Economic Modelling 157 (2026) 107480

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.journals.elsevier.com/economic-modelling

Tax evasion and the productivity distribution

Francesco Menoncin [a], Andrea Modena [b], Luca Regis [c,*]

[a] Department of Economics and Management, University of Brescia, Italy
[b] Department of Economics, University of Mannheim, Germany
[c] ESOMAS Department, University of Torino & Collegio Carlo Alberto, corso Unione Sovietica 218/bis, 10134, Italy

A R T I C L E I N F O

Dataset link: Replication Package for the article "Tax evasion and the productivity distribution" on Economic Modeling (Original data)

Keywords:
Distribution
Equilibrium
Leverage constraints
Public debt
Evasion

A B S T R A C T

We develop a heterogeneous-firm macroeconomic model to investigate how tax evasion affects the productivity distribution in general equilibrium. In our model, entrepreneurs choose capital and labor to produce with their firms, invest in bonds, and evade taxes to maximize their intertemporal utility, derived from dividends. Firms face leverage constraints and uninsurable productivity shocks. The results reveal that tax evasion redistributes capital toward low-productivity firms, relaxing their leverage constraints. It also increases public debt, raising the cost of capital and crowding out firms at the margin. As a result of these forces, we demonstrate that (i) the decline in high-productivity firms' average productivity drives the negative correlation between the size of the shadow economy and aggregate productivity, and (ii) the productivity gains from reduced tax evasion are smaller in economies with higher public debt and stricter leverage constraints.

1. Introduction

Empirical studies on tax evasion have reached consensus on two stylized facts: (i) a negative correlation exists between the size of a country's shadow economy and its aggregate productivity (Loayza and Rigolini, 2006; Dabla-Norris et al., 2019), and (ii) tax evasion rates are higher in countries where firms face greater financial constraints due to less developed financial markets (Beck et al., 2014; La Porta and Shleifer, 2014).[1]

Based on these observations, several studies have examined the macroeconomic consequences of tax evasion (e.g., Di Nola et al., 2021; Franjo et al., 2022; Erosa et al., 2023). However, neither empirical nor theoretical literature has examined the impact of tax evasion on firms' productivity distribution, which is the subject of this study. Understanding this impact is crucial for designing effective tax enforcement policies, as firms' tax evasion practices vary significantly with their productivity (Gradstein et al., 2019).

The study constructs a macroeconomic model in which leverage-constrained firms with heterogeneous productivity use capital and labor for production while evading income taxes. The government relies on tax revenue and debt to finance its (unproductive) spending. Within this framework, we analyze the mechanisms driving the heterogeneous impact of tax evasion across firms and characterize the resulting endowment–productivity distribution in general equilibrium. Similar to previous studies (e.g., Franjo et al., 2022; Erosa et al., 2023), we find that tax evasion exacerbates capital misallocation caused by leverage constraints, replicating stylized facts (i) and (ii). However, our contribution advances the literature in three dimensions.

First, we derive firms' optimal dynamic tax evasion decisions in closed form, separating their dependence on idiosyncratic and aggregate economic productivity. Second, we analytically demonstrate how heterogeneous tax evasion practices indirectly affect relative and aggregate capital, as well as public debt accumulation, by *redistributing* resources from high- to low-productivity firms, since the latter have stronger incentives to evade taxes. Third, we disentangle this effect from the direct impact of tax evasion on public debt and show how this feeds back into firms' decisions through equilibrium prices. In particular, we uncover a novel channel – the *debt crowding-out effect* – through which the additional public debt generated by a larger shadow

* Corresponding author.
E-mail addresses: francesco.menoncin@unibs.it (F. Menoncin), andrea.modena@uni-mannheim.de (A. Modena), luca.regis@unito.it (L. Regis).

[1] These stylized facts align with firm-level studies (e.g., Fajnzylber et al., 2011; Amin and Okou, 2020) showing that productivity gaps between firms that comply with taxes range between 25%–50%. Dabla-Norris et al. (2008) demonstrated the connection between financing constraints and informality, particularly among small firms.

https://doi.org/10.1016/j.econmod.2026.107480
Received 25 July 2025; Received in revised form 8 January 2026; Accepted 8 January 2026
Available online 10 January 2026

economy counteracts the redistribution effect by raising the equilibrium cost of capital and crowding out low-productivity firms at the margin.

Our model builds on Moll (2014), where firms are run by risk-averse entrepreneurs who use their capital endowment, hire labor, and operate a production technology subject to idiosyncratic productivity shocks, while buying or issuing bonds to maximize their intertemporal utility from dividends. Firms' leverage is limited to a multiple of their endowments. Firms pay taxes on their capital and bond income, which they can partially reduce through tax evasion. Tax evasion is chosen optimally, balancing the benefits of tax savings and the risk of being audited and fined by the government, as in Allingham and Sandmo (1972). The government collects tax revenue and audit fines and provides a fixed level of public spending, covering its deficit by issuing debt. Government and firms' debt are traded in a competitive market. Both are risk-free and, therefore, indistinguishable in equilibrium. Markets are incomplete because idiosyncratic productivity shocks cannot be contracted upon, as in Gersbach et al. (2023).[2]

We solve the firms' stochastic control problem in closed form, revealing that producing is only optimal if idiosyncratic productivity exceeds an endogenous threshold, which depends on the size of the aggregate shadow economy. The optimal share of evaded taxes varies across firms, depending negatively on idiosyncratic productivity, which aligns with Gradstein et al. (2019)'s findings. Next, we show that tax evasion in equilibrium *indirectly* impacts aggregate productivity in two ways. First, tax evasion affects the rate at which firms with different productivity levels accumulate capital over time; that is, it influences their endowment–productivity distribution. Second, tax evasion impacts the dynamics of aggregate capital and public debt, which subsequently affects optimal production decisions through prices.

To evaluate the magnitude of these mechanisms and qualitatively verify their consistency with empirical evidence, we parametrize the model and conduct three numerical exercises to assess different scenarios. Our first analysis compares our tax evasion economy with a benchmark economy where tax compliance is perfectly enforced. Eradicating evasion eliminates the capital redistribution effect, improving aggregate productivity by about 1%. At the same time, perfect tax enforcement mitigates the crowding-out effect, reducing public debt and, consequently, the equilibrium cost of capital. Lower production costs allow low-productivity firms' entry, which offsets the productivity gains from tax compliance. When this second channel is sterilized by fixing the government debt level, the productivity gains from eliminating tax evasion are four times larger. This demonstrates that the magnitude of the identified debt crowding-out effect is substantial.

Our second exercise compares equilibrium aggregates and corresponding productivity distributions across economies with different tax evasion levels. We find that aggregate productivity is negatively correlated with the size of the shadow economy. We also explain this relationship with the fact that tax evasion disproportionately reduces high-productivity firms' average productivity, leaving those at the lower end almost unaffected. We use empirical data to verify that this prediction is consistent with the pattern observed in a panel of 14 Organization for Economic Cooperation and Development (OECD) member countries over the past 20 years.

Our third exercise compares the effects of tax evasion across economies with different levels of public debt and leverage constraints. We find that tighter leverage constraints and higher debt-to-GDP ratios reduce the benefits of reducing tax evasion, as the capital redistribution effect weakens and the debt crowding-out effect strengthens. This finding indicates that achieving productivity gains by reducing tax evasion in financially underdeveloped countries with larger public debts may require more aggressive measures than in more advanced economies.

The remainder of this paper is organized as follows. Section 2 connects our study to the relevant studies on tax evasion. Section 3 describes the model. Section 4 analytically characterizes the endowment redistribution and debt crowding-out effects of tax evasion. Section 5 uses numerical simulations to explore the model's mechanisms and compare its key predictions to the OECD data. Section 6 concludes.

2. Related literature

This study is related to several articles that have investigated optimal tax evasion in dynamic (but partial equilibrium) settings (e.g., Levaggi and Menoncin, 2013; Bernasconi et al., 2020). In contrast to these studies, we construct a model in which agents make heterogeneous tax evasion decisions depending on their idiosyncratic productivity, and explore their interactions within a general equilibrium framework. Therefore, we connect with a broader body of research on the macroeconomic effects of tax evasion in representative agent macroeconomic models.

Early contributions in this field include Chen (2003) and Kafkalas et al. (2014), who analyzed how tax evasion, public spending, and economic growth interact in representative agent economies. Gillman and Kejak (2014) developed a model in which human capital boosts productivity and reduces tax evasion incentives. Ordonez (2014) and López (2017) focused on capital misallocation in environments where firms deliberately limit their size to avoid detection due to weak tax enforcement. The authors found that tax enforcement increases public revenue, but also introduces inefficiencies. Our work differs by examining how tax evasion interacts with leverage constraints in a heterogeneous-agent economy where firms' distribution varies endogenously over time. This connects our research to the extensive literature on financial frictions and capital misallocation (Cooley et al., 2004; Cagetti and De Nardi, 2006; Buera and Moll, 2015; Bassetto et al., 2015) and its link to corporate taxation (Macnamara, 2019; Árpád et al., 2023; Dávila and Hébert, 2023). In particular, we construct our model extending (Moll, 2014)'s framework to account for tax evasion and public debt.

In examining tax evasion within a heterogeneous agent model, this study is closely related to the more recent works of Di Nola et al. (2021), Franjo et al. (2022), Erosa et al. (2023), and Fernandez-Bastidas (2023). Di Nola et al. (2021) showed that the self-employed benefit from tax evasion at the expense of firm-employed workers. Franjo et al. (2022) argued that formal and informal firms have different leverage constraints and that removing these constraints reduces tax evasion and increases productivity. Erosa et al. (2023) found that eliminating payroll taxes reduces business informality, benefiting large (less constrained) employers, and the productivity gains from reducing constraints grow with the economy's informality rate. Fernandez-Bastidas (2023) highlight the crucial role of considering entrepreneurs' heterogeneous tax evasion motives when evaluating tax policy reforms. Our contributions to this literature are twofold.

First, we characterize the mechanisms through which a heterogeneous relationship between entrepreneurs' tax evasion decisions and productivity arises endogenously, affecting both aggregate output and public debt. Second, our framework enables us to extend the analysis of the economic impact of tax evasion by Di Nola et al. (2021), characterizing the effects analytically rather than only numerically. In doing so, we uncover two novel (and opposing) mechanisms: the endowment redistribution effect, which complements the subsidy channel described in Di Nola et al. (2021), and the debt crowding-out effect. The latter effect arises indirectly through the endogenous level of public debt and its interaction with tax evasion, a dimension that all previous studies have overlooked.

The finding that high-productivity enterprises evade fewer taxes aligns with a standard argument in the firm informality literature (see

[2] Accordingly, firms can be considered as small-sized enterprises. Our focus on small firms is motivated by the empirical evidence that roughly half of the aggregate evasion consists of small business underreporting profits (Slemrod, 2019) and about one-third of self-employed activities are misreported (Hurst et al., 2014).

e.g., Fajnzylber et al., 2011) and is consistent with the empirical evidence of Dabla-Norris et al. (2019), who demonstrated that productivity improvements reduce firm-level tax evasion.[3] Our model also reproduces two regularities identified in previous studies, i.e. that informality is concentrated among small, low-productivity firms (Di Caro and Sacchi, 2020) and among financially constrained firms (La Porta and Shleifer, 2014). By capturing these factors within a unified framework, our model offers a novel perspective on the macroeconomic implications of Di Caro and Sacchi (2020) and La Porta and Shleifer (2014), that increasing productivity by reducing tax evasion among low-productivity firms, where informality and fiscal costs are most pronounced (Di Caro and Sacchi, 2020), may be less effective when public debt is higher, and productivity and public debt gains from eliminating tax evasion are smaller in economies with stronger leverage constraints (La Porta and Shleifer, 2014).

3. Model

Time $t \in [0, \infty)$ is continuous. A unit mass of entrepreneurs and the government populate the economy. Each entrepreneur owns one firm. Each entrepreneur–firm pair (henceforth, "firm") is indexed by its productivity (z_t) and net worth (n_t). One capital good (the numéraire) can be consumed or used for production. Firms and the government trade private and public debt securities in a competitive financial market.

