this stream of research in that we study how the deposit-refund system can be designed to maximize the return rate, with the ultimate objective of moving to a closed-loop supply chain.

The third stream of research we rely on considers retail pricing decisions and their impact on the demand for RTI. This are of research is relevant to our investigation since the demand rate is generally sensitive to the retail price. In this vein, [23] contended that cost-plus pricing mechanisms can be used for developing fair (or reasonable) prices without precise knowledge of market demand or marginal cost conditions. In addition, [24] investigated pricing and production decisions involving reusable containers under conditions of stochastic customer demand, assuming that the return quantity depends on both customer demand and the manufacturer's acquisition fee. Similarly, [25] analyzed the impact of a packaging tax policy on the decisions of food manufacturers and retailers, considering the effects on output reduction and input substitution. They also examined the sensitivity of material prices to reductions at the source of product packaging.

We build on this stream of research in that we consider a generalized retail pricing structure that includes factors for handling RTI. With this approach, we can discern the impacts of RTI-related dynamics. Building upon this context, this contribution encompasses three primary investigations. Firstly, as the deposit amount is expected to have a positive association with the return rate, we explore what the optimal deposit amount should be for different supply chain actors to maximize the RTI return rate. In addition, we examine what the optimal return rate should be for maximizing profits given a predetermined deposit amount. Secondly, we investigate several options for composing the retail price based on RTI deposits and study the impact on supply chain actors' profits. Finally, typical patterns in the supply chain actors' profits based on the changes in return rate will be examined. Overall, to the best of our knowledge, this is the first paper that studies a deposit-refund system coupled with the operational performance of RTI to improve supply chain profitability.

3. Problem description

We consider a business situation where a single vendor supplies a single type of finished goods to a retailer using RTI as depicted in ***Figure 1***. The return of used RTI is encouraged by a deposit-refund system, which allows the vendor to use the RTI again (thus saving money for not having to purchase single-use transport items) or to compensate for lost or damaged RTI. In this setting, a retailer pays a deposit τ to a vendor for the receipt of the finished goods that are transported with the RTI. The retailer then sells the finished goods to customers with a selling price p, which is assumed to be a function of the wholesale price p_0 and the costs associated with using the RTI (the deposit τ). The customer's demand rate D is assumed to be a function of the retail price, i.e., the demand is price-sensitive. A retailer obtains a refund of the deposit τ when s/he returns the used (empty) RTI to the vendor. If it is not returned (e.g., due to loss or damage), the deposit remains unredeemed. As a result, the return rate of used RTI is an important measure for determining the profits of both vendors and retailers. In particular, the return rate can also determine the retailer's selling price, since the foregone deposit amount of unreturned RTI needs to be recouped.

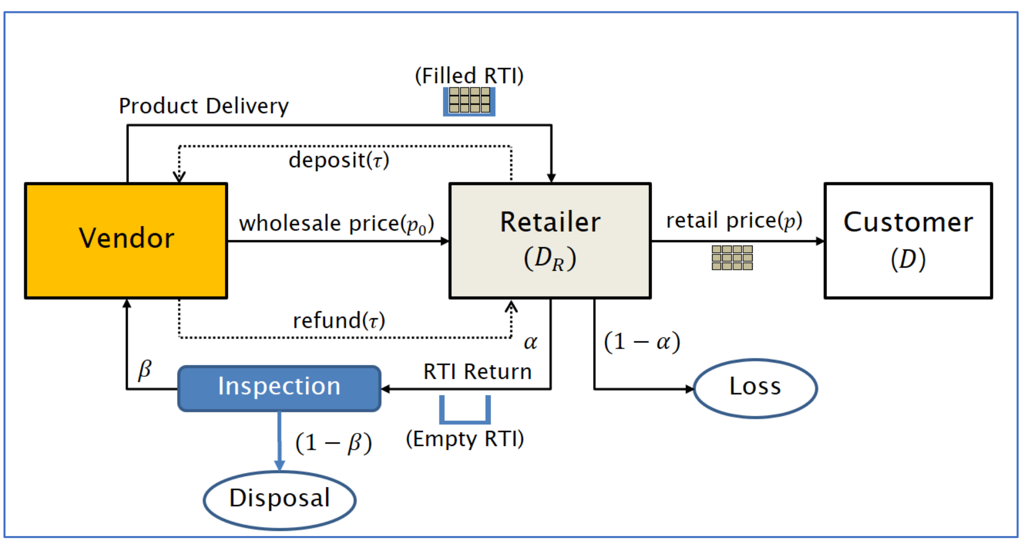


Figure 1: The deposit-refund system for RTI operations

In developing our model, we make the following assumptions.

1. All parameters are deterministic, known, and constant over time. In addition, we assume that a single vendor supplies a single type of finished goods to a single retailer (according to [12]).
2. A retailer pays a deposit (i.e.,τ) for each RTI and receives a refund of the same amount when the retailer returns the used RTI to the vendor (according to [18]).
3. A vendor purchases new RTI from an RTI manufacturer if the used RTI are not returned or damaged RTI cannot be repaired (according to [12]).
4. Based on the cost-plus pricing policy, a retailer determines its selling price (i.e.,p) based on the wholesale price (i.e.,p_0) and the RTI deposit-related cost due to lost or damaged RTI. This means that the retailer's selling price (i.e.,p) would be dependent on the cost of utilizing the RTI (i.e.,θ_r) as well as a markup rate (i.e.,θ_m) (according to [26]).
5. The customer's demand rate is assumed to be sensitive to the retailer's selling price (i.e.,p).
6. A vendor's unit wholesale price(i.e.,p_0) is given enough to compensate for the cost of operating and maintaining an RTI, i.e., $p_0 \geq K_q$

Further, in developing our model, we use the following notations.

Given parameter:	
p_0	: Vendor's wholesale price for one unit of product (in dollars)
c_a	: Inspection cost for one RTI unit (in dollars)
c_r	: Repair cost for one RTI unit (in dollars)
c_p	: Procurement cost for one RTI unit (in dollars), where $c_p > c_r$
K	: Handling cost for one RTI unit (in dollars)
q	: RTI size (i.e., capacity) (in number of items)
θ_m	: Markup rate on the wholesale price p_0 (in percentage)
θ_r^{max}	: Customer's maximally acceptable economic burden for an RTI deposit (in dollars)
α	: Fraction of returned RTI (in percentage), where $0 \leq \alpha \leq 1$ and $\rho = 1 - \alpha$
β	: Fraction of returned but damaged RTI that can be repaired, where $0 \leq \beta \leq 1$
Decision variable:	
τ_q	: Deposit for one unit of product (in dollars), where $\tau_q = \tau/q$ and $\tau_q \geq 0$
Derived quantities:	
u_R	: Unredeemed deposit (in dollars), where $u_R = \tau(1-\alpha)D_R$
θ_r	: RTI deposit-related cost, which is imposed on the retail price (in dollars), where $\theta_r = (c_1\tau_q + c_2 u_R)$ and $\theta_r \leq \theta_r^{max}$. Note that the parameter of c_1 presents the weight of the unit deposit which is paid for one RTI unit (i.e., τ_q) while c_2 is one for the aggregated unredeemed deposit considering the return performance (i.e., u_R), where $c_1, c_2 \geq 0$.
τ	: Deposit for one RTI unit (in dollars), where $\tau = q\tau_q$
K_q	: Unit handling cost for one unit of product (in dollars), where $K_q = K/q$
$p(\tau_q, \alpha)$	: Retailer's selling price (i.e., retail price) (in dollars), where $p(\tau_q, \alpha) = (1+\theta_m)p_0 + \theta_r$.
$D(\tau_q, \alpha)$	: Demand rate for the product (in number of items), expressed as a retail price-sensitive demand function, i.e., $D(\tau_q, \alpha) = a - bp(\tau_q, \alpha)(a > b > 0)$, where a represents the

market size and b the price sensitivity, with the latter being assumed to have a non-zero positive value (according to [27]).

D_R : Demand rate for RTI (in number of RTI), where $D_R = D(\tau_q, \alpha)/q$

As noted above, the retailer's selling price (i.e., $p(\tau_q, \alpha)$) is assumed to be determined by considering the margin rate (i.e., θ_m) as well as the deposit-related costs (i.e., θ_r). As such, the retailer considers the costs of utilizing the RTI, which include the deposit cost τ_q and the unredeemed deposit u_R, together with the markup, to determine the retail price. Thus, the components determining the retail price would be changed according to the values of c_1 and c_2. In other words, the retailer's selling price changes according to whether the values for c_1 or c_2 are greater than zero or not. As a result, a retailer adopts a cost-plus pricing mechanism, where the deposit-related cost of using the RTI (i.e., θ_r) and the markup (i.e., $\theta_m p_0$) is added to the cost of procuring the product (i.e., p_0) (according to [26]). In this work, it is assumed that both c_1 and c_2 are arbitrarily given to analyze the properties of the optimal policies for an individual supply chain actor. In this work, we assume a generalized form of the retail price when the RTI-related factors are included thereby incorporating both c_1 and c_2. Indeed, both c_1 and c_2 are used to take into account the retailer's generalized pricing policy when the RTI-related cost factors are considered. In other words, for the generalizability of the developed models, we consider the following whether these two individual components (i.e., c_1 and c_2) are included or not. Thus, there would be three possible cases in formulating the retail price, i.e., (i) $\theta_r = c_1\tau_q$ (i.e., $c_1 > 0$ and $c_2 = 0$), (ii) $\theta_r = c_2 u_R$ (i.e., $c_1 = 0$ and $c_2 > 0$), and (iii) $\theta_r = (c_1\tau_q + c_2 u_R)$ (i.e., $c_1, c_2 > 0$). The case with $\theta_r = c_1\tau_q$ represents the ***deposit-based pricing*** by taking into account the deposit for one unit of product while the case with $\theta_r = c_2 u_R$ for the ***performance-based pricing*** based on the return performance of the used RTI. In addition, the case with $\theta_r = (c_1\tau_q + c_2 u_R)$ represents the ***cost-performance pricing*** schemes, i.e., the mixture of deposit cost and return performance.

Hereafter, for the brevity of the presentation, please note that we omit two variables, i.e., τ_q and α when both $D(\tau_q, \alpha)$ and $p(\tau_q, \alpha)$ are presented. As noted above, the handling cost for an RTI unit (i.e., K) for the vendor can be captured by Eq. (1), where c_a is the fixed inspection cost, c_r is the repair cost, and c_p is the purchasing cost for an RTI unit (according to [12]).

$$K = (c_a + \beta c_r)\alpha + c_p(1 - \beta\alpha) = c_p + c_\beta\alpha \text{ , where } c_\beta = c_a - (c_p - c_r)\beta \tag{1}$$

As stated earlier, we further capture the demand rate (i.e., D is also assumed to be a function of the retail price (i.e.,p) as shown in Eq. (2).

$$D = a - bp = a - b(1 + \theta_m)p_0 - b\theta_r = a - b(1 + \theta_m)p_0 - (\delta_1\tau_q + \delta_2 u_R) \tag{2}$$
$$\text{, where } \delta_1 = bc_1 \text{ and } \delta_2 = bc_2$$

As presented in Eq. (2), the parameter δ_1 accounts for the sensitivity of the demand rate based on the unit deposit (i.e., τ_q), while δ_2 accounts for the sensitivity of total unredeemed deposit (i.e., u_R). Further, the function of D in Eq. (2) can be generalized according to three different pricing policies, i.e., (i) $\theta_r = c_1\tau_q$, (ii) $\theta_r = c_2 u_R$, and (iii) $\theta_r = (c_1\tau_q + c_2 u_R)$. From Eq. (2), we observe that the upper bound of p, i.e.,

p_{max}, is given as (a/b) to ensure the non-negative demand rate at the retailer regardless of both δ_1 and δ_2. The equation of D in Eq. (2) could be further re-arranged as captured in Eq. (3).

$$D = \left(\frac{d_0 - \delta_1 \tau_q}{1+\delta_2 \tau_q \rho}\right), \text{ where } d_0 = a - b(1+\theta_m)p_0 \text{ and } \rho = (1-\alpha) \tag{3}$$

In addition, from the definition of d_0 in Eq. (3), the inequality of $p_0 \le \left(\frac{a-\delta_1 \tau_q}{b(1+\theta_m)}\right)$ is further derived to assure the upper bound of the wholesale price (i.e., p_0) if the other factors are given. First, it would be necessary to investigate the basic properties of the demand function with the key factors for the deposit-refund system, i.e., τ_q and α. As captured in Eqs. (4.1) and (4.2), the demand rate is a strictly decreasing function based on τ_q - this is due to the reimbursed deposit increasing the retail price. In other words, the consumer would be reluctant to buy the finished goods if the portion of the deposit in retail price becomes larger. It also means that the retailer should carefully design the retail pricing framework if s/he wants to maximize the obtainable profit.

$$\frac{\partial D}{\partial \tau_q} = -\frac{(\delta_1 + \delta_2 d_0 \rho)}{\left(1+\delta_2 \tau_q \rho\right)^2} \le 0 \tag{4.1}$$

$$\frac{\partial^2 D}{\partial {\tau_q}^2} = \frac{2(\delta_1 + \delta_2 d_0 \rho)\delta_2 \rho}{\left(1+\delta_2 \tau_q \rho\right)^3} \ge 0 \tag{4.2}$$

In addition, we now consider the effects of the return rate on customer demand. The demand rate (i.e.,D) can be expressed as an increasing function of α (i.e., the return rate of used RTI). As such, the retailer is willing to buy the goods due to there being less burden associated with unredeemed deposits as the return rate of used RTI increases. This is captured in Eqs. (5.1) and (5.2).

$$\frac{\partial D}{\partial \alpha} = \left(\frac{\delta_2 \tau_q}{1+\delta_2 \tau_q \rho}\right) D \ge 0 \tag{5.1}$$

$$\frac{\partial^2 D}{\partial \alpha^2} = 2\left(\frac{\delta_2 \tau_q}{1+\delta_2 \tau_q \rho}\right)^2 D \ge 0 \tag{5.2}$$

As presented in ***Appendix A***, it is necessary to satisfy the allowable range of τ_q for keeping the relevance of the developed model as shown in Eq. (6).

$$0 \le \tau_q \le \tau_q^{max} = min\,(\tau_{q,d_0}^{max}, \tau_{q,\theta_r}^{max}) \tag{6}$$

$$\text{, where } \tau_{q,d_0}^{max} = \left(\frac{d_0}{\delta_1}\right) \text{ and } \tau_{q,\theta_r}^{max} = \frac{\theta_r^{max}}{(c_1 + \rho c_2(d_0 - \theta_r^{max} b))}$$

Therefore, the value of τ_q^{max} in Eq. (6) would be a significant reference in establishing the deposit for a single product. From Eq. (6), the upper bound of τ can be derived as $\tau \le \tau^{max}$, where $\tau^{max} = q\tau_q^{max}$.

4. Model development

Our model aims to investigate the effects of deposit-related factors to determine the optimal deposit amount and its impact on profit. As such, the problem of determining the optimal deposit amount (i.e.,τ_q^*) is studied under a given RTI return rate (i.e.,α) in ***Section 4.1***. This is then followed by the derivation of the optimal return rate (i.e.,α^*) in ***Section 4.2***, where the deposit amount (i.e.,τ_q) is given as a pre-determined constant.

