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MODELS AND METHODS IN ELECTRICITY MARKET: STOCHASTIC PROGRAMMING

Candidato: AREGA GETANEH ABATE

NOME DEL RELATORE:

Prof. ROSSANA RICCARDI

NOME DEL CORRELATORE:

Prof. CARLOS RUIZ

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Candidate: AREGA GETANEH ABATE

Name of Advisor:

Prof. ROSSANA RICCARDI

Name of Co-Advisor:

Prof. CARLOS RUIZ

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Abstract

Una progettazione del mercato elettrico utilizzando metodi e modelli efficienti è essenziale per la costruzione di un sistema affidabile sia per i partecipanti che per la sua funzionalità. In questa tesi, si sviluppano metodi e modelli per il design di mercati elettrici con elevata penetrazione delle fonti rinnovabili, la presenza di un mercato delle emissioni, per lo studio di mercati demand-response in condizioni di incertezza. Tali modelli vengono analizzati sia con un approccio neutrale al rischio che in situazione di avversione al rischio ed è anche esplorato l'utilizzo di contratti derivati in condizioni di incertezza. L'obiettivo della tesi, pertanto, è quello di sviluppare modelli e metodi di supporto a generatori, pianificatori, retailers e consumatori finali per identificare le decisioni ottime attraverso l'utilizzo di modelli matematici e strumenti di mercato. Come primo passo, si propone un modello di equilibrio che studia le interazioni tra generatori oligopolistici che operano in due mercati: il mercato spot ed il mercato future. Si introducono vari contratti derivati nel mercato future e se ne analizza l'impatto complessivo in termini di profitti e quantità scambiate nei due mercati, in presenza di una penetrazione delle fonti rinnovabili sempre maggiore. Un secondo lavoro introduce il mercato delle emissioni, European Union Emission Trading Scheme (EU ETS), in cui vengono attribuiti i permessi di emissione con un meccanismo d'asta allo scopo di ridurre le emissioni di CO_2. Il modello di equilibrio definito è un modello stocastico a due stadi utilizzato per effettuare un'analisi

parametrica volta a determinare se l'allocazione con il meccanismo d'asta porti alla riduzione dei windfall profits ottenuti con il modello di allocazione basato sullo storico. I modelli introdotti nel secondo lavoro vengono analizzati sia in condizioni di neutralità al rischio sia in condizione di avversione al rischio. Infine, in un terzo lavoro viene introdotto un modello demand-response dove gli operatori possono gestire l'incertezza considerando il lato della domanda. Viene, pertanto, sviluppato un modello retailer-consumer in cui entrambi gli operatori massimizzano le proprie funzioni obiettivo soggette a vincoli di capacità sia in condizioni di market power sia in competizione perfetta. Il modello viene risolto determinando le condizioni di ottimalità del problema del consumatore (Karush-Kuhn-Tucker conditions) ed inserando tali condizioni come vincolo nel problema del distributore. Il risultato è un problema di programmazione matematica con vincoli di equilibrio noto come "mathematical program with equilibrium constraints" (MPEC). Il problema, nel caso di competizione perfetta, viene poi risolto dopo opportune trasformazioni, come un problema di ottimizzazione non lineare misto intero (MINOP). Per tutti i modelli considerati le soluzioni, gli approcci pratici per le soluzioni e le relative tecniche risolutive sono basate su dati reali, opportunamente calibrati. I risultati ottenuti, quindi, propongono strumenti effettivi per l'operatività del mercato elettrico e analizzano gli impatti di tali strumenti. I contratti finanziari hanno un forte impatto sui risultati del mercato elettrico, poiché l'avversione al rischio, utilizzando questi contatti, aumenta i profitti dei generatori, il che è un risultato controintuitivo nei nostri modelli. Il modello di demand-response mostra che i consumatori finali aumentano la loro flessibilità, massimizzando così la loro utilità e minimizzando il costo dell'elettricità. Pertanto, tale modello è in grado di ridurre le fluttuazioni dei prezzi pur mantenendo l'affidabilità del sistema.

Abstract

Designing the electricity market with efficient models and methods is essential for market participants and for the operation of the electricity system reliably. In this dissertation, we develop electricity market models and methods with high renewable penetration, emission trading, demand response, and address risk aversion with financial contracts under uncertainty. The goal of the thesis work is to develop models and methods to help electricity generators, system planners, retailers, and end-consumers to identify optimal decisions using mathematical models and market instruments. In the first chapter, we propose a game-theoretical equilibrium model that characterizes the interactions between oligopolistic generators in a two-stage electricity market with high penetration of renewable resources. The spot market equilibrium outcomes are derived analytically, and this allows the posterior numerical solution of the overall equilibrium problem. We introduce different types of contracts in the futures market to evaluate their performance and impact on the equilibrium market outcomes and how the outcomes depend on the levels of renewable penetration in the system. In the second chapter, the European Union Emission Trading Scheme (EU ETS) is introduced with a market-based auctioning with financial contracts in the equilibrium electricity market model aiming greenhouse gas (GHG) emission reduction. The auction-based allocation of emissions with allowance trading is examined and explored whether it brings economic efficiency by negating windfall profits that have been resulted from grandfathered allocation of allowances. Moreover, a coherent risk measurement is applied to model both risk averse and risk neutral generators and a stochastic optimization setting is introduced to deal with the uncertainty. Finally, we apply a demand response program in the electricity market where market entities can manage uncertainty considering the demand-side. We develop a retailer-consumer electricity market model so that both players optimize their respective objective functions with respect to certain constraints both with market power and with perfect competition. The retailer's problem with market power is solved using the consumer's Karush-Kuhn-Tucker (KKT) optimally conditions entered as constraints in the retailer's problem so that a mathematical program with equilibrium constraints (MPEC) problem that can be solved as nonlinear MPEC is formulated. The perfect competition equilibrium model is then transformed into an equivalent mixed integer nonlinear problem. The solution sets, the practical approaches for solutions, the required techniques to test and to compare the performance of all models are undertaken with calibrated and realistic data. Our results show contracts to integrate renewable energy resources in the system and market-based emissions trading are effective tools for the operation of electricity system. Financial contracts have a strong impact on electricity market outcomes, as risk aversion using these contacts increases generators' profit, which is a counter-intuitive result in our models. Our demand response model shows that end-consumers increase their flexibility thereby maximizing welfare from their utility and cost of electricity consumption. Thus, the demand response model effectively manages price fluctuations and keeps system reliability.

This is dedicated to my father Getaneh Abate

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I had been in Madrid at the University of Carlos III of Madrid (UC3M) for 18 months, and Prof. Carlos has tutored, guided, and showed me what it takes to be an independent researcher. He had eased some of my frustrations while working on simulation. Besides, he had been hospitable and helpful in difficult times.

My parents are the strongest of all from the area where I was born and grown. They had gone through, unimaginably, difficult times to educate me. I cannot overemphasize the pains my father had endured to help me study and had been proud of me. This thesis is dedicated to him, and my gratitude is immense for my mother as well.

Finally, this dissertation wouldn't have been possible without the determination of my wife Alem Tsehay. Indeed, my son Emmahus and my daughter Ekklesia are always my inspirations.

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Chapter 1: Introduction

The goal of the research work reported in this dissertation is to provide models and methods for agents participating in the electricity market so that they can identify their optimal decisions under uncertainty. Due to the impending future energy shortage and envisioned consequences of climate change, integration of renewable energy sources (RES) in the electricity system has come to occupy a prominent place in most industrialized nations agenda during the recent years. We use RES penetration to solve the trade-off between the rising energy demand and the objective to reduce greenhouse gas emission with a market-based allowance trading. Within the framework of an electricity market affected by uncertainty, the demand response program is considered to tackle price fluctuations with carefully designed game-theoretical models. Therefore, the optimal decision for electricity generators with contract designs under high RES penetration, economically efficient and cost-effective models to combat CO_2 emissions, securing system reliability and managing price fluctuation are the topics addressed in this thesis.

This chapter introduces the thesis work. Section 1.1 provides some background about models and methods in the electricity market. In Section 1.2, the challenges to modeling the electricity market under uncertainty that motivate the work reported in this document are described. Section 1.3 presents the objectives pursued and the approaches used in the thesis. Finally, the structure of the document is provided in Section 1.4.

1.1 Some Background

Electricity is a fundamental resource in modern economies. The deregulation and restructuring of the electricity sector have led researchers to develop models and methods that support the whole chain of processes from the generation to the delivery of power to the end-consumers. For that reason, the electricity market must be designed to provide efficient theoretical models and practical approaches for generators and other market participants to manage their respective risks associated with the chains of activities. Moreover, the transition to a green economy using renewable deployment which is favored by energy planners poses new problems to the operation and management of the electricity system, as supply must match demand at all times (Street et al., 2009; Zugno et al., 2013; Zou et al., 2017). Given its intermittent and stochastic nature, integrating RES into the electricity system with risk management models has been dealt with, by researchers, using financial derivatives and emission allowance trading schemes (Allaz, 1992; Allaz and Vila, 1993; Mendelson and Tunca, 2007; Morales et al., 2013). Uncertainty increases the need for backup capacity to cope with unpredictable fluctuations of power generation. This increases the demand for the use of forward/futures contracts (due to its liquidity) in the electricity market. Since the level of power production is not known beforehand with certainty due to production cost, or weather dependent variables, measures need to be taken to ensure the reliability of the electricity system. One of the methods to manage short-run price fluctuations is using the flexibility of consumers' with demand response programs (Albadi and El-Saadany, 2008; Lau et al., 2015; Hu et al., 2016).

Given this framework, the thesis considers contract designs in the electricity market with the integration of conventional and RES generators under uncertainty. As an extension of this modeling framework, the EU ETS is introduced with market-based allowance auctioning aiming to greenhouse gas (GHG) reduction and promoting green economic path. Apart from financial derivatives, other price-based risk management models using consumers' flexibility under a demand response programs are considered as well.

1.2 Motivation, Challenges and Work Carried Out

The path to a sustainable electricity system with high penetration of RES requires the use of electricity modeling tools that account for the risk associated with generation and electricity prices. Besides, public policy regulations and international agreements urge electricity generators to reduce CO_2 emission, which in turn, invites the financial market to play a key role with allowance trading. The integration of RES generation capacity requires modeling techniques to manage uncertainties (Allaz, 1992; Botterud et al., 2011; Möller et al., 2011; Bruno et al., 2016). From the demand-side, demand response has been an effective tool and resource for system reliability, and price hedging which requires models and methods to help market agents (Deng et al., 2015; Afşar et al., 2016; Soares et al., 2019; Aussel et al., 2020; Luo et al., 2020). In this regard, the growth in the use of big-data and model-based solutions for the electricity market are some of the factors that motivated this thesis. The research topics addressed in this dissertation and the challenges they pose are describe below:

1. Uncertainty Modeling of key market variables

2. Contracts and RES integration in the electricity market modeling

3. EUA trading and electricity market modeling

4. Demand Response and market competitiveness

The integration of weather-dependent RES and conventional generation either with financial derivatives or through demand response programs is performed under great levels of uncertainty, both in the short and in the long-run. The generation mix is changing due to the increasing integration of RES generators. Since characterizing the relationship between futures market and spot market contracts is crucial, mainly for short-term decision-making problems, agents need to make their optimal decisions considering cost, demand, and generation capacity with uncertainty using better models and methods. For instance, electricity market participants need models to accurately predict the day-ahead electricity market outcomes so that consumers' flexibility can be exploited to maximize social welfare. However, an appropriate uncertainty representation is critical to optimally implement financial contracts, to achieve CO_2 emission reduction, and to secure system reliability. The risk presented by fluctuating market prices has been dealt with financial contracts where the uncertainties are modeled using stochastic programming, or robust optimization approaches. Moreover, in this thesis we use stochastic programming because: i) it provides a powerful framework to model and include parameters' uncertainty in an optimization problem, via a plausible set of scenarios; and ii) the CVaR can be easily incorporated to the model with a linear formulation (Rockafellar and Uryasev, 2000) for the first two models and to deal with different market configuration in the last model.

Generators usually use contracts for buying and selling electricity where they can hedge the volatile prices. These contracts affect how generators with different risk profiles operate in the market. These challenges are addressed in this dissertation. For the first two models, we apply a game-theoretical framework with a general formulation of stochastic optimization using scenario aggregation as proposed by Rockafellar and Uryasev (2000) to deal with some of the challenges. Though spot market equilibrium is solved with a closed-form solution, due to the quadratic production cost and the bilinear products in the profit formulation, the global solution is solved numerically. The global problem is solved numerically by concatenating the KKT optimality conditions and an NLP reformulation which is proposed by Leyffer and Munson (2010). Similarly, the introduction of emission allowance contracts in the electricity market is considered with a stochastic optimization approach. Given the availability of MILP and NLP optimization solvers, demand response program, which we formulated as a nonlinear MPEC is solved with a market power and a perfect competitive setting (equilibrium) with MILP and NLP reformulations.

1.3 Objectives, Models and Methods

In this thesis, we propose models and methods to assist electricity market participants to make their respective optimal decisions under generation cost, demand, and renewable capacity uncertainties. The main objectives pursued are three:

1. to develop electricity market models and methods with financial derivatives taking into account conventional and RES generators, focusing on RES integration into the electricity system under uncertainty.

2. to develop an electricity market model under the EU ETS framework to address the goal of reducing GHG emission and pursuing a long-run carbon-free economic path while integrating RES and conventional generators.

3. to develop a demand response model that optimizes electricity retailer's and end-consumers' respective objectives with market power and with a perfect competition (equilibrium model).

The three main objectives are addressed by developing three models as presented in the next three chapters. We apply an NLP reformulation adopted from Leyffer and Munson (2010), and an MPEC, that is, solved by employing a standard commercial NLP solver. We use Knitro to solve all the three models but the MILP. The solver implements interior-point and active-set methods for solving continuous, nonlinear optimization problems). For the NLP reformulation, we minimize the sum of the non-negative complementarity products subject to the remaining KKT conditions. Thus, the objective function value equal to zero guarantees that all the original KKTs are met (each complementarity product needs to be zero, and the remaing KKTs are met as constraints), and hence it is a solution to the original system and the solution obtained is a global optimum.