3.1. Preferences and technologies

Firms are infinitely lived. The entrepreneur of each firm derives utility from intertemporal dividend payments (c_t) according to the following recursive preferences (Duffie and Epstein, 1992)[4]:

$$\mathbb{E}_t\left[\int_t^{\infty}(1-\gamma)\rho V_s\left(\log c_s - \frac{1}{1-\gamma}\log((1-\gamma)V_s)\right)ds\right] \tag{1}$$

where V_t is the value function at time t and $(\gamma, \rho) \in \mathbb{R}_+^2$ parametrize risk aversion and subjective discounting. Referencing Moll (2014), each firm's productivity level is subject to idiosyncratic shocks and satisfies the following stochastic differential equation:

$$d\ln z_t = -\nu \ln z_t dt + \sigma\sqrt{\nu}dW_t, \text{ with } z_0 = z, \tag{2}$$

where (W_t) is a standard Brownian motion independent across firms, and $(\nu, \sigma) \in \mathbb{R}_+^2$ parametrize auto-correlation and volatility. Firms cannot insure themselves against productivity shocks, making the market incomplete. Therefore, we interpret each firm as a small or medium-sized enterprise.[5]

Firms use capital (k_t) and hire labor (l_t) to produce output (y_t) with the following Cobb–Douglas technology:

$$y_t = (Az_tk_t)^{\alpha} l_t^{1-\alpha}, \tag{3}$$

where $\alpha \in (0,1)$ is the output elasticity of capital, and $A \in \mathbb{R}_+$ parametrizes aggregate total factor productivity (TFP). In addition to using their own endowment, firms can raise capital by issuing bonds, $b_t = n_t - k_t$, up to a multiple of their endowment, as follows:

$$k_t \in [0, n_t \cdot \phi(z_t)], \tag{4}$$

where $\phi(\cdot) : \mathbb{R}_+ \to [1, \infty)$ is a bounded function of z_t, that captures the economy's financial development.[6] Capital depreciates at the exogenous rate $\delta \in [0,1]$ and is remunerated at the competitive rate R_t.

Labor is supplied inelastically by a representative worker, who earns (and consumes) competitive wage w_t and has no further role in the economy.[7] Since bonds provide instantaneous payments and productivity shocks cancel out in the aggregate, private and public debt securities are risk-free and indistinguishable. To prevent arbitrage, the risk-free rate must be $r_t = R_t - \delta$.

3.2. Government, taxation, and tax evasion

The government finances a constant level of unproductive public spending (G) by levying corporate income taxes. For simplicity, proceeds from bonds and capital are taxed at the same rate. We let the marginal tax rate be any bounded function of firms' productivity: $\tau_k(\cdot) : \mathbb{R}_+ \to [0,1]$. This assumption enables us to capture size and/or income-dependent tax rates while preserving analytical tractability because, in equilibrium, firms' income and size distribution will be fully characterized as a function of z_t. Labor income is taxed at a constant rate $\tau_l \in [0,1]$.

Firms can reduce their tax burden by concealing a fraction (e_t) of their income from the tax authority, which exposes them to the possibility of being audited and fined. Auditing events are i.i.d. and exponentially distributed with rate $\lambda(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ between time t and $t + dt$, as in Levaggi and Menoncin (2013). After being audited, firms face fines equal to a proportion $\eta(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ of evaded income. The auditing rate and the fine can be any bounded function of z_t.

Government revenue (T_t) is the sum of all taxes and fines for evasion that the government collects. Any shortfall between T_t and G is covered by issuing public debt, which yields the risk-free rate r_t. As a result, the aggregate stock of public debt (B_t) solves the following ordinary differential equation (ODE):

$$\frac{dB_t}{dt} = r_tB_t + G - T_t, \text{ with } B_0 = B. \tag{5}$$

3.3. Firms' optimization problem

Firms choose dividends (c_t), production factors (k_t, l_t), and tax evasion (e_t) to maximize their expected intertemporal utility Eq. (1) subject to Eq. (4) and the following dynamic endowment:

$$dn_t = \left(1-\tau_k(z_t)\left(1-e_t\right)\right)\left(n_tr_t + \Pi(k_t,l_t)\right)dt - c_tdt - e_t\eta(z_t)\left(n_tr_t + \Pi(k_t,l_t)\right)dJ_t, \tag{6}$$

with $n_0 = n$, where $\Pi(\cdot) := y_t - l_tw_t - k_tR_t$ denotes the gross profits of production, and J_t is a standard Poisson process with intensity $\lambda(z_t)$. Eq. (6) indicates that evading taxes for a given z_t is, on average, profitable only if the following holds:

$$\tau_k(z_t) \geq \eta(z_t)\lambda(z_t). \tag{7}$$

Therefore, we assume that Eq. (7) holds for all z_t.

[3] Other related studies (e.g., Kanbur, 2017) have examined alternative and complementary mechanisms, such as more productive firms' greater use of legal tax avoidance or size-dependent regulation, which we abstract from in this study.

[4] As we show in the next section, this assumption enables us to derive the firms' optimal tax evasion decisions in closed form and separate them from their optimal dividend decisions without assuming a risk aversion of 1 (log utility).

[5] "Large" enterprises could hedge their risks using derivatives linked to the broader economy.

[6] Following Gertler and Kiyotaki (2010), this assumption can be micro founded as the incentive-compatible constraint of firms allowed to divert a fraction $1/\phi(z) \in [0,1]$ of their assets and default on their liabilities. The constraint dispels diversion incentives by ensuring that the value of diversion revenue is equal to or less than the continuation value of the firm.

[7] This study's supplementary material verifies that the worker exhibits an endogenous "hand-to-mouth" behavior, assuming that the worker's inter-temporal discount rate is large enough.

Using standard stochastic control arguments (e.g., Pham, 2009, Chapter 2), the problem of each firm can be expressed recursively using the following Hamilton–Jacobi–Bellman (HJB) equation:

$$0=\sup_{c,k,l,e}\left\{\begin{array}{c}(1-\gamma)\,\rho V_t(n,z)\left(\log c-\frac{1}{1-\gamma}\log((1-\gamma)V_t(n,z))\right)+\\+\partial_n V_t(n,z)\left[\left(1-\tau_k(z)(1-e)\right)\left(nr_t+\Pi(k,l)\right)-c\right]+\\+\lambda(z)\left[V_t\left(n\left(1-e\eta(z)\Pi(k,l)\right),z\right)-V_t(n,z)\right]+\\+\partial_z V_t(n,z)z\left(\frac{1}{2}\sigma^2\nu-\nu\ln z\right)+\\+\partial_t V_t(n,z)+\frac{1}{2}\partial^2_{zz}V_t(n,z)z^2\sigma^2\nu\end{array}\right\} \tag{8}$$

subject to Eq. (4), with transversality condition $\lim_{s\to\infty}\mathbb{E}_t\left[V_s(n,z)e^{-(\rho+\lambda(z))(s-t)}\right]=0,\ \forall(n,z)\in\mathbb{R}^2_+$. With a slight abuse of notation, we use a time subscript ($V_t(n,z)$) rather than an explicit functional dependence (i.e., $V(t,n,z)$) to emphasize that the value function directly depends on time only through the (time-varying) levels of competitive wages (w_t) and the risk-free rate (r_t). We adopt this convention for all quantities in the remainder of the paper.

As shown in Appendix A.1, solving the optimal control problem in Eq. (8) yields the following optimal control:

Proposition 1 (*Firms' Optimal Controls*)**.** *For a given pair of factor prices (R_t, w_t) and endowment and productivity levels (n,z), the optimal controls that maximizes Eq. (1) subject to Eqs. (4) and (6) are as follows:*

1 *Production factors demand:*

$$k^*_t(n,z)=\phi(z)\cdot n\cdot\mathbb{I}\left\{z\ge z^*_t\right\}, \tag{9}$$

$$l^*_t(n,z)=k^*_t(n,z)\cdot z\cdot A\left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}}, \tag{10}$$

where $\mathbb{I}\{X\}$ denotes the indicator function of the event X, and

$$z^*_t:=\frac{1}{A}\frac{R_t}{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{\frac{1-\alpha}{\alpha}}. \tag{11}$$

Moreover, optimal production profits are a linear function of capital and equal to the following:

$$\Pi^*_t(n,z):=\Pi(k^*_t(n,z),l^*_t(n,z))=k^*_t(n,z)\cdot R_t\cdot\left(\frac{z-z^*_t}{z^*_t}\right). \tag{12}$$

2 *Dividends:*

$$c^*(n)=n\rho. \tag{13}$$

3 *Tax evasion:*

$$e^*_t(n,z)=\frac{1-\left(\frac{\lambda(z)\eta(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}}{\eta(z)\left(R_t-\delta+\frac{\Pi^*_t(n,z)}{n}\right)}. \tag{14}$$

The capital and labor demand functions are of the bang–bang type and are independent of dividends and tax evasion decisions.[8] In particular, firms find it optimal to produce only if their z exceeds the endogenous threshold z^*_t defined in Eq. (11). When this is the case, firms issue bonds up to their maximum allowed limit and allocate their proceeds and entire endowment to capital. Conversely, firms use their endowment to buy bonds when production is not profitable ($z\le z^*_t$). Since labor demand is unconstrained and the production technology has constant returns to scale, optimal labor demand is proportional to capital (and, thus, to n and z).

Due to the assumption of unit elasticity of intertemporal substitution, firms pay a constant share (ρ) of their endowment as dividends, independently of z.

Keeping all else constant, tax evasion increases with the tax rate ($\tau_k(\cdot)$) and decreases with the frequency of audits ($\lambda(\cdot)$) and fines ($\eta(\cdot)$), due to agents' risk aversion, as demonstrated by Levaggi and Menoncin (2013, 2016). Additionally, tax evasion depends negatively on firms' gross income per unit of endowment ($R_t-\delta+\Pi^*_t(n,z)/n$). As a result of this outcome, tax evasion decreases with firms' production profits ($\Pi^*_t(\cdot)$), which subsequently increases with the idiosyncratic productivity (z). This occurs because firms are risk-averse and have diminishing marginal utility. Therefore, for a given tax rate and auditing parameters, tax evasion becomes less beneficial as income increases. Notably, a milder leverage constraint (i.e., a higher level of $\phi(\cdot)$) also reduces tax evasion incentives, as it allows firms to earn higher profits per unit of capital if their z is sufficiently high. By the same logic, tax evasion decreases for *all* firms when the rental rate of capital R_t (and the productivity threshold z^*_t) is higher, as capital and bonds yield higher returns. Additionally, by substituting Eq. (12) into Eq. (14) and rearranging, it is straightforward to verify that a higher rental rate reduces the heterogeneity in tax evasion behavior across different productivity levels.

The prediction that firms with higher productivity have less incentive to evade taxes rationalizes the empirical evidence documented by Dabla-Norris et al. (2019). More broadly, it aligns with the consolidated evidence of a negative relationship between productivity and informality (La Porta and Shleifer, 2014; Kanbur, 2017; Di Caro and Sacchi, 2020). Empirical evidence supporting the positive relationship between financial frictions and tax evasion was also provided by Beck et al. (2014).

3.4. Equilibrium

Next, we define and derive the economy's competitive equilibrium referring to Radner (1972). For this purpose, let $f_t(z,n)$ denote the joint density function of firms' endowment and productivity at time t. We also normalize the aggregate labor supply to one for simplicity. Then, we can define the competitive equilibrium as follows.

Definition 1 (*Competitive Equilibrium*)**.** A competitive equilibrium of the economy described in Sections 3.1–3.3 is a collection $\left[\left(c^*_t(n,z),e^*_t(n,z),k^*_t(n,z),l^*_t(n,z)\right),f_t(n,z),(w_t,R_t)\right]_{t\ge0}$ of optimal controls (consumption, tax evasion, and production factor demand), a density function, and a factor price system (capital rental rate and wage) such that, for all $t\ge0$:

1. For a given factor price system, firms solve the optimal control problem using Eq. (8).
2. For a given public debt level, optimal controls, and a productivity net-worth density function the following holds:
   (a) The rental rate of capital clears the capital market as follows:
   $$\int_0^\infty\int_0^\infty nf_t(n,z)\cdot dn\cdot dz=B_t+\int_0^\infty\int_0^\infty k^*_t(n,z)f_t(n,z)\cdot dn\cdot dz. \tag{15}$$
   (b) Wage clears the labor market as follows:
   $$\int_0^\infty\int_0^\infty l^*_t(n,z)f_t(n,z)\cdot dn\cdot dz=1. \tag{16}$$
   (c) Government tax revenue equals aggregate income taxes minus evasion plus (average) auditing revenue as follows:
   $$\begin{aligned}T_t=&\int_0^\infty\int_0^\infty\tau_k(z)\left(1-e^*_t(n,z)\right)\left(nr_t+\Pi^*_t(n,z)\right)\\&\times f_t(n,z)\cdot dn\cdot dz+\\&+w_t\tau_l+\int_0^\infty\int_0^\infty\eta(z)\lambda(z)e^*_t(n,z)\left(nr_t+\Pi^*_t(n,z)\right)\\&\times f_t(n,z)\cdot dn\cdot dz.\end{aligned} \tag{17}$$

[8] However, tax evasion will affect production decisions in equilibrium by influencing factor prices and the firms' endowment–productivity distribution.

3. Given the government tax revenue, the public debt level satisfies the ODE in Eq. (5).
4. For a given set of factor prices and optimal controls, the productivity net-worth density function satisfies the following Fokker–Planck equation[9]:

$$\frac{d f_t(n,z)}{dt} = -\frac{\partial}{\partial n}\left[f_t(n,z)\left(1-\tau_k(z)\left(1-e_t^*(n,z)\right)\right)\left(nr_t+\Pi_t^*(n,z)\right)\right] + -\frac{\partial}{\partial z}\left[f_t(n,z)z\left(0.5\sigma^2\nu-\nu\ln z\right)\right]+\frac{\sigma^2\nu}{2}\frac{\partial^2}{\partial z^2}\left[f_t(n,z)z^2\right] +\lambda(z)\left[f_t(z,n-e_t^*(n,z)\eta(z)(nr_t+\Pi_t^*(n,z)))-f_t(n,z)\right]. \tag{18}$$

Eq. (15) requires the rental rate on capital to be such that entrepreneurs' aggregate endowment, defined as follows:

$$N_t := \int_0^\infty\int_0^\infty n f_t(n,z)\cdot dn\cdot dz, \tag{19}$$

equals the aggregate stock of capital

$$K_t := \int_0^\infty\int_0^\infty k_t^*(n,z)\cdot dn\cdot dz \tag{20}$$

plus debt, B_t. Similarly, Eq. (16) sets the competitive wage such that aggregate labor demand equals one.