4.1 Optimal deposit amount (i.e.,τ_q^*) with a given value of α

We formulate a corresponding profit function for both the vendor (***Section 4.1.1***) and the retailer (***Section 4.1.2***), followed by an analysis of the system profit function (***Section 4.1.3***) to identify an optimal policy with a given set of parameters. As noted earlier, we consider three possible case when formulating the demand function, capturing the effect of deposit-related cost factors, i.e., (i) $\theta_r = c_1\tau_q$, (ii) $\theta_r = c_2 u_R$, and (iii) $\theta_r = \left(c_1\tau_q + c_2 u_R\right)$. We, therefore, need to consider the different mechanisms based on to whether c_1 and c_2 are applicable in determining the retail price.

4.1.1 Vendor's Problem (Model-V)

We assume that a vendor aims to maximize its obtainable profit by considering the retailer's return behavior for RTI. As such, we need to assess the pattern of the vendor's profit function (i.e.,$\pi_V(\tau_q|\alpha)$) in τ_q with a given value of α, where α presents the retailer's return performance. We thus formulate the vendor's problem as aiming to maximize their expected profit as shown in Eq. (7).

$$\boldsymbol{Maximize}\ \pi_V(\tau_q|\alpha) = p_0 D + \tau D_R \rho - D_R K = D\left(p_0 + \tau_q\rho - K_q\right) \tag{7}$$

In determining the value of the optimal deposit amount for maximizing the vendor's profit, we consider three possible cases when formulating the demand function, as noted above, i.e., (i) $\theta_r = c_1\tau_q$, (ii) $\theta_r = c_2 u_R$, and (iii) $\theta_r = \left(c_1\tau_q + c_2 u_R\right)$. These are denoted in the following as ***Case V1)***, ***Case V2)***, and ***Case V3)***, respectively. Specifically, we aim to determine the optimal value of $\tau_q(\alpha)$, maximizing the vendor's profit under the assumption that the value of α is given. ***Appendix B*** offers details on how the value for $\tau_q^*(\alpha)$ is derived, together with the conditions that ensure the optimality of the developed solutions.

Case V1) Deposit-based Pricing (i.e., $\theta_r = c_1\tau_q$)

In the first case, from Eq. (2), we consider the case where the unredeemed deposit is assumed to not affect the retailer's selling price, i.e., $c_2 = 0$. Considering the demand rate in Eq. (2), the optimal deposit amount, i.e., $\tau_{q,V1}^*(\alpha)$, can then be defined as captured in Eq. (8).

$$\tau_{q,V1}^*(\alpha) = max\left(0, \tau_{q,V1}^0(\alpha)\right) \tag{8}$$

$$\text{, where } \tau_{q,V1}^0(\alpha) = \frac{1}{2}\left(\tau_{q,d_0}^{max} - \frac{(p_0 - K_q)}{(1-\alpha)}\right) \text{ and } \tau_{q,d_0}^{max} = \left(\frac{d_0}{\delta_1}\right)$$

From Eq. (8), it is known that a non-negative value for $\tau_{q,V1}^0(\alpha)$ can then be achieved given that $p_0 \leq \left(K_q + \tau_{q,d_0}^{max}(1-\alpha)\right)$ is satisfied. The parameter K_q captures the needed operating cost associated with the RTI for a single product, analogous to K being defined in Kim and Glock (2014) as the handling cost for a single RTI. Eq. (8) also recognizes that the deposit amount increases with decreasing RTI return rates, based on $\frac{d\tau_{q,V1}^{0}(\alpha)}{d\alpha} = -\frac{1}{2(1-\alpha)^2}\left(p_0 - \frac{(c_p+c_\beta)}{q}\right) < 0$, if $p_0 > \frac{(c_p+c_\beta)}{q} = \left(K_q + \frac{c_\beta \rho}{q}\right)$ because $\frac{d\rho}{d\alpha} < 0$, where $\rho = 1-\alpha$. As such, the vendor aims to recover the economic burden from unreturned RTI by increasing the deposit amount. From Eq. (8), it is noted that the wholesale price (i.e., p_0) is a critical criterion in observing the pattern of $\tau_{q,V1}^{0}(\alpha)$ with the given return performanc (i.e., α).

Case V2) Performance-based Pricing (i.e., $\theta_r = c_2 u_R$)

For the second case, as explained in ***Appendix B***, the vendor's profit is either an increasing or a decreasing function of τ_q according to the sign of $(1-\delta_2(p_0-K_q))$ as shown in Eq. (9).

$$\frac{\partial \pi_V(\tau_q|\alpha)}{\partial \tau_q} = \frac{d_0\rho\left(1-\delta_2(p_0-K_q)\right)}{\left(1+\delta_2\tau_q\rho\right)^2} = -\frac{d_0\rho\left(p_0-(K_q+\delta_2^{-1})\right)}{\delta_2\left(1+\delta_2\tau_q\rho\right)^2} \tag{9}$$

In other words, this condition implies that there is no stationary point for the deposit amount (i.e.,τ_q) that maximizes the vendor's profit function. As such, the vendor's profit would increase as the deposit amount increases, given that $p_0 \le \left(K_q + \delta_2^{-1}\right)$, also since the demand rate and the number of RTI would increase. In other words, if $p_0 \le \left(K_q + \delta_2^{-1}\right)$, then $\tau_{q,V1}^{*}$ should be set as τ_q^{max} whereas $\tau_{q,V1}^{*} = 0$ if $p_0 > \left(K_q + \delta_2^{-1}\right)$. As a result, the wholesale price (i.e., p_0) is so critical factor in determining the behaviors of the vendor according to the changes in the unit deposit(i.e., τ_q).

Case V3) Cost-Performance Pricing (i.e., $\theta_r = c_1\tau_q + c_2 u_R$)

The third case considers that the pattern of the vendor's profit function depends on whether $p_0 \le \left(K_q + \delta_2^{-1}\right)$ is satisfied or not, as demonstrated in ***Appendix B***. However, the stationary point satisfying the first-order condition can be re-arranged as follows.

$$\tau_{q,V3}^{*}(\alpha) = max\left(0, \tau_{q,V3}^{0}(\alpha)\right), \text{ where } \tau_{q,V3}^{0}(\alpha) = \frac{-\delta_1 + \sqrt{\delta_1(\delta_1+\delta_2 d_0\rho)\left(1-\delta_2(p_0-K_q)\right)}}{\delta_1\delta_2\rho} \tag{10}$$

We note that the inequality of $\tau_{q,V3}^{0}(\alpha) \le \tau_{q,d_0}^{max}$ is always satisfied if $p_0 \ge K_q$ as assumed above, given that the stationary value for maximizing the vendor's profit can be found. As a result, as shown in Eqs. (8) and (9), the effects of the wholesale price would be so significant in analyzing the vendor's obtainable profit. Thus, we must perform the numerical analyses taking into account the different values of the wholesale price.

4.1.2 Retailer's Problem (Model-R)

Similar to the vendor's case, the retailer aims to maximize its profit, which is a function of the deposit amount as presented in Eq. (11):

$$\boldsymbol{Maximize}\ \pi_R(\tau_q|\alpha) = pD - \tau\rho D_R = \left(p - \tau_q\rho\right)D \tag{11}$$

Analogous to the vendor's profit function, we aim to determine the optimal value of τ_q, which maximizes the retailer's profit with the assumption that the value of α is given. Depending on the signs of δ_1 and δ_2, there are three possible cases for determining the optimal deposit amount that maximizes the retailer's profit, similar to what was done above for the determination of the vendor's profit. ***Appendix C*** offers details on how the properties for $\pi_R(\tau_q|\alpha)$ were determined.

Case R1) Deposit-based Pricing (i.e., $\theta_r = c_1\tau_q$)

Similar to the vendor's problem, the optimal deposit amount (i.e.,$\tau^*_{q,R1}(\alpha)$), which maximizes the retailer's profit, can be obtained based on Eq. (12).

$$\tau^*_{q,R1}(\alpha) = max\left(0, \tau^0_{q,R1}(\alpha)\right) \tag{12}$$

$$\text{, where } \tau^0_{q,R1}(\alpha) = \frac{b\left(p_{max} - \tau^{max}_{q,d_0}\rho\right)}{2(b\rho - \delta_1)} + \tau^{max}_{q,d_0}, \tau^{max}_{q,d_0} = \left(\frac{d_0}{\delta_1}\right), \text{ and } p_{max} = \left(\frac{a}{b}\right)$$

From Eq. (12), it is definite that the inequality of $\tau^0_{q,R1}(\alpha) \leq \tau^{max}_{q,d_0}$ is always satisfied if $p_{max} \geq \tau^{max}_{q,d_0}\rho$. Additionally, the value of $\tau^0_{q,R1}(\alpha)$ can be considered as an increasing function of α, based on the property of $\frac{d\tau^0_{q,R1}(\alpha)}{d\alpha} = \frac{b}{2}\left(\frac{a - \tau^{max}_{q,d_0}\delta_1}{(b\rho - \delta_1)^2}\right) = \frac{b(a - d_0)}{2(b\rho - \delta_1)^2} > 0$, since $a > d_0$ is always satisfied by the definition of d_0 in Eq. (3). As such, the retailer seeks to decrease the deposit amount with decreasing RTI return performance, to reduce the economic burden inherent to the unredeemed deposit.

Case R2) Performance-based Pricing (i.e., $\theta_r = c_2u_R$)

If the unredeemed deposit is only considered for determining the retail price, the retailer's profit can be maximized if the value of the deposit amount is determined as shown in Eq. (13).

$$\tau^*_{q,R2}(\alpha) = max\left(0, \tau^0_{q.R2}(\alpha)\right) \text{, where } \tau^0_{q,R2}(\alpha) = \frac{1}{\rho}\left(\frac{2d_0}{\delta_2 a + b} - \frac{1}{\delta_2}\right) \tag{13}$$

We further note that the retailer's profit function is concave if $\tau_q \leq \tau^b_{q,R2}(\alpha)$, where $\tau^b_{q,R2}(\alpha) = \frac{1}{\rho}\left(\frac{3d_0}{\delta_2 a + b} - \frac{1}{\delta_2}\right)$ and the inequality of $\tau^0_{q,R2}(\alpha) \leq \tau^b_{q,R2}(\alpha)$ being always satisfied. Details on this derivation are included in ***Appendix C***.

Case R3) Cost-Performance Pricing (i.e., $\theta_r = c_1\tau_q + c_2u_R$)

Considering both c_1 and c_2 simultaneously when formulating the retail price makes the problem more complex, similar to ***Case V3***. As presented in ***Appendix C***, it is indeed so hard to derive the closed-form satisfying the first-order optimality from the structure of $\frac{\partial\pi_R(\tau_q|\rho)}{\partial\tau_q}$. However, it is also known that the retailer's profit function would be either concave or convex according to the value of $\tau^b_{q,R3}(\alpha)$. In other words, the retailer's profit function would be concave if $\tau_q < \tau^b_{q,R3}(\alpha)$ in Eq. (14).

$$\tau^b_{q,R3}(\alpha) = \left(\frac{(3\delta_2 d_0 - \delta_2 a - b)\rho + \delta_1}{(\delta_2 a\rho + 2\delta_1 + b\rho)\delta_2\rho}\right) \tag{14}$$

Thus, it is useful to find out the optimal solution which maximizes the retailer's profit using other analytical approaches such as cubic formula, methods for the numerical analyses if $\tau^b_{q,R3}(\alpha) < \tau^{max}_q$. In this work, we use a bi-section method to find out the solution satisfying the first-order optimality condition in Eq. (C.11).

As a result, the following ***Table 1*** is developed to summarize the optimal values of τ_q for both a vendor and a retailer with different settings of the deposit-refund system.

Table 1: Properties of the optimal solutions for both a vendor and a retailer

Case	**Vendor's profit ($\pi_V(\tau_q\|\alpha)$)**	**Retailer's profit ($\pi_R(\tau_q\|\alpha)$)**
Case 1) Deposit-based Pricing	**Case V1**) Concave function for all τ_q $\tau_{q,V1}^* = max(0, min(\tau_{q,V1}^0, \tau_q^{max}))$, where $\tau_{q,V1}^0 = \frac{1}{2}\left(\tau_{q,d_0}^{max} - \frac{1}{\rho}(p_0 - K_q)\right)$	**Case R1-1)** Concave function if $c_1 \geq \rho$ $\tau_{q,R1}^* = max\left(0, min(\tau_{q,R1}^0, \tau_q^{max})\right)$, where $\tau_{q,R1}^0 = \frac{(a-2d_0)\delta_1 + d_0\rho b}{2(b\rho - \delta_1)\delta_1}$ **Case R1-2)** Convex function if $c_1 < \rho$ $\tau_{q,R1}^* \in \{0, \tau_q^{max}\}$
Case 2) Performance-based Pricing	**Case V2-1)** Increasing function if $p_0 \leq (K_q + \delta_2^{-1})$ $\tau_{q,V1}^* = \tau_q^{max}$ **Case V2-2)** Decreasing function if $p_0 > (K_q + \delta_2^{-1})$ $\tau_{q,V1}^* = 0$	**Case R2)** The mixture of a concave and convex function $\tau_{q,R2}^* = max\left(0, min(\tau_{q,R2}^0, \tau_q^{max})\right)$, where $\tau_{q,R2}^0 = \frac{2d_0}{\rho(\delta_2 a + b)} - \frac{1}{\rho\delta_2}$
Case 3) Cost-Performance Pricing	**Case V3-1)** Concave function if $p_0 \leq (K_q + \delta_2^{-1})$ $\tau_{q,V3}^* = max\left(0, min(\tau_{q,V3}^0, \tau_q^{max})\right)$, where $\tau_{q,V3}^0 = \frac{-\delta_1 + \sqrt{\delta_1(\delta_1 + \delta_2 d_0 \rho)\left(1 - \delta_2(p_0 - K_q)\right)}}{\delta_1\delta_2\rho}$ **Case V3-2)** Convex function if $p_0 > (K_q + \delta_2^{-1})$ $\tau_{q,V1}^* \in \{0, \tau_q^{max}\}$	The mixture of a concave and convex function **Case R3-1)** if $\left.\frac{\partial \pi_R(\tau_q\|\rho)}{\partial \tau_q}\right\|_{\tau_q=0} \geq 0$, i.e., $\rho \leq \frac{\delta_1(2d_0 - a)}{d_0\left(b - \delta_2(2d_0 - a)\right)}$ Using a one-dimensional search, $\tau_{q,R3}^* \in [0, min\ (\tau_{b,R3}, \tau_q^{max})]$, where $\tau_{q,R3}^b = \frac{(3\delta_2 d_0 - \delta_2 a - b)\rho + \delta_1}{(\delta_2 a\rho + 2\delta_1 + b\rho)\delta_2\rho}$ **Case R3-2)** if $\left.\frac{\partial \pi_R(\tau_q\|\rho)}{\partial \tau_q}\right\|_{\tau_q=0} < 0$, i.e., $\rho > \frac{\delta_1(2d_0 - a)}{d_0\left(b - \delta_2(2d_0 - a)\right)}$ $\tau_{q,R3}^* \in \{0, \tau_q^{max}\}$

4.1.3 System Problem (Model-S)

In the third problem, as presented in Eq. (15), we consider the problem involving both a vendor and a retailer to determine the best policy to maximize the system performance (instead of individual performance before). Specifically:

$$\boldsymbol{Maximize}\ \pi_S(\tau_q|\alpha) = \pi_V(\tau_q|\alpha) + \pi_R(\tau_q|\alpha) = D(p_0 - K_q + p) \tag{15}$$