A summary of the details of these models and methods are provided in the remainder of this section.

1.3.1 Contract Design in Electricity Market with High penetration of RES

We propose a game-theoretical equilibrium model that characterizes the interactions between oligopolistic generators in a two-stage electricity market under the presence of high RES penetration. Given conventional generators with generation cost uncertainty and RES generators with intermittent and stochastic nature, we consider a single futures contract market that is cleared prior to a spot market, where the energy delivery takes place. We introduce physical and financial contracts to evaluate their performance and assess their impact on the electricity market outcomes and examine how these depend on the level of RES penetration. Since market participants are usually risk averse, a coherent risk measure is introduced to deal with both risk neutral and risk averse generators. We derive analytical relationships between contracts, study the implications of uncertainties, test the performance of the proposed equilibrium model and its main properties through numerical examples. Our results show that overall electricity prices, generation costs, profits, and quantities, for conventional generators, decrease whereas quantities and profits for RES generators increase with RES penetration. Hence, both physical and financial contracts efficiently mitigate the impact of uncertainties and help the integration of RES into the electricity system. Moreover, risk aversion increases profit and decreases generators' risk under different levels of competition, as it shields them from the risk associated with cost, demand, and RES uncertainties.

1.3.2 Contract Design in Electricity Market under EU ETS

In this chapter, we analyze and simulate the interaction of oligopolistic generators with a game-theoretical framework where the electricity and the emissions markets interact in a two-stage electricity market. For analytical simplicity, we assume a single futures market where the electricity is committed at the futures price, and the emissions allowance is contracted in advance, prior to a spot market where the energy and the allowances delivery takes place. Moreover, a coherent risk measure is applied (CVaR) to model both risk averse and risk neutral generators, and a two-stage stochastic optimization setting is introduced to deal with the uncertainty of renewable capacity, demand, generation, and emission costs. The performance of the proposed equilibrium model and its main properties are examined through realistic numerical simulations. Our results show that RES generators are increasing and substituting conventional generators without compromising social welfare. Hence, both RES deployment and emission allowance auctioning are effectively reducing GHG emissions and promoting low-carbon economic path.

1.3.3 Retailer-Consumer Electricity Market Model under Demand Response

Demand response (DR) programs have gained much attention during the last three decades to optimize the decisions of the electricity market participants considering demand-side management (DSM). It can potentially enhance system reliability and manage price volatility by modifying the amount, or time of electricity consumption. We proposes a novel game-theoretical model accounting for the relationship between retailers (leaders) and consumers (followers) in a dynamic price environment under uncertainty. The model is solved under two frameworks: by considering the retailer's market power and by accounting for an equilibrium setting based on a perfect competitive game. These are formulated in terms of a mathematical program with equilibrium constraints (MPEC) and with a mixed-integer linear program (MILP), respectively. In particular, the retailers' market power model is first formulated as a bi-level optimization problem, and the MPEC is subsequently derived by replacing the consumers' problem (lower level) with its Karush-Kuhn-Tucker (KKT) optimality conditions. In contrast, the equilibrium model is solved as a MILP by concatenating the retailer's and consumers' KKT optimality conditions. The solution sets, the practical approaches for solutions, the required techniques to test and compare the performance of the model are undertaken with realistic data. Numerical simulations show the applicability and effectiveness of the proposed model to explore the interactions of market power and DR programs. The results confirm that consumers are better off in an equilibrium framework while the retailer increases its expected profit when exercising its market power. However, we show how these results are highly affected by the levels of consumers' flexibility.

1.4 Thesis Structure

The remainder of the thesis is organized as follows. Chapter 2 describes contract designs in the electricity market with high RES penetration. A game-theoretical model based on conventional and RES generators production and delivery are provided with futures and spot market contracts. A closed-form solution for the spot market and a CVaR nonlinear reformulation of the global optimization solution are derived with different risk profiles and different contracts under uncertainty. Then, a case study based on realistic electricity market data is discussed and analyzed to demonstrate the effectiveness of the models and competitive strategies for both generators with different market configurations. Chapter 3 provides the details of financial contracts in the electricity market under EU ETS with a stochastic programming framework. The solution technique adopted to solve the model, the allowance market with risk profiles, the sensitivity analyses for RES penetration, and CO_2 increase, and policy implication of the essential parameters are described in detail with illustrated examples and a realistic case study. Chapter 4 presents a demand response program that coordinates a retailer that serves as an intermediary between the wholesale electricity market and end-consumers. The detailed formulation and solution technique are first provided with a MPEC, which characterizes retailer market power, and with a MILP and/or NLP, which characterizes a perfect competition market setting. Then, the two market configurations are analyzed by using electricity market data and by modeling consumer flexibility via utility functions. Finally, Chapter 5 closes by summarizing the thesis, providing concluding remarks, and suggesting future research works.

The results provided in chapters 2, 3 and 4 are collected in three different papers:

1. A. G. Abate, R. Riccardi, C. Ruiz, *Contract designs in electricity markets with high Renewable penetration: A two stages approach*, *submitted to OMEGA.*

2. A. G. Abate, R. Riccardi, C. Ruiz, *Contracts in electricity markets under EU ETS: A stochastic programming approach*, *Energy Economics* (2021) 105309.

3. A. G. Abate, R. Riccardi, C. Ruiz, *Retailer-consumers model in electricity market under demand response*, *submitted to European Journal of Operational Research.*

Chapter 2: Contract Design in Electricity Market with High penetration of Renewables: A Two Stage Approach

2.1 Introduction

The increasing concern about climate change, limited electricity sources, reliability of electricity supply, and security of electricity service while integrating renewable energy sources (RES) into the electricity system have been extensively important topics in energy economics. Such models have emerged after the liberalization of the electricity sector in several countries since the early 1990s. Liberalization has not only brought the active role of demand and supply in the electricity market but also has increased inter-regional trades so that greater competition in electricity generation has spurred efficient production, distribution, and transmission of electricity (Eydeland and Wolyniec, 2003; Agency, 2005; Wang et al., 2018).

Moreover, the transition from fossil-based to RES and sustainable energy is favored by international agreements and national plans (Jurasz and Ciapała, 2017). However, the transition and integration of RES in the electricity system not only address climate change targets but also complicate the electricity system management to maintain supply-demand balance through market mechanisms. The inherent risk at all levels of activities (production, transmission, and distribution) cannot be dealt with standard risk management techniques given the characteristics of electricity and the intermittent and stochastic nature of RES (Bruno et al., 2016; Falbo and Ruiz, 2019). Empirical literature in the electricity market has provided tools and models to deal with the distinctive electricity features and risk management through derivatives market (Bushnell, 2007; Carrion et al., 2007; Anderson and Hu, 2008; Baringo and Conejo, 2013; Philpott et al., 2016). For instance, Philpott et al. (2016) show how a risk averse social plan is equivalent to competitive equilibrium when agents use dynamic coherent risk measures and there are enough market instruments to hedge inflow uncertainty.

For electricity generators, the derivatives markets work by allowing them to commit part of their production through forward or/and bilateral contracts to reduce the impact of major uncertainties on electricity outcomes. Generators, retailers, and consumers can agree on prices, quantities, delivery times, and other conditions within the framework of the contracts. There are a vast number of different contracts, from plain forwards to interruptible contracts which can improve competition in the spot market by applying sophisticated risk management modeling (Unger, 2002; Willems, 2006; Bushnell, 2007).

In particular, futures contracts become more liquid and relevant in electricity trading to hedge price risks given the non-storability of electrical energy and its inelastic demand. Futures markets allow trading products spanning a large time horizon, such as one month or a quarter. However, the electricity market depends on key variables such as fuel prices, electricity demand, and weather, introducing financial contracts in generators' profit function with different generation technologies (conventional and RES) under uncertainty is not an easy task.

From a methodological perspective, different mathematical models have been proposed for determining optimal strategies in the electricity market with RES penetration using financial contracts. Some of these consider deterministic models, and others deal with uncertainty by using stochastic programming or robust optimization approaches either considering market power and/or with Nash competitive games (Allaz, 1992; Allaz and Vila, 1993; Botterud et al., 2011; Möller et al., 2011; Bruno et al., 2016). After the pioneering work of Allaz (1992) that deals with oligopolistic generation traded on a forward market for risk hedging, extensions with uncertainty and with different modeling approaches have been introduced (Allaz and Vila, 1993; Willems, 2006; Bushnell, 2007; Jurasz and Ciapała, 2017).

However, there are still gaps in the literature regarding the design of appropriate financial products that help integrating large amounts of RES in oligopolistic power markets. To this end, in this chapter, we propose a two-stage game-theoretical model, where each oligopolistic generator optimizes its profit by integrating the futures (physical and financial) and spot markets while conjecturing the impact its production may have on other generators' quantities and market prices. Uncertainties are managed with contracts and a coherent risk measure is introduced in the generators' decision processes. The proposed equilibrium model characterizes the interactions between generators with different levels of market power (à la Cournot, a perfect competition) in a two-stage electricity market. We consider a single futures market that is cleared before a spot market, where the energy delivery takes place, by introducing both physical and financial contracts to evaluate their impact and performance in the equilibrium market outcomes with high RES penetration.

First, we develop a general model (GM) with a futures market that requires physical delivery when the spot market takes place. Then, we consider a market, where the futures market makes use of contracts for differences (CFD), a financial product that serves as a market coordination mechanism. We also analyze the spot market without the presence of futures trading for comparison. To deal with both risk averse and risk neutral generators, we introduce a coherent risk measure (Conditional Value at Risk (CVaR)) which has been extensively used in the literature after its introduction by Rockafellar and Uryasev (2000) for portfolio optimization problems[1]. We derive analytical relationships between contracts and study the implications of demand, production cost, and RES capacity uncertainties on electricity market outcomes. Finally, the performance of the proposed equilibrium model and its main properties are tested with extensive numerical simulations. Our results show that overall electricity prices, generation costs, profits, and quantities (for conventional generators) decrease whereas quantities and profits (for RES generators) increase with the level of RES penetration. Physical and financial contracts exhibit different impacts on the resulting equilibrium market outcomes. However, they both efficiently mitigate the impact of uncertainties and facilitate the integration of RES into the electricity system. Moreover, risk aversion increases profit and decreases generators' risk in the general model for both the Cournot and perfect competition behaviors, which is a counter-intuitive

[1] Although VaR is a well-known risk measure in economic problems, it is a non-coherent risk measure suffering from non-convexity, non-smoothness, subadditivity, etc., which makes it undesirable in optimization programs. To avoid this problem, there is an attractive alternative risk measure identified as CVaR also known as average value at risk or mean shortfall. CVaR is a risk assessment technique often used to reduce the probability a portfolio will incur large losses. This is performed by assessing the likelihood (at a specific confidence level α) that a specific loss will exceed the value at risk. That is, CVaR is derived by taking a weighted average between the VaR and losses exceeding the VaR.

result. That is because contracts increase market participants' confidence and shield them from the risk associated with cost, demand, and RES uncertainties.

The chapter is organized as follows. Section 2.2 reviews the current state-of-the-art in the electricity market equilibrium models with renewables and underlines our contribution. Section 2.3 formulates the model for the two-stage problem, showing how it can be reformulated from stage-two to stage-one in a stochastic programming framework. Section 2.4 discusses how the parameters are generated, presents the simulation results, and comments on the numerical results. Finally, Section 2.5 concludes with some relevant remarks.

2.2 Literature Review and Contributions

2.2.1 Literature Review

In this section, we review the literature on futures and spot market contracts in the electricity market with high RES penetration, focusing on risk averse and risk neutral stochastic models. In the post-liberalized electricity market, the economics and regulation of electrical energy system that includes an integral amount of stochastic renewable generation have been studied via mathematical models using financial derivatives under uncertainty (Collins, 2002; Hasan et al., 2008; Kettunen et al., 2009; Martín et al., 2014; Dueñas et al., 2014; Morales et al., 2014). These financial instruments are traded in the over the counter (OTC) and/or exchange markets as energy commodity and guarantee the delivery of an established amount of electricity over a specific future period, settled with physical, or financial delivery. The literature on electricity market equilibrium modeling with financial derivatives has rapidly developed because of the growing need for obtaining models that describe electricity market behavior accurately due to the need to integrate RES in the electricity system and the challenge posed by its intermittent nature. In the following, we review standard electricity literature, starting from contracts.

Futures and Spot Markets in Electricity

The futures market has become relevant for trading electricity as it helps to hedge the volatility of spot market prices based on prior agreements. The futures contracts have the spot price as an underlying reference in both physically and financially settled contracts, where generators can reduce their risk exposure for later delivery (Allaz, 1992; Allaz and Vila, 1993; Kettunen et al., 2009; OMIE, 2020; EEX, 2020). Futures contracts assure a fixed price of electricity in the future while spot market contracts are subject to uncertainties.

Allaz (1992), with a simple two-period classical model of an oligopoly producing a homogeneous good, shows forward contracting can be an effective tool in the hands of noncompetitive producers. Allaz and Vila (1993) develop a model with two Cournot duopolies and forward market contracts. They show that at equilibrium, each Cournot player will sell forward, which may worsen their position, if they do not fully understand when and how to use these contracts prior to their production. Powell (1993) develops a model of contracting in the electricity industry, where the generators are price setters in the futures market and quantity setters in the spot market with risk aversion. He concludes that the futures price is higher than the expected spot market price.

Mendelson and Tunca (2007) analyze forward and spot trading under uncertainties among supply chain participants by utilizing new demand and cost information. Bushnell (2007) extends the Allaz and Vila model to multiple firms with increasing costs to demonstrate that suppliers' market power plays a key role in the interaction between forward and spot trading. de Maere d'Aertrycke and Smeers (2013), which endogenously account for payoffs and an incomplete risk market, analyze the interaction between financial instruments with spot electricity markets, with a generalized Nash equilibrium framework.