According to Eq. (21), aggregate tax revenue is obtained by averaging firms' individual expected tax revenue across productivity levels. This follows because there is a continuum of firms subject to i.i.d. auditing shocks for each z. Hence, by the law of large numbers, individual randomness averages out across firms, and we can compute the following:

$$\int_0^\infty\int_0^\infty e_t^*(n,z)\left(nr_t+\Pi_t^*(n,z)\right)dJ_t\cdot f_t(n,z)\cdot dn\cdot dz = =\int_0^\infty\int_0^\infty e_t^*(n,z)\left(nr_t+\Pi_t^*(n,z)\right)\lambda(z)f_t(n,z)\cdot dn\cdot dz\cdot dt. \tag{21}$$

Consistent with Section 3.3, we label the quantities in our definition of the competitive equilibrium with a time subscript rather than stating their explicit functional dependence on t. This choice is intended to emphasize that all equilibrium objects only indirectly depend on time, through the (time-varying) levels of macroeconomic aggregates and the productivity net-worth density function, as we will verify in Section 4.

4. Analytical results

This section examines the equilibrium dynamics of economic aggregates and firms' endowment–productivity distribution. We then analytically demonstrate how tax evasion affects them. For these purposes, we define the following function:

Definition 2 (*Endowment Share*)**.** For a given density function $f_t(n,z)$, the share of aggregate endowment owned by firms with productivity z is as follows:

$$\theta_t(z) := \frac{\int_0^\infty n f_t(z,n)\cdot dn}{\int_0^\infty\int_0^\infty n f_t(z,n)\cdot dn\cdot dz} \geq 0,\ \text{for all } z\in[0,\infty) \tag{22}$$

This function is non-negative everywhere by construction, and satisfies $\int_0^\infty \theta_t(z)dz = 1$ for all $t \in [0,\infty)$. Therefore, it can be interpreted as a probability density function.[10] Adopting $\theta_t(z)$ in place of $f_t(n,z)$ for aggregation is convenient, as it represents the firms' endowment–productivity distribution as a one-dimensional object. This simplification is possible without losing information about the original distribution because firms' optimal controls are linear in their endowment.

[9] For a formal derivation of the Fokker–Planck equation for a generic diffusion process, we refer the reader to Stokey (2008).

[10] This claim can be verified by rearranging Eq. (22) and using Eq. (19), and integrating the resulting equation over z.

Substituting Eqs. (9) and (22) into Eq. (15) and rearranging yields the following relationship:

$$\underbrace{\frac{\int_{z_t^*}^\infty \phi(z)\theta_t(z)\cdot dz}{\int_{z_t^*}^\infty \theta_t(z)\cdot dz}}_{:=\mathbb{E}_t^\theta[\phi(z)|z\geq z_t^*]}\cdot\underbrace{\int_{z_t^*}^\infty \theta_t(z)\cdot dz}_{:=\mathbb{P}_t^\theta\{z\geq z_t^*\}} = \frac{K_t}{K_t+B_t}, \tag{23}$$

which connects the economy's leverage capacity (i.e., the average leverage constraint among firms with $z \geq z_t^*$) to aggregate capital, public debt, and the share of active firms, $\mathbb{P}_t^\theta\{z\geq z_t^*\}$. This equation indicates that, ceteris paribus, an economy with a higher (lower) leverage capacity has a larger share of its endowment allocated into capital (bonds) and/or a smaller (larger) fraction of active firms. In other words, economies with higher leverage capacity must have a higher z_t^* in equilibrium, which implies higher average productivity among active firms.

We then use $\theta_t(z)$ to express production factor prices as follows (details are presented in the online Supplementary material):

$$w_t = (1-\alpha)\cdot\mathbb{E}_t^{\theta^\phi}\left[z|\,z\geq z_t^*\right]^\alpha\left(AK_t\right)^\alpha, \tag{24}$$

$$R_t = z_t^*\cdot A\alpha\cdot\mathbb{E}_t^{\theta^\phi}\left[z|\,z\geq z_t^*\right]^{\alpha-1}\left(AK_t\right)^{\alpha-1}, \tag{25}$$

where $\mathbb{E}_t^{\theta^\phi}\left[z\middle|z\geq z_t^*\right]$ denotes active firms' average productivity under the probability measure $\theta_t^\phi(z) := \phi(z)\theta_t(z)\mathbb{I}\{z\geq z_t^*\}/(\int_{z_t^*}^\infty \phi(z)\theta_t(z)\cdot dz)$. Eq. (25) reveals that a higher break-even productivity threshold z_t^* is associated with higher marginal productivity of capital and, consequently, higher returns in equilibrium.

Notably, while tax evasion does not directly affect the economy's leverage capacity in Eq. (23) or the production factor prices in Eqs. (24) and (25), it *indirectly* influences them in two ways. First, by altering the rate at which firms with differing productivity levels accumulate endowments relative to one another, i.e., through $\theta_t(z)$. Second, by impacting the *aggregate* accumulation of capital and public debt. The following proposition characterizes the mechanisms behind the former channel.

Proposition 2 (*Endowment Share Dynamics*)**.** *The endowment share of firms with productivity z satisfies the following partial differential equation:*

$$\frac{d\theta_t(z)}{dt} = \underbrace{\left[\left(1-\tau_k(z)\right)\left(R_t-\delta+\mathbb{I}\{z\geq z_t^*\}\left(Az\alpha\left(\frac{1-\alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}}-R_t\right)\right)-\rho\right]\theta_t(z)+}_{(i)} \underbrace{+\left(\frac{\tau_k(z)}{\eta(z)}-\lambda(z)\right)\left[1-\left(\frac{\eta(z)\lambda(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}\right]\theta_t(z)}_{(ii)}-\underbrace{\frac{dN_t}{dt}\frac{1}{N_t}}_{(iii)}\theta_t(z)+ \underbrace{-\partial_z\left[z\nu\left(\frac{\sigma^2}{2}-\ln z\right)\theta_t(z)\right]+\frac{\nu\sigma^2}{2}\partial_{zz}^2\left(z^2\theta_t(z)\right)}_{(iv)}, \tag{26}$$

where

$$\frac{dN_t}{dt}\frac{1}{N_t} = \mathbb{E}_t^\theta\left[\left(\frac{\tau_k(z)}{\eta(z)}-\lambda(z)\right)\left(1-\left(\frac{\lambda(z)\eta(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}\right)\right]+\left(R_t-\delta\right)\mathbb{E}_t^\theta\left[\left(1-\tau_k(z)\right)\right]+ +R_t\,\mathbb{P}_t^\theta\{z\geq z_t^*\}\,\mathbb{E}_t^\theta\left[\phi(z)\left(1-\tau_k(z)\right)\left(\frac{z-z_t^*}{z_t^*}\right)\middle|\,z\geq z_t^*\right]-\rho \tag{27}$$

is the growth rate of the economy's aggregate endowment.

Proof. See Appendix A.2. □

The right-hand side of Eq. (26) can be decomposed into four terms. Term *(i)* shows that firms' endowment share increases with their after-tax income, net of consumption, as intuition suggests. Term *(ii)*, which is always positive under the parametric restriction of Eq. (7), indicates that $\theta_t(z)$ increases with the (net) benefits of tax evasion. Term *(iii)* indicates that higher aggregate growth leads to lower $\theta_t(z)$, as it is associated with a uniformly higher income rate (R_t) across all firms. Term *(iv)* captures the effect of randomness in individual productivity, as described in Eq. (2).

We refer to term (ii) as the *endowment redistribution effect* of tax evasion, which complements the subsidy channel described by Di Nola et al. (2021), through which tax evasion subsidizes self-employed businesses' savings. The magnitude of the redistribution effect and its overall impact on the distribution of firms' net worth across productivity levels depends on the (relative) size of the term as follows:

$$\left(\frac{\tau_k(z)}{\eta(z)} - \lambda(z)\right)\left(1 - \left(\frac{\eta(z)\lambda(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}\right) \geq 0. \tag{28}$$

Under the natural assumptions that firms' nominal income tax rate is constant and both auditing frequency and fines do not decrease with productivity (as a proxy for firm size), the relative benefit from tax evasion is non-increasing in z as follows:

$$\begin{aligned}
&\frac{\partial}{\partial z}\left[\left(\frac{\tau_k}{\eta(z)} - \lambda(z)\right)\left(1 - \left(\frac{\eta(z)\lambda(z)}{\tau_k}\right)^{\frac{1}{\gamma}}\right)\right] \\
&= -\left(\frac{\tau_k}{\eta(z)}\frac{\eta'(z)}{(z)} + \lambda'(z)\right)\left(1 - \left(\frac{\eta(z)\lambda(z)}{\tau_k}\right)^{\frac{1}{\gamma}}\right) + \\
&\quad -\left(\frac{\tau_k}{\eta(z)} - \lambda(z)\right)\left(\frac{\eta(z)\lambda(z)}{\tau_k}\right)^{\frac{1-\gamma}{\gamma}}\frac{\eta'(z)\lambda(z) + \eta(z)\lambda'(z)}{\gamma\tau_k} \leq 0.
\end{aligned} \tag{29}$$

The same argument holds even when all fiscal parameters are independent of z, in which case the relative impact of the (constant) Term (ii) on the firm's overall growth rate (Term (i) plus Term (ii)) becomes progressively smaller as z rises.

Another remark concerns the effect of tax evasion along the economy's transition toward its steady state (i.e., as long as $dN_t/dt \neq 0$). In this regard, we can substitute Eq. (27) into Eq. (26) to verify whether tax evasion favors or curbs entrepreneurs' net worth accumulation, depending on the sign of the following inequality:

$$\left(\frac{\tau_k(z)}{\eta(z)} - \lambda(z)\right)\left(1 - \left(\frac{\lambda(z)\eta(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}\right) \gtreqless \mathbb{E}_t^{\theta}\left[\left(\frac{\tau_k(z)}{\eta(z)} - \lambda(z)\right)\left(1 - \left(\frac{\lambda(z)\eta(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}\right)\right], \tag{30}$$

that is, depending on whether a firm with productivity z evades more or less than the average. Notably, this effect vanishes in the steady state ($dN_t/dt = 0$) and is absent when fiscal parameters are independent of z; thus Eq. (30) holds with equality.

Combined with Eq. (27), the following proposition describes the mechanism by which tax evasion affects public debt and aggregate capital accumulation.

Proposition 3 (*Public Debt Dynamics*). *For a given aggregate endowment level (N_t) and density function, $\theta_t(z)$, the equilibrium level of public debt solves the following ODE:*

$$\begin{aligned}
\frac{dB_t}{dt}\frac{1}{B_t} &= R_t - \delta - \frac{N_t}{B_t}\mathbb{E}_t^{\theta}\left[\tau_k(z)\left(R_t\left[1 + \phi(z)\cdot\mathbb{I}\left\{z \geq z_t^*\right\}\cdot\left(\frac{z - z_t^*}{z_t^*}\right)\right] - \delta\right)\right] + \\
&+ \frac{G - w_t\tau_l}{B_t} + \frac{N_t}{B_t}\mathbb{E}_t^{\theta}\left[\left(\frac{\tau_k(z)}{\eta(z)} - \lambda(z)\right)\left(1 - \left(\frac{\lambda(z)\eta(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}\right)\right],
\end{aligned} \tag{31}$$

with $B_0 = B$.

Proof. See Appendix A.3. □

Comparing Eqs. (27) and (31) reveals that the right-hand side of Eq. (30), which captures firms' average tax evasion benefit, exhibits a positive sign in both equations; therefore, tax evasion fosters both public debt and aggregate endowment accumulation. The same comparison also reveals that tax evasion affects the former more than the latter since $N_t \geq B_t$ according to Eq. (15).

The fact that tax evasion increases public debt more than aggregate endowment, combined with the market-clearing condition in Eq. (15), implies that tax evasion curbs aggregate capital accumulation, leading to higher capital prices in equilibrium, based on Eq. (25). A higher R_t crowds out some low-productivity firms by making their production technology unviable. We label this mechanism the debt *crowding-out effect* of tax evasion. This effect is somewhat reminiscent of the *selection channel* proposed by Di Nola et al. (2021), through which the opportunity to evade taxes encourages less-productive agents to launch self-employed businesses. However, unlike this channel, the debt crowding-out effect of tax evasion arises indirectly through public debt and discourages low-z firms from producing.