Apart from the above two problems, i.e., ***Model-V*** and ***Model-R***, it is noted that there is no need to categorize the solution procedure according to the values of c_1 and c_2 since the derived optimal solution can be used for all the cases. Details for deriving the properties of the optimal deposit amount are provided in ***Appendix D***. As such, the optimal value of τ_q maximizing the system profit (i.e., $\pi_S(\tau_q|\alpha)$) can be obtained as seen in Eq. (16).

$$\tau_{q,S}^*(\alpha) = max\left(0, \tau_{q,S}^0(\alpha)\right), \text{ where } \tau_{q,S}^0(\alpha) = \left(\frac{2d_0 - \varphi}{2\delta_1 + \delta_2\varphi\rho}\right) \text{ and } \varphi = b(p_0 - K_q) + a \tag{16}$$

Further, we can see from Eq. (16) that the inequality of $\frac{d\tau_{q,S}^0(\alpha)}{d\alpha} = \frac{(2d_0 - \varphi)\delta_2\varphi}{(2\delta_1 + \delta_2\varphi\rho)^2} > 0$ is satisfied if $\tau_{q,S}^0(\alpha) > 0$, i.e., $\varphi < 2d_0$. Thus, it is better to reduce the deposit amount as the return rate of used RTI becomes lower to maximize the system's profit. In other words, we can say that the decrement in the deposit amount leads to a lowering of the retail price, subsequently increasing the total profit with an increase in the demand rate. Furthermore, if $\varphi > 2d_0$, the value of $\tau_q = 0$ is an optimal point maximizing the value of $\pi_S(\tau_q|\alpha)$, since $\pi_S(\tau_q|\alpha)$ has a unique stationary point as presented in Eq. (16). Note that the stationary value satisfying the first-order optimality condition is valid for all cases regardless of the signs of both δ_1 and δ_2. Also, we know that the system profit function is concave when $\tau_q \leq \tau_{q,S}^b(\alpha)$, where the value of $\tau_{q,S}^b(\alpha)$ is given in Eq. (17).

$$\tau_{q,S}^b(\alpha) = \frac{\delta_2\rho(3d_0-\varphi)+\delta_1}{\delta_2\rho(2\delta_1+\delta_2\varphi\rho)} \tag{17}$$

As shown in Eq. (18), the value of $\tau_{q,S}^b(\alpha)$ in Eq. (17) can be represented with the value of $\tau_{q,S}^0(\alpha)$ in Eq. (16).

$$\tau_{q,S}^b(\alpha) = \tau_{q,S}^0(\alpha) + \frac{(\delta_1+\delta_2\rho d_0)}{\delta_2\rho(2\delta_1+\delta_2\varphi\rho)} \tag{18}$$

In other words, we know that the unique stationary value of $\tau_{q,S}^0(\alpha)$ can be considered as the optimal value maximizing the system profit while satisfying the concavity condition.

4.2 Optimal return rate (i.e.,α^*) with the given value of τ_q

In this section, the optimal return rate (i.e.,α^*) is analyzed under the assumption that the value of the deposit amount (i.e.,τ_q) is fixed. ***Appendix E*** details the effects of the return rate (i.e.,α) on the profit functions since it can be one of the decision variables under the assumption that the deposit amount is arbitrarily given. As derived in ***Appendix E***, the values of α, which satisfies the first-order optimality condition maximizing the retailer's profit, are obtained as follows:

$$\alpha_R^0(\tau_q) = 1 - \frac{1}{\tau_q}\left(\frac{2(d_0-\delta_1\tau_q)}{(a\delta_2+b)} - \frac{1}{\delta_2}\right) \tag{19}$$

We obtain the stationary value maximizing the corresponding profit functions, except for the vendor's profit, since the vendor's profit function is a monotonically increasing or decreasing function. Furthermore, from Eq. (19), we note that the inequality of $\frac{d\alpha_R^0(\tau_q)}{d\tau_q} = \left((1-\alpha_R^0) + \frac{2\delta_1}{(a\delta_2+b)}\right)/\tau_q > 0$ is always satisfied if $\alpha_R^0(\tau_q) < 1$. In addition, the stationary value of α satisfying the first-order optimality condition for maximizing the system profit is derived as seen below:

$$\alpha_S^0(\tau_q) = 1 - \left(\frac{2q(d_0-\delta_1\tau_q)}{(\tau_q\delta_2 a_S q - bc_\beta)} - \frac{1}{\delta_2\tau_q}\right), \text{ where } a_S = a + b\left(p_0 - \frac{c_p+c_\beta}{q}\right) \tag{20}$$

Also, from Eqs. (19) and (20), the retailer's return policy is identical to the one for maximizing the system profit, i.e., $\alpha_R^0(\tau_q) = \alpha_S^0(\tau_q)$ when the following condition in Eq. (21) could be satisfied.

$$p_0 = \frac{1}{q}\left((c_p + c_\beta) + \frac{c_\beta}{\delta_2\tau_q}\right) + \frac{1}{\delta_2} \tag{21}$$

In other words, if the vendor's wholesale price is determined as presented in Eq. (21), then the retailer's return policy is the same as in the situation where the system profit can be maximized. As explained in ***Appendix E***, we note that the first-order optimality condition of α for maximizing the vendor's profit in Eq. (E.1) is the same as the condition for making the retailer's preference identical to one for the return rates considering a system perspective. Thus, in a case where the deposit is arbitrarily given, it is possible to make all supply chain actors profitable at the same time if the wholesale price (i.e.,p_0) is set as presented in Eq. (21).

5. Numerical analyses

In this section we provide several illustrative examples that apply the developed models, considering both demand-related and RTI-related parameters. This is followed by sensitivity analyses characterizing system behaviors when the values of key factors change. As presented up to now, there are a lot of parameters to determine the optimal deposit such that we assume the situation where there is no restriction on the value of θ_r^{max}, i.e., $\theta_r^{max} = \infty$, because the value of τ_{q,d_0}^{max} could be used to ensure a non-negative customer demand rate. Hereafter, Please note that the following sections will be developed with a set of illustrative parameters.

5.1 Illustrative examples

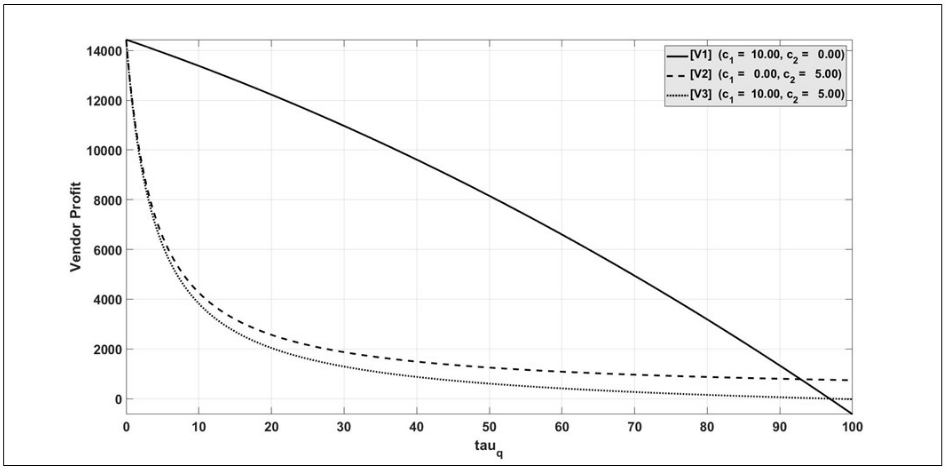
We will use the parameters shown in ***Table 2*** to develop the illustrative examples. These values can be referred to as a real case in the sector of automotive parts distribution, in particular in this case the RTI are referred to as returnable packaging racks for shipping casted brake disks. RTI are metal racks with wheels to facilitate the manual movement of the racks within the facility. The RTI are used to store and transport semifinished casted brake disks at the foundry which is a second-tier automotive supplier, then RTI are used to ship the products to the first-tier automotive supplier that is in charge of assembling the entire disk brake system. In this case, $p_0 q \gg c_p$, *i.e. 3000*≫ 20. In other words, the procurement cost for a single RTI is negligible compared with the vendor's sales revenue per unit RTI. This situation can be referred to as a vendor-driven supply chain. Thus, a vendor would be indifferent to the amount of the deposit since s/he could guarantee a larger revenue by decreasing the retail price as much as possible (i.e., indeed zero deposit). We structure our analysis analogous to ***Sections 4.1*** and ***4.2*** for studying the possible interactions between the value of the deposit amount (i.e.,τ_q) and the return rate (i.e.,α). We proceed with an analysis of the optimal deposit amount when the return rate is given, followed by the optimal return rate when the deposit amount is given since those are the two key parameters for managing the circulation of RTI.

Table 2: Data for Illustrative Examples

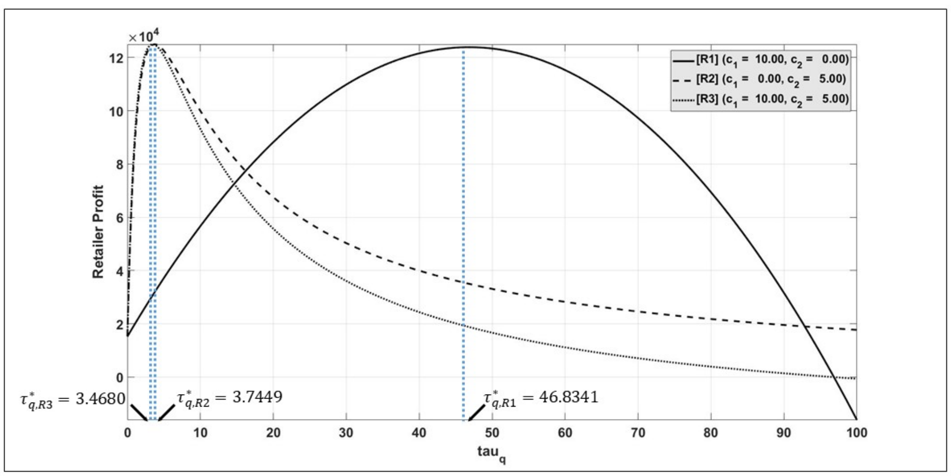
Demand-related parameters						RTI-related parameters					
a	b	c_1	c_2	p_0	θ_m	q	β	c_a	c_p	c_r	α
500	0.5	10.0	5.0	30.0	0.05	100	0.1	0.1	20	0.2	0.9

5.1.1 Optimal deposit amount (i.e.,τ_q^*) with a given value of α

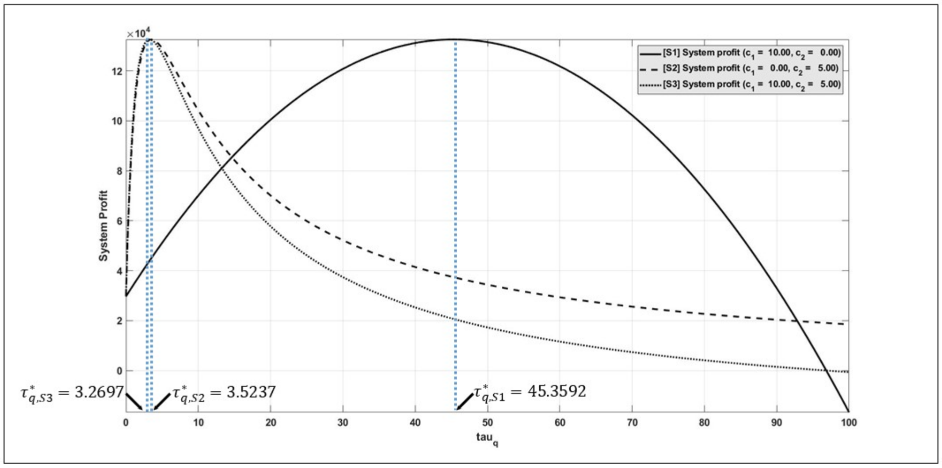
Using the data in ***Table 2***, we first obtain the patterns of corresponding profit functions for a vendor, a retailer, and the overall system, as shown in ***Figure 2***. As derived in ***Section 4***, the properties of the optimal deposit amount can be different from each other according to whether the positivities of both c_1 and c_2— these are the parameters that determine the retail price. In other words, as presented in ***Figure 2***, there are several different patterns of profit functions in τ_q such that the optimal value of τ_q can be explicitly obtained. As summarized in ***Table 3***, the optimal value of the deposit amount is significantly larger compared to the cases when the return rate (i.e.,α) is not considered in determining the retail price. However, if the return rate is valid in determining the retail price, i.e.,$c_2 > 0$, the optimal deposit amount maximizing the corresponding profit is lower than the value for ***Case 1)***. We can also see that the values of τ_q^* become lower as we consider more parameters in formulating the retail price. In other words, the case with two parameters, i.e., c_1 and c_2, has the lowest deposit amount in comparison to the other cases.



(a) Vendor's profits



(b) Retailer's profits



(c) System profits

Figure 2: Profit profiles in τ_q (with the data in Table 2)

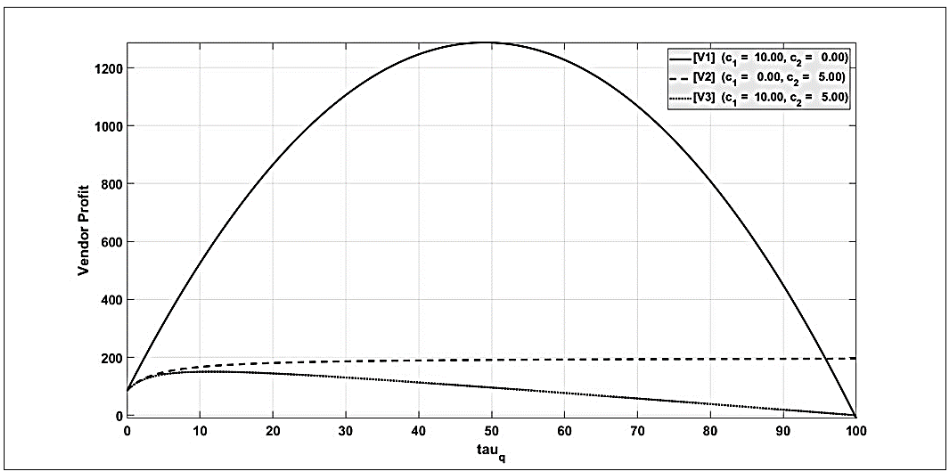
As shown in Table 3, there are no significant differences in obtainable profits between ***Model-R*** and ***Model-S***. However, there are noticeable changes in profits for all parties if ***Model-V*** is deployed. In other words, the vendor's revenue with a single unit of RTI (i.e., $p_0 \cdot q$) is extremely larger than the procurement cost for one RTI unit(i.e., c_p). In addition, ***Model-V*** shows the same results regardless of the retail pricing mechanism. In other words, the vendor's profit could have no effects from the pricing schemes by setting the value of the unit deposit to zero since the wholesale price is significantly large as explained earlier for Eqs. (8) and (9).