There are other examples in the literature, where financial derivatives have been used extensively in the electricity market (Le Coq and Orzen, 2002; Willems, 2006; Su, 2007; Anderson and Hu, 2008; Ruiz et al., 2012; de Maere d'Aertrycke and Smeers, 2013; Martín et al., 2014; Ralph and Smeers, 2015). These contracts have been used to achieve different objectives from different modeling perspectives: for instance, physical and financial contracts with different levels of competition in electricity market (Willems, 2006), social welfare and futures trading with retailers with market power (Anderson and Hu, 2008), electricity markets without price volatility (Su, 2007), market equilibrium model with asymmetric producers (Le Coq and Orzen, 2002), the equilibrium in futures and spot markets with oligopolistic generators and conjectural variations (Ruiz et al., 2012), effects of futures markets on the investment decisions of a strategic electricity producer (Martín et al., 2014), a risky design game (a stochastic Nash game) problem in a complete risk market using financial products (Ralph and Smeers, 2015), are some relevant contributions.

High RES Penetration

Because of the growing concern on conventional generation's impact on the environment and its high generation cost, the electricity generation mix has been significantly changing with strict policies towards sustainable energy adaptation. This adaptation and penetration of RES in the electricity system lead towards a green economy and lower electricity prices. Nevertheless, due to high uncertainty on RES capacity, integrating RES into the modern electricity system requires a risk averse dispatch of resources to account for the stochastic availability of renewable capacity and to hedge the price volatility in the system (Zhang and Giannakis, 2015; Zou et al., 2017).

In relation to this, governments have been creating better conditions (with specific contracts, green certificates, feed-in-tariffs, and other market-based approaches) for investment in RES and integrating it with conventional electricity system (Street et al., 2009; Boomsma et al., 2012; Verbruggen and Lauber, 2012).

Recently, the electricity market with high RES penetration has been studied for different purposes and approaches: with a two-stage energy management strategy under uncertainty (Wang et al., 2018), with multistage stochastic programming by combining real options and forward contracts for risk management in renewable investment (Bruno et al., 2016), risk-based energy management of renewable-based microgrid using information gap decision theory (Mehdizadeh et al., 2018), are some of the contributions. Since risk and uncertainty are the main challenges considering RES in the system, Nguyen and Le (2014a) design a CVaR model that enables the microgrid aggregator to exploit the flexibility of the load to mitigate the negative impacts of the uncertainties due to RES generation and electricity prices. Hence, CVaR is a well-known risk measure that has been widely used in various energy management problems for different entities in the electricity market such as for retailers (Carrion et al., 2007; García-Bertrand, 2013), for producers (Conejo et al., 2008; Morales et al., 2010; Baringo and Conejo, 2013), for distributors (Safdarian et al., 2013), for optimal power flow (Zhang and Giannakis, 2013) and for coordinated energy trading problems

(Al-Awami and El-Sharkawi, 2011; de la Nieta et al., 2013). However, knowing how uncertainty management is translated into contracts that enable market participants to act upon the received information and understanding the challenges entailed with RES integration, requires better models and methods particularly, in the electricity market. In the following, we explore some equilibrium models and our departure to fill the gap in the literature.

Stochastic Equilibrium Models in the Electricity Market

In the optimization research community, various equilibrium models in the electricity market and related problems under uncertainty have drawn increasing attention, particularly on short-term operations, capacity expansion, optimal dispatch, optimal power flow, RES integration, risk management, and decarbonization, which are very important topics to analyze the electricity market designs (Gürkan et al., 1999; Vehviläinen and Pyykkönen, 2005; Galiana and Khatib, 2010; Lin and Fukushima, 2010; Pozo and Contreras, 2011; Aïd et al., 2011; Zhang and Giannakis, 2013; Schröder, 2014; Dvorkin, 2019). Novel mathematical model formulations and numerical methods have been proposed to deal with those problems. For example, Gürkan et al. (1999) explore the application of sample-path to the investments in gas production by formulating a stochastic variational inequality problem and Vehviläinen and Pyykkönen (2005) present a stochastic factor-based approach to mid-term modeling of spot market prices in deregulated electricity markets. Galiana and Khatib (2010) propose a model on emission trading that can be viewed as a mathematical program (the allowance auction) subject to a Nash equilibrium problem, which in turn is subject to the Cournot-Nash equilibrium conditions of an hourly oligopolistic electricity market. Aïd et al. (2011) use an equilibrium model to study the relationship between forward, spot, and retail markets, where they conclude both vertical integration and forward hedging retail prices decrease under demand uncertainty. Finally, Dvorkin (2019) develop a chance-constrained stochastic market design that enables to produce a robust competitive equilibrium and internalize uncertainty of the RES in the price formation process.

However, modeling uncertainty in the electricity market by integrating conventional and RES oligopolistic generators, considering generators with risk hedging via physical and financial assets is still an important challenge.

2.2.2 Contributions

In this chapter, we analytically develop an electricity market equilibrium model with risk aversion that combines conventional and RES generators under uncertainty. The equilibrium model is solved based on a pre-existing futures market contract assuming each generator has an estimation (conjecture) of the impact that its decision may have on the other generators strategies. Analytical relationship between futures and spot contracts and implications of uncertainties on market outcomes are examined from risk behaviors, generation technologies, and levels of competition perspectives. The model is tested with multiple cases based on calibrated data, and the comparison of physical and financial contracts is made with high RES penetration. Though cost, demand and RES uncertainties are properly managed through contracts, RES penetration displaces conventional generators as it decreases the overall prices, given their production cost. Consumers' welfare is not affected as more generation is delivered by RES generators with less prices.

Thus, the major contributions of this chapter are: (1) to consider the simultaneous optimization of conventional and RES generators' profit both in the futures and spot markets, with risk aversion using the CVaR, given physical and financial contracts. (2) to propose a game-theoretical model, where each player solves a two-stage stochastic model, and to examine the impacts of demand, production cost, and RES capacity uncertainties on market outcomes under different levels of competition. (3) to derive analytically the spot market equilibrium outcomes, which allows the posterior numerical solution of the overall equilibrium problem, by applying the Leyffer and Munson (2010) nonlinear problem (NLP) reformulation. (4) to account for quadratic generation cost functions for conventional generators. (5) to relax and generalize the standard non-arbitrage condition (futures price equals expected spot price) by endogenously defining futures and spot markets demand curves. (6) to illustrate the analytical results using numerical examples that analyze the quantitative and qualitative impacts of RES penetration, level of competition, and players' risk aversion on market outcomes.

2.3 Problem Formulation

We consider the profit optimization of two types of electricity generators (conventional and RES, including solar, wind, or both) that trade their electricity in a two-stage electricity market. In the first stage, generators sell their electricity (to be settled physically or financially) in a futures market. In the second stage, they participate in a spot market, where they trade the remaining electricity. We derive two main models. The first model is the general model (GM) with physical delivery, and the second model is based on contracts for differences (CFD)-settled financially.

Finally, for comparison, we compute a third equilibrium model without the presence of the futures market (only spot market).

2.3.1 The General Model (GM) Formulation

We consider $|I|$ conventional and $|J|$ RES generators producing and trading their electricity in the futures and spot markets. We analyze a two-stage single futures market, where generators using both technologies can trade their futures quantities (q_i^F/q_j^F) at a futures price P^F and trade the remaining quantities in a subsequent single spot market at a spot price P_ω^S, which depends on scenario ω. Conventional generators have a quadratic generation cost function to generate q_i^F and $q_{i\omega}^S$ in the futures market and in the spot market, respectively. Electricity demand, conventional generators production costs, and availability of RES are the sources of uncertainty in the model. Under this market setting, generators have to make their first stage decision (amount of electricity to sell in the futures market, price, and delivery time) anticipating those uncertainties. These uncertainties are thus characterized by scenarios $\omega \in \Omega$ that represents possible realizations of the final consumers' demand, generation cost, and availability of RES. Given this market setting, conventional generators' profit is expressed as:

$$\Pi_{i\omega} = P^F q_i^F + P_\omega^S q_{i\omega}^S - [a_{i\omega} + b_{i\omega}(q_i^F + q_{i\omega}^S) + \frac{1}{2}c_{i\omega}(q_i^F + q_{i\omega}^S)^2] \quad \forall i, \forall \omega \qquad (2.3.1)$$

where the first term stands for revenue from futures market selling, the second term revenue from spot market selling, and the last term expresses the quadratic conventional generator i's cost function with $a_{i\omega} \geq 0$, $b_{i\omega} \geq 0$ and $c_{i\omega} \geq 0$ characterizing cost uncertainties. The total generation of conventional generator i, with scenario ω, is the sum of the electricity traded in the futures market and in the spot market.

Futures market decision variables q_i^F and P^F do not depend on the scenario ω, as the futures market is settled before the uncertain parameters are observed.

Similarly, the profit for RES generator with zero generation cost assumption is expressed as:

$$\Pi_{j\omega} = P^F q_j^F + P_\omega^S (Q_{j\omega} - q_j^F) = (P^F - P_\omega^S) q_j^F + P_\omega^S Q_{j\omega} \quad \forall j, \forall \omega \tag{2.3.2}$$

where the first term stands for revenue in the futures market, the second term revenue in the spot market, and with $Q_{j\omega}$, which is a stochastic parameter representing the total amount of RES generator j production with scenario ω which is traded in the futures and spot markets. RES generators are nondispatchable so that $Q_{j\omega} = q_j^F + q_{j\omega}^S$, and they trade $q_{j\omega}^S = Q_{j\omega} - q_j^F$ in the spot market. The inverse demand curve in the spot market is expressed as:

$$P_\omega^S = \gamma_\omega^S - \beta_\omega^S [\underbrace{\sum_{i \in I} (q_{i\omega}^S + q_i^F)}_{\text{conv.gen.}} + \underbrace{\sum_{j \in J} (q_{j\omega}^S + q_j^F)}_{\text{RES gen.}}] = \gamma_\omega^S - \beta_\omega^S [\sum_{i \in I} (q_{i\omega}^S + q_i^F) + \sum_{j \in J} Q_{j\omega}] \tag{2.3.3}$$

with $\gamma_\omega^S > 0$ and $\beta_\omega^S > 0$ for the price-demand function to be well-behaved. Since $Q_{j\omega}$ is a parameter, by rearranging, we can simplify (2.3.3) as:

$$P_\omega^S = \hat{\gamma}_\omega^S - \beta_\omega^S \sum_{i \in I} (q_{i\omega}^S + q_i^F) \quad \forall \omega \tag{2.3.4}$$

where $\hat{\gamma}_\omega^S = \gamma_\omega^S - \beta_\omega^S \sum_{j \in J} Q_{j\omega} \quad \forall \omega$.

Though it would be realistic to consider the futures market over time (monthly, quarterly) and the spot market over time (day-ahead, real-time), due to the nature of our model and analytical tractability, we assume a single futures market followed by a subsequent single spot market where the electricity generation and delivery takes place. Besides, instead of taking the usual assumption that the futures price is the expected spot price, we endogenously define it with an inverse demand curve as:

$$P^F = \gamma^F - \beta^F(\sum_{i \in I} q_i^F + \sum_{j \in J} q_j^F). \tag{2.3.5}$$

To characterize the futures market decision variables (stage-one variables), we compute the equilibrium in the spot market (stage-two), assuming the futures market has already settled. Hence, the resulting spot market prices and quantities, per scenario ω, are parametrized with the futures market decision. To deal with the futures market equilibrium, we move one step backward to stage-one and use the spot market equilibrium decision variables to characterize the global solution.

Spot Market Equilibrium (Stage-Two)

In this section, we analytically derive the equilibrium conditions in the spot market starting from the equilibrium price-demand curve of generators in Proposition 1 followed by the spot market equilibrium quantities in Proposition 2.

Assumption 1 *Each generator has an estimation of the impact that its generation $q_{k\omega}^S, q_k^F$ may have in the spot market price and rival quantities $P_\omega^S = P_\omega^S(q_{k\omega}^S, q_k^F)$ and $q_{-k\omega}^S = q_{-k\omega}^S(q_{k\omega}^S) \quad \forall k, \forall \omega$.*

This is an important assumption to derive both the equilibrium price and quantity in the spot market. The assumption is based on the idea of conjectural variations, an approach that is used widely to study oligopolistic markets. It is used to model an oligopolistic market wherein firms react to price by conjecturing how changes in their decisions may impact the production decisions of their competitors, which is a generalization of standard Nash–Cournot models to represent imperfect competition (Day et al., 2002; Aïd et al., 2011; Mousavian et al., 2020).