The endowment redistribution and debt crowding-out effects interact in equilibrium, affecting firms' optimal decisions and all macroeconomic variables simultaneously. This makes a full analytical investigation of their magnitude infeasible. Therefore, in the next section we parametrize the model and analyze the equilibrium using numerical simulation.

5. Numerical results

In this section, we numerically solve the model for its steady state. We then simulate the model to evaluate the macroeconomic consequences of tax evasion through the following analyses.

First, we compare the model's equilibrium to an economy with full tax enforcement. Second, we vary the tax evasion parameters to examine how changes in the shadow economy's size (i.e., the aggregate share of evaded taxes) influence firms' individual tax evasion decisions and the resulting endowment–productivity distribution. Third, we demonstrate that our model's predictions are consistent with empirical evidence from 20 years of panel data across 14 OECD countries. Fourth, we examine the sensitivity to the leverage constraint parameter to assess how financial development influences the size of the shadow economy and the potential productivity gains from reducing it. Finally, we consider an alternative parametrization representing a developing country economy, compare the aggregate effect of tax enforcement with that of the benchmark parametrization, and discuss the model's main policy implications.

5.1. Model parameters and steady state

Unless otherwise specified, our simulations use the parameters summarized in Table 1 ("benchmark" parametrization). The subjective discount rate (ρ), relative risk aversion (γ), and capital share (α) assume standard values in the heterogeneous-agent macroeconomic literature (e.g., Krusell and Smith, Jr., 1998; Benhabib et al., 2016). The capital depreciation rate (δ) equals the aggregate depreciation rate, as reported in the National Income and Product Accounts (NIPAs). The idiosyncratic TFP autocorrelation and volatility parameters (ν and σ) are based on the estimates in Gilchrist et al. (2014) that are similar to González et al. (2022). Note that we do not choose the parameters of the productivity law of motion to match the moments of the firm's productivity distribution. This choice is inconsequential, as aggregate outcomes are characterized in terms of $\theta_t(z)$, which is endogenously determined in equilibrium.

For simplicity, firms' leverage constraint (ϕ) is assumed to be constant and matches the average corporate debt-to-equity ratio from United States (US) Federal Reserve Economic Data (FRED). Fiscal and auditing parameters are also constant. In particular, τ_k and τ_l equal the average corporate and wage income tax rates across OECD countries. These values are consistent with related studies such as Bernasconi et al. (2020). Referencing Yitzhakt (1974), auditing fines are proportional to the amount of evaded taxes: $\eta = \tau_k \cdot \eta_1$. The value of

Table 1
Benchmark parametrization.

Parameter	Meaning	Value	Source
		Externally set parameters	
ρ	Subjective discount	0.015	Standard
γ	Risk aversion	3	Standard
α	Capital share	0.33	Standard
δ	Capital depreciation	0.065	National Income and Product Accounts (NIPAs)
ν	TFP shock (autocorrelation)	0.14	Gilchrist et al. (2014)
σ	TFP shock (volatility)	0.3	González et al. (2022)
ϕ	Leverage constraint	1.46	US Federal Reserve Economic Data (FRED)
τ_k	Tax rate (capital)	0.23	OECD, Bernasconi et al. (2020)
τ_l	Tax rate (labor)	0.35	OECD, Bernasconi et al. (2020)
η	Auditing fine	1.45	IRS (2019)
		Internally set parameters	
λ	Auditing intensity	0.15	Target: Shadow economy
G	Public expenditure	0.54	Target: Debt-to-GDP ratio
A	Total factor productivity	0.16	Target: Risk-free rate

η_1 is the simple average between the minimum and the maximum administrative fines across OECD countries (10% in France and 75% in the US, according to the IRS, 2019).[11]

We set the auditing intensity parameter (λ) so that the size of the shadow economy equals 18% in the steady state. This value aligns with the World Bank data reported in the online Supplementary material. The average estimated share of the shadow economy across OECD countries between 2000 and 2020 is 17.71%, ranging from 7.1% to 36.9%. Public spending and TFP parameters (G and A) replicate the average debt-to-GDP ratio in the Euro Area (about 73%, according to FRED) and a risk-free rate of 4.5% in the steady state.

The complete characterization of the economy's steady state, when fiscal and financial friction parameters are independent of firms' idiosyncratic productivity, is as follows.

Proposition 4 (*Steady State Characterization*)**.** *When parameters τ_k, λ, η, and ϕ are independent of z, the steady-state equilibrium can be fully characterized by the collection $\{\bar{w}, \bar{R}, \bar{N}, \bar{B}, \bar{\theta}(z), \bar{z}^*\}$ (respectively representing wage, capital rental rate, aggregate net worth and debt levels, productivity–net-worth density function, and a productivity threshold) that solves the following system of integro-differential–algebraic equations:*

$$\begin{cases} \phi \cdot \int_{\bar{z}^*}^{\infty} \bar{\theta}(z) \cdot dz - \left(1 - \frac{\bar{N}}{\bar{B}}\right) = 0, \\ \bar{w} - (1-\alpha) \cdot \left(\int_{\bar{z}^*}^{\infty} z\bar{\theta}(z) \cdot dz / \int_{\bar{z}^*}^{\infty} \bar{\theta}(z) \cdot dz\right)^{\alpha} \left(A\left(\bar{N} - \bar{B}\right)\right)^{\alpha} = 0, \\ \bar{R} - \bar{z}^* \cdot A\alpha \cdot \left(\int_{\bar{z}^*}^{\infty} z\bar{\theta}(z) \cdot dz / \int_{\bar{z}^*}^{\infty} \bar{\theta}(z) \cdot dz\right)^{\alpha-1} \left(A\left(\bar{N} - \bar{B}\right)\right)^{\alpha-1} = 0, \\ (1-\tau_k)\left(\bar{R}\left(1 + \phi \int_{\bar{z}^*}^{\infty} \left(\frac{z-\bar{z}^*}{\bar{z}^*}\right) \bar{\theta}(z) \cdot dz\right) - \delta\right) \\ \quad + \left(\frac{\tau_k}{\eta} - \lambda\right)\left(1 - \left(\frac{\lambda\eta}{\tau_k}\right)^{\frac{1}{\gamma}}\right) - \rho = 0, \\ \left(\bar{R} - \delta\right)\bar{B} + G - \bar{N}\tau_k\left(\bar{R}\left(1 + \phi \int_{\bar{z}^*}^{\infty} \left(\frac{z-\bar{z}^*}{\bar{z}^*}\right) \bar{\theta}(z) \cdot dz\right) - \delta\right) \\ \quad + \bar{w}\tau_l + \left(\lambda - \frac{\tau_k}{\eta}\right)\left(1 - \left(\frac{\lambda\eta}{\tau_k}\right)^{\frac{1}{\gamma}}\right) = 0, \\ \left[(1-\tau_k)\left(\bar{R} - \delta + \mathbb{I}\{z \geq \bar{z}^*\}\left(Az\alpha\left(\frac{1-\alpha}{\bar{w}}\right)^{\frac{1-\alpha}{\alpha}} - \bar{R}\right)\right) - \rho \right. \\ \quad \left. + \left(\frac{\tau_k}{\eta} - \lambda\right)\left(1 - \left(\frac{\eta\lambda}{\tau_k}\right)^{\frac{1}{\gamma}}\right)\right] \bar{\theta}(z) - \\ + \partial_z \left[z\nu\left(\frac{\sigma^2}{2} - \ln z\right)\bar{\theta}(z)\right] + \frac{\nu\sigma^2}{2}\partial^2_{zz}\left(z^2\bar{\theta}(z)\right) = 0. \end{cases} \tag{32}$$

Proof. The system is obtained by simplifying Eqs. (23)–(24), and setting the left-hand sides of Eqs. (26)–(31) to zero. □

In the steady state, tax evasion directly affects the equilibrium levels of aggregate net worth and public debt in the fourth and fifth equations in Eq. 4 through the (constant) term $(\tau_k/\eta - \lambda)(1-(\eta\lambda/\tau_k)^{1/\gamma})$. As discussed above, the same term also influences the distribution of entrepreneurs' net worth through the sixth equation of the system, with its impact diminishing as individual productivity increases. Through these forces, tax evasion indirectly affects production factors' prices and the endogenous productivity threshold, as determined by the first three equations of the system described by Eq. 4

The numerical results discussed in the following sections are based on an approximation of the solution to Eq. 4. Details of the algorithm used to obtain this solution are provided in the online Supplementary material.

5.2. Tax evasion and the productivity distribution

Column (1) of Table 2 reports the steady-state values of key aggregate variables, denoted with an upper bar, under the benchmark parametrization detailed in Table 1. To evaluate the macroeconomic effects of tax evasion, Column (2) presents their percentage variations from the benchmark when taxes are fully enforced.[12] $\bar{E}$ and $\bar{Z}$ denote the economy's respective average tax evasion rate and the average productivity across "active" firms. They are computed as follows:

$$\bar{E} := \frac{1 - \left(\frac{\lambda\eta}{\tau_k}\right)^{\frac{1}{\gamma}}}{\eta} \int_0^{\infty} \frac{\bar{\theta}(z)}{\bar{R}\left[1 + \phi \cdot \mathbb{I}\{z \geq \bar{z}^*\}\right]\left(\frac{z-\bar{z}^*}{\bar{z}^*}\right) - \delta} \cdot dz, \tag{33}$$

$$\bar{Z} := \int_{\bar{z}^*}^{\infty} z\bar{\theta}(z) \cdot dz.$$

Since aggregate output and tax evasion are linear functions of aggregate net worth, and the tax rate is constant across all productivity levels, the average tax evasion rate in Eq. (33) represents the shadow economy's size.

5.2.1. Discussion

Preventing firms from evading taxes generates higher tax revenue, which reduces public debt to a negligible level. While the aggregate endowment ($\bar{N}$) decreases slightly, the economy's capital stock ($\bar{K} = \bar{N} - \bar{B}$) increases. Consequently, its equilibrium price ($\bar{R}$) declines. Since $\bar{R}$ also represents the marginal firm's entry cost, the productivity threshold $\bar{z}^*$ decreases, fostering an increased proportion of active firms. As labor demand rises, the equilibrium wage ($\bar{w}$) increases by the same magnitude because labor supply is constant.

While eliminating tax evasion from the economy increases aggregate productivity, it also lowers the cut-off productivity threshold

[11] The values of administrative tax evasion fines in France can be found at *www.europarl.europa.eu*.

[12] The full-enforcement economy is obtained by setting the auditing intensity parameter $\lambda = \tau_k/\eta$, which ensures that Eq. (14) equals zero for all productivity levels.

Table 2
Aggregate effects of tax evasion. This table compares macroeconomic aggregates (benchmark levels and $\Delta\%$ from the benchmark in the steady state) for economies with tax evasion and full tax enforcement. $\bar{N}$ and $\bar{B}$ represent aggregate endowment and public debt. $\bar{Z}$ and $\bar{z}^*$ denote active firms' average productivity and the corresponding cut-off threshold. $\bar{\Theta}$ indicates the share of active firms. $\bar{E}$, $\bar{R}$, and $\bar{w}$ represent the share of the shadow economy and factor prices.

Aggregate variable	Model (1) Tax evasion benchmark levels	(2) Full enforcement $\Delta\%$ from Model (1)	(3) Full enforcement, fixed debt $\Delta\%$ from Model (1)
$\bar{N}$	**8.71**	−4.25	4.46
$\bar{B}$	**1.48**	−99.97	–
$\bar{K}$	**7.23**	15.35	4.46
$\bar{Z}$	**1.22**	1.09	4.26
$\bar{z}^*$	**0.94**	−10.26	5.13
$\bar{\Theta}$	**0.57**	20.76	0.41
$\bar{E}$	**0.18**	–	–
$\bar{R}$	**0.11**	−20.20	−6.68
$\bar{w}$	**1.36**	5.96	6.04

($\bar{z}^*$). This counterintuitive outcome arises from the combined effect of the two mechanisms described in Section 4. On the one hand, when public debt rises due to tax evasion, the cost of capital increases. Consequently, production becomes infeasible for some low-z firms that find it optimal to exit production (debt *crowding-out effect*). Conversely, evasion reallocates endowment from high-to low-z firms because the latter evades more taxes than the former (capital *redistribution effect*). In our simulations, although it enables a larger share of low-productivity firms to remain active, full tax enforcement enhances the economy's average productivity, indicating that the redistribution effect outweighs the crowding-out effect.

Column (3) of Table 2 examines an economy in which all taxes are enforced, but the crowding-out effect is neutralized by adjusting public spending (G) to keep $\bar{B}$ at its benchmark level. When the effect is removed, the productivity threshold $\bar{z}^*$ shifts to the right, and aggregate productivity increases threefold compared with Column (2). At the same time, the share of active firms remains unchanged from the benchmark.

To visualize the endowment redistribution effect, Fig. 1 compares firms' tax evasion strategy ($\bar{e}^*(z)$) in the benchmark and full tax enforcement economies. The shaded areas depict the corresponding endowment–productivity densities ($\bar{\theta}(z)$) in the steady state.