Table 3: Details for the optimal deposit policy with illustrative example data

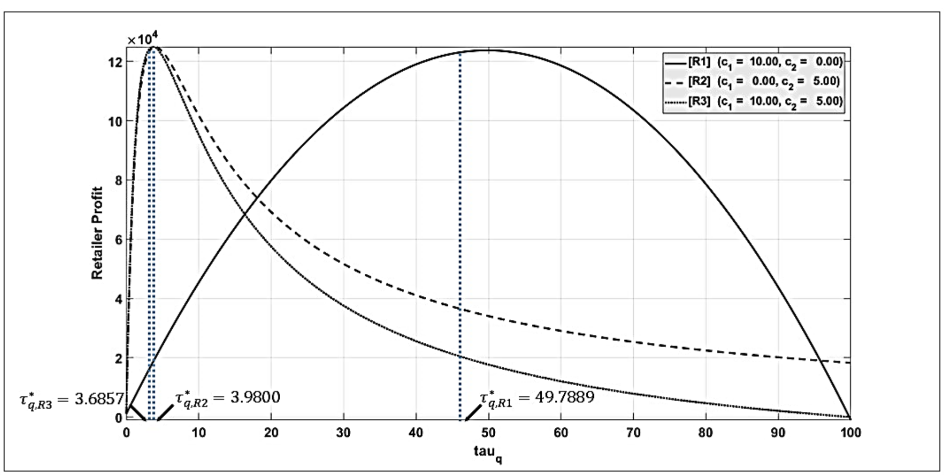
<table>
<tr><th>Case</th><th colspan="2">Model-V</th><th>Model-R</th><th>Model-S</th></tr>
<tr><td rowspan="2">Case 1)
Deposit-based Pricing</td><td rowspan="6">$\tau^*_{q,V1} = \tau^*_{q,V2}$
$= \tau^*_{q,V3} = 0.0$
(Decreasing profit function in τ_q)</td><td rowspan="6">π_V=14,438.8
π_R=15,253.9.
π_S=29,692.7</td><td>$\tau^*_{q,R1} = 46.8341$</td><td>$\tau^*_{q,S1} = 45.3592$</td></tr>
<tr><td>π_V= 8,627.8
π_R=123,828.8
π_S=132,456.6</td><td>π_V=8,844.3
π_R=123,721.1
π_S=132,565.4</td></tr>
<tr><td rowspan="2">Case 2)
Performance-based Pricing</td><td>$\tau^*_{q,R2} = 3.7449$</td><td>$\tau^*_{q,S2} = 3.5237$</td></tr>
<tr><td>π_V=7,550.9
π_R=124,906.3
π_S=132,457.2</td><td>π_V=7,767.2
π_R=124,798.2
π_S=132,565.4</td></tr>
<tr><td rowspan="2">Case 3)
Cost-Performance Pricing</td><td>$\tau^*_{q,R3} = 3.4680$
$(\tau^b_{q,R3} = 7.2014)$</td><td>$\tau^*_{q,S3} = 3.2697$
$(\tau^b_{q,S3} = 6.9045)$</td></tr>
<tr><td>π_V=7,543.5
π_R=124,913.3
π_S=132,456.8</td><td>π_V=7,760.6
π_R=124,804.7
π_S=132,565.4</td></tr>
</table>

Regarding the previously introduced case of racks for shipping casted brake disks, the vendor aims at reducing as much as possible the deposit, while the optimal deposit for maximizing supply chain profit is very close to the one to optimize the retailer profit.

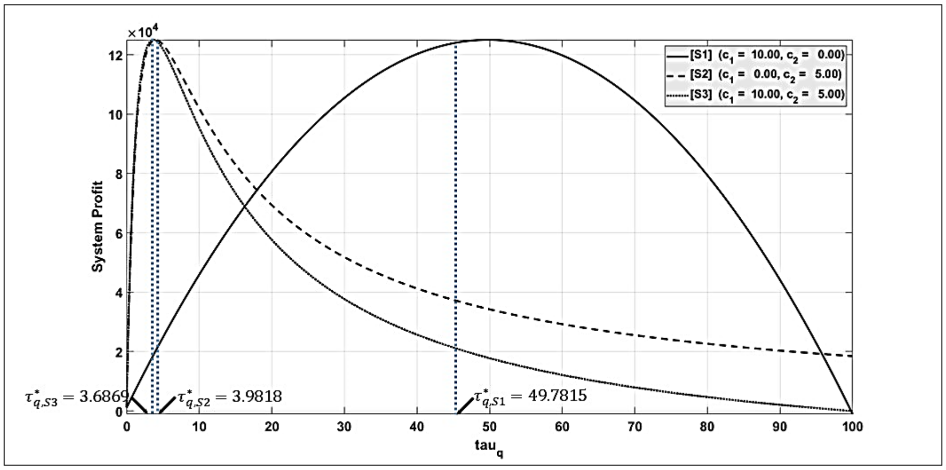
In addition, we study the results for different values of p_0= 2 and q= 10, representing a low wholesale price and a small RTI capacity, respectively, for assessing other system behaviors. These modified values can be referred to as a real case in the sector of beer distribution, in particular in this case the RTI are referred to as beer crates for 10 x 0,33 l bottles. The beer crate is personalized with the colors and name of the beer producer and the value of the crate is comparable to that of the goods it holds, i.e. in this case, $p_0 q = c_p$, *i.e. 20* $= 20$. This situation can be referred to as a retailer-driven supply chain. In this case, the vendor's sales revenue with a single RTI is indeed identical to the procurement cost of a unit RTI. In this modified scenario, as shown in ***Figure 3*** and ***Table 4***, the vendor's profit functions have a completely different pattern when compared to the ones presented in ***Figure 2*** when the wholesale price is very low (compared to the procurement cost for one RTI unit), as explained in ***Section 4.1.1***. As such, the vendor seeks to determine the optimal value of the deposit amount to maximize their profit while managing the economic burden associated with the RTI, i.e., procurement, inspection, and repair. As explained in ***Section 4.1.1***, the overall pattern of the vendor's profit function changes according to whether the inequality of $p_0 \leq \left(K_q + \delta_2^{-1}\right)$ is satisfied or not. In the given illustrative examples, the value of $\left(K_q + \delta_2^{-1}\right)$ is evaluated as 2.2308, rendering $p_0 \leq \left(K_q + \delta_2^{-1}\right)$ as not satisfied since p_0= 2.



(a) Vendor's profits



(b) Retailer's profits



(c) System profits

Figure 3: Profit profiles in τ_q with p_0=2 and $q = 10$

As presented in Table 4, the vendor's profit is insignificantly smaller than other parties for all cases. In addition, there are no differences in profits between ***Model-R*** and ***Model-S***. In other words, the lower wholesale price has no significant effect on the performance in terms of the corresponding profits. In this case, the retailer has more significant influence in developing the system profit.

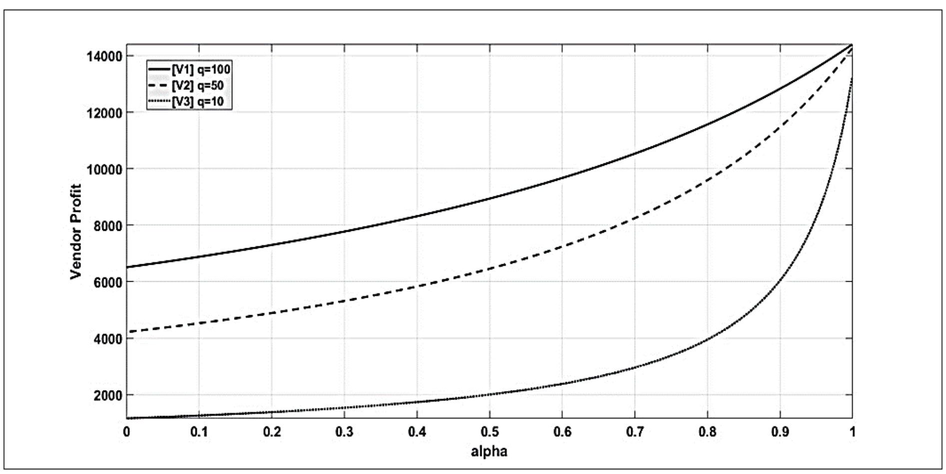
Table 4: Details of the optimal deposit policy with p_0=2 and $q = 10$

Case	Model-V	Model-R	Model-S
Case 1) Deposit-based Pricing	$\tau^*_{q,V1} = 49.0490$	$\tau^*_{q,R1} = 49.7889$	$\tau^*_{q,S1} = 49.7815$
	π_V= 1,287.3 π_R=123,728.1 π_S=125,015.5	π_V=1,287.1 π_R=123,755.3 π_S=125,042.3	π_V= 1,287.1 π_R= 123,755.2 π_S=125,042.3
Case 2) Performance-based Pricing	Increasing profit function in τ_q	$\tau^*_{q,R2} = 3.9800$	$\tau^*_{q,S2} = 3.9818$
		π_V=141.9 π_R=124,900.4 π_S=125,042.3	π_V=141.9 π_R=124,900.4 π_S=125,042.3
Case 3) Cost-Performance Pricing	$\tau^*_{q,V3} = 11.4773$	$\tau^*_{q,R3} = 3.6857$ ($\tau^b_{q,R3} = 7.5280$)	$\tau^*_{q,S3} = 3.6869$ ($\tau^b_{q,S3} = 7.5304$)
	π_V=150.3 π_R=87,941.7 π_S=88,092.0	π_V=134.5 π_R=124,907.8 π_S=125,042.3	π_V=134.5 π_R=124,907.8 π_S=125,042.3

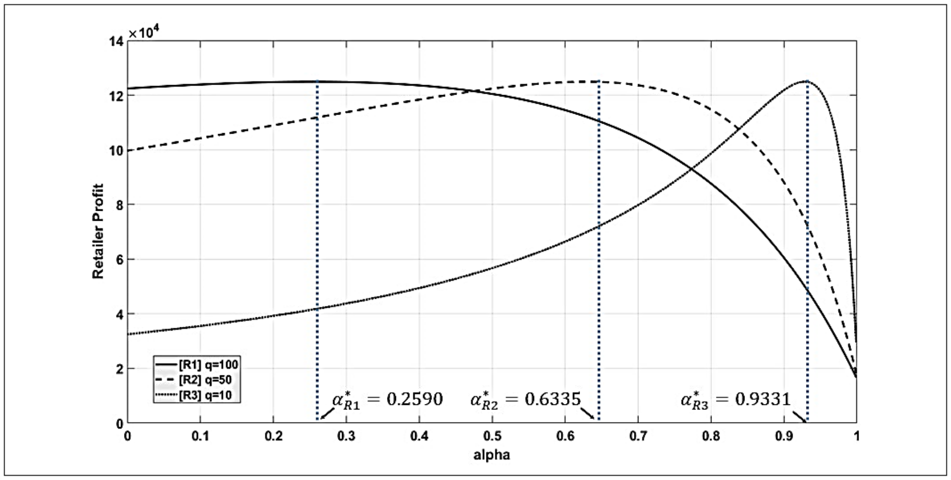
Concerning the previously introduced case of the beer crate, the vendor to optimize its profit aims at a specific deposit that is three times higher than the optimal deposit for maximizing retailer profit that is very close to the one to optimize the supply chain profit. Generally speaking, there are no noticeable changes in the corresponding profit according to the owner of the RTI-related decision-making. In other words, the adopted retail pricing scheme has a strictly significant impact on the obtainable profits.

5.1.2 Optimal return rate (i.e.,α^*) with a given value of τ_q

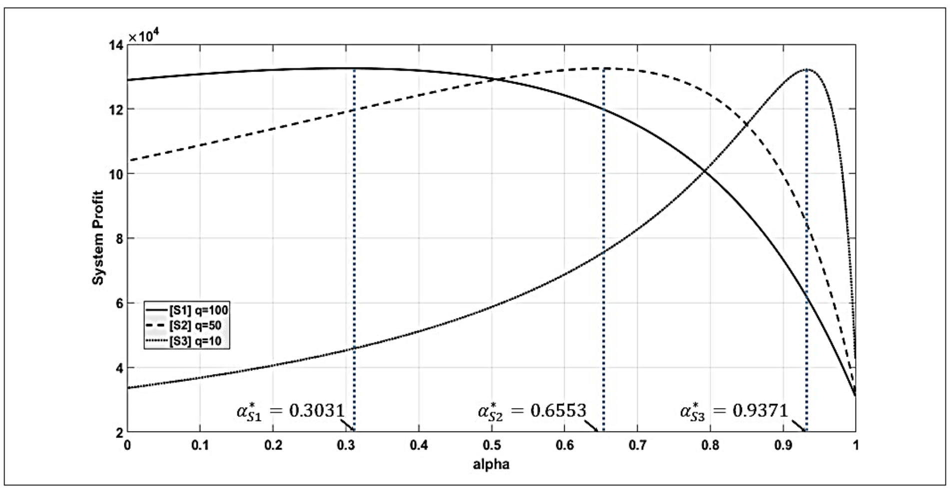
This section analyzes illustrative numerical examples for the optimal return rate as studied in ***Section 4.2***. For purposes of this illustration, the value of the deposit for a single RTI (i.e.,τ) is assumed to be given as 50. As explained earlier, the value of the deposit for a single unit of finished goods (i.e.,τ_q) can be different based on the value of the handling capacity for a single RTI (i.e.,q). We develop the corresponding profit function based on the value of the return rate (i.e.,α) while fixing other parameters as constant with three different values of q (i.e., 100, 50, and 10). As presented in ***Figure 4***, the vendor's profit function is an increasing function in α, such that it is straightforward to determine the optimal return rate if values of both τ_q and q are given. However, as explained above, the optimal return rate that maximizes the retailer's profit and the system profit needs to be carefully established. The optimal values of α are obtained as α_R^*= 0.2590 and α_S^*= 0.3031 when the value of q is given as 100, while α_R^*= 0.6335 and α_S^*= 0.6553 are obtained for q= 50. As seen in ***Figure 4***, the vendor has a strict policy in determining the optimal return rate as the profit function must be an increasing function in α. As such, the vendor desires the value of the unreturned rate of used RTI to be either 0 or 1, based on the pattern of the profit function. As demonstrated in ***Appendix E***, the value of the wholesale price (i.e.,p_0) can be a significant measure of the patterns in the vendor's profit function. Thus, with the fixed value of τ_q, the vendor's profit function tends to be a decreasing function when the value of q increases. Also, as presented in ***Figure 4***, there are similar patterns in both the retailer's profit and the system's profit when the value of the return rate changes. Furthermore, there are more rapid changes in both cases when the value of q becomes less. Thus, the value of α^* increases as the value of q changes in sequence from 100 to 50 and 10. We note that the profit function is closely dependent on the return rate based on the given deposit amount. A critical consideration is also sustainable RTI operations, which are dependent on the design of the RTI and the economical RTI capacity.