Proposition 1 *Given Assumption 1 and the futures market quantities* $q_k^F, \forall k \in I \cup J$, *at each scenario* ω, *the equilibrium spot market price is expressed as:*

$$P_\omega^S = \varphi_\omega \left[\hat{\gamma}_\omega^S - \beta_\omega^S \sum_{i \in I} q_i^F + \beta_\omega^S \sum_{i \in I} \tau_{i\omega}(b_{i\omega} + c_{i\omega} q_i^F) \right] \quad \forall \omega$$

where $\tau_{i\omega} = \frac{1}{\beta_\omega^S(1+\delta_i)+c_{i\omega}}$ *and* $\varphi_\omega = \frac{1}{1+\beta_\omega^S \sum_{i \in I} \tau_{i\omega}}$. *The parameter* $\delta_i = \sum_{i \neq k} \frac{\partial q_{k\omega}^S}{\partial q_{i\omega}^S}$ *measures the level of competition in the spot market, as analyzed in Lindh (1992), where* $\delta_i = -1$ *means generator i behaves as a competitive generator,* $\delta_i = 0$ *means generator i behaves as a Cournot generator and* $\delta_i = m - 1$ *means that producer i behaves as à la Cournot generator that is part of a cartel, where m is the number of identical cartel members.*

Proof 1 *The profit for conventional generator i, in scenario* ω, *in the spot market is:*

$$\Pi_{i\omega}^S = P_\omega^S q_{i\omega}^S - a_{i\omega} - b_{i\omega}(q_i^F + q_{i\omega}^S) - \frac{1}{2} c_{i\omega}(q_i^F + q_{i\omega}^S)^2. \quad (2.3.6)$$

From (2.3.6), we can derive the spot market equilibrium by computing the first-order optimality conditions for all generators simultaneously as:

$$\frac{\partial \Pi_{i\omega}}{\partial q_{i\omega}^S} = 0 = \frac{\partial P_\omega^S}{\partial q_{i\omega}^S} q_{i\omega}^S + P_\omega^S - b_{i\omega} - c_{i\omega}(q_i^F + q_{i\omega}^S) = 0 \quad \forall i, \forall \omega \quad (2.3.7)$$

where $\frac{\partial P_\omega^S}{\partial q_{i\omega}^S} = -\beta_\omega^S \left(1 + \sum_{i \neq k} \frac{\partial q_{k\omega}^S}{\partial q_{i\omega}^S}\right) = -\beta_\omega^S(1+\delta_i)$. *By substituting* $-\beta_\omega^S(1+\delta_i)$ *into (2.3.7), we can simplify and rearrange it to solve for* $q_{i\omega}^S$*:*

$$q_{i\omega}^S = \frac{1}{\beta_\omega^S(1+\delta_i)+c_{i\omega}}(P_\omega^S - b_{i\omega} - c_{i\omega} q_i^F) = \tau_{i\omega}(P_\omega^S - b_{i\omega} - c_{i\omega} q_i^F) \quad \forall i, \forall \omega. \quad (2.3.8)$$

At this level, we can substitute P_ω^S *from (2.3.4) into (2.3.8) to obtain the clearing price in the spot market in terms of futures market decision variables which can be rearranged as:* $P_\omega^S = \hat{\gamma}_\omega^S - \beta_\omega^S \sum_{i \in I} q_{i\omega}^S - \beta_\omega^S \sum_{i \in I} q_i^F$, *where we can collect price as:* $P_\omega^S +$

$\beta^S_\omega \sum_{i\in I} \tau_{i\omega} P^S_\omega = \hat{\gamma}^S_\omega - \beta^S_\omega \sum_{i\in I} q^F_i + \beta^S_\omega \sum_{i\in I} \tau_{i\omega}\left(b_{i\omega} + c_{i\omega} q^F_i\right)$. *Finally, the equilibrium spot market price is:*

$$P^S_\omega = \varphi_\omega \left[\hat{\gamma}^S_\omega - \beta^S_\omega \sum_{i\in I} q^F_i + \beta^S_\omega \sum_{i\in I} \tau_{i\omega}\left(b_{i\omega} + c_{i\omega} q^F_i\right)\right] \quad \forall\omega. \tag{2.3.9}$$

Proposition 2 *At each scenario ω, the optimal spot quantity $q^S_{i\omega}$ of conventional generator i, given the futures market quantity q^F_k, with knowledge of* (2.3.3)*, and P^S_ω from* (2.3.9) *is expressed as:*

$$q^S_{i\omega} = \tau_{i\omega}\varphi_\omega \left[\hat{\gamma}^S_\omega - \beta^S_\omega \sum_{i\in I} q^F_i + \beta^S_\omega \sum_{i\in I} \tau_{i\omega}(b_{i\omega} + c_{i\omega} q^F_i)\right] - \tau_{i\omega}\left(b_{i\omega} + c_{i\omega} q^F_i\right). \tag{2.3.10}$$

Proof 2 *The explicit formula for conventional generator i's quantity in the spot market is $q^S_{i\omega} = \tau_{i\omega}(P^S_\omega - b_{i\omega} - c_{i\omega} q^F_i)$ from* (2.3.8)*. Then, substituting the equilibrium spot price, which is expressed in (2.3.9) into* (2.3.8) *gives the optimal $q^S_{i\omega}$.*

Given $Q_{j\omega}$ is total RES production exogenously expressed for RES generators, the optimal generation in the spot market is:

$$q^S_{j\omega} = Q_{j\omega} - q^F_j \quad \forall j, \forall\omega. \tag{2.3.11}$$

Futures Market Analysis (Stage-One)

Once the spot market equilibrium conditions are obtained in closed-form, we can go one step backward in time to deal with the futures market equilibrium outcomes using the optimal spot market decision variables ($P^S_\omega, q^S_{i\omega}$ and $q^S_{j\omega}$). Then, we characterize the joint maximization of all generators' expected, or risk averse profit (with both technologies). To that end, first we start from the conventional generator's profit function expressed in (2.3.1), RES generator's profit expressed in (2.3.2), and compute the derivative with respect to futures market quantities.

$$\frac{\partial \Pi_{i\omega}}{\partial q_i^F} = \frac{\partial P^F}{\partial q_i^F} q_i^F + P^F + \frac{\partial P_\omega^S}{\partial q_i^F} q_{i\omega}^S + P_\omega^S \frac{\partial q_{i\omega}^S}{\partial q_i^F} - b_{i\omega}\left(1 + \frac{\partial q_{i\omega}^S}{\partial q_i^F}\right) - c_{i\omega}\left[q_i^F + q_{i\omega}^S\right]\left(1 + \frac{\partial q_{i\omega}^S}{\partial q_i^F}\right) \forall i, \forall \omega \tag{2.3.12a}$$

$$\frac{\partial \Pi_{j\omega}}{\partial q_j^F} = \left(\frac{\partial P^F}{\partial q_j^F} - \frac{\partial P_\omega^S}{\partial q_j^F}\right) q_j^F + P^F - P_\omega^S + \frac{\partial P_\omega^S}{\partial q_j^F} Q_{j\omega} \quad \forall j, \forall \omega. \tag{2.3.12b}$$

Assumption 2 *$\forall k \in I \cup J$, each generator k has an estimation of the impact its generation q_k^F may have in the futures price and competitors' generation, where $P^F = P^F(q_k^F)$ and $q_{-k}^F = q_{-k}^F(q_k^F)$.*

This is considered to compute $\frac{\partial \Pi_{k\omega}}{\partial q_k^F}$ which now depend on $\psi_k = \frac{\partial q_{-k}^F}{\partial q_k^F}$. Therefore, the different levels of competition in the futures market can be modeled as: $\psi_k = -\frac{1}{I+J-1}$ for competitive generator,$\psi_k = 0$ for Cournot generator, and $\psi_k > 0$ for generator part of a cartel. From (2.3.12), we can observe there are several partial derivatives ($\frac{\partial P^F}{\partial q_i^F}$, $\frac{\partial P_\omega^S}{\partial q_i^F}$ and $\frac{\partial q_{i\omega}^S}{\partial q_i^F}$ (for conventional generators) and $\frac{\partial P^F}{\partial q_j^F}$, $\frac{\partial P_\omega^S}{\partial q_j^F}$ and $\frac{\partial q_{j\omega}^S}{\partial q_j^F}$ (for RES generators), which are derived using (2.3.5), (2.3.9),(2.3.10) and (2.3.11)), that can influence the futures market hence, the global solution in the CVaR formulation, as we will see in the next section.

Given the equilibrium spot market outcomes, the partial derivatives needed to be computed for both sets of generators are:

$$\frac{\partial P^F}{\partial q_i^F} = -\beta^F \left(1 + \sum_{i \neq k} \frac{\partial q_k^F}{\partial q_i^F} + \sum_{j \in J} \frac{\partial q_j^F}{\partial q_i^F}\right) = -\beta^F(1 + (I + J - 1)\psi_i) \quad \forall i \quad (2.3.13a)$$

$$\frac{\partial P_\omega^S}{\partial q_i^F} = \varphi_\omega \left[\left(-\beta_\omega^S(1 + (I - 1)\psi_i)\right)\right] + \varphi_\omega \left[\beta_\omega^S c_{i\omega}\tau_{i\omega} + \beta_\omega^S \sum_{k \neq i}^{I} c_{k\omega}\psi_k\tau_{k\omega}\right] \quad \forall i, \forall \omega \quad (2.3.13b)$$

$$\frac{\partial q_{i\omega}^S}{\partial q_i^F} = \tau_{i\omega}\left(\frac{\partial P_\omega^S}{\partial q_i^F} - c_{i\omega}\right) \quad \forall i, \forall \omega \quad (2.3.13c)$$

$$\frac{\partial P^F}{\partial q_j^F} = -\beta^F\left(1 + (I + J - 1)\psi_j\right) \quad \forall j \quad (2.3.13d)$$

$$\frac{\partial P_\omega^S}{\partial q_j^F} = 0 \quad \forall j, \forall \omega \quad (2.3.13e)$$

$$\frac{\partial q_{j\omega}^S}{\partial q_j^F} = -1 \quad \forall j, \forall \omega. \quad (2.3.13f)$$

Futures Market Equilibrium under CVaR

By replacing the equilibrium spot price P_ω^S and quantities $q_{i\omega}^S$ and $q_{j\omega}^S$ in the profit functions $\Pi_{i\omega}$ and $\Pi_{j\omega}$, we can obtain parameterized profits in the futures market decision variables:

$$\Pi_{i\omega} = \Pi_{i\omega}(q_i^F, q_{-i}^F, q_{j \in J}^F) \quad \forall i, \forall \omega \quad (2.3.14a)$$

$$\Pi_{j\omega} = \Pi_{j\omega}(q_j^F, q_{-j}^F, q_{i \in I}^F) \quad \forall j, \forall \omega. \quad (2.3.14b)$$

In this chapter, the risk aversion level of generators is considered by using the CVaR measure. CVaR is a coherent[2] risk measure that calculates VaR and optimizes CVaR simultaneously. The CVaR at α confidence level (CVaR_α) can be defined as the

[2]A functional $\mathcal{R} : \mathcal{L}^2 \to (\infty, \infty]$ is called a coherent measure of risk in the extended sense if (1) $\mathcal{R}(C) = C$ for all constants C, (2) $\mathcal{R}((1-\lambda)X + \lambda X') \leq (1-\lambda)\mathcal{R}(X) + \lambda\mathcal{R}(X)$ for $\lambda \in (0,1)$ ("convexity"),
(3) $\mathcal{R}(X) \leq \mathcal{R}(X')$ ("monotonicity"), (4) $\mathcal{R}(X) \leq 0$ when $||X^k - X||_2 \to 0$ ("closedness"), (5) $\mathcal{R}(\lambda X) = \lambda\mathcal{R}(X)\text{for}\lambda > 0$ ("positive homogeneity").

expected value of the profit smaller than the $(1-\alpha)$-quantile of the profit distribution. In the scenario-based stochastic optimization method, CVaR_α measures the expected profit in the $(1-\alpha) \times 100\%$ worst scenarios. The $(1-\alpha)-$quantile of the profit distribution is known as VaR, which is the largest value ensuring that the probability of obtaining a profit less than that value is lower than $1-\alpha$, $\forall \alpha \in [0,1]$. Thus, the profit maximization problem solved by the risk (neutral and averse) generators is formulated in (2.3.15), where $\sigma_{k\omega}$ is the probability assigned by generator k to scenario ω, and $1-\alpha$ represents the level of significance associated with the CVaR. The risk averse problem solved by each generator is (Rockafellar and Uryasev, 2000):

$$\max_{\xi_k, \eta_{k\omega}, q_k^F} (1-\phi) \sum_{\omega \in \Omega} \sigma_{k\omega} \Pi_{k\omega} + \phi \left[\xi_k - \frac{1}{1-\alpha} \sum_{\omega \in \Omega} \sigma_{k\omega} \eta_{k\omega} \right] \tag{2.3.15a}$$

s.t.

$$\eta_{k\omega} + \Pi_{k\omega} - \xi_k \geq 0 \quad : \mu_{k\omega} \quad \forall \omega \tag{2.3.15b}$$

$$\eta_{k\omega} \geq 0 \quad : \theta_{k\omega} \quad \forall \omega \tag{2.3.15c}$$

$$q_k^{F_{min}} \leq q_k^F \leq q_k^{F_{max}} \quad : \nu_k^{min}, \nu_k^{max}. \tag{2.3.15d}$$

The objective function is represented by the expected profit (first term for risk neutral generators) and the CVaR (second term for risk averse generators) multiplied by a factor $\phi \in [0,1]$, that regulates a trade-off between the expected profit and the CVaR for a given level α. In this setting, $\phi = 0$ corresponds to a risk neutral generator (maximization of the expected profits) and $\phi = 1$ to the most risk averse setting in which all the weight in the objective $\xi_k - \frac{1}{1-\alpha} \sum_{\omega \in \Omega} \sigma_{k\omega} \eta_{k\omega}$ (maximization of the CVaR) is considered. Parameter $\sigma_{k\omega}$ represents the scenario occurrence probability which is assumed to be $\frac{1}{\Omega}$. At the optimal solution, ξ_k represents the VaR at the optimal, and the auxiliary variable $\eta_{k\omega}$ equals (for each scenario) the positive difference between the VaR and the profit ($\Pi_{k\omega}$), which is expressed with constraint (2.3.15b). Constraints (2.3.15d) enforce the lower and upper bounds for electricity schedule of generator k. The dual variables associated with constraints (2.3.15b), (2.3.15c) and (2.3.15d) are $\mu_{k\omega}, \theta_{k\omega}, \nu_k^{min}$, and ν_k^{max}, which are treated as variables in the model.

The equilibrium is then obtained by solving simultaneously (2.3.15) for all generators. This is done by replacing (2.3.15) by its associated Karush-Kuhn-Tucker (KKT) system of optimality conditions. Therefore, the KKT system associated with each generator is:

$$\frac{\partial \mathcal{L}}{\partial q_k^F} = -(1-\phi)\sum_{\omega\in\Omega}\sigma_{k\omega}\frac{\partial \Pi_{k\omega}}{\partial q_k^F} - \sum_{\omega\in\Omega}\mu_{k\omega}\frac{\partial \Pi_{k\omega}}{\partial q_k^F} - \nu_k^{min} + \nu_k^{max} = 0 \quad \forall k \tag{2.3.16a}$$

$$\frac{\partial \mathcal{L}}{\partial \eta_{k\omega}} = \phi\frac{1}{1-\alpha}\sigma_{k\omega} - \mu_{k\omega} - \theta_{k\omega} = 0 \quad \forall k, \forall \omega \tag{2.3.16b}$$

$$\frac{\partial \mathcal{L}}{\partial \xi_k} = -\phi + \sum_{\omega}^{\Omega}\mu_{k\omega} = 0 \quad \forall k \tag{2.3.16c}$$

$$0 \leq \eta_{k\omega} + \Pi_{k\omega} - \xi_k \perp \mu_{k\omega} \geq 0 \quad \forall k, \forall \omega \tag{2.3.16d}$$

$$0 \leq \eta_{k\omega} \perp \theta_{k\omega} \geq 0 \quad \forall k, \forall \omega \tag{2.3.16e}$$

$$0 \leq q_k^F - q_k^{F_{min}} \perp \nu_k^{min} \geq 0 \quad \forall k \tag{2.3.16f}$$

$$0 \leq q_k^{F_{max}} - q_k^F \perp \nu_k^{max} \geq 0 \quad \forall k. \tag{2.3.16g}$$

The complementarity conditions are denoted by $0 \leq x \perp y \geq 0$ is equivalent to: $x \geq 0, y \geq 0$ and $xy = 0$. One way to solve system (2.3.16) for all k is by using the technique discussed in Leyffer and Munson (2010), where a nonlinear programming reformulation of KKT systems is proposed. This technique is based on minimizing the sum of the complementarity products subject to the remaining KKT conditions.