Consistent with the analytical results in Section 3.3, optimal tax evasion decreases with z. This means that, in equilibrium, lower-productivity firms use tax evasion to accumulate larger endowments at the expense of their (higher-productivity) peers. Comparing the shaded areas in the figure confirms that implementing full tax enforcement removes this advantage. Due to the crowding-out effect, keeping public debt constant mitigates the positive impact of the redistribution channel, as illustrated by the dotted black line in Fig. 1.

5.2.2. Examining the mechanisms

Next, we analyze the relationship between shadow economy size and endowment–productivity distribution. To do so, we compare steady states obtained by varying the auditing parameter λ above and below the benchmark value. The range of variation is selected so that the resulting $\bar{E}$ falls between the empirical minimum and maximum observed across OECD countries from January 2000 to December 2020, according to the World Bank.[13]

To summarize our results, Fig. 2 illustrates the variations in steady state density $\bar{\theta}(z)$ from the benchmark that correspond to different shadow economy sizes. Table 3 presents the corresponding steady-state relationship between $\bar{E}$ and $\bar{Z}$.

Table 3
Size of the shadow economy and aggregate productivity. This table presents the size of the shadow economy ($\bar{E}$) and the corresponding aggregate productivity level ($\bar{Z}$) for different the auditing parameter λ values.

λ	−3.0%	−2.0%	Benchmark	+1.4%	+2.8%
$\bar{E}(\lambda)$	0.34	0.22	0.18	0.13	0.07
$\bar{Z}(\lambda)$	1.18	1.21	1.22	1.24	1.25

Our simulations predict a negative relationship between shadow economy size and aggregate productivity, aligning with Di Nola et al. (2021), Franjo et al. (2022), and Erosa et al. (2023). However, in contrast to previous studies, we demonstrate how this effect is driven by changes in firms' endowment–productivity distribution. In particular, we show that aggregate productivity declines when the shadow economy expands because the share of low-productivity firms grows. In contrast, the mass of medium- and high-productivity firms declines more than proportionally.

In the next section, we estimate the relationship between different percentiles of the empirical productivity distribution and a country's shadow-economy share to verify whether our model's predictions align with real-world data.

5.3. Model predictions vs OECD data

Our empirical analysis uses a panel of 14 OECD countries from January 2000 to December 2020. Tax evasion data are sourced from the World Bank and Schneider et al. (2010), and TFP percentiles data are from CompNet (2023). The IMF Financial Development Index measures the extent of leverage constraints. The remaining variables are taken from the OECD dataset. Descriptive statistics for the dataset are presented in Table S.1. in the online Supplementary material.

Using these data, we estimate the following regression:

$$TFP^p_{i,t} = \beta^p_0 + \beta^p_1 \times \text{Shadow}_{i,t} + \beta^p_2 \times \text{Shadow}_{i,t} \times \text{FinD}_{i,t-1} + \\ +\beta^p_3 \times \text{FinD}_{i,t-1} + \mathbf{X}^T_{i,t-1}\mathbf{B} + u_{i,t} + \epsilon_{i,t}, \quad (34)$$

where $TFP^p_{i,t}$ denotes the p-percentile of firms' TFP in country i at time t, $\text{Shadow}_{i,t}$ is the share of a country's shadow economy, $\text{FinD}_{i,t}$ denotes financial development, and $X_{i,t-1}$ is a vector of lagged macroeconomic controls.[14] $u_{i,t}$ denotes a country-time fixed effect (FE), and $\epsilon_{i,t}$ are heteroskedasticity and autocorrelation consistent (HAC) standard errors. Table 4 presents the β^p coefficient values. Further details of the estimates for other percentiles of the productivity distribution and the

[13] Descriptive statistics are presented in Table S.1 in the online Supplementary material.

[14] We adopt one-period-lagged controls to mitigate simultaneity issues in the regression.

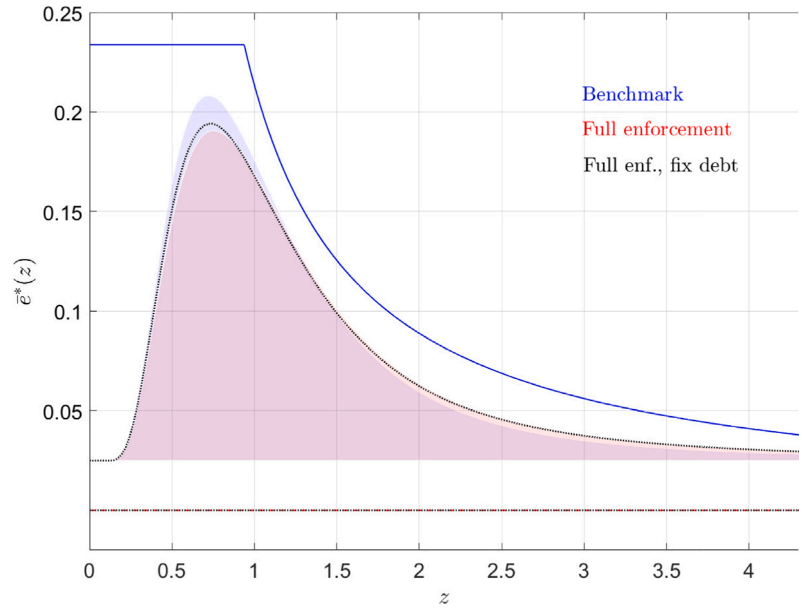

Fig. 1. Tax evasion and the productivity distribution. This figure compares firms' optimal tax evasion decisions ($\bar{e}^*(z)$), and the corresponding endowment–productivity density function ($\bar{\theta}(z)$) in the steady state of the benchmark economy (solid blue) with that of the full tax enforcement economy (dashed red). The black dotted line represents the steady-state density under full tax enforcement but with public spending adjusted to maintain $\bar{B}$ at the benchmark level.

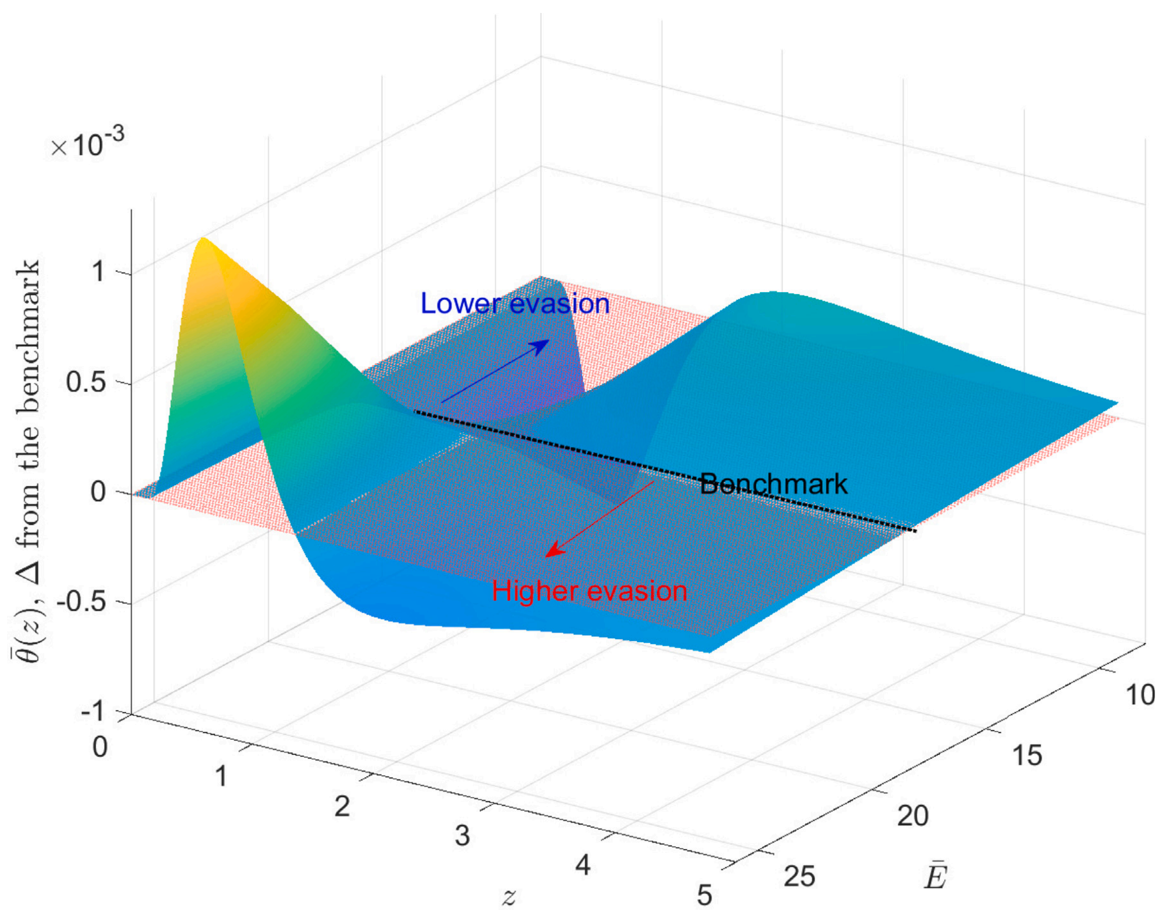

Fig. 2. Size of the shadow economy and the productivity distribution. This figure illustrates the (absolute) variations of the steady-state density function $\bar{\theta}(z)$ and the corresponding share of the shadow economy $\bar{E}$.

Table 4
Size of the shadow economy and productivity distribution in the data. This table presents the coefficients of the linear relationship between different productivity distribution percentiles and the share of the shadow economy. Newey–West standard errors are in parentheses. The coefficients of other percentiles and the estimates without controls are presented in the online Supplementary material. Notes: *** p < 0.01, ** p < 0.05, * p < 0.1. Controls include: ln(GDP), public debt/GDP, GDP growth, TFP volatility and ln(Public spending).

	Percentile					
TFP	1st	5th	Average	75th	90th	95th
$Shadow$	−0.25	−1.16	**−1.10***	**−1.29****	**−4.51****	**−7.86****
	(0.53)	(0.28)	(0.59)	(0.94)	(1.78)	(3.71)
$Shadow \times FinD$	0.68	−0.41	−0.57	−0.49	−1.21	−3.37
	(0.85)	(0.37)	(0.77)	(1.18)	(2.26)	(4.34)
Fixed effects	✓	✓	✓	✓	✓	✓
Controls	✓	✓	✓	✓	✓	✓
Observations	216	216	216	216	216	216
R^2	0.27	0.41	0.47	0.52	0.45	0.36
Number of groups	14	14	14	14	14	14

coefficients of the main control variables appear in the supplementary material.

Column (3) of Table 4 supports the prediction of a negative relationship between *average* TFP and shadow economy size, with a significant coefficient at the 10% level. However, more importantly, the estimates reveal that the relationship between the shadow economy and TFP is relatively minimal and statistically insignificant for the lowest productivity percentiles in Columns (1) and (2) but becomes progressively more negative and significant at higher percentiles in Columns (4)–(6). To demonstrate that this pattern is consistent with our predictions, the solid blue and dashed red lines in Fig. 3 compare the estimated TFP sensitivity to the size of the shadow economy from Eq. (34), computed as follows:

$$\frac{\partial TFP^p}{\partial \text{Shadow}} = \hat{\beta}_1^p + \hat{\beta}_2^p \times \overline{\text{FinD}}, \tag{35}$$

where $\overline{\text{FinD}}$ denotes the average financial development in the sample, with those implied by the simulations in Section 5.2.2. The model's coefficients are obtained by computing the linear relationship between

$$\bar{z}(p) := \left\{ y : \int_0^y \bar{\theta}(z)dz = p \right\} \approx \text{TFP}^p, \tag{36}$$

and the corresponding level of $\bar{E} \approx$ Shadow level produced by the simulations of Section 5.2.2. The theoretical levels in Eq. (36) are normalized to match the scale of the estimated sensitivity in the 50th percentile.

The second row of Table 4 presents the coefficient of the interaction term between the size of the shadow economy and financial development. The coefficients for low productivity percentiles are either positive or minimal and statistically insignificant. Although the coefficients are statistically insignificant, they become progressively larger moving towards higher percentiles. This pattern suggests that, ceteris paribus, a larger shadow economy is associated with greater TFP losses in more financially developed economies, with these effects concentrated among the most productive firms.

To demonstrate that this observation aligns with the model's predictions, the dotted black line in Fig. 3 shows the TFP sensitivity to shadow economy size predicted by the model when the leverage constraint parameter is one standard deviation (SD) below its empirical mean ($\phi = 1.29$). In summary, when leverage constraints are tighter, higher percentiles of productivity distribution are *more* sensitive to variations in shadow economy size. To further explore this channel, we next examine how financial development affects firms' tax evasion decisions and how these decisions, in turn, shape their equilibrium distribution and macroeconomic aggregates.

Table 5
Shadow economy and leverage constraints. This table compares macroeconomic aggregates (benchmark levels and Δ% from the benchmark in the steady state) of economies with different leverage constraints. $\bar{N}$ and $\bar{B}$ represent aggregate endowment and public debt. $\bar{Z}$ and $\bar{z}^*$ denote active firms' average productivity and the corresponding cut-off threshold. $\bar{\Theta}$ indicates the share of active firms. $\bar{E}$, $\bar{R}$, and $\bar{w}$ label the share of the shadow economy and factor prices.