(a) Vendor's profits



(b) Retailer's profits



(c) System Profits

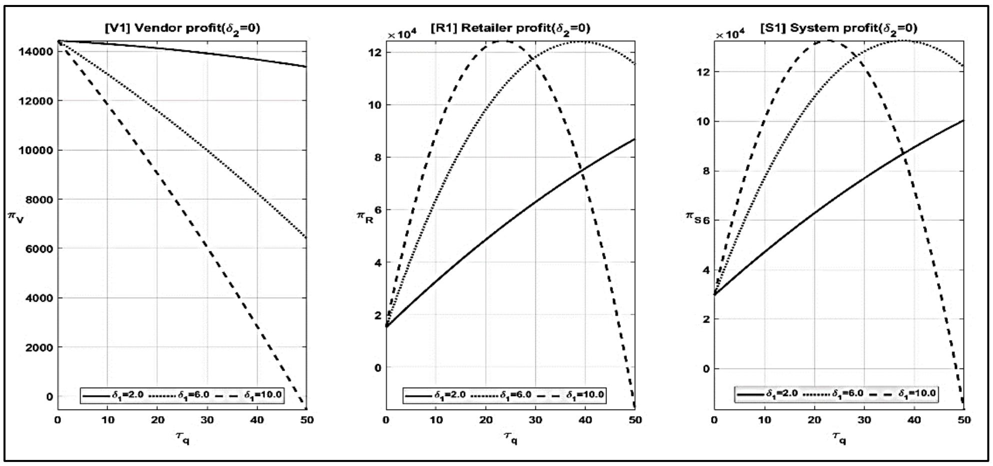
Figure 4: Profit profiles in α (with the data in Table 2)

5.2 Sensitivity analyses

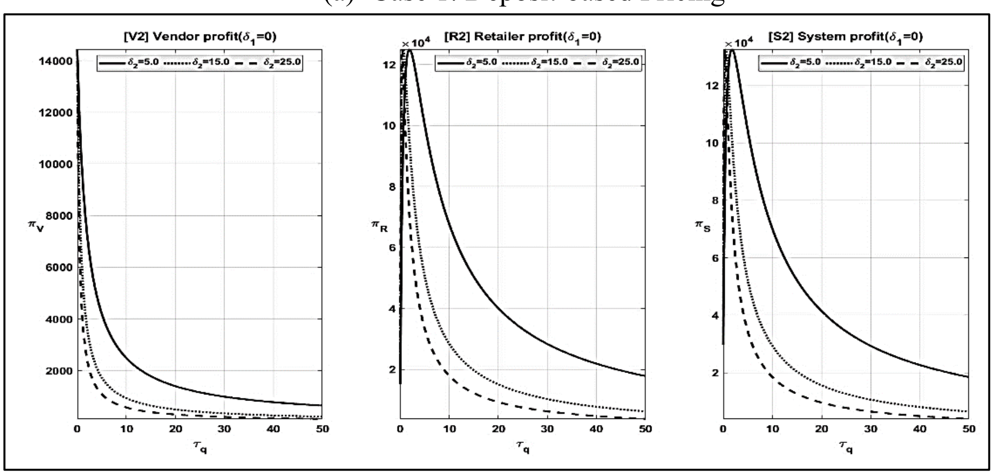
In this section, we investigate the effects of three key parameters on the profits of the vendor, the retailer, and the system overall. The first factor we consider is the retail price structure, i.e., c_1 and c_2, which is discussed in ***Section 5.2.1***. As a second factor, we investigate price sensitivity (i.e.,b) and its impact on profits, while considering other parameters as fixed, which is done in ***Section 5.2.2***. Obtaining insight into these factors seems prudent since both the retail price structure and price sensitivity influence the product's demand rate, which in turn triggers RTI requirements. As a last factor look at the return rate (i.e.,α) and its impact on profits, which we do in ***Section 5.2.3***. We expect useful insight to be derived from this sensitivity analysis since the return rate is a key determinant for further RTI requirements. Please note that the following works would be done using the parameter values in ***Table 2*** while selectively changing the values of the parameter to be analyzed.

5.2.1 The effects of the retail price structure (i.e.,$\boldsymbol{\delta_1}$ and $\boldsymbol{\delta_2}$) on the profit functions

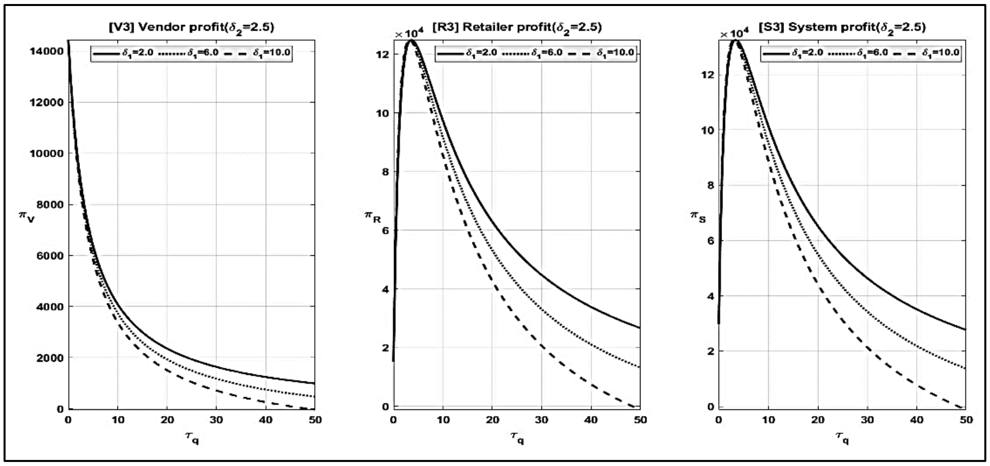
We first investigate the sensitivity of the parameters (i.e., δ_1 and δ_2) to profit functions, using a fixed value of $q = 100$. As defined above, three different cases are considered for which the sensitivity analyses are performed by changing the values of δ_1 and δ_2 according to the given case. Specifically, for ***Case 1)***, the values of δ_1 are given as 2.0, 6.0, and 10.0; for ***Case 2)*** the values of δ_2 are given as 5.0, 15.0, and 25.0; for ***Case 3)*** the value of δ_2 is given as 2.5 and the value of δ_1 changes as done for ***Case 1)***. As presented in ***Figure 5***, the profit functions for ***Case 1)*** vary significantly as the value of δ_1 gradually changes. However, in other cases, the patterns of the profit function themselves do not change noticeably when the parameters of δ_1 and δ_2 change. Above all, the optimal value of τ_q maximizing the retailer's and the overall system's profit seems to be invariant with different values of δ_1 in ***Case 3)***.



(a) Case 1: Deposit-based Pricing



(b) Case 2: Performance-based Pricing

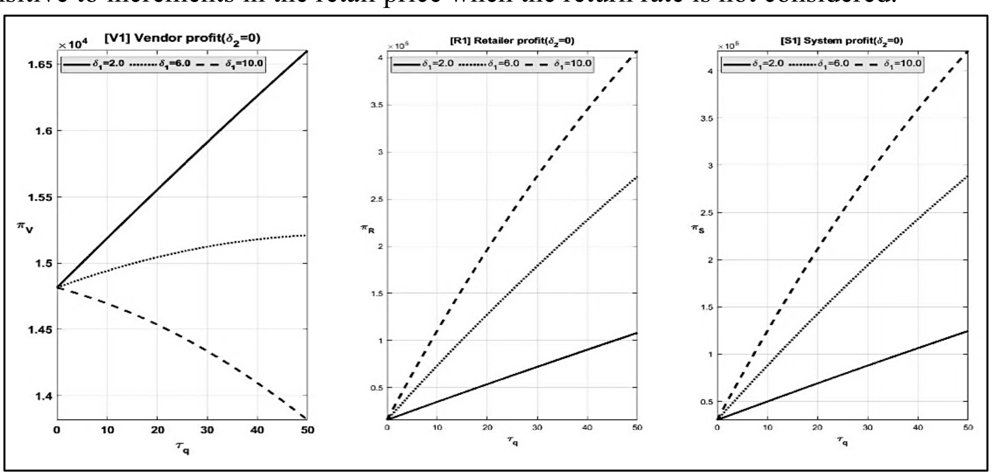


(c) Case 3: Cost-Performance Pricing

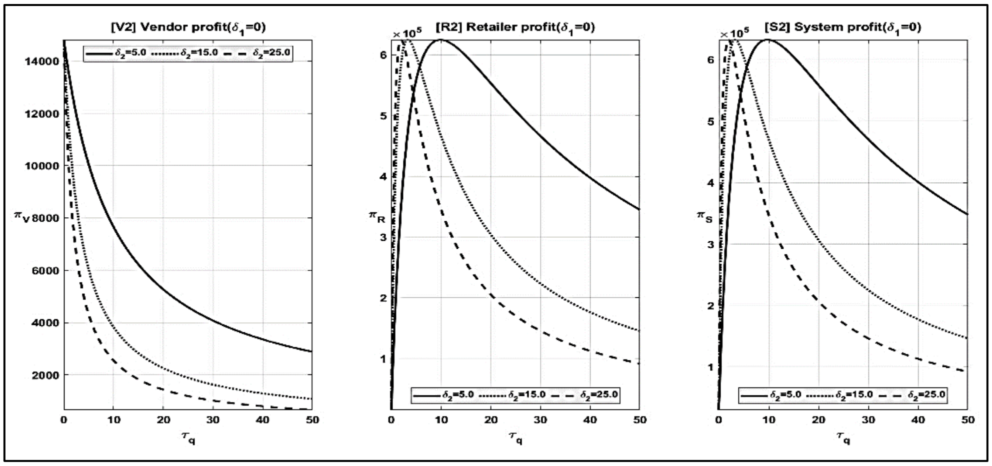
Figure 5: Profit profiles in τ_q with given values of δ_1 and δ_2 (α=0.9 and b=0.5)

5.2.2 The effects of the price sensitivity (i.e., b) on the profit functions

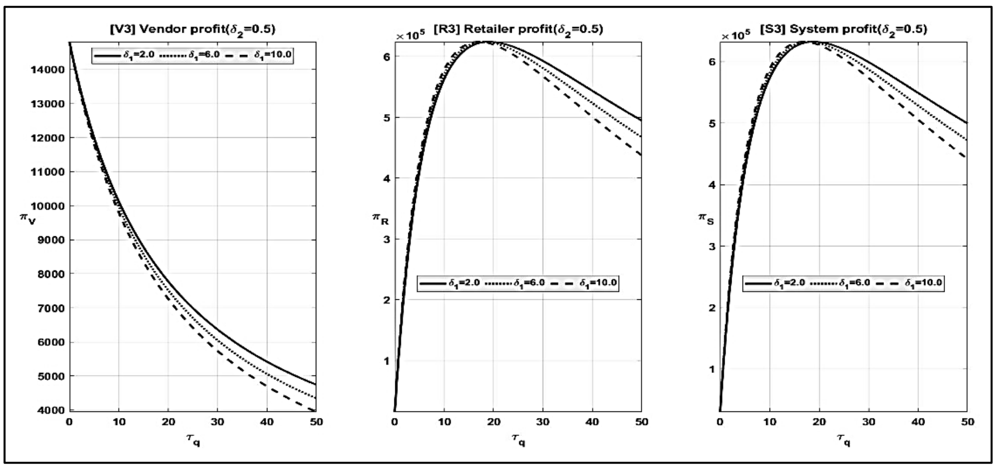
We now investigate the effects of changes in price sensitivity (i.e., b), since it can be a significant determinant for the needed RTI requirements. We consider two different values of b (i.e., 0.5 and 0.1) and a fixed value of q= 100, together with the other parameters in ***Table 2***. The results are presented in ***Figure 6***. When comparing these profit function patterns to ***Figure 5***, the changes in b across identical cases(considering both δ_1 and δ_2) can be observed. One noticeable change is when the value of b changes from 0.5 to 0.1 i.e. when the customer demand is less sensitive to retail price changes. As can be seen in ***Figure 6***, the drops in profits across the three cases seem to be less significant as the value of the deposit becomes larger. In addition, we observe that the vendor can increase the deposit amount since the consumers are less sensitive to increments in the retail price when the return rate is not considered.



(a) Case 1: Deposit-based Pricing



(b) Case 2: Performance-based Pricing

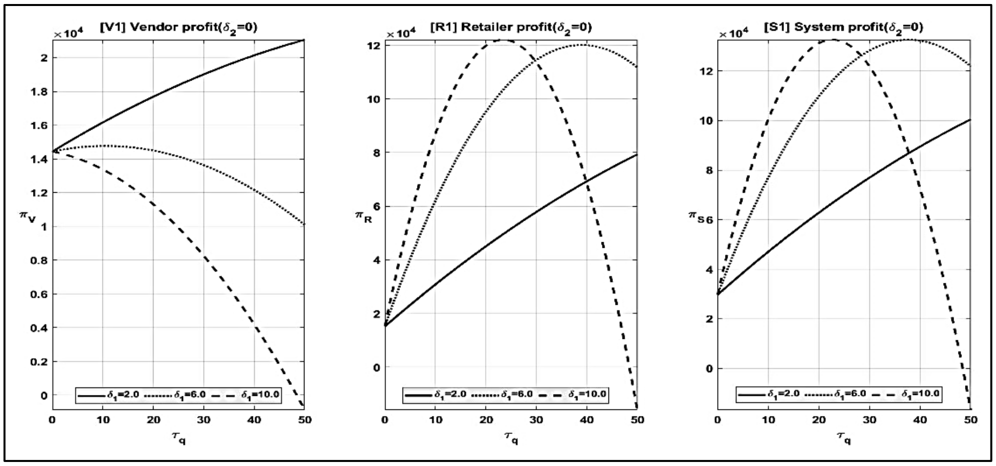


(c) Case 3: Cost-Performance Pricing

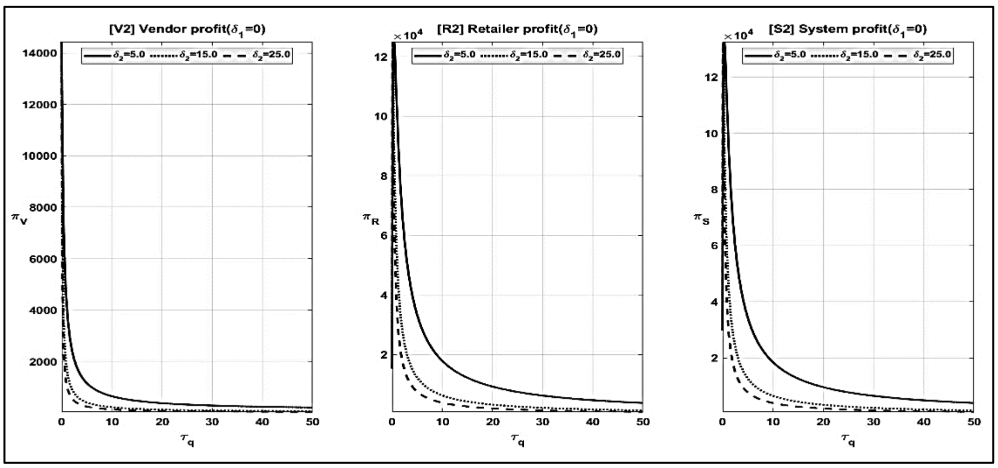
Figure 6: Profit profiles in τ_q with given values of δ_1 and δ_2 (α=0.9 and b=0.1)

5.2.3 The effects of the return rate (i.e., α) on the profit functions

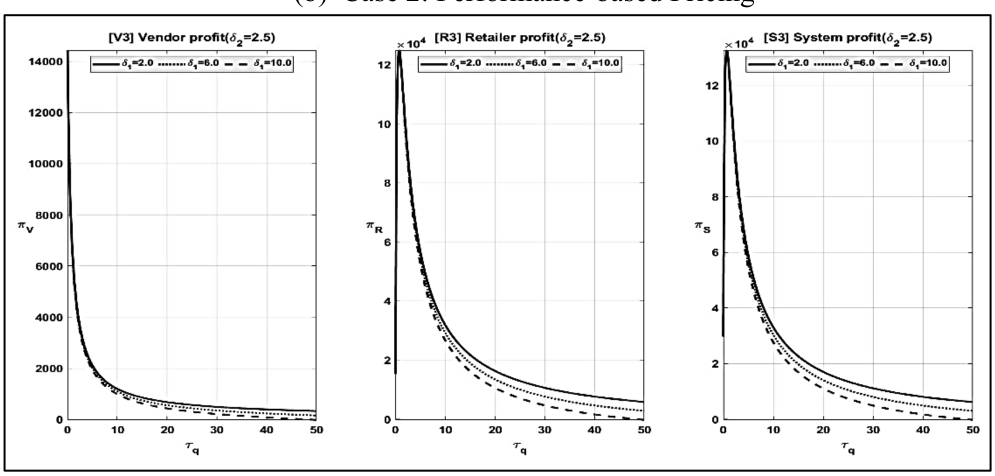
In a third set of sensitivity analyses, we consider the return rate (i.e.,α), which is a key operational characteristic in yielding profits for the vendor, the retailer, and the system overall. We obtain the profit function patterns by changing the value of α from 0.9 to 0.5 such that the return rate is reduced. When comparing ***Figure 5*** (where all plots were developed with the value of α= 0.9) with ***Figure*** *7*, we can see that the profit functions in ***Case 1)*** do not change since the return rate is not considered. However, we observe that the profit functions for both ***Case 2)*** and ***Case 3)*** have a similar pattern. We can thus conclude that the profit effects of δ_1 become insignificant as the return rate reduces when the value of δ_2 is considered. As such, there are no noticeable gaps among the profit functions with different values of δ_1 and δ_2.