Solutions where the objective function is zero guarantee that all the original KKT conditions are met. Thus, can consider the problem in extended form:

$$\begin{aligned}
&\min \sum_{i\in I}\sum_{\omega\in\Omega}\left[\mu_{i\omega}(\eta_{i\omega}+\Pi_{i\omega}-\xi_i)+\eta_{i\omega}\theta_{i\omega}\right]+\sum_{j\in J}\sum_{\omega\in\Omega}\left[\mu_{j\omega}(\eta_{j\omega}+\Pi_{j\omega}-\xi_j)+\eta_{j\omega}\theta_{j\omega}\right]+\\
&\quad+\sum_{i\in I}\left[\left(q_i^F-q_i^{F_{min}}\right)\nu_i^{min}+\left(q_i^{F_{max}}-q_i^F\right)\nu_i^{max}\right]\\
&\quad+\sum_{j\in J}\left[\left(q_j^F-q_j^{F_{min}}\right)\nu_j^{min}+\left(q_j^{F_{max}}-q_j^F\right)\nu_j^{max}\right] && (2.3.17a)\\
&\quad\text{subject to}\\
&\quad\text{equalities}\quad (2.3.16a)-(2.3.16c) && (2.3.17b)\\
&\quad\text{inequalities}\quad (2.3.16d)-(2.3.16g) && (2.3.17c)\\
&\quad(2.3.1),(2.3.2),(2.3.9)-(2.3.13) && (2.3.17d)
\end{aligned}$$

where the objective function is formulated by the complementarity conditions, except those with the type $xy = 0$ and all the KKT optimality conditions entered as constraints. Moreover, (2.3.1) and (2.3.2) are the profit definitions of the conventional and RES generators, respectively. (2.3.9) is equilibrium price in the spot market, whereas, (2.3.10) and (2.3.11) are the equilibrium quantities of conventional and RES generators, respectively. Finally, (2.3.12) and (2.3.13) are the partial derivative with respect to the futures market quantities and their definitions, respectively.

2.3.2 Contracts for Differences (CFD) Model Formulation

Our second model deals with a futures market which settles financially based on contracts for differences (CFD), that can be used by generators to protect themselves from the price and quantity fluctuations mainly arise in the spot electricity market (Unger, 2002; Kristiansen, 2004; Oliveira et al., 2013). The structure of the electricity market with CFD is similar to the market in the GM except in the CFD generators use the strike price as a reference price and there is a financial settlement at expiry rather than a physical delivery. For all generators, $k \in I \cup J$, the contract signed is a CFD, that fixes the electricity amount q_k^F, and the delivery price P^F (the strike price) in the first stage. When the spot market takes place, the spot price P_ω^S can be either greater than or less than the futures price P^F. Therefore, if the spot market price P_ω^S is lower than P^F, generator k earns the difference between the two price times the amount of electricity agreed in the contract $(P^F - P_\omega^S)q_k^F$. Otherwise, generator k will pay the difference between the two prices times the amount of energy agreed in the contract at time-one $(P_\omega^S - P^F)q_k^F$, if the spot market price surpasses the fixed (futures market) price at the spot market. Therefore, profit for conventional generator is expressed as:

$$\Pi_{i\omega} = (P^F - P_\omega^S)q_i^F + P_\omega^S q_{i\omega}^S - a_{i\omega} - b_{i\omega} q_{i\omega}^S - \frac{1}{2} c_{i\omega} (q_{i\omega}^S)^2 \quad \forall i, \forall \omega \tag{2.3.18}$$

and

$$\Pi_{j\omega} = (P^F - P_\omega^S)q_j^F + P_\omega^S Q_{j\omega} \quad \forall j, \forall \omega \tag{2.3.19}$$

for RES generator. Note that the profit formulation for conventional generators in the CFD is different from the GM profit formulation. The analytical derivation follows a similar procedure carried out in Section 2.3.1, where we need to start from the spot market and then come back to the futures market to compute the market equilibrium.

Spot Market Equilibrium (Stage-Two)

Since we do not have futures production (q_i^F and q_j^F are financial quantities) in the CFD, the spot market price is described as:

$$P_\omega^S = \gamma_\omega^S - \beta_\omega^S \sum_{i\in I} q_{i\omega}^S - \beta_\omega^S \sum_{i\in I} Q_{j\omega} = \hat{\gamma}_\omega^S - \beta_\omega^S \sum_{i\in I} q_{i\omega}^S \quad \forall\omega. \tag{2.3.20}$$

Proposition 3 *Taking definition of φ_ω and $\tau_{i\omega}$ from Proposition 1, the equilibrium spot price and spot quantities in the CFD are described as:*

$$P_\omega^S = \varphi_\omega \left[\hat{\gamma}_\omega^S + \beta_\omega^S \sum_{i\in I} \tau_{i\omega} b_{i\omega} - \beta_\omega^S \sum_{i\in I} q_i^F \beta_\omega^S (1+\delta_i)\tau_{i\omega}\right] \quad \forall\omega \tag{2.3.21}$$

and

$$q_{i\omega}^S = \tau_{i\omega}\left\{\varphi_\omega[\hat{\gamma}_\omega^S + \beta_\omega^S \sum_{i\in I} \tau_{i\omega} b_{i\omega} - \beta_\omega^S \sum_{i\in I} q_i^F \beta_\omega^S (1+\delta_i)\tau_{i\omega}] - b_{i\omega} + q_i^F \beta_\omega^S (1+\delta_i)\right\} \quad \forall i, \forall\omega. \tag{2.3.22}$$

Proof 3 *From the profit function expressed in (2.3.18), we can compute the spot equilibrium by deriving the first-order optimality conditions for all generators:*

$$\frac{\partial \Pi_{i\omega}}{\partial q_{i\omega}^S} = -q_i^F \frac{\partial P_\omega^S}{\partial q_{i\omega}^S} + \frac{\partial P_\omega^S}{\partial q_{i\omega}^S} q_{i\omega}^S + P_\omega^S - b_{i\omega} - c_{i\omega} q_{i\omega}^S = 0 \tag{2.3.23}$$

Rearranging (2.3.23) and solving for $q_{i\omega}^S$ as:

$$q_{i\omega}^S = \tau_{i\omega}\left(P_\omega^S - b_{i\omega} + q_i^F \beta_\omega^S (1+\delta_i)\right). \tag{2.3.24}$$

Then, substituting (2.3.24) into (2.3.20) and simplifying it gives: $P_\omega^S + \beta_\omega^S \sum_{i\in I} \tau_{i\omega} P_\omega^S = \hat{\gamma}_\omega^S + \beta_\omega^S \sum_{i\in I} \tau_{i\omega} b_{i\omega} - \beta_\omega^S \sum_{i\in I} q_i^F \beta_\omega^S (1+\delta_i)\tau_{i\omega}$, from which the optimal price at the spot market is expressed as:

$$P_\omega^S = \varphi_\omega \left[\hat{\gamma}_\omega^S + \beta_\omega^S \sum_{i\in I} \tau_{i\omega} b_{i\omega} - \beta_\omega^S \sum_{i\in I} q_i^F \beta_\omega^S (1+\delta_i)\tau_{i\omega}\right] \quad \forall\omega. \tag{2.3.25}$$

Finally, by substituting (2.3.25) *into* (2.3.24)*, the optimal spot quantity for generator i, at scenario ω, is expressed as:*

$$q_{i\omega}^S = \tau_{i\omega}\left\{\varphi_\omega[\hat{\gamma}_\omega^S + \beta_\omega^S\sum_{i\in I}\tau_{i\omega}b_{i\omega} - \beta_\omega^S\sum_{i\in I}q_i^F\beta_\omega^S(1+\delta_i)\tau_{i\omega}] - b_{i\omega} + q_i^F\beta_\omega^S(1+\delta_i)\right\} \quad \forall i, \forall\omega.$$

Futures Market Analysis (Stage-One)

Similar to the GM, the partial derivatives with respect to futures quantities, needed to characterize conventional generators and RES generators in CFD, are expressed as follows:

$$\begin{aligned}\frac{\partial\Pi_{i\omega}}{\partial q_i^F} &= \frac{\partial P^F}{\partial q_i^F}q_i^F + P^F - \frac{\partial P_\omega^S}{\partial q_i^F}q_i^F - P_\omega^S + \frac{\partial P_\omega^S}{\partial q_i^F}q_{i\omega}^S = 0 + \frac{\partial q_{i\omega}^S}{\partial q_i^F}P_\omega^S - b_{i\omega} - c_{i\omega}q_{i\omega}^S\\ &= \left(\frac{\partial P^F}{\partial q_i^F} - \frac{\partial P_\omega^S}{\partial q_i^F}\right)q_i^F + q_{i\omega}^S\left(\frac{\partial P_\omega^S}{\partial q_i^F} - c_{i\omega}\right) + P_\omega^S\left(\frac{\partial q_{i\omega}^S}{\partial q_i^F} - 1\right) + P^F - b_{i\omega} \quad \forall i, \forall\omega\end{aligned} \tag{2.3.26}$$

and

$$\frac{\partial\Pi_{j\omega}}{\partial q_j^F} = \left(\frac{\partial P^F}{\partial q_j^F} - \frac{\partial P_\omega^S}{\partial q_j^F}\right)q_j^F + P^F - P_\omega^S + \frac{\partial P_\omega^S}{\partial q_j^F}Q_{j\omega} \quad \forall j, \forall\omega, \tag{2.3.27}$$

respectively.

By recalling the futures market price expressed in (2.3.5) and considering Assumption 2, the partial derivatives needed to complete (2.3.26) and (2.3.27) are:

$$\frac{\partial P^F}{\partial q_i^F} = -\beta^F(1 + (I + J - 1)\psi_i) \quad \forall i \tag{2.3.28a}$$

$$\frac{\partial P^F}{\partial q_j^F} = -\beta^F\left(1 + (I + J - 1)\psi_j\right) \quad \forall j \tag{2.3.28b}$$

$$\frac{\partial P_\omega^S}{\partial q_i^F} = \varphi_\omega[-\beta_\omega^S\beta_\omega^S(1 + \delta_i)\tau_{i\omega} - \beta_\omega^S \sum_{i \neq k} \beta_\omega^S \psi_i(1 + \delta_i)\tau_{i\omega}] \quad \forall i, \forall \omega \tag{2.3.28c}$$

$$\frac{\partial q_{i\omega}^S}{\partial q_i^F} = \tau_{i\omega}\frac{\partial P_\omega^S}{\partial q_i^F} + \tau_{i\omega}\beta_\omega^S(1 + \delta_i) \quad \forall i, \forall \omega \tag{2.3.28d}$$

$$\frac{\partial P_\omega^S}{\partial q_j^F} = 0 \quad \forall j, \forall \omega \tag{2.3.28e}$$

$$\frac{\partial q_{j\omega}^S}{\partial q_j^F} = -1 \quad \forall j, \forall \omega. \tag{2.3.28f}$$

Risk-Averse Futures Market Equilibrium under CVaR

Once we know the equilibrium spot price P_ω^S and quantities, $q_{i\omega}^S$ and $q_{j\omega}^S$, we can obtain parameterized the total profits in the futures decision variables:

$$\Pi_{i\omega} = \Pi_{i\omega}(q_i^F, q_{-i}^F, q_{j\in J}^F) \quad \forall i, \forall \omega \tag{2.3.29a}$$

$$\Pi_{j\omega} = \Pi_{j\omega}(q_j^F, q_{-j}^F, q_{i\in I}^F) \quad \forall j \forall \omega. \tag{2.3.29b}$$

Each player $k \in I \cup J$ maximizes its total profit (risk neutral, or risk averse) by solving an optimization problem similar to (2.3.15) in the futures market considering the CVaR. The market equilibrium can be done by following the procedure described in Section 2.3.1 and concatenate the KKT conditions for each generator to solve with an equivalent NLP formulation.

2.3.3 Spot Market without the Presence of Futures

Finally, we derive the equilibrium market outcomes of a market without the presence of futures trading (a single spot market) for comparison purposes. Thus, the profit for conventional generator in the only spot configuration is expressed as:

$$\Pi_{i\omega} = P_{\omega}^{S} q_{i\omega}^{S} - a_{i\omega} - b_{i\omega} q_{i\omega}^{S} - \frac{1}{2} c_{i\omega} (q_{i\omega}^{S})^{2} \quad \forall i, \forall \omega \tag{2.3.30}$$

and the profit for generator j as:

$$\Pi_{j\omega} = P_{\omega}^{S} Q_{j\omega} \quad \forall \omega. \tag{2.3.31}$$

The price-demand curve is:

$$P_{\omega}^{S} = \gamma_{\omega}^{S} - \beta_{\omega}^{S} \left(\sum_{i \in I} q_{i\omega}^{S} + \sum_{j \in J} Q_{j\omega} \right) \quad \forall \omega \tag{2.3.32}$$

which can be further simplified as: $P_{\omega}^{S} = \hat{\gamma}_{\omega}^{S} - \beta_{\omega}^{S} \sum_{i \in I} q_{i\omega}^{S}$.