Aggregate variable	Model (1) $\phi = 1.46$ Benchmark levels	(2) $\phi = 1.29$ Δ% from Model (1)	(3) $\phi = 1.29$, fix debt Δ% from Model (1)
$\bar{N}$	**8.71**	6.40	−4.88
$\bar{B}$	**1.48**	92.71	–
$\bar{K}$	**7.23**	−10.90	−5.88
$\bar{Z}$	**1.22**	−0.86	−3.58
$\bar{z}^*$	**0.94**	2.56	−10.26
$\bar{\Theta}$	**0.57**	−8.69	10.76
$\bar{E}$	**0.18**	21.25	26.25
$\bar{R}$	**0.11**	13.05	0.43
$\bar{w}$	**1.36**	−4.68	−5.39

5.4. Tax evasion and leverage constraints

Column (1) of Table 5 reports macroeconomic aggregates in the steady state when $\phi = 1.46$, as in the benchmark parametrization. Column (2) details the percentage deviations from the benchmark when the leverage constraint is one SD below the average ($\phi = 1.29$). Column (3) presents the variations when $\phi = 1.29$ but debt is maintained at its benchmark level by changing the public spending parameter G. Panels (a) and (b) of Fig. 4 illustrate firms' optimal tax evasion and the endowment–productivity density function corresponding to each column.

Consistent with the empirical evidence motivating this study (Loayza and Rigolini, 2006; La Porta and Shleifer, 2014; Dabla-Norris et al., 2019), tighter leverage constraints foster tax evasion across all productivity levels (see Fig. 4, Panel (a)). Consequently, when ϕ is lower, the shadow economy grows (row 7), public debt increases (row 2), and average productivity decreases (row 4). A closer comparison between Columns (2) and (3) reveals that the adverse effect of leverage constraints on the shadow economy and aggregate productivity, driven by the endowment redistribution effect, is mitigated by the debt crowding-out effect (rows 5, 6, and 8).

Fig. 4 illustrates the effect of tightening the leverage constraints on optimal tax evasion decisions across different z values. While tax evasion increases for all z when endowment redistribution and debt crowding-out effects are in place (Panel (a)), this is not the case when the latter is neutralized (Panel (b)). In that case, a lower ϕ only fosters tax evasion among low- and high-productivity firms but curbs it among those with a z close to $\bar{z}^*$. This occurs because the profitability gains from entering production outweigh the losses incurred from facing higher capital costs for those firms (on this point, see the discussion following Eq. (14)).

To conclude this subsection, Table 6 compares the benefits of full tax enforcement across economies for different ϕ levels, representing the economy's financial development. This analysis reveals that the benefits of eliminating tax evasion in terms of productivity and public debt reduction are milder in economies with stronger leverage constraints (rows 2 and 4) despite a larger decrease in overall tax evasion

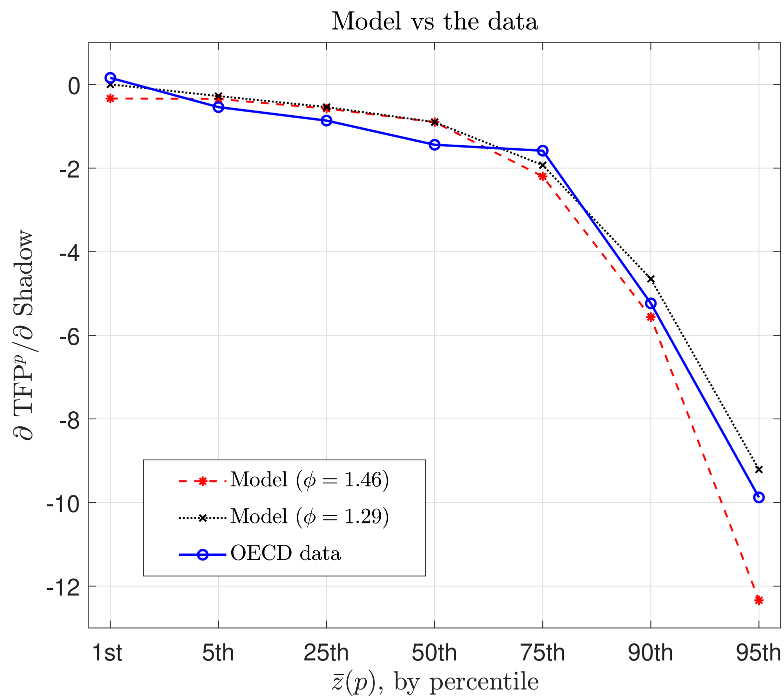

Fig. 3. Model vs the data. This figure compares the linear relationship between different percentiles (p) of firms' productivity distribution and the size of their country's shadow economy, as predicted by our model and observed in the data. Sources: World Bank data, Schneider et al. (2010) (shadow economy share), and CompNet (2023) (productivity by percentiles).

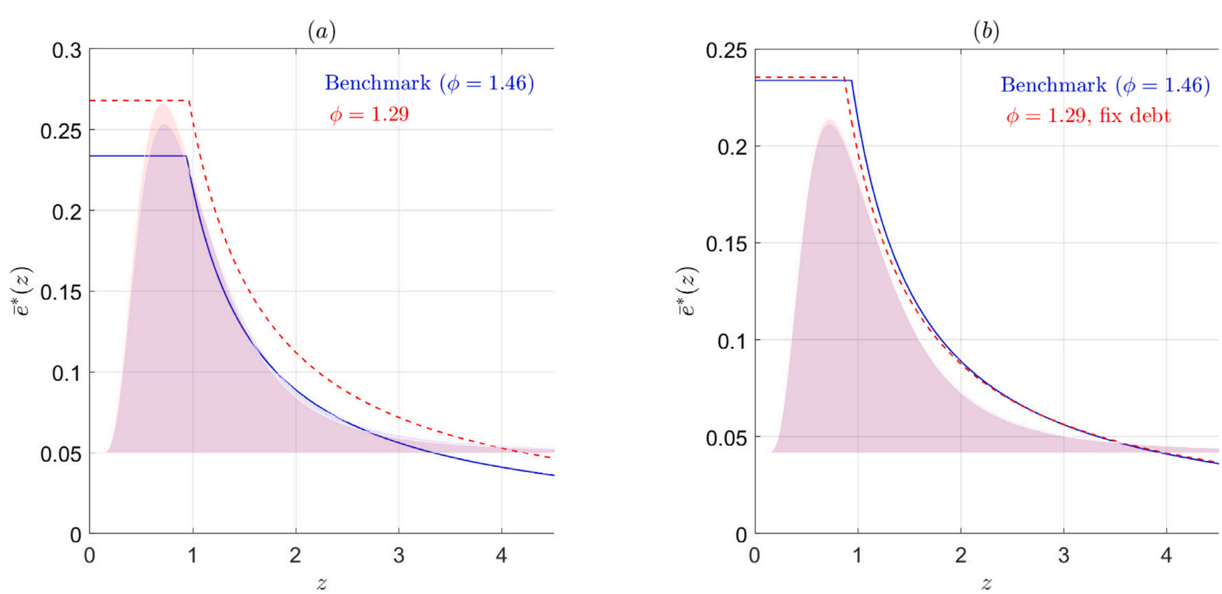

Fig. 4. Tax evasion distribution and leverage constraints. Panel (a) compares firms' optimal tax evasion strategy ($\bar{e}^*(z)$) and the corresponding endowment–productivity density ($\bar{\theta}(z)$) in the steady state for different leverage constraint parameter (ϕ) levels. Panel (b) conducts the same comparison while adjusting public spending to maintain public debt at the benchmark level.

(row 7). The reason for this is that lower financial development reduces the impact of the endowment redistribution effect (rows 5 and 6). Furthermore, the implications of our model are broadly consistent with the data (see Table 4), which indicate a stronger effect of tax evasion among highly productive firms when financial development is higher. Indeed, the regression results suggest that financial development does not, per se, appear to affect TFP at any percentile, but that the effects of tax evasion are comparatively milder for highly productive firms when financial constraints are tighter.

5.5. Tax enforcement in developed vs developing countries

The majority of our numerical results are based on a parametrization representing the average OECD (i.e., developed) member country. In this section, we verify the robustness of the results and further explore the model's main policy implications.

Similar to Section 5.2, we compare the effect of reducing tax evasion under our benchmark parametrization with that under an alternative set of values that capture the features of an emerging economy.

Table 6
Full tax enforcement and leverage constraints. This table summarizes the effect of full tax enforcement on macroeconomic aggregates in economies with different financial development levels (Δ% relative to the respective benchmark). $\bar{N}$ and $\bar{B}$ respectively represent the aggregate endowment and public debt. $\bar{Z}$ and $\bar{z}^*$ denote active firms' average productivity and the corresponding cutoff threshold. $\bar{\Theta}$ indicates the share of active firms, and $\bar{E}$, $\bar{R}$, and $\bar{w}$ refer to the share of the shadow economy and factor prices.

	Model			
Aggregate variable	(1) $\phi = 1.46$ Levels	(2) $\phi = 1.46$, Full enf. Δ% from (1)	(3) $\phi = 1.29$ Levels	(4) $\phi = 1.29$, Full enf. Δ% from (3)
$\bar{N}$	**8.71**	−4.25	**9.26**	−12.62
$\bar{B}$	**1.48**	−99.97	**2.85**	−96.84
$\bar{K}$	**7.23**	15.35	**6.42**	24.13
$\bar{Z}$	**1.22**	1.09	**1.21**	0.38
$\bar{z}^*$	**0.94**	−10.26	**0.96**	−22.5
$\bar{\Theta}$	**0.57**	20.76	**0.52**	46.15
$\bar{E}$	**0.18**	–	**0.21**	–
$\bar{R}$	**0.11**	−20.20	**0.12**	−14.66
$\bar{w}$	**1.36**	5.96	**1.29**	6.99

Following Franjo et al. (2022), we consider the case of Brazil and set the financial development parameter $\phi = 1.32$ and $\tau_k = 0.135$. We adjust the auditing intensity parameter λ so that the steady-state size of the shadow economy equals 36% and set the TFP shock parameter $\sigma = 0.45$. This is consistent with the empirical evidence in Collard-Wexler et al. (2011), who reported that Brazil has a TFP dispersion that is about 1.5 times that of Poland, the only OECD country considered in the study. Finally, in line with the IMF database, we adjust public spending G to match a 90% debt-to-GDP ratio.

Columns (1) and (2) of Table 7 compare the changes in key macroeconomic aggregates resulting from 5% and 10% reductions in tax evasion under the developed- and developing-economy parameterizations. The simulations confirm that our baseline results still hold under the parametrization for a developing economy; that is, reducing the size of the shadow economy fosters productivity by improving capital allocation.

Comparing Columns (1) and (2) of Table 7 also reveals that the magnitude of the effect increases with improved financial development. However, the analysis indicates that reducing tax evasion in developing economies with higher public debt lowers the debt-to-GDP ratio more but increases productivity less than in financially developed economies with lower initial public debt. The rational for this is twofold. First, the benefits from the endowment reallocation effect are smaller in the latter than in the former. Second, the debt crowding-out effect is larger. Column (3) validates this intuition by examining the effect of reducing tax evasion in a developing economy with an initial debt-to-GDP ratio equal to that of the developed-economy parametrization. When developed and developing economies begin from the same debt-to-GDP level, the benefits of reducing tax evasion are smaller in the latter, as the reduction primarily mitigates productivity losses from low-z entrepreneurs entering the economy.

Table 7
Aggregate effects of tax enforcement in developing vs developed economies. This table compares the macroeconomic effects of reducing tax evasion by 5% and 10% (Δ% from the initial steady state) in developed vs developing economies. $\bar{B}$ and $\bar{Y}$ represent aggregate debt and GDP. $\bar{Z}$ and $\bar{z}^*$ denote the active firms' average productivity and the corresponding cut-off threshold. $\bar{\Theta}$ indicates the share of active firms. $\bar{R}$ and $\bar{w}$ denote the shadow economy share and factor prices.

	Model		
Aggregate variable	(1) Developed Δ% from s.s.	(2) Developing Δ% from s.s.	(3) Developing (debt-GDP) Δ% from s.s.
	−5 p.p. of tax evasion		
$\bar{B}/\bar{Y}$	−1.61	−2.00	−1.01
$\bar{Z}$	1.50	0.12	0.13
$\bar{z}^*$	2.55	0.61	0.94
$\bar{\Theta}$	−1.45	0.17	−0.84
	−10 p.p. of tax evasion		
$\bar{B}/\bar{Y}$	−3.00	−4.36	−2.85
$\bar{Z}$	2.41	0.27	0.26
$\bar{z}^*$	2.56	0.62	0.27
$\bar{\Theta}$	−0.78	0.40	−0.90

5.6. Policy implications

Unlike previous studies, our framework reveals additional nuances in the interplay among tax enforcement, public debt, and financial development, with a few noteworthy policy implications.