(a) Case 1: Deposit-based Pricing



(b) Case 2: Performance-based Pricing



(c) Case 3: Cost-Performance Pricing

Figure 7: Profit profiles in τ_q with given values of δ_1 and δ_2 (α=0.5 and b=0.5)

As shown in this section, it is noted that there would be an influence on profit profiles when either the return performance(i.e., α) or the price sensitivity(i.e., b) changes. Thus, it would be necessary to establish the decisions considering the given operating environment, such as the operational performance, and the customer's perception of the changes in the retail price, in a sophisticated manner.

6. Managerial insights and implications

An extensive analysis has been conducted, investigating into a variety of real-world cases that can be categorized as either vendor-driven or retailer-driven, depending upon the settings of the wholesale price, container price, and capacity variables. Through this analysis, it has been elucidated that the significance of the vendor's position within these scenarios hinges upon subtle differentiations in the relative worth between the RTI and the goods it contains, underscoring the multifaceted nature of their relationship and its implications on supply chain dynamics. In the case of a vendor-driven supply chain, the vendor aims at reducing as much as possible the deposit, while the optimal deposit for maximizing supply chain profit is very close to the one to optimize the retailer profit. In the case of retailer-driven system, the retailer's overall behaviors are so similar to ones for maximizing the system profits. Therefore the vendor wants to optimize its profit by aiming at a specific deposit higher than the optimal deposit for maximizing retailer profit, which is very close to the one to optimize the supply chain profit. As general findings, it should be noted that the optimal return rate that maximizes the retailer's profit and the system profit needs to be carefully established since the profit function is closely dependent on the return rate based on the given deposit amount. Furthermore, an essential factor to consider critically is the sustainability of RTI operations, a facet reliant upon both the configuration of the RTI and their economic capacity. Sustainable RTI operations hinge not only on the initial design of the systems but also on their ongoing economic viability, emphasizing the importance of meticulous planning and resource allocation to ensure their long-term effectiveness.

7. Concluding remarks

We developed a profitable deposit-refund system for the circulation of RTI to maximize the supply chain actor's corresponding profitability in a single-vendor-single-retailer supply chain system. A retailer is assumed to determine the retail price with the cost-plus pricing policy when the deposit-refund system is used to circulate the RTI. Under the cost-plus pricing scheme, three different and optional profit functions were considered to study the effects of RTI-related parameters, including the deposit amount and the RTI return rate, and to determine optimal policies under different retail price structures. Specifically, we investigated what the optimal deposit amount should be to maximize the RTI return rate, what the effect of the retail price (which includes the deposit amount) is on the supply chain actors' profits given the RTI return rate, and what the supply chain actors' profit patterns are based on changes in the retailer's return performance of the used RTI. The contributions of this work are three-fold. First, we developed solution mechanisms for the three cases based on the composition of the retail price. A key finding included that the vendor's wholesale price (i.e.,p_0) plays a significant role in determining the decision-maker's optimal policy in designing a DRS for RTI. Second, based on the numerical analyses, we observed that the pricing based on the return performance (i.e., (i.e.,u_R) would be more significant in determining the value of the deposit than the pricing with the unit deposit cost (i.e.,τ_q), since it is directly related to the return rate. The vendor's profit function was able to be determined straightforwardly for most cases, based on the amount of the wholesale price, while other profit functions were more complex in their structure and even in the optimal deposit amount. Thus, it would be also interesting work to determine the vendor's wholesale price in designing the sustainable and profitable closed-loop supply chain. And third, we identified the retailer's profit function to be similar to the system's profit in most cases. It thus seems prudent to maximize the retailer's profit as it would lead to a maximization of the overall system profit as well.

Despite these contributions, our work is not void of limitations. First, in modeling the supply chain, we considered a single retailer. Future research should seek to develop a model with multiple retailers that can share RTI between themselves. Considering the multiple heterogeneous characteristics of retailers (e.g., demand structures, cost factors, return rates) would be a worthwhile extension of our work. In other words, there would be some unavoidable conflicts in establishing the deposit-refund system, which is commonly applied to multiple retailers. Thus, it would be valuable to develop a stable and satisfactory deposit-refund system for accelerating the adoption of RTI sharing for multiple retailers. Secondly, this work was based on the development of our models on static parameters, which included the return rate and the demand. Surely, it is also necessary to study the effects of variability in those key factors on the robust deposit-refund system circulating the RTI for a sustainable supply chain. Also, in this work, we did not explicitly define the customer's willingness-to-pay when the deposit cost is imbursed on the retail price. Thus, as a valuable extension of this work, it is necessary to incorporate the customer's willingness-to-pay for adopting the profitable deposit-refund system. As a result, future research is encouraged to extend our work by studying the stochastic dynamics of RTI-related operations for the design of the deposit-refund system. Finally, a possible extension of this work can consider a situation where RTI can be reused only a finite number of times, i.e. according to [28] which considered items with a finite number of recycling.

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Appendix A: A feasibility condition of τ_q

There would be two conditions to be satisfied to keep the relevance of the value of τ_q for the developed model. The first condition is needed to satisfy the non-negative demand rate in Eq. (3), i.e., $D = \left(\frac{d_0 - \delta_1 \tau_q}{1+\delta_2 \tau_q \rho}\right)$. From Eq. (3), the following condition of τ_q could be derived as shown in Eq. (A.1).

$$\tau_q \leq \tau_{q,d_0}^{max} = \left(\frac{d_0}{\delta_1}\right) \tag{A.1}$$

And, the second condition is to establish the feasible value of τ_q within the customer's acceptable burden for a deposit, i.e., θ_r^{max}, as shown in Eq. (A.2).

$$\theta_r = \left(c_1 \tau_q + c_2 \tau_q \rho D\right) \leq \theta_r^{max}, \text{ where } D = \left(\frac{d_0 - \delta_1 \tau_q}{1+\delta_2 \tau_q \rho}\right) \tag{A.2}$$

From the condition in Eq. (A.2), the following condition of τ_q is further developed as presented in Eq. (A.3).

$$\tau_q \leq \tau_{q,\theta_r}^{max} = \frac{\theta_r^{max}}{\left(c_1 + \rho c_2 (d_0 - \theta_r^{max} b)\right)} \tag{A.3}$$

As a result, the feasibility condition of τ_q would be developed as follows:

$$0 \leq \tau_q \leq \tau_q^{max} = min\ (\tau_{q,d_0}^{max}, \tau_{q,\theta_r}^{max}) \tag{A.4}$$

$$\text{, where } \tau_{q,d_0}^{max} = \left(\frac{d_0}{\delta_1}\right) \text{ and } \tau_{q,\theta_r}^{max} = \frac{\theta_r^{max}}{\left(c_1 + \rho c_2 (d_0 - \theta_r^{max} b)\right)}$$

Appendix B: Properties of the vendor's profit function, i.e., $\pi_V(\tau_q|\alpha)$

First, from Eq. (7), the first-order derivative of $\pi_V(\tau_q|\alpha)$ in τ_q can be derived as seen in Eq. (B.1).

$$\frac{\partial \pi_V(\tau_q|\alpha)}{\partial \tau_q} = \frac{\partial D}{\partial \tau_q}\left(p_0 + \tau_q \rho - K_q\right) + D\rho = -\left(\frac{\delta_1 \delta_2 \rho^2 \tau_q^2 + 2\delta_1 \rho \tau_q - \left(d_0 \rho - (\delta_1 + \delta_2 d_0 \rho)(p_0 - K_q)\right)}{\left(1+\delta_2 \tau_q \rho\right)^2}\right) \tag{B.1}$$

With this, the following condition is satisfied to keep the first-order optimality condition, i.e., $\frac{\partial \pi_V(\tau_q|\alpha)}{\partial \tau_q} = 0$:

$$\delta_1 \delta_2 \rho^2 \tau_q^2 + 2\delta_1 \rho \tau_q - \left(d_0 \rho - (\delta_1 + \delta_2 d_0 \rho)(p_0 - K_q)\right) = 0 \tag{B.2}$$

In addition, the equation of $\frac{\partial^2 \pi_V(\tau_q|\alpha)}{\partial {\tau_q}^2}$ can be derived as follows:

$$\frac{\partial^2 \pi_V(\tau_q|\alpha)}{\partial {\tau_q}^2} = \frac{\partial^2 D}{\partial {\tau_q}^2}\left(p_0 + \tau_q \rho - K_q\right) + 2\rho \frac{\partial D}{\partial \tau_q} = \frac{2\delta_2 \rho (\delta_1 + \delta_2 d_0 \rho)\left(p_0 - (K_q + \delta_2^{-1})\right)}{\left(1+\delta_2 \tau_q \rho\right)^3} \tag{B.3}$$

As presented in Eqs. (B.2) and (B.3), the optimality condition for maximizing the vendor's profit can be differently developed according to the signs of both δ_1 and δ_2, which are defined as the key determinants of the retail price.

Case V1) Deposit-based Pricing (i.e., $\theta_r = c_1 \tau_q$)

The first-order optimality condition in Eq. (B.2) can be further simplified as seen in Eq. (B.4) when the retail price is unrelated to the unredeemed deposit (i.e., u_R), i.e., $c_2 = 0$.

$$2\delta_1 \rho \tau_q - d_0 \rho + \delta_1 (p_0 - K_q) = 0 \tag{B.4}$$

And then, from Eq. (B.4), the optimal deposit maximizing the vendor's profit is obtained as follows:

$$\tau_{q,V1}^{0}(\alpha) = \frac{1}{2}\left(\tau_{q,d_0}^{max} - \frac{(p_0 - K_q)}{\rho}\right), \text{ where } \tau_{q,d_0}^{max} = \left(\frac{d_0}{\delta_1}\right) \quad \text{(B.5)}$$

Furthermore, from Eq. (B.1), the second-order derivative can be obtained as shown in Eq. (B.6) in case that $\delta_2 = 0$.

$$\frac{\partial^2 \pi_V(\tau_q|\alpha)}{\partial {\tau_q}^2} = -2\delta_1\rho < 0 \quad \text{(B.6)}$$

This means that the vendor's profit function is strictly concave in τ_q if $\delta_1 > 0$ and $\delta_2 = 0$. In addition, from Eq. (B.5), it is known that the inequality of $\frac{d\tau_{q,V1}^{0}(\alpha)}{d\alpha} = \frac{p_0 - (c_p + c_\beta)/q}{2(1-\alpha)^2} > 0$ is always valid such that a vendor wants to increase the value of the deposit amount as the return rate increases if $p_0 > (c_p + c_\beta)/q = \left(K_q + \frac{c_\beta \rho}{q}\right)$.

Case V2) Performance-based Pricing (i.e., $\theta_r = c_2 u_R$)

The equation of $\frac{\partial \pi_V(\tau_q|\alpha)}{\partial \tau_q}$ in Eq. (B.2) can be simplified based on the condition of $c_1 = 0$:

$$\frac{\partial \pi_V(\tau_q|\alpha)}{\partial \tau_q} = -\left(\frac{d_0 \rho \delta_2 \left(p_0 - (K_q + \delta_2^{-1})\right)}{(1 + \delta_2 \tau_q \rho)^2}\right) \geq 0 \text{ if } p_0 \leq (K_q + \delta_2^{-1}) \quad \text{(B.7)}$$

Further, we know that there is no stationary point satisfying the first-order optimality condition in Eq. (B.7), i.e., $\frac{\partial \pi_V(\tau_q|\alpha)}{\partial \tau_q} = 0$. As noted in Eq. (B.7), the function of $\pi_V(\tau_q|\alpha)$ is an increasing function of τ_q if the wholesale price, i.e., p_0, satisfies the given condition, i.e., $p_0 \leq (K_q + \delta_2^{-1})$. Note that the value of K_q represents the unit handling cost of an RTI per item to be sold and δ_2 denotes the sensitivity of the unredeemed deposit, i.e., $u_R = \tau(1-\alpha)D_R$. With this, it is necessary to check the additional condition which makes the profit function concave in τ_q for the cases considered. Specifically, from Eq. (B.3), the condition for a concave profit function to be obtained is shown in Eq. (B.8).

$$\frac{\partial^2 \pi_V(\tau_q|\alpha)}{\partial {\tau_q}^2} = \frac{2\rho^2 d_0 \delta_2^2 \left(p_0 - (K_q + \delta_2^{-1})\right)}{(1 + \delta_2 \tau_q \rho)^3} \leq 0 \text{ if } p_0 \leq (K_q + \delta_2^{-1}) \quad \text{(B.8)}$$

Thus, the vendor's profit function can be either a concave or a convex function based on whether the inequality condition of $p_0 \leq (K_q + \delta_2^{-1})$ is satisfied. As a result, according to the sign of $\frac{\partial \pi_V(\tau_q|\alpha)}{\partial \tau_q}$, the value of τ_q, which maximizes the vendor's profit should be determined. In other words, if $\pi_V(\tau_q|\alpha)$ is an increasing function of τ_q, i.e., $\frac{\partial \pi_V(\tau_q|\alpha)}{\partial \tau_q} \geq 0$, then the vendor's profit can be maximized at $\tau_q = \tau_q^{max}$. Otherwise, the vendor's profit is maximized at $\tau_q = 0$ if $\frac{\partial \pi_V(\tau_q|\alpha)}{\partial \tau_q} < 0$.