The equilibrium in the spot market is reached when, all conventional generators maximize their profits simultaneously, which is expressed by the first-order optimality conditions as:

$$\frac{\partial \Pi_{i\omega}}{\partial q_{i\omega}^{S}} = \frac{\partial P_{\omega}^{S}}{\partial q_{i\omega}^{S}} q_{i\omega}^{S} + P_{\omega}^{S} - b_{i\omega} - c_{i\omega} q_{i\omega}^{S} = 0. \tag{2.3.33}$$

Simplifying (2.3.33), gives $q_{i\omega}^{S} = \tau_{i\omega}(P_{\omega}^{S} - b_{i\omega})$, where we can substitute $q_{i\omega}^{S}$ into (2.3.32), and solving for the spot price as $P_{\omega}^{S} \left(1 + \beta_{\omega}^{S} \sum_{i \in I} \tau_{i\omega}\right) = \hat{\gamma}_{\omega}^{S} + \beta_{\omega}^{S} \sum_{i \in I} \tau_{i\omega} b_{i\omega}$ gives the equilibrium price:

$P_{\omega}^{S} = \varphi_{\omega} \left[\hat{\gamma}_{\omega}^{S} + \beta_{\omega}^{S} \sum_{i \in I} \tau_{i\omega} b_{i\omega}\right]$, which can be expressed by substituting $\hat{\gamma}_{\omega}^{S}$ as:

$$P_{\omega}^{S} = \varphi_{\omega} \left[\gamma_{\omega}^{S} - \beta_{\omega}^{S} \sum_{i=1}^{J} Q_{j\omega} + \beta_{\omega}^{S} \sum_{i \in I} \tau_{i\omega} b_{i\omega} \right] \quad \forall \omega. \tag{2.3.34}$$

Finally, substituting (2.3.34) into (2.3.33) gives the equilibrium quantity for conventional generators as:

$$q_{i\omega}^{S} = \tau_{i\omega}\varphi_{\omega}\left[\hat{\gamma}_{\omega}^{S} + \beta_{\omega}^{S}\sum_{i\in I}\tau_{i\omega}b_{i\omega}\right] - \tau_{i\omega}b_{i\omega} \quad \forall i, \forall\omega. \tag{2.3.35}$$

2.4 Numerical Results

In this section, we analyze and discuss numerical examples using calibrated data inspired by the Spanish electricity market that combines futures contracts and pool market structures. In the Spanish electricity market, generators can sign physical or financial futures contracts which are composed of an energy quantity, a price, and a delivery date. Then, they participate in a daily market and the market operator maintains the bid-offer balance of electricity power to determine the market price as well as the generation of electricity quantities corresponding to each generator for each hour in the schedule (OMIE, 2020).

2.4.1 Data

As the Spanish electricity market is an oligopolistic structure with handful large generators, we consider three conventional generators and one RES generator who can produce and retail their generation in the futures and spot markets. Table 2.1 presents the expected values of the calibrated data for the cost, demand and RES parameters along with the corresponding standard deviations. From the cost parameters, $a_{i\omega}$ are fixed to arbitrary values $a_{1\omega} = a_{2\omega} = a_{3\omega} = 0$, as they do not have a direct influence on the market results. The values of $b_{i\omega}$ and $c_{i\omega}$ are randomly generated by estimating their expected values, which are close to the real market values (see Ruiz et al. (2012); Oliveira et al. (2013); Mousavian et al. (2020) for similar approach). Each realization

Parameters	$i=1$	$i=2$	$i=3$	CV	Parameters	mean	σ
$a_{i\omega}$	0.00	0.00	0.00	-	γ^F	180.00	18
$b_{i\omega}$	37.00	40.00	43.00	0.09	β^F	0.005	0.0005
$c_{i\omega}$	0.013	0.003	0.019	0.05	$Q_{j\omega}$	0-10,000	1000

Table 2.1: Mean, CV and standard deviation used to calibrate the data applied in our simulations for conventional generators and the RES parameter.

of $b_{i\omega}$ is generated from a multivariate normal distribution with mean $\mu_b = [37, 40, 43]$ [€/MWh] and standard deviation $\sigma_b = [3.5, 4.55, 5.59]$ [€/MWh], which is calculated using 9% coefficient of variation (CV), as $\sigma = \mu \times CV$. Similarly, $c_{i\omega}$ is generated randomly from a normal distribution with $\mu_c = [0.013, 0.003, 0.019]$ [€/MWh^2] and standard deviation $\sigma_c = [0.000125, 0.0002, 0.000195]$ [€/MWh^2], calculated using 5% CV. 150 and 200 equiprobable scenarios are considered for the risk neutral case and for the CVaR, respectively. The significance level is fixed at $\alpha = 0.90$.

The demand curve parameters are generated by approximating the aggregated step-wise demand curve in the spot market using the mean futures market intercept $\mu_{\gamma^F} = 180$ and mean slope $\mu_{\beta^F} = 0.005$. We assume that the parameters of the inverse demand curve in the spot market equal, on expectation, those parameters of the futures inverse demand, i.e, $\gamma^F = E[\gamma^S_\omega]$ and $\beta^F = E[\beta^S_\omega]$. Note that the uncertainty associated with consumer's behavior is represented by both the demand intercept (γ^S_ω) and its slope (β^S_ω), which are scenario dependent. Thus, the value of γ^S_ω for each scenario is randomly generated with a normal distribution so that $\gamma^S_\omega \sim N(\mu_{\gamma^F}, \sigma_{\gamma^F})$ and $\beta^S_\omega \sim N(\mu_{\beta^F}, \sigma_{\beta^F})$, where σ_{γ^F} and σ_{β^F} are calculated as $\sigma = \mu \times CV$, assuming CV to be 10% for the corresponding mean values for both parameters. Total RES parameter is randomly generated with its expected value at $5,000$MWh and a standard deviation of 1000 with CV of 20%.

The simulation is done for the RES parameter that ranges from 0 to $10,000$ MWh (we consider the amount of RES from 0-10,000MWh in a range of 1000MWhs given the expectation of RES to be integrated into the system is increasing realistically). The maximum amount that conventional generators are allowed to generate in the model is arbitrarily set at 6,000, 7,000, and 5,000MWh for the three generators, respectively.

According to the specification of problem (2.3.15), two risk profiles ($\phi = 0$ for the risk neutral and $\phi = 1$ for the risk averse generators) are considered. We analyze how the relationship between futures and spot markets, risk aversion, and level of competition are affected with respect to high RES penetration (level of renewables).

The model is implemented in JuMP version 0.21.1 (Dunning et al., 2017) under the open-source Julia language version 1.5.2 (Bezanson et al., 2017). We use Artelys Knitro solver version 12.2 (Byrd et al., 2006) on a CPU E5-1650v2@3.50GHz and 64.00 GB of RAM running workstation. Since there are multiple simulation cases, we report the computational performance of one case as a representative reference: For instance, the overall computational time for the CVaR definition with $|\Omega| = 200$ equiprobable scenarios is 917.177904 seconds (156.46 M allocations: 3.199 GiB, 0.10% gc time).

2.4.2 Result Presentation and Discussion

We present the numerical simulations for risk neutral and risk averse generators. The electricity market outcomes (electricity price, quantity, and generators' profit)

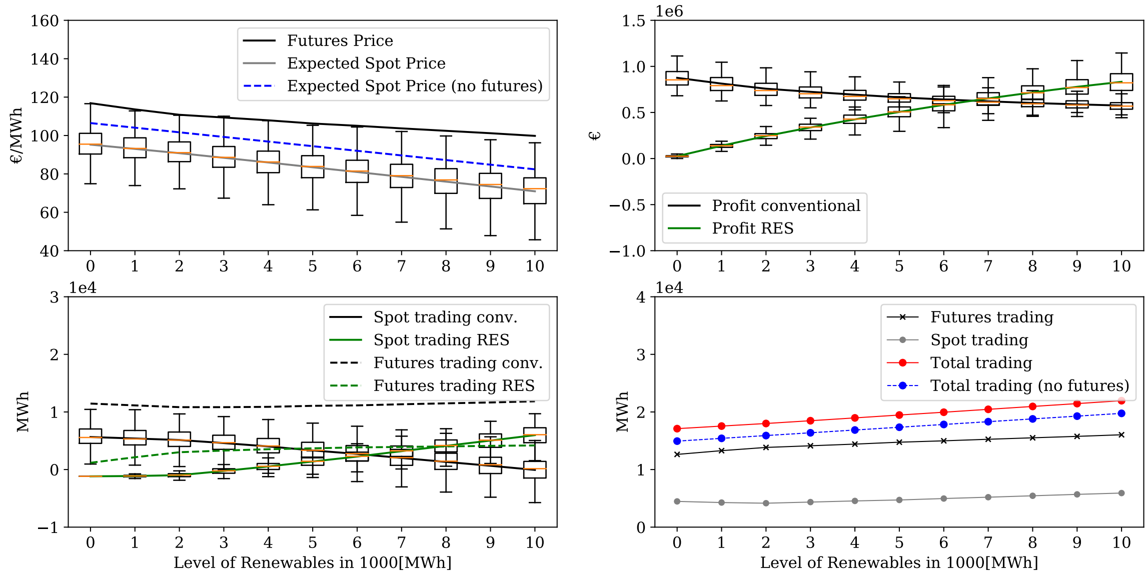


Figure 2.1: Risk averse Cournot model in the GM

are discussed for two market structures (Cournot and perfect competition) and for two models (GM and CFD) with respect to RES penetration. The x-axis in all the plots, but one represents RES penetration that ranges from 0 to 10,000MWh, whereas the y-axis may be price, quantity, or profit.

2.4.3 Risk Averse Generators' Numerical Results ($\phi = 1$)

For the risk averse generators, the numerical results are obtained by simulating the model with the parameter $\phi = 1$. The results are futures market and spot market equilibrium outcomes when generators maximize their CVaR from our spectral risk measure representation. We analyze the influence of generators' risk aversion behavior on equilibrium market outcomes based on levels of competitiveness (Cournot and perfect competition) and based on two contracts (physical, or financial).

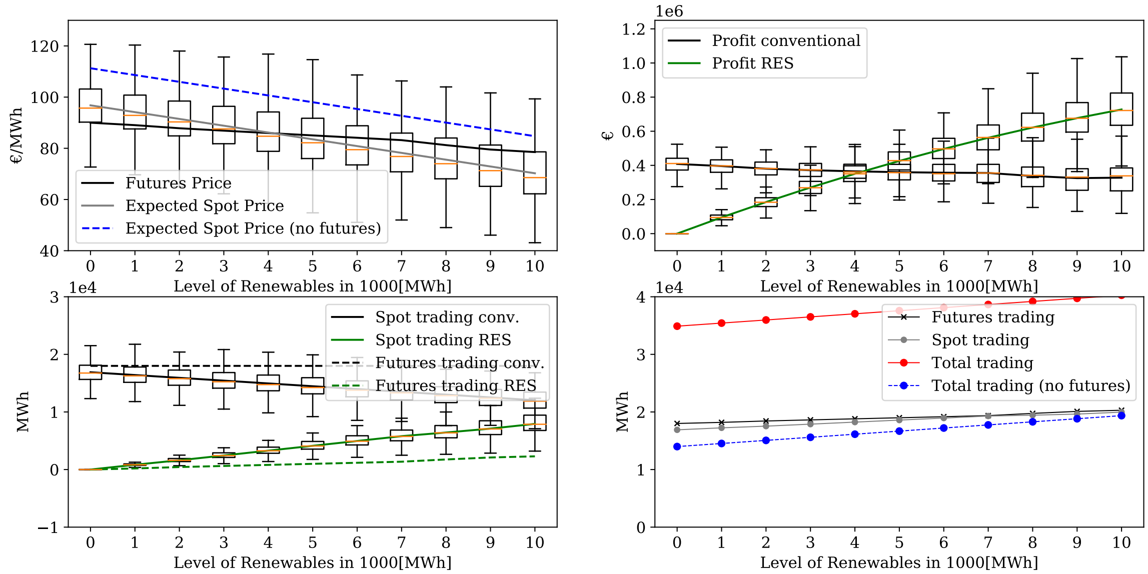


Figure 2.2: Risk averse Cournot CFD

Let's examine the results starting from the Cournot model in the general model. Looking at Figure 2.1, Figure 2.2, and Table 2.2, overall price decreases (downward sloping) with respect to RES penetration. That is because when RES penetration increases, conventional generation with generation cost decreases and is replaced by RES generation(with no generation cost), which may transfer part of the cost saved to consumers. If we look at the general model results, we see that consumers must pay as much as 116€/MWh with zero level of renewables, and as less as 101€/MWh in the futures market with higher (10,000MWh) RES penetration. The expected spot market price without the presence of futures contracts is always higher (ranges from 84 to 111€/MWh) than the expected spot price in the presence of futures contracts (ranges from 75 to 100€/MWh), which overlapped in the CFD model. In the CFD, the expected spot price without the presence of the futures market is always higher than the futures price (coincides with normal backwardation over time) and decreases with respect to RES penetration.

Therefore, buyers benefit from RES penetration as it decreases electricity prices in both stages and motivates RES generators to generate and trade higher quantities of electricity in the market. Regarding competition, there is a counter-argument that attaining a certain level of renewable with no marginal cost in a liberalized electricity market distorts market competitiveness and creates contango (Blazquez et al., 2018). The argument is that the further reduction of expected spot prices beyond the speed of the futures price with respect to RES penetration is due to market inefficiency created by the dispatchability and zero generation cost of RES generators. The decrease in price with RES penetration corroborates with classical economic theory, where the higher the competition (because of RES generators competitiveness increases) the larger is the fall in price even by conventional generators to compete with RES generators. In perfect competition, generators are price takers and they have no influence on the future market prices.

As far as trading quantities are concerned, conventional generators trade quantities that range from 11,414 to 12,761MWh with RES penetration. Futures market trading for conventional generators is not significantly affected as much as the expected spot market quantities, which decrease across RES penetration (see Figure 2.1). Following the market prices trend, the CVaR decreases for conventional generators with respect to RES penetration, which shows left tailed profit distributions with a low probability of low profits with high RES penetration. The opposite is true for RES generators with RES penetration. This is evident as the RES generators trading quantities increase both in the futures market and spot market with respect to RES penetration (see Figure 2.2). As profit is the reflection of price and quantity, the higher the RES penetration, the lower the conventional production, which in turn lowers their profit.