First, our model predicts that easing financing constraints makes tax evasion less attractive to firms at all productivity levels, enabling the government to collect additional revenue. This mechanism aligns with the empirical results of Besley and Persson (2009), who demonstrated that financial development is positively associated with government revenue. Moreover, it is consistent with several studies showing that financial development increases the benefits of income or asset disclosure (Blackburn et al., 2012; Capasso and Jappelli, 2013) and mitigates the risk of disintermediation (Gordon and Li, 2009).

Second, the results indicate that policies intended to curb tax evasion can enhance aggregate productivity by reallocating capital toward more productive firms. However, it also reveals that the effect of such policies is amplified in economies with higher financial development and lower public debt.

Third, our model warns that policies intended to lower public debt and its financing costs can lead to higher aggregate tax evasion as capital misallocation worsens, particularly when financial constraints are tight. This occurs because the policy enables lower-productivity (i.e., more evasive) firms to enter production by improving their financing conditions. This outcome squares nicely with empirical evidence from developing countries (Ağca and Celasun, 2012).

6. Conclusions

This study developed a macroeconomic model featuring heterogeneous firms and a public sector to examine how tax evasion and public debt affect the distribution of firms' productivity in general equilibrium.

Our theoretical model identifies two key channels through which tax evasion affects the economy: the endowment redistribution effect and the debt crowding-out effect. The first explains how tax evasion shifts endowment from high- to low-productivity firms. The second shows that tax evasion increases public debt, raising the cost of capital and ultimately forcing marginal firms out of the market.

We analyzed how these two effects interact in equilibrium using a parameterized model and numerical simulations. The findings reveal that the negative link between the shadow economy and TFP does not hold for low-productivity firms but becomes progressively stronger and more significant at higher productivity levels. This prediction is validated using OECD panel data from the past 20 years.

We also examined the relationships between financial development, productivity, public debt, and shadow economy size. Our findings demonstrate that reducing tax evasion yields smaller productivity gains in economies with higher debt-to-GDP ratios and/or where firms face tighter leverage constraints. This has significant policy implications, indicating that financially underdeveloped and high-debt countries may require more substantial government interventions to achieve the same benefits from reducing tax evasion relative to more developed, low-debt economies. Although we do not explicitly model corruption, it has been found to exacerbate financial constraints and further reduce the effectiveness of tax enforcement policies (see Pappa et al., 2015).

To keep the model tractable, we assumed a fixed labor supply and a simplified representation of workers, which prevents us from analyzing how tax evasion redistributes resources between workers and businesses. As a result, our model does not fully capture the welfare implications of different fiscal and tax auditing policies. Addressing these questions is a significant direction for future research.

Funding

Modena acknowledges support from the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 (Project C03). Regis gratefully acknowledges support from the “Dipartimenti di Eccellenza 2023–2027” grant and the PRIN 2022 project grant “Stochastic control and games and the role of information” from the Italian Ministry of University.

Declaration of competing interest

I have nothing to declare.

Acknowledgments

The authors thank Manuel Amador, Giorgio Ferrari, Yuriy Gorodnichenko, Philipp Grübener (discussant), Debarya Jana (discussant), Jonathan Heathcote, Leo Kaas, Galo Nuno, Caterina Pavese, B. Ravikumar, Almuth Scholl, Jan Schymik, Elu von Thadden, seminar participants at the University of Brescia, the University of Bielefeld, the Catholic University (Milan), the University of Siena, and conference participants at the AMASES 2022 (Palermo), the ASSET 2022 (Crete), the 1st INFER Workshop on Macroeconomic Dynamics (Rome), the Frankfurt–Mannheim Macro Work-shop 2022, the WQF workshop (Gaeta), the CRC224 YRW (Bonn), the CEF 2023 (Nice), and the 2nd DISEI Workshop (Florence) for the helpful discussions and comments. Modena is grateful to LTI@UniTO and the Collegio Carlo Alberto for their hospitality and support during his research fellowship. All authors contributed equally to this work.

Appendix A. Proofs and derivations

A.1. Firms' optimal control problem

To solve the firms' stochastic control problem, it is convenient to express their controls as (omit time subscripts and functional dependence) $\{k, l, c, e\} \to \left\{\kappa := \frac{k}{n}, \mu := \frac{l}{k}, c, e\right\}$. Accordingly, we can rewrite Eq. (6) as follows:

$$\frac{dn}{n} = \left[\left(1-\tau_k(1-e)\right) r + \kappa\left((Az)^\alpha \mu^{1-\alpha} - \mu w - R\right) - \frac{c}{n}\right] dt \\ - e\eta\kappa\left((Az)^\alpha \mu^{1-\alpha} - \mu w - R\right) dJ.$$

Then, the HJBE Eq. (8) becomes the following:

$$0 = \sup_{\kappa,\mu,e}\left\{ \begin{array}{c} \frac{\partial V}{\partial n} n\left[\left(1-\tau_k(1-e)\right)\left(r+\kappa\left((Az)^\alpha \mu^{1-\alpha}-\mu w - R\right)\right)\right] + \\ +\lambda V\left(n\left(1-e\eta\left(r+\kappa\left((Az)^\alpha \mu^{1-\alpha}-\mu w-R\right)\right)\right)\right) \end{array}\right\} \\ + \sup_c\left\{(1-\gamma)\rho V\left(\log c - \frac{1}{1-\gamma}\log((1-\gamma)V)\right) - c\frac{\partial V}{\partial n}\right\} + \frac{\partial V}{\partial z}\mu_z \\ + \frac{1}{2}\frac{\partial^2 V}{\partial z^2}\sigma_z^2 - \lambda V, \tag{A.1}$$

subject to

$$0 < \kappa \le \phi. \tag{A.2}$$

This entails the following first order conditions:

$$c^* : \frac{(1-\gamma)\rho V}{c} = \frac{\partial V}{\partial n}, \tag{A.3}$$

$$\mu^* : \frac{\frac{\partial V}{\partial n}}{\frac{\partial \bar V}{\partial \bar n}}\frac{\kappa}{\lambda\eta}\left(1-\tau_k(1-e)\right)\left((Az)^\alpha(1-\alpha)\mu^{-\alpha} - w\right) = e\kappa\left((Az)^\alpha(1-\alpha)\mu^{-\alpha} - w\right), \tag{A.4}$$

in which

$$\frac{\partial \bar V}{\partial \bar n} := \frac{\partial V\left(n\left(1-e\eta\left(r+\kappa\left((Az)^\alpha\mu^{1-\alpha}-\mu w - R\right)\right)\right)\right)}{\partial\left(n\left(1-e\eta\left(r+\kappa\left((Az)^\alpha\mu^{1-\alpha}-\mu w-R\right)\right)\right)\right)},$$

and

$$e^* : \frac{\frac{\partial V}{\partial n}}{\frac{\partial \bar V}{\partial \bar n}}\frac{\tau_k}{\lambda\eta}\left(r+\kappa\left((Az)^\alpha\mu^{1-\alpha}-\mu w - R\right)\right) = r + \kappa\left((Az)^\alpha \mu^{1-\alpha} - \mu w - R\right), \tag{A.5}$$

$$\kappa^* : \frac{\partial V}{\partial n}\left(1-\tau_k(1-e)\right)\left((Az)^\alpha\mu^{1-\alpha}-\mu w - R\right) \\ \ge \frac{\partial \bar V}{\partial \bar n}\lambda e\eta\left(\left((Az)^\alpha\mu^{1-\alpha}-\mu w - R\right)\right), \tag{A.6}$$

where (A.6) holds slack (with equality) depending on whether the upper (lower) constraint Eq. (A.2) binds.

To derive the optimal controls as they appear in the main text, we guess and verify that the value function takes the following form:

$$V(n,z) := v(z)\frac{n^{1-\gamma}}{1-\gamma}, \tag{A.7}$$

where $v(\cdot)$ is an unknown function of z. Substituting Eq. (A.7) into Eqs. (A.3) and (A.4) yields $c^* = \rho n$ and the following:

$$(Az)^\alpha(1-\alpha)\mu^{-\alpha} = w \Longleftrightarrow \mu^* = Az\left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}}, \tag{A.8}$$

$$l^* = k^* Az\left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}}. \tag{A.9}$$

Substituting Eq. (A.8) into Eq. (A.5) and using $r + \kappa^*\left((Az)^\alpha(\mu^*)^{1-\alpha} - \mu^* w - R\right) \ge 0$ (i.e., capital revenue is greater than or equal to zero), optimal tax evasion is as follows:

$$e_t^* = \frac{1-\left(\frac{\tau_k}{\lambda\eta}\right)^{-\frac{1}{\gamma}}}{\eta\left[r_t+\kappa_t^*\left(Az\alpha\left(\frac{1-\alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}}-R_t\right)\right]}. \tag{A.10}$$

To obtain κ^*, we substitute Eqs. (A.8), (A.10), and (A.7) in Eq. (A.6) and rearrange to obtain the following:

$$\left(Az\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}-R\right)\left(1-\tau_k+\frac{\tau_k}{\eta}\frac{1-\left(\frac{\tau_k}{\lambda\eta}\right)^{-\frac{1}{\gamma}}}{r+\kappa^*\left(Az\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}-R\right)}\right)\geq \left(Az\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}-R\right)\frac{\tau_k}{\eta}\frac{1-\left(\frac{\tau_k}{\lambda\eta}\right)^{-\frac{1}{\gamma}}}{r+\kappa^*\left(Az\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}-R\right)}. \tag{A.11}$$

When z is larger (lower) than the following:

$$z^* = \left\{z\in[0,\infty):\ Az\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}=R\right\}, \tag{A.12}$$

and under the parametric condition that $\tau_k<1$, then Eq. (A.11) holds slack (with equality) and $\kappa^*=\phi$ ($\kappa^*=0$). Plugging k^*/n into Eq. (A.10) yields e^*. Substituting $\{k^*,l^*,c^*,e^*\}$ and Eq. (A.7) in Eq. (A.1) and rearranging, one obtains the following ODE:

$$v\left(\rho(1-\log\rho)-\lambda\frac{\left(\frac{\eta\lambda}{\tau_k}\right)^{\frac{1-\gamma}{\gamma}}}{1-\delta}-\frac{\tau_k}{\eta}\left(1-\left(\frac{\eta\lambda}{\tau_k}\right)^{\frac{1}{\gamma}}\right)\right)=$$
$$=v\left((1-\tau_k)\left(r+\phi\max\left\{Az\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}-R,0\right\}\right)+\frac{\log v}{\gamma-1}\right)+\frac{\partial v}{\partial z}\mu_z+\frac{1}{2}\frac{\partial^2 v}{\partial z^2}\sigma_z^2,$$

for the value of $v(z)$. The solution can be numerically approximated by using an up-wind finite difference scheme with the boundary conditions $\lim_{z\to 0}\frac{\partial v(z)}{\partial z}=\lim_{z\to\infty}\frac{\partial v(z)}{\partial z}=0$ (“reflecting barriers”). See Dixit (2013) for a formal derivation of these conditions.

A.2. Proof of Proposition 2

As discussed in the main text, the density function $f(z,n)$ satisfies the following PDE (omitting time subscripts):

$$\frac{df(z,n)}{dt}=-\frac{\partial}{\partial n}\left[nf(z,n)\mu_n^*(z)\right]-\frac{\partial}{\partial z}\left[f(z,n)\mu_z(z)\right]+\frac{1}{2}\frac{\partial^2}{\partial z^2}\left[f(z,n)\sigma_z^2(z)\right]+$$
$$-\lambda(z)f(z,n)+\lambda(z)f(z,n(1-(\eta(z)\lambda(z)/\tau_k(z))^{1/\gamma})), \tag{A.13}$$

where $\mu_n(z)\cdot n$ denotes the drift of Eq. (6) under the optimal controls in Eqs. (9)–(14), and $\mu_z(z)$ and $\sigma_z(z)$ are the drift and diffusion of $dz=z\left(\frac{\sigma^2}{2}v-v\ln z\right)dt+z\sigma\sqrt{v}dW$.

We use Eq. (A.13) to derive (26) through the following steps.