Case V3) Cost-Performance Pricing (i.e., $\theta_r = c_1\tau_q + c_2 u_R$)

The value of τ_q satisfying the first-order optimality condition in Eq. (B.2) can be obtained as shown in Eq. (B.9).

$$\tau_{q,V3}^{0}(\alpha) = \frac{-\delta_1 + \sqrt{\delta_1(\delta_1+\delta_2 d_0\rho)\left(1-\delta_2(p_0-K_q)\right)}}{\delta_1\delta_2\rho} \tag{B.9}$$

As explained in ***Case V2***), from Eq. (B.3), if the condition for a concave profit function, i.e., $p_0 \leq \left(K_q + \delta_2^{-1}\right)$, is satisfied, then the value of $\tau_{q,V3}^{0}(\alpha)$ in Eq. (B.9) is an optimal value maximizing the vendor's profit function. Otherwise, if the condition of $p_0 \leq \left(K_q + \delta_2^{-1}\right)$ is not satisfied, then the vendor's profit function can be maximized at one of the available boundary values, i.e., 0 or τ_q^{max}, since the vendor's profit function is a convex function. Thus, the value of τ_q can be differently established according to whether the inequality of $p_0 \leq \left(K_q + \delta_2^{-1}\right)$ is satisfied or not as shown in ***Case V2)***. Furthermore, it is known that the inequality of $\tau_{q,V3}^{0}(\alpha) \leq \tau_{q,d_0}^{max}$ is always satisfied if the assumption of $p_0 \geq K_q$ is satisfied as presented below:

$$\tau_{q,V3}^{0}(\alpha) \leq \tau_{q,d_0}^{max} \rightarrow \frac{-\delta_1 + \sqrt{\delta_1(\delta_1+\delta_2 d_0\rho)\left(1-\delta_2(p_0-K_q)\right)}}{\delta_1\delta_2\rho} \leq \left(\frac{d_0}{\delta_1}\right) \tag{B.10}$$

The inequality in Eq. (B.10) could be further simplified as shown in Eq. (B.11).

$$\rho d_0 + \frac{\delta_2(\rho d_0)^2}{\delta_1} + (\delta_1 + \delta_2 d_0\rho)\left(p_0 - K_q\right) \geq 0 \tag{B.11}$$

As assumed earlier, the inequality in Eq. (B.11) is always valid in cases when $p_0 \geq K_q$ since the other terms are given as nonnegative-valued ones.

Appendix C: Properties of the retailer's profit function, i.e., $\pi_R(\tau_q|\alpha)$

The first-order derivative of $\pi_R(\tau_q|\alpha)$, i.e.,$\frac{\partial\pi_R(\tau_q|\alpha)}{\partial\tau_q}$, is obtained as shown in Eq. (C.1).

$$\frac{\partial\pi_R(\tau_q|\alpha)}{\partial\tau_q} = \left(\frac{\partial p}{\partial\tau_q}D + p\frac{\partial D}{\partial\tau_q} - D\rho - \tau_q\rho\frac{\partial D}{\partial\tau_q}\right) = \frac{\partial D}{\partial\tau_q}\left(\frac{a-2D}{b} - \tau_q\rho\right) - D\rho \tag{C.1}$$

$$\text{, where } p = \frac{a-D}{b} \text{ and } \frac{dp}{d\tau_q} = -\frac{1}{b}\left(\frac{\partial D}{\partial\tau_q}\right)$$

Hereafter, as done for the vendor's profit function in ***Appendix B***, the optimality properties of $\pi_R(\tau_q|\alpha)$ are developed according to the signs of both δ_1 and δ_2**.**

Case R1) Deposit-based Pricing (i.e., $\theta_r = c_1\tau_q$)

In the case where $\delta_1 > 0$ and $\delta_2 = 0$, Eq. (C.1) can be re-arranged with the condition of $\delta_2 = 0$ as follows:

$$\frac{\partial\pi_R(\tau_q|\alpha)}{\partial\tau_q} = \left(-\frac{\delta_1}{b}\right)\left((a-2d_0) + \frac{b\rho d_0}{\delta_1} + 2(\delta_1 - b\rho)\tau_q\right) \tag{C.2}$$

With this, the stationary value of $\tau_{q,R1}^{0}(\alpha)$, which satisfies the first-order optimality condition, can be obtained as seen below:

$$\tau_{q,R1}^{0}(\alpha) = \frac{\delta_1(a-2d_0)+d_0\rho b}{2\delta_1(b\rho-\delta_1)} = \frac{b\left(p_{max}-\tau_{q,d_0}^{max}\rho\right)}{2(b\rho-\delta_1)} + \tau_q^{max} \tag{C.3}$$

$$, \text{ where } \tau_{q,d_0}^{max} = \left(\frac{d_0}{\delta_1}\right) \text{ and } p_{max} = \left(\frac{a}{b}\right)$$

Furthermore, from Eq. (C.2), the second-order derivative of $\pi_R(\tau_q|\alpha)$ can be derived as seen in Eq. (C.4).

$$\frac{\partial^2 \pi_R(\tau_q|\alpha)}{\partial {\tau_q}^2} = 2\delta_1(\rho - c_1) \text{ , where } \delta_1 = bc_1 \tag{C.4}$$

Thus, from Eq. (C.4), the retailer's profit function can be identified as either convex or concave according to whether $\rho \leq c_1$ or not. As a result, if $\rho \leq c_1$, the retailer's profit function is concave such that the stationary point in Eq. (C.3) is a unique optimal point. In addition, if $\rho > c_1$, the retailer's profit function must be a convex function, and the optimal value of τ_q is one of two extreme points, i.e., $\tau_{q,R1}^* \in \{0, \tau_q^{max}\}$.

Case R2) Performance-based Pricing (i.e., $\theta_r = c_2 u_R$)

With the assumption of $\delta_1 = 0$ and $\delta_2 > 0$, the first-order derivative of $\pi_R(\tau_q|\alpha)$ is developed as shown in Eq. (C.5).

$$\frac{\partial \pi_R(\tau_q|\alpha)}{\partial \tau_q} = \frac{\partial D}{\partial \tau_q}\left(\frac{a}{b} - \frac{2D}{b} - \tau_q \rho\right) - D\rho = -\frac{d_0 \rho\left((\delta_2(a-2d_0)+b) + \delta_2 \rho(\delta_2 a + b)\tau_q\right)}{b(1+\delta_2 \tau_q \rho)^3} \tag{C.5}$$

With this, the unique stationary value satisfying the condition of $\frac{\partial \pi_R(\tau_q|\alpha)}{\partial \tau_q} = 0$ in Eq. (C.5) can be presented in Eq. (C.6).

$$\tau_{q,R2}^0(\alpha) = \frac{\delta_2(2d_0 - a) - b}{\delta_2 \rho(\delta_2 a + b)} = \frac{1}{\rho}\left(\frac{2d_0}{\delta_2 a + b} - \frac{1}{\delta_2}\right) \tag{C.6}$$

Furthermore, from Eq. (C.5), the second-order derivative of $\pi_R(\tau_q|\alpha)$ can be derived as follows.

$$\frac{\partial^2 \pi_R(\tau_q|\alpha)}{\partial {\tau_q}^2} = -2\delta_2\left(\frac{d_0 \rho^2 (1+\delta_2 \tau_q \rho)^2}{b}\right)\left(\frac{3\delta_2 d_0 - (\delta_2 a + b) - \rho\delta_2(\delta_2 a + b)\tau_q}{(1+\delta_2 \tau_q \rho)^6}\right) \tag{C.7}$$

Thus, with the condition that ensures the concavity of the retailer's profit function, i.e., $\frac{\partial^2 \pi_R(\tau_q|\alpha)}{\partial {\tau_q}^2} \leq 0$, the value of τ_q should satisfy the following condition:

$$\tau_q \leq \tau_{q,R2}^b(\alpha), \text{ where } \tau_{q,R2}^b(\alpha) = \frac{1}{\rho}\left(\frac{3d_0}{\delta_2 a + b} - \frac{1}{\delta_2}\right) \tag{C.8}$$

From Eqs. (C.6) and (C.8), note that the inequality of $\tau_{q,R2}^0(\alpha) \leq \tau_{q,R2}^b(\alpha)$ is always satisfied such that there must be a stationary point satisfying the first-order optimality condition in $[0, \tau_{q,R2}^b(\alpha)]$. In other words, it is certain that the stationary value in Eq. (C.6), i.e., $\tau_{q,R2}^0(\alpha)$, always satisfies the condition for the existence of the optimal deposit amount, since the profit function is concave, i.e., $\tau_{q,R2}^0(\alpha) \leq \tau_{q,R2}^b(\alpha)$. Thus, it is necessary to check whether the retailer's profit function is either a concave increasing function or a concave decreasing function at $\tau_q = \tau_{q,R2}^b(\alpha)$. As shown in Eq. (C.9), the value of $\frac{\partial \pi_R(\tau_q|\alpha)}{\partial \tau_q}$ at $\tau_q = \tau_{q,R2}^b(\alpha)$ is always negative such that the retailer's profit function is a decreasing function at $\tau_q = \tau_{q,R2}^b(\alpha)$.

$$\left.\frac{\partial \pi_R(\tau_q|\alpha)}{\partial \tau_q}\right|_{\tau_q = \tau_{q,R2}^b} = \frac{-\delta_2 d_0^2 \rho}{b(1+\delta_2 \tau_q \rho)^3} < 0 \tag{C.9}$$

Case R3) Cost-Performance Pricing (i.e., $\theta_r = c_1\tau_q + c_2 u_R$)
Eq. (C.1) can be further re-arranged when both δ_1 and δ_2 are significant as shown in Eq. (C.10).

$$\frac{\partial \pi_R(\tau_q|\alpha)}{\partial \tau_q} = \frac{\partial D}{\partial \tau_q}\left(\frac{a-2D}{b} - \tau_q\rho\right) - D\rho = \left(-\frac{(\delta_1 + d_0\delta_2\rho)}{\left(1+\delta_2\tau_q\rho\right)^2}\right)\left(\frac{a}{b} - \frac{2\left(d_0 - \delta_1\tau_q\right)}{b\left(1+\delta_2\tau_q\rho\right)} - \tau_q\rho\right) - \frac{\left(d_0 - \delta_1\tau_q\right)\rho}{\left(1+\delta_2\tau_q\rho\right)}$$
$$= \left(\frac{z_1\tau_q^3 + z_2\tau_q^2 + z_3\tau_q + z_4}{b\left(1+\delta_2\tau_q\rho\right)^3}\right) \tag{C.10}$$

, where $z_1 = b\delta_1\delta_2^2\rho^3$, $z_2 = 3b\delta_1\delta_2\rho^2$, $z_3 = \left((\delta_1 + \delta_2 d_0\rho)(b\rho - 2\delta_1 - \delta_2 a\rho) - (2\delta_2 d_0\rho - \delta_1)b\rho\right)$, and $z_4 = (\delta_1 + \delta_2 d_0\rho)(2d_0 - a) - bd_0\rho$

It is hard to handle the closed-form of τ_q, which satisfies the condition of $\frac{\partial \pi_R(\tau_q|\alpha)}{\partial \tau_q} = 0$, since Eq. (C.10) is the polynomial of degree three when τ_q is set as an unknown variable. Thus, one possibility is to use an analytical approach in finding the value of τ_q, for instance via the bi-section method, after having investigated the pattern of the retailer's profit function. Also, the equation of $\frac{\partial^2 \pi_R(\tau_q|\alpha)}{\partial {\tau_q}^2}$ can be further re-arranged as follows:

$$\frac{\partial^2 \pi_R(\tau_q|\alpha)}{\partial {\tau_q}^2} = \frac{\partial^2 D}{\partial {\tau_q}^2}\left(\frac{a-2D}{b} - \tau_q\rho\right) - \frac{2}{b}\left(\frac{\partial D}{\partial \tau_q}\right)^2 - 2\rho\frac{\partial D}{\partial \tau_q}$$
$$= \frac{2(\delta_1 + \delta_2 d_0\rho)\left((\delta_2 a\rho + 2\delta_1 + b\rho)\delta_2\rho\tau_q + (a\delta_2 + b - 3\delta_2 d_0)\rho - \delta_1\right)}{b\left(1+\delta_2\tau_q\rho\right)^4} \tag{C.11}$$

From Eq. (C.11), it is known that $\pi_R(\tau_q|\alpha)$ is a concave function, i.e., $\frac{\partial^2 \pi_R(\tau_q|\alpha)}{\partial {\tau_q}^2} \le 0$, if $\tau_q \le \tau_{q,R3}^b(\alpha)$, where the value of $\tau_{q,R3}^b(\alpha)$ is presented as follows:

$$\tau_{q,R3}^b(\alpha) = \frac{(3\delta_2 d_0 - a\delta_2 - b)\rho + \delta_1}{(a\delta_2\rho + 2\delta_1 + b\rho)\delta_2\rho} \tag{C.12}$$

In other words, the inequality of $\frac{\partial^2 \pi_R(\tau_q|\alpha)}{\partial {\tau_q}^2} > 0$ is always satisfied if $\tau_q > \tau_{q,R3}^b(\alpha)$. Thus, it is definite that $\pi_R(\tau_q|\alpha)$ is a concave-convex function with a reflection point of $\tau_{q,R3}^b(\alpha)$. As a result, $\tau_{q,R3}^b(\alpha)$ can be used as the upper bound when we search for the value of τ_q, which satisfies the first-order optimality condition, i.e., $\frac{\partial \pi_R(\tau_q|\alpha)}{\partial \tau_q} = 0$. So, it is necessary to check the sign of $\left.\frac{\partial \pi_R(\tau_q|\alpha)}{\partial \tau_q}\right|_{\tau_q=0}$ to investigate the pattern of $\pi_R(\tau_q|\alpha)$ because it is known that the function of $\pi_R(\tau_q|\alpha)$ is a concave-convex function.

Based on the property that $\pi_R(\tau_q|\alpha)$ is a concave-convex function, we could develop the following rules in finding out the optimal deposit for a retailer with the following steps. Above all, the function of $\frac{\partial \pi_R(\tau_q|\alpha)}{\partial \tau_q}$ in Eq. (C.10) can be further simplified with $\tau_q = 0$ as shown in Eq. (C.13).

$$\left.\frac{\partial \pi_R(\tau_q|\alpha)}{\partial \tau_q}\right|_{\tau_q=0} = \frac{(\delta_1 + \delta_2 d_0\rho)(2d_0 - a)}{b} - d_0\rho \tag{C.13}$$

Thus, we can say that the function of $\pi_R(\tau_q|\alpha)$ is an increasing function at $\tau_q = 0$ if the following condition is satisfied:

$$\left.\frac{\partial\pi_R(\tau_q|\alpha)}{\partial\tau_q}\right|_{\tau_q=0} > 0 \to \rho < \frac{\delta_1(2d_0-a)}{d_0\big(b-\delta_2(2d_0-a)\big)} \tag{C.14}$$

As a result, according to the return performance, i.e., $\rho = (1-\alpha)$, the retailer's profit function has a different pattern as shown in Eq. (C.14). Thus, we can say that the value of τ_q maximizing the retailer's profit exists between 0 and $\tau_{q,R3}^{b}(\alpha)$ if the condition in Eq. (C.14) is satisfied. Otherwise, if $\left.\frac{\partial\pi_R(\tau_q|\alpha)}{\partial\tau_q}\right|_{\tau_q=0} < 0$, the boundary values of τ_q, i.e., 0 and τ_q^{max}, should be selected as an optimal value of τ_q, since the retailer's profit function can be maximized at the available extreme points.