Comparing the models, conventional generators' profit is higher in the GM compared to CFD, but it decreases in both models with respect to RES penetration. This is due to the higher futures electricity price in the GM than the relatively lower futures price in the CFD with the RES penetration. RES generators' profit, on the other hand, increases in both models as generators trade higher electricity with respect to RES penetration.

When generators react in the market as perfect competition, the competition squeezes their profits compared to Cournot competition. This is an intuitive result as it shows when generators act in the market as perfect competition, the pattern of profits follows the shape of prices for both in the GM and CFD, which is consistent with existing literature (Jabr, 2005; Oliveira et al., 2013).

Total trading quantities in the CFD are much higher than the GM. This is because the CFD model includes physical and financial quantities (see Figure 2.2 and Figure 2.4). In other words, the CFD model entails larger electricity transactions in the spot market, which is explained by the lack of a physical settlement in the futures market that requires all quantities must be delivered in the spot market. In addition, total trading without the presence of futures market is higher in the GM and lower in the CFD, where the expected spot market trading is higher. Conversely, the spot market total trading with the futures market decreases in the GM the higher the RES penetration is.

Figure 2.3 and Figure 2.4 show the market outcomes with perfect competition in the GM and in the CFD, respectively. With perfect competition, the overall price is

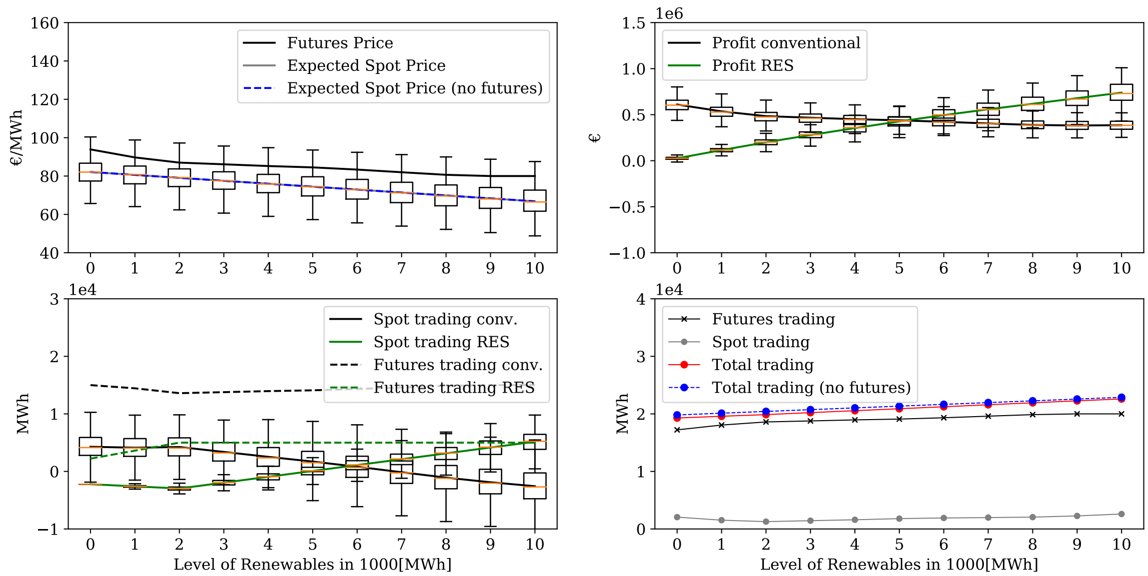


Figure 2.3: Risk averse perfect competition in the GM

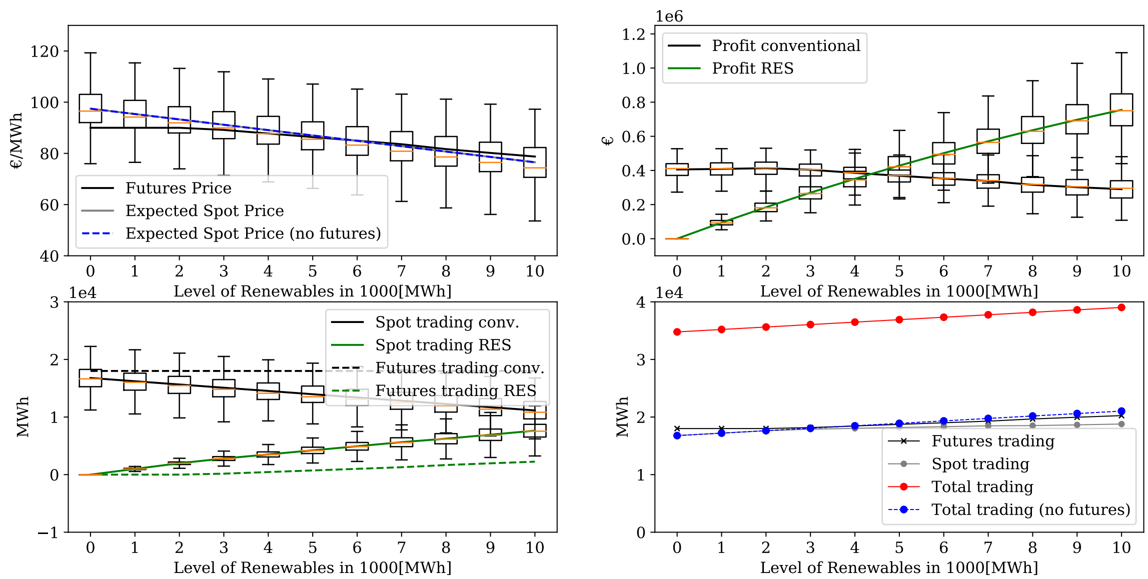


Figure 2.4: Risk averse perfect competition in the CFD

Price	General Model (GM)		Contract for differences (CFD)	
	Cournot model	Perfect Competition	Cournot model	Perfect Competition
P^F	101-116	83-100	78-90	79-90
P^S_ω	75-100	76-97	70-96	76-97
P^S_ω only	84-111	76-97	84-111	76-97

Table 2.2: The minimum and the maximum electricity prices [€/MWh] with the two models and the two levels of competitions in the risk averse case.

lower in both models with respect to RES penetration (ranges from 76 to 100€/MWh in both contracts). Besides, the expected spot prices are overlapped in both the GM and the CFD (with and without the presence of the futures market). In the GM, total trading quantity with and without the presence of futures contracts shows no significant difference as highlighted in Figure 2.3. However, this is the opposite as the CFD model encompasses physical and financial quantities. In the CFD, total trading sharply increases with no futures market despite the trading volume in the spot market with the presence of the futures market is not as low as it is in the GM.

2.4.4 Risk Neutral Generators' Numerical Results ($\phi = 0$)

In the risk neutral case, the numerical simulation is done by setting the risk parameter $\phi = 0$. In the risk neutral case, generators maximize their expected profits, which is expressed in (2.3.16) of the spectral risk measure representation. Figure 2.5 and Figure 2.6 show the risk neutral results for Cournot competition, and Figure 2.7 and Figure 2.8 perfect competition ones with respect to high RES penetration. Similar to the risk averse case, the overall electricity price decreases with respect to RES penetration. With Cournot competition, the market is in contango in the GM, where

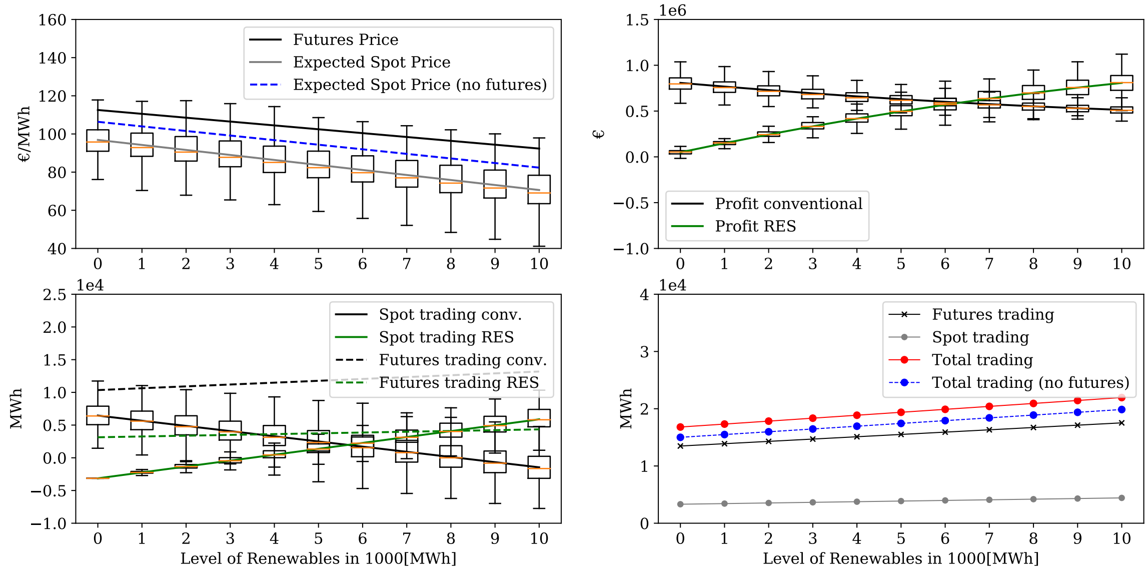


Figure 2.5: Risk neutral Cournot competition in the GM with RES penetration.

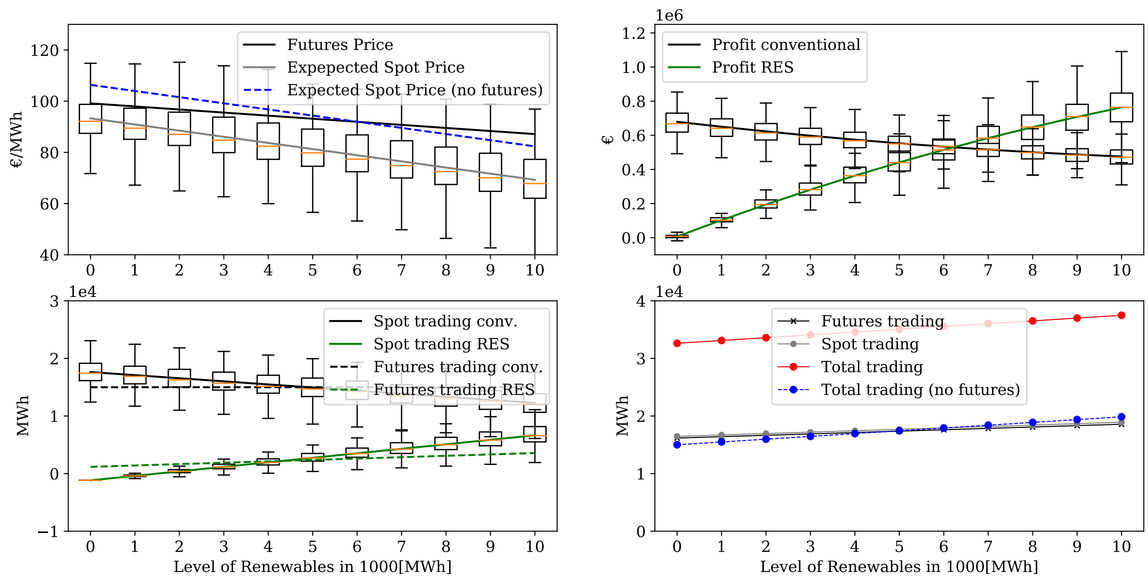


Figure 2.6: Risk neutral Cournot competition in the CFD with RES penetration.

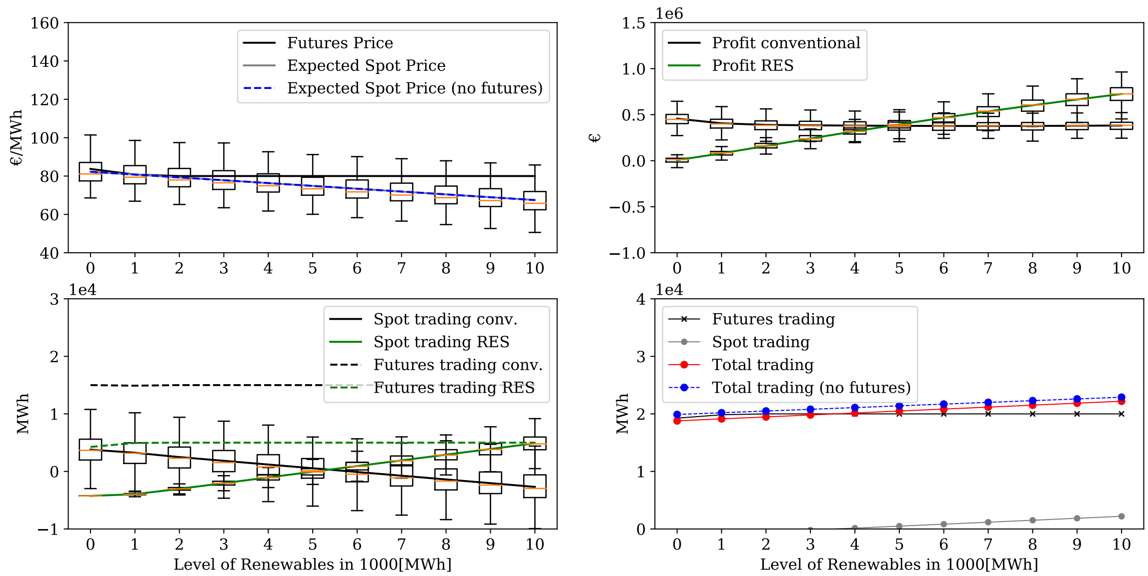


Figure 2.7: Risk neutral perfect competition in the GM with RES penetration.