1. Differentiate Eq. (22) to obtain the following

$$\frac{d\theta(z)}{dt}=\frac{N\int_0^\infty n\cdot\frac{df(n,z)}{dt}\cdot dn-\dot N\int_0^\infty n\cdot f(n,z)\cdot dn}{N^2}. \tag{A.14}$$

2. Substitute Eq. (A.13) into Eq. (A.14) and rearrange to obtain the following:

$$\frac{d\theta(z)}{dt}=-\frac{\int_0^\infty n\left[\frac{\partial}{\partial n}\left(f(n,z)\cdot n\mu_n(z)\right)+\frac{\partial}{\partial z}\left(f(n,z)\cdot\mu_z(z)\right)\right]dn}{N}+$$
$$+\frac{1}{2}\frac{\int_0^\infty n\frac{\partial^2}{\partial z^2}\left(f(n,z)\cdot\sigma_z^2(z)\right)\cdot dn}{N}-\frac{\int_0^\infty n\lambda(z)f(n,z)\cdot dn}{N}+$$
$$-\frac{d\ln N}{dt}\frac{\int_0^\infty nf(n,z)\cdot dn}{N}+\frac{\int_0^\infty n\lambda(z)\cdot f(n,z)\cdot\left(n(1-(\eta(z)\lambda(z)/\tau_k(z))^{1/\gamma})\right)\cdot dn}{N}. \tag{A.15}$$

3. Integrate by parts the first three terms of Eq. (A.15) to obtain the following:

$$\int_0^\infty n\frac{\partial\left(nf(n,z)\mu_n(z)\right)}{\partial n}\cdot dn=-\mu_n(z)\int_0^\infty nf(n,z)\cdot dn, \tag{A.16}$$

$$\frac{\partial}{\partial n}\left(\int_0^\infty nf(n,z)\cdot\mu_z(z)\cdot dn\right)=N\frac{\partial\left(\theta(z)\cdot\mu_z(z)\right)}{\partial z}, \tag{A.17}$$

$$\int_0^\infty n\frac{\partial^2}{\partial z^2}\left(f(n,z)\cdot\sigma_z^2(z)\right)dn=N\frac{\partial^2}{\partial z^2}\left(\theta(z)\cdot\sigma_z^2(z)\right). \tag{A.18}$$

4. Since the optimal controls are linear in n, rewrite the fourth and fifth equations of Eq. (A.15) as follows:

$$\frac{\lambda(z)\int_0^\infty nf(n,z)\left(n\left(1-\left(\frac{\eta(z)\lambda(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}\right)\right)\cdot dn}{N}=\theta(z)\cdot\left(1-\left(\frac{\eta(z)\lambda(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}\right)\lambda(z). \tag{A.19}$$

5. Substitute Eqs. (A.16)–(A.19) into Eq. (A.15), use Eq. (22), and rearrange to obtain Eq. (A.15) as it appears in the main text.

We adopt the following steps to obtain Eq. (27).

1. Substitute the optimal controls into Eqs. (12)–(14) into Eq. (6) and rearrange to obtain the following:

$$\frac{dn}{n}=(1-\tau_k(z))\left(r_t+\mathbb{I}\left\{z\geq z_t^*\right\}R_t\left(\frac{z-z_t^*}{z_t^*}\right)\right)dt+$$
$$+\left(\frac{\tau_k(z)}{\eta(z)}dt-dJ\right)\left(1-\left(\frac{\lambda(z)\eta(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}\right)-\rho dt \tag{A.20}$$

2. Integrate Eq. (A.20) using $f(n,z)$ and use Eq. (22) to simplify the resulting equation as follows:

$$\frac{dN_t}{dt}\frac{1}{N_t}=\int_0^\infty(1-\tau_k(z))\left(R_t-\delta+\mathbb{I}\left\{z\geq z_t^*\right\}R_t\left(\frac{z-z_t^*}{z_t^*}\right)\right)\theta(z)\cdot dz+$$
$$+\int_0^\infty\left(\frac{\tau_k(z)}{\eta(z)}-\lambda(z)\right)\left(1-\left(\frac{\lambda(z)\eta(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}\right)\theta(z)\cdot dz-\rho, \tag{A.21}$$

where we impose the no-arbitrage condition $r_t=R_t-\delta$ and used the following:

$$\int_0^\infty\left(1-\left(\frac{\lambda(z)\eta(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}\right)\theta(z)\lambda(z)\cdot dz\cdot dt=\int_0^\infty\left(1-\left(\frac{\lambda(z)\eta(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}\right)\theta(z)\cdot dz\cdot dJ,$$

as in Eq. (21).

Rearranging Eq. (A.21) and using $\mathbb{P}_t^\theta\left\{z\geq z_t^*\right\}:=\int_0^\infty\mathbb{I}\left\{z\geq z_t^*\right\}\cdot\theta(z)\cdot dz$ yields dN_t/dt as it appears in the main text.

A.3. Proof of Proposition 3

The proof entails the following steps.

1 Plug Eqs. (9)–(14) in Eq. (21) and use Eq. (22) to rewrite the aggregate tax revenue as follows:

$$T_t=N_t\int_0^\infty\tau_k(z)\left(r_t+\phi(z)\cdot\mathbb{I}\left\{z\geq z_t^*\right\}R_t\left(\frac{z-z_t^*}{z_t^*}\right)\right)\theta_t(z)\cdot dz+$$
$$+w_t\tau_l+\int_0^\infty\left(\lambda(z)-\frac{\tau_k(z)}{\eta(z)}\right)\left(1-\left(\frac{\lambda(z)\eta(z)}{\tau_k(z)}\right)^{\frac{1}{\gamma}}\right)\theta_t(z)\cdot dz. \tag{A.22}$$

2 Plug Eq. (A.22) into Eq. (5) and impose the no-arbitrage condition $r_t=R_t-\delta$ to obtain dB_t/dt as in the main text.

Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.econmod.2026.107480.

Data availability

We have shared the link to our replication package in the attach files section

Replication Package for the article "Tax evasion and the productivity distribution" on Economic Modeling (Original data) (Mendeley Data)

References

Ağca, c., Celasun, O., 2012. Sovereign debt and corporate borrowing costs in emerging markets. J. Int. Econ. 88 (1), 198–208.
Allingham, M., Sandmo, A., 1972. Income tax evasion: A theoretical analysis. J. Public Econ. 1, 323–338.
Amin, M., Okou, C., 2020. Casting a shadow: Productivity of formal firms and informality. Rev. Dev. Econ. 24 (4), 1610–1630.
Árpád, Gottardi, P., Hubmer, J., Mayr, L., 2023. Tax wedges, financial frictions and misallocation. J. Public Econ. 227 (105000).
Bassetto, M., Cagetti, M., De Nardi, M., 2015. Credit crunches and credit allocation in a model of entrepreneurship. Rev. Econ. Dyn. 18 (1), 53–76.
Beck, T., Lin, C., Ma, Y., 2014. Why do firms evade taxes? The role of information sharing and financial sector outreach. J. Financ. 69 (2), 763–817.
Benhabib, J., Bisin, A., Zhu, S., 2016. The distribution of wealth in the Blanchard–Yaari model. Macroecon. Dyn. 20 (2), 466–481.
Bernasconi, M., Levaggi, R., Menoncin, F., 2020. Dynamic tax evasion with habit formation in consumption. Scand. J. Econ. 122 (3), 966–992.
Besley, T., Persson, T., 2009. The origins of state capacity: Property rights, taxation, and politics. Am. Econ. Rev. 99 (4), 1218–1244.
Blackburn, K., Bose, N., Capasso, S., 2012. Tax evasion, the underground economy and financial development. J. Econ. Behav. Organ. 83 (2), 243–253.
Buera, F., Moll, B., 2015. Aggregate implications of a credit crunch: The importance of heterogeneity. Am. Econ. J.: Macroecon. 7 (3), 1–42.
Cagetti, M., De Nardi, M., 2006. Entrepreneurship, frictions, and wealth. J. Political Econ. 114 (5), 835–870.
Capasso, S., Jappelli, T., 2013. Financial development and the underground economy. J. Dev. Econ. 101, 167–178.
Chen, B.-L., 2003. Tax evasion in a model of endogenous growth. Rev. Econ. Dyn. 6 (2), 381–403.
Collard-Wexler, A., Asker, J., De Loecker, J., 2011. Productivity volatility and the misallocation of resources in developing economies. NBER working paper No. w17175.
CompNet, 2023. User guide for the 9th vintage CompNet dataset. Accessible via: https://www.comp-net.org/data/7th-vintage/.
Cooley, T., Marimon, R., Quadrini, V., 2004. Aggregate consequences of limited contract enforceability. J. Political Econ. 112 (4), 817–847.
Dabla-Norris, E., Gradstein, M., Inchauste, G., 2008. What causes firms to hide output? The determinants of informality. J. Dev. Econ. 85 (1–2), 1–27.
Dabla-Norris, E., Gradstein, M., Miryugin, F., Misch, F., 2019. Productivity and tax evasion. IMF Working Paper.
Dávila, E., Hébert, B., 2023. Optimal corporate taxation under financial frictions. Rev. Econ. Stud. 90 (4), 1893–1933.
Di Caro, P., Sacchi, A., 2020. The heterogeneous effects of labor informality on VAT revenues: Evidence on a developed country. J. Macroecon. 63, 103190.
Di Nola, A., Kocharkov, G., Scholl, A., Tkhir, A.-M., 2021. The aggregate consequences of tax evasion. Rev. Econ. Dyn. 40, 198–227.
Dixit, A., 2013. The Art of Smooth Pasting. Routledge.
Duffie, D., Epstein, L., 1992. Stochastic differential utility. Econometrica 353–394.
Erosa, A., Fuster, L., Martinez, T., 2023. Public financing with financial frictions and underground economy. J. Monet. Econ. 135, 20–36.
Fajnzylber, P., Maloney, W., Montes-Rojas, G., 2011. Does formality improve micro-firm performance? Evidence from the Brazilian SIMPLES program. J. Dev. Econ. 94 (2), 262–276.
Fernandez-Bastidas, R., 2023. Entrepreneurship and tax evasion. Econ. Model. 128, 106488.
Franjo, L., Pouokam, N., Turino, F., 2022. Financial frictions and firm informality: a general equilibrium perspective. Econ. J. 132 (645), 1790–1823.
Gersbach, H., Rochet, J.-C., von Thadden, E.-L., 2023. Public debt and the balance sheet of the private sector.
Gertler, M., Kiyotaki, N., 2010. Financial intermediation and credit policy in business cycle analysis. Handb. Monet. Econ. 3, 547–599.
Gilchrist, S., Sim, J., Zakrajsek, E., 2014. Uncertainty, financial frictions, and investment dynamics. NBER Working paper w20038.
Gillman, M., Kejak, M., 2014. Tax evasion, human capital, and productivity-induced tax rate reduction. J. Hum. Cap. 8 (1), 42–79.
González, B., Nuño, G., Thaler, D., Albrizio, S., 2022. Firm heterogeneity, capital misallocation and optimal monetary policy. Working paper.
Gordon, R., Li, W., 2009. Tax structures in developing countries: Many puzzles and a possible explanation. J. Public Econ. 93 (7–8), 855–866.
Gradstein, M.M., Misch, F., Miryugin, F., Dabla-Norris, M.E., et al., 2019. Productivity and Tax Evasion. Technical report, International Monetary Fund.
Hurst, E., Li, G., Pugsley, B., 2014. Are household surveys like tax forms? Evidence from income underreporting of the self-employed. Rev. Econ. Stat. 96 (1), 19–33.
IRS, 2019. Federal income tax compliance research: Tax gap estimates for tax years 2011–2013. Int. Revenue Serv..
Kafkalas, S., Kalaitzidakis, P., Tzouvelekas, V., 2014. Tax evasion and public expenditures on tax revenue services in an endogenous growth model. Eur. Econ. Rev. 70, 438–453.
Kanbur, R., 2017. Informality: Causes, consequences and policy responses. Rev. Dev. Econ. 21 (4), 939–961.
Krusell, P., Smith, Jr., A.A., 1998. Income and wealth heterogeneity in the macroeconomy. J. Political Econ. 106 (5), 867–896.
La Porta, R., Shleifer, A., 2014. Informality and development. J. Econ. Perspect. 28 (3), 109–126.
Levaggi, R., Menoncin, F., 2013. Optimal dynamic tax evasion. J. Econom. Dynam. Control 37 (11), 2157–2167.
Levaggi, R., Menoncin, F., 2016. Optimal dynamic tax evasion: a portfolio approach. J. Econ. Behav. Organ. 124, 115–129.
Loayza, N., Rigolini, J., 2006. Informality trends and cycles. World Bank Working paper 4078.
López, J.J., 2017. A quantitative theory of tax evasion. J. Macroecon. 53, 107–126.
Macnamara, P., 2019. Taxes and financial frictions: Implications for corporate capital structure. J. Econom. Dynam. Control 101, 82–100.
Moll, B., 2014. Productivity losses from financial frictions: Can self-financing undo capital misallocation? Am. Econ. Rev. 104 (10), 3186–3221.
Ordonez, J.C.L., 2014. Tax collection, the informal sector, and productivity. Rev. Econ. Dyn. 17 (2), 262–286.
Pappa, E., Sajedi, R., Vella, E., 2015. Fiscal consolidation with tax evasion and corruption. J. Int. Econ. 96, S56–S75.
Pham, H., 2009. Continuous-Time Stochastic Control and Optimization with Financial Applications, vol. 61, Springer.
Radner, R., 1972. Existence of equilibrium of plans, prices, and price expectations in a sequence of markets. Econometrica 289–303.
Schneider, F., Buehn, A., Montenegro, C., 2010. New estimates for the shadow economies all over the world. Int. Econ. J. 24 (4), 443–461.
Slemrod, J., 2019. Tax compliance and enforcement. J. Econ. Lit. 57 (4), 904–954.
Stokey, N., 2008. The Economics of Inaction. Princeton University Press.
Yitzhaklt, S., 1974. A note on income tax evasion: A theoretical analysis. J. Public Econ. 3, 201–202.