Appendix D: Properties of the system profit function, i.e.,$\pi_S(\tau_q|\alpha)$

From the function of $\pi_S(\tau_q|\alpha)$ in Eq. (15), the first-order derivative, i.e.,$\frac{\partial\pi_S(\tau_q|\alpha)}{\partial\tau_q}$, can be presented as follows:

$$\frac{\partial\pi_S(\tau_q|\alpha)}{\partial\tau_q} = \frac{\partial D}{\partial\tau_q}\left(p_0 - K_q + \frac{a-D}{b}\right) - \frac{D}{b}\frac{\partial D}{\partial\tau_q} = \frac{\partial D}{\partial\tau_q}\frac{\big(b(p_0-K_q)+a-2D\big)}{b} \tag{D.1}$$

Note that there is no need to categorize the possible cases according to the signs of δ_1 and δ_2, since some cost terms are void when it comes to the system profit. Thus, the stationary value satisfying $\frac{\partial\pi_S(\tau_q|\alpha)}{\partial\tau_q} = 0$ can be arranged as seen in Eq. (D.2).

$$\tau_{q,S}^{0}(\alpha) = \frac{2d_0-\varphi}{2\delta_1+\delta_2\varphi\rho}\text{, where } \varphi = b(p_0-K_q)+a \tag{D.2}$$

From Eq. (D.2), it is known that the condition of $\varphi \le 2d_0$ should be satisfied to ensure that the value of $\tau_{q,S}^{0}(\alpha)$ is non-negative. In addition, the function of $\frac{\partial^2\pi_S(\tau_q|\alpha)}{\partial{\tau_q}^2}$ can be further presented as follows:

$$\begin{aligned}\frac{\partial^2\pi_S(\tau_q|\alpha)}{\partial{\tau_q}^2} &= \frac{\partial^2 D}{\partial{\tau_q}^2}\left(\frac{(p_0-K_q)b+a-2D(\tau_q,\alpha)}{b}\right) - \frac{2}{b}\left(\frac{\partial D}{\partial\tau_q}\right)^2 \\ &= \frac{2(\delta_1+\delta_2 d_0\rho)}{b\big(1+\delta_2\tau_q\rho\big)^4}\Big(\delta_2\varphi\rho - 3\delta_2\rho d_0 - \delta_1 + \delta_2\tau_q\rho(\delta_2\rho\varphi+2\delta_1)\Big)\end{aligned} \tag{D.3}$$

Thus, under the condition that the concavity of the system's profit function is assured, i.e.,$\frac{\partial^2\pi_S(\tau_q|\alpha)}{\partial{\tau_q}^2} \le 0$, the value of τ_q should satisfy the following:

$$\tau_q \le \tau_{q,S}^{b}(\alpha)\text{, where } \tau_{q,S}^{b}(\alpha) = \frac{\delta_2\rho(3d_0-\varphi)+\delta_1}{\delta_2\rho(2\delta_1+\delta_2\varphi\rho)} \tag{D.4}$$

From Eq. (D.4), it is also shown that the function of $\pi_S(\tau_q|\alpha)$ is a concave-convex function, which can be characterized by the value of $\tau_{q,S}^b(\alpha)$. Also, it is necessary to check the sign of $\frac{\partial \pi_S(\tau_q|\alpha)}{\partial \tau_q}\Big|_{\tau_q=0}$ to obtain the details as captured in Eq. (D.5).

$$\frac{\partial \pi_S(\tau_q|\alpha)}{\partial \tau_q}\Big|_{\tau_q=0} = \frac{(\delta_1+d_0\delta_2\rho)(2d_0-\varphi)}{b} > 0 \text{ if } \varphi \leq 2d_0 \tag{D.5}$$

It is also noted that the condition of $\frac{\partial \pi_S(\tau_q|\alpha)}{\partial \tau_q}\Big|_{\tau_q=0} > 0$ in Eq. (D.5) is identical to the one for a positive value of $\tau_{q,S}^0(\alpha)$ in Eq. (D.2). Thus, when the inequality of $\varphi \leq 2d_0$ is satisfied, we can say that the function of $\pi_S(\tau_q|\alpha)$ is an increasing function at $\tau_q = 0$ from Eq. (D.5), having a positive stationary value of $\tau_{q,S}^0(\alpha)$ as shown in Eq. (D.2). Also, one additional property of $\pi_S(\tau_q|\alpha)$ is the sign of $\frac{\partial \pi_S(\tau_q|\alpha)}{\partial \tau_q}\Big|_{\tau_q=\tau_{q,S}^b}$, which we need to check the pattern of $\pi_S(\tau_q|\alpha)$ using $\tau_{q,S}^b(\alpha)$ in Eq. (D.4).

$$\frac{\partial \pi_S(\tau_q|\alpha)}{\partial \tau_q}\Big|_{\tau_q=\tau_{q,S}^b} = \frac{\partial D}{\partial \tau_q}\left(\frac{d_0\delta_2\rho+\delta_1}{b\delta_2\rho(1+\delta_2\tau_q\rho)}\right) = -\frac{(\delta_1+d_0\delta_2\rho)}{(1+\delta_2\tau_q\rho)^2}\left(\frac{d_0\delta_2\rho+\delta_1}{b\delta_2\rho(1+\delta_2\tau_q\rho)}\right) < 0 \tag{D.6}$$

From Eqs. (D.5) and (D.6), it is said that there is a unique stationary value of $\tau_{q,S}^0(\alpha)$ maximizing the value of $\pi_S(\tau_q|\alpha)$ if a positive value of $\tau_{q,S}^0(\alpha)$ exists with the condition of $\varphi \leq 2d_0$. As shown in Eq. (D.2), if $\varphi > 2d_0$, then the function of $\pi_S(\tau_q|\alpha)$ has a negative stationary point and a downward slope at $\tau_q = 0$, i.e., $\tau_{q,S}^0(\alpha) < 0$ and $\frac{\partial \pi_S(\tau_q|\alpha)}{\partial \tau_q}\Big|_{\tau_q=0} < 0$. Thus, the value of $\tau_q = 0$ is the optimal value for maximizing the function of $\pi_S(\tau_q|\alpha)$ since it has a single and unique stationary point as shown in Eq. (D.2). Indeed, the inequality condition of $\varphi \leq 2d_0$ can be re-arranged as the condition of p_0 as shown in Eq. (D.7).

$$\varphi \leq 2d_0 \rightarrow p_0 \leq \frac{a+bK_q}{b(2\theta_m+3)} \tag{D.7}$$

As a result, the pattern of $\pi_S(\tau_q|\alpha)$ is characterized according to the value of p_0 and other given parameters, including the unit handling cost of RTI(i.e., K_q) and the mark-up rate(i.e., θ_m).

Appendix E: Effects of the return rate on the profit functions

In this section, we study how the return rate affects the profit functions under the condition that the value of the deposit is given. As assumed earlier, the unredeemed deposit amount is a component influencing the retail price such that the return rate is a critical factor in calculating it. Thus, we consider only the case where $c_2 > 0$, because the retail price (or equivalently, the profit functions) is not dependent on the return rate if $c_2 = 0$.

[E1] The effects of return rates on the vendor's profit

We need to investigate the effect of α on $\pi_V(\alpha|\tau_q)$ since the return rate of used RTI to the vendor for subsequent deliveries is critical. Thus, the first-order derivative of $\pi_V(\alpha|\tau_q)$, using the equality of $\frac{\partial D}{\partial \alpha} = \left(\frac{\delta_2\tau_q}{1+\delta_2\tau_q\rho}\right) D$ concerning ρ, can be composed as follows:

$$\frac{\partial \pi_V(\alpha|\tau_q)}{\partial \alpha} = \frac{\partial D}{\partial \alpha}\left(p_0 + \tau_q\rho - K_q\right) - D\left(\tau_q + \frac{dK_q}{d\alpha}\right) = D\left(\frac{\delta_2\tau_q\left(p_0 + \tau_q\rho - K_q\right)}{\left(1+\delta_2\tau_q\rho\right)} - \left(\frac{\tau_q q + c_\beta}{q}\right)\right)$$
$$= D\left(\frac{\delta_2\tau_q\left(qp_0 - \left(c_p + c_\beta\right)\right) - \left(\tau_q q + c_\beta\right)}{\left(1+\delta_2\tau_q\rho\right)q}\right) \tag{E.1}$$

, where $K_q = \frac{c_p + c_\beta \alpha}{q}$ and $\frac{dK_q}{d\alpha} = \left(\frac{c_\beta}{q}\right)$

As seen in Eq. (E.1), the vendor's profit function is either an increasing function or a decreasing function according to the value of p_0. Thus, from Eq. (E.1), the following condition can be derived, which maximizes the vendor's profit.

$$p_0 = \left(\frac{c_\beta + c_p}{q} + \frac{c_\beta + q\tau_q}{\delta_2 q\tau_q}\right) = \frac{1}{q}\left(\left(c_p + c_\beta\right) + \frac{c_\beta}{\delta_2\tau_q}\right) + \frac{1}{\delta_2} \tag{E.2}$$

In other words, the vendor's profit function is an increasing function when the vendor's selling price is less than a specific limit in Eq. (E.2). In addition, we obtain the second-order derivative using $\frac{\partial \pi_V(\alpha|\tau_q)}{\partial \alpha}$ in Eq. (E.1) as follows:

$$\frac{\partial^2 \pi_V(\alpha|\tau_q)}{\partial \alpha^2} = \frac{\partial D}{\partial \alpha}\left(\frac{\delta_2\tau_q\left(p_0 + \tau_q\rho - K_q\right)}{\left(1+\delta_2\tau_q\rho\right)} - \left(\tau_q + \frac{c_\beta}{q}\right)\right) + D\left(\frac{\left(\delta_2\tau_q\right)^2}{\left(1+\delta_2\tau_q\rho\right)^2}\left(p_0 + \tau_q\rho - K_q\right) - \frac{\delta_2\tau_q}{\left(1+\delta_2\tau_q d\rho\right)}\left(\tau_q + \frac{c_\beta}{q}\right)\right)$$
$$= 2\frac{\partial D}{\partial \alpha}\left(\frac{\delta_2\tau_q\left(p_0 + \tau_q\rho - K_q\right)}{\left(1+\delta_2\tau_q\rho\right)} - \left(\tau_q + \frac{c_\beta}{q}\right)\right) \tag{E.3.1}$$

In addition, $\frac{\partial^2 \pi_V(\alpha|\tau_q)}{\partial \alpha^2}$ in Eq. (E.3.1) can be further re-arranged as presented in Eq. (E.3.2).

$$\frac{\partial^2 \pi_V(\alpha|\tau_q)}{\partial \alpha^2} = \left(\frac{2}{D}\right)\frac{\partial D}{\partial \alpha}\frac{\partial \pi_V(\alpha|\tau_q)}{\partial \alpha} = \left(\frac{2\delta_2\tau_q}{1+\delta_2\tau_q\rho}\right)\frac{\partial \pi_V(\alpha|\tau_q)}{\partial \alpha} \tag{E.3.2}$$

As a result, the vendor's profit function depends on the condition for p_0 in Eq. (E.2), since both $\frac{\partial \pi_V(\alpha|\tau_q)}{\partial \alpha}$ and $\frac{\partial^2 \pi_V(\alpha|\tau_q)}{\partial \alpha^2}$ have the same sign as shown in Eq. (E.3.2).

[E2] The effects of return rate on the retailer's profit

As done for the vendor's profit function, the following equations of the retailer's profit function are used to analyze the properties of the retailer's profit functions.

$$\frac{\partial \pi_R(\alpha|\tau_q)}{\partial \alpha} = \frac{\partial D}{\partial \alpha}\left(\frac{a}{b} + \frac{1}{\delta_2} - \frac{2D}{b}\right) \tag{E.4.1}$$

$$\frac{\partial^2 \pi_R(\alpha|\tau_q)}{\partial \alpha^2} = \frac{\partial^2 D}{\partial \alpha^2}\left(\frac{a}{b} + \frac{1}{\delta_2} - \frac{3D}{b}\right) \tag{E.4.2}$$

From Eq. (E.4.1), the stationary value of α_R^0, which satisfies the first-order condition, is obtained as seen in Eq. (E.5).

$$\alpha_R^0(\tau_q) = 1 - \frac{1}{\tau_q}\left(\frac{2(d_0 - \delta_1\tau_q)}{(a\delta_2 + b)} - \frac{1}{\delta_2}\right) \tag{E.5}$$

Also, we can say that $\frac{\partial^2 \pi_R(\alpha|\tau_q)}{\partial\alpha^2} = -\frac{\partial^2 D}{\partial\alpha^2}\left(\frac{D}{b}\right) < 0$ at $\alpha = \alpha_R^0(\tau_q)$, where $\left(\frac{a}{b} + \frac{1}{\delta_2}\right) = \frac{2D(\tau_q, \alpha_R^0)}{b}$. As a result, the value of $\alpha_R^0(\tau_q)$ in Eq. (E.5) can maximize the retailer's profit when the value of τ_q is fixed.

[E3] The effects of return rate on the system's profit

Similar to previous sections, the system profit has the following properties in α as presented below:

$$\frac{\partial \pi_S(\alpha|\tau_q)}{\partial\alpha} = D\left(\frac{\delta_2\tau_q\big(b(p_0 - K_q) + (a - 2D)\big)}{b(1 + \delta_2\tau_q\rho)} - \frac{c_\beta}{q}\right) \tag{E.6.1}$$

$$\frac{\partial^2 \pi_S(\alpha|\tau_q)}{\partial\alpha^2} = \left(\frac{2\delta_2\tau_q}{1 + \delta_2\tau_q\rho}\right)\left(\frac{\partial \pi_S(\alpha|\tau_q)}{\partial\alpha} - \frac{1}{b}\left(\frac{\delta_2\tau_q D^2}{1 + \delta_2\tau_q\rho}\right)\right) \tag{E.6.2}$$

From Eq. (E.6.1), a stationary value of ρ, which satisfies the first-order condition, can be presented as follows:

$$\alpha_S^0(\tau_q) = 1 - \left(\frac{2q(d_0 - \delta_1\tau_q)}{\tau_q\delta_2 a_S q - bc_\beta} - \frac{1}{\delta_2\tau_q}\right), \text{ where } a_S = a + b\left(p_0 - \frac{c_p + c_\beta}{q}\right) \tag{E.7}$$

Thus, we can say that $\frac{\partial \pi_S(\alpha|\tau_q)}{\partial\alpha} < 0$ if $\alpha < \alpha_S^0(\tau_q)$ and $\frac{\partial \pi_S(\alpha|\tau_q)}{\partial\alpha} > 0$ if $\alpha > \alpha_S^0(\tau_q)$. In addition, the inequality of $\frac{\partial^2 \pi_S(\alpha|\tau_q)}{\partial\alpha^2} < 0$ is satisfied if $\frac{\partial \pi_S(\alpha|\tau_q)}{\partial\alpha} < 0$. We can further rearrange the function of $\frac{\partial^2 \pi_S(\alpha|\tau_q)}{\partial\alpha^2}$ at the stationary point, i.e., $\alpha = \alpha_S^0(\tau_q)$, into $\left.\frac{\partial^2 \pi_S(\alpha|\tau_q)}{\partial\alpha^2}\right|_{\alpha=\alpha_S^0} = -\frac{2}{b}\left(\frac{\delta_2\tau_q D}{1 + \delta_2\tau_q\rho}\right)^2 < 0$ from the condition of $\frac{\partial \pi_S(\alpha|\tau_q)}{\partial\alpha} = 0$. Thus, the value of α_S^0 can maximize the system profit when the value of τ_q is arbitrarily given.