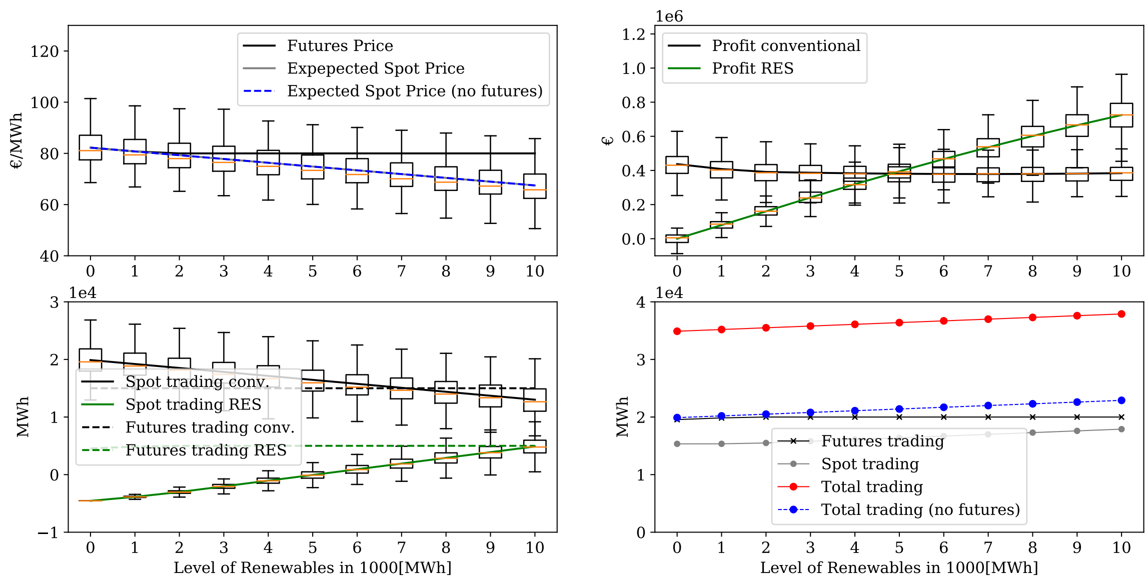


Figure 2.8: Risk neutral perfect competition in the CFD with RES penetration.

Market Outcomes	Cournot Model				Perfect competition			
	Risk neutral		Risk averse		Risk neutral		Risk averse	
	GM	CFD	GM	CFD	GM	CFD	GM	CFD
P^F	108.28	108.28	107.68	107.68	87.26	87.26	91.46	91.46
$P^S_{i\omega}$	90.64	90.64	88.48	88.48	87.26	87.26	86.99	86.99
$\sum_{i\in I} q_i^F$	17999.99	10815.77	11791.94	17999.99	17999.99	14658.56	13924.75	17999.99
$\sum_{i\in I} q_i^S$	14737.43	2740.58	1656.61	14467.78	14106.44	-550.57	19.05	13961.32
$\sum_{j\in J} q_j^F$	667.27	3527.72	2671.51	1065.89	900.89	3888.64	3782.41	863.93
$\sum_{j\in J} q_{j\omega}^S$	4321.95	1461.51	2441.14	4046.76	4251.07	1263.32	1160.34	4078.83
$\sum_{i\in I} \Pi_i$	394779.9	634928.70	636130.7	362298.2	361638.9	393461.36	446717.43	362277.3
$\sum_{j\in J} \Pi_j$	397829.4	483218.95	478013.2	401020.6	424595.6	424595.63	426842.08	408029.3

Table 2.3: Expected market outcomes with levels of competition in the GM and CFD for risk neutral and risk averse cases.

the futures price is higher than the expected spot prices, and in normal backwardation in the CFD model, where the futures price is lower than the expected spot market price with respect to RES penetration.

Contrary to the risk averse generators case in the GM, conventional generators' futures market trading in the risk neutral case slightly increases with RES penetration, whereas, RES generators futures market trading is higher in the risk averse case with Cournot competition (comparing Figure 2.1 and Figure 2.5). In the CFD, conventional generators' trade with Cournot competition is always higher than the RES generators' trading with respect to RES penetration despite RES generator profit is as high as conventional generators' profit in the GM.

For the perfect competition, futures prices overlapped with expected spot market prices with respect to RES penetration, in both the GM and the CFD model, which is inline with the standard non-arbitrage condition that explains futures price is equal to the expected spot market prices, particularly with risk neutrality. Total trading and profit exhibit similar pattern with the Cournot model discussed above.

2.4.5 Overall Comparison

Table 2.3 shows the market outcomes, mainly the overall expected outcomes for comparison between risk neutrality and risk aversion, levels of competition, and the two contract settlements. Futures market and expected spot market prices are slightly higher for risk neutral generators in the Cournot competition than in the perfect competition. Expected spot market trading varies in all the cases presented in Table 2.3 expected for the futures market trading, which matches the expectation. RES generators trade more electricity in the spot market compared to their trade in the futures market. The sum of one large RES generator's profit is almost as close as the three conventional generators' profit as the RES penetration displaces conventional ones from competition. Moreover, it is evident from our simulation results that RES generators trade much of their electricity in the spot market as contracts manage the uncertainties raised from demand and RES capacity availability. Another reason is due to non-dispatchability of RES, they have to trade all the remaining quantities in the spot market so that that may contribute to the spot market trading increments.

2.4.6 The effect of Risk Parameter on Market Outcomes

Finally, we analyze the impact of risk parameter (ϕ) increase on market outcomes with perfect competition, and in the GM as a representative reference. This is done by keeping the RES parameter at its expected value (5,000MWh) and moving the CVaR parameter (from extreme risk neutrality ($\phi = 0$) to extreme risk aversion ($\phi = 0$)). With this setting, Figure 2.9 shows that risk aversion increases electricity market prices both in the futures market and spot market, though the effect is higher in the futures market. The expected spot price overlapped both with and without the

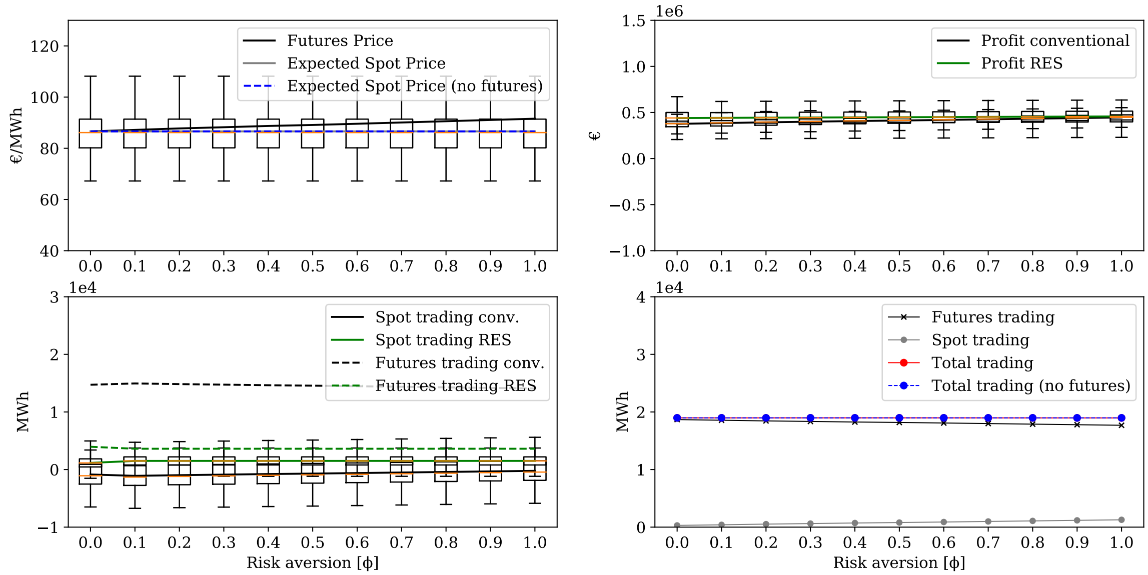


Figure 2.9: The effect of varying risk parameter on market outcomes in perfect competition.

presence of a futures contracts. For conventional generators, spot market quantity slightly increases with respect to the CVaR parameter, which is reflected in their improved profit as well. This is a counter-intuitive result as risk neutrality is expected to render higher profit. This implies contracts give a shield for generators from the risk of cost, demand, and RES capacity uncertainties, as the more risk averse they are the better the confident they become to produce, and retail their electricity.

2.5 Summary and Conclusion

This chapter proposes a game-theoretical framework to model an electricity market that considers different futures market contract designs in a two-stage approach, with high RES penetration.

We model an electricity market with risk averse (using the CVaR) and risk neutral under uncertainty between oligopolistic generators whose decisions are made based on the assumption that each generator has an estimation (conjecture) of their impact on the other generators' decisions, and they act simultaneously both in the spot market and in the futures market. Generators have the option to trade their generation first, in the futures market (stage-one), and subsequently in the spot market (stage-two) given the inherent uncertainties. Since demand, conventional generators' production cost, and RES generation availability are uncertain, a coherent risk measure with the CVaR is introduced to model risk aversion. We also introduce different types of contracts in the futures market (physical and financial) to evaluate their performance and impact in the equilibrium market outcomes, which in turn depend on the levels of RES generation in the system.

The analytical derivation of the market equilibrium starts at the second stage (spot market) where we parameterize the spot market equilibrium as a function of the futures market decision variables. In the first stage, the global equilibrium of the market is computed from the joint solution of all the generators' profit maximization problems using the CVaR. The global optimization is solved by concatenating and minimizing the product of the complementarity constraints subject to the remaining equality and inequality KKT optimality conditions from the CVaR formulation, and the closed-form solutions obtained from the spot market equilibrium, which becomes a nonconvex nonlinear optimization problem.

The analytical results obtained are tested with numerical results by analyzing different levels of competition (Cournot and perfect competition), different contracts (physical and financial) with risk neutral, or risk averse strategies for players.

We can conclude that the level of risk aversion and competition have a strong impact on electricity market outcomes with respect to RES penetration. Therefore, RES penetration increases overall quantity traded and decreases overall electricity prices, which is better for social welfare. RES generators' profit increases as their trading quantities increase for RES penetration and contracts give confidence for generators so that they can increase their profit acting as risk averse in the market, which is a counter-intuitive result of the model. Comparing contracts, the CFD (financial trading) model renders better consumer surplus as it has lower electricity prices while the GM (physical delivery) is better for electricity generators as it renders higher electricity prices so that higher profits.

There are several possible extensions to the model and methods proposed in this chapter. First, the conventional generators' profit function could be modified to study emission trading scheme in the electricity market in a bid to plummet greenhouse gas emission. Second, by relaxing conventional generators' production cost function to be linear, the model could be extended to study the interaction between market power, and risk aversion considering different contracts over time. Finally, the model can be applied for problems that incorporate financial contracts, and risk aversion to study equilibrium solutions.

Chapter 3: Contracts in Electricity Markets under EU ETS: A Stochastic Programming Approach

3.1 Introduction

Global warming has become one of the crucial environmental issues that has derived the attention of researchers and politicians for the last three decades. To curb the serious damage of global warming, the United Nations Framework Convention on Climate Change (UNFCCC) established the Kyoto Protocol in 1997 and entered into force on 16 February 2005. The EU had committed to reducing GHG emissions by 8%, compared to the 1990 level, by the years 2008-2012 through a trading scheme in sectors with an installed capacity of more than 20 MW (Thema et al., 2013). The EU ETS is the largest single market for CO_2 emission allowances, accounting for approximately 84% of the global carbon market value (Action, 2013) and aimed to aid the EU's fight against the global warming.

GHG emissions, especially carbon dioxide (CO_2), and climate change are closely related to economic growth and social development. One of the main mechanisms of GHG reduction dictated by the protocol was to trade human-related emission allowances or permits, primarily CO_2, in organized financial markets[3]. Thus, several

[3]Since the electricity industry is the largest contributor of GHG, it should be considered in any global emissions combating policy initiative to reach decarbonized economy.

national and regional emission markets have been established where a variety of specialized financial instruments are traded (Lau et al., 2012; Moore et al., 2016).

Being the largest emissions trading system, the EU ETS covers more than 11,000 power factories and other installations in 31 countries, and flights between airports of participating countries which cover around 45% of the EU's GHG emissions (Action, 2013; European Commission, 2020). According to the European Commission, to achieve the EU's overall GHG emissions reduction target for 2030, the sectors covered by the EU ETS must reduce their emissions by 43% compared to 2005 levels. This target is supposed to be achieved by putting a price on carbon and thereby giving a financial value to each ton of emissions saved. Therefore, CO_2 is priced and leads electricity generators to invest in low-carbon technologies which, in turn discourage carbon-intensive generators by increasing their production cost[4].

The details of the ETS application have been an important issue since the beginning. It commenced as cap-and-trade[5] principle where the overall volume of GHG that can be emitted by the power plants and other companies covered by the system was subject to a cap set at the EU level. The emission cap was allocated for free of charge to power producers in the form of carbon permits, the so-called European emission allowances (EUAs), where one EUA gives the owner the right to release one ton of CO_2 in the atmosphere[6]. If electricity generators have emission discrepancy, they should turn to the EU ETS market to sell/buy any surplus/shortage of permits.

[4]From its commencement, the EU ETS has progressed four distinct trading compliance phases to date: Phase I 2005-2007, Phase II 2008-2012, Phase III 2013-2020, and Phase IV 2021-2030.

[5]Cap-and-trade is the most discussed scheme to control CO_2 emissions in the European Union trading system.

[6]Within the cap, companies receive or buy emission allowances which they can trade, if they wish to do so in the later phases.

For instance, if they emit less than their cap, they can sell the surplus of permits. Otherwise, they pay the penalties if they emit above the cap. However, studies from the earlier phases show that allocating emissions based on the cap-and-trade strategy is not as efficient as expected (Veith et al., 2009; Ellerman et al., 2016).

The main issue concerning the operational allocation method based on grandfathering emitters by assigning emission allowances free of charge (based on historical emissions) had enabled compliant electricity producers to generate large carbon rents or windfall profits (Veith et al., 2009; Ellerman et al., 2016). Moreover, the allocation of allowances to member states based on grandfathering is criticized for the alleged competitive distortions (Ellerman et al., 2016). One of the distortions was that companies generated windfall profits by reporting costs above reality. Windfall profits stifle investments in low-carbon technologies and in the worst-case encourage investment in carbon-intensive installations. As a solution, with Directive 2009/29/EC, a significant shift towards an exclusively auction-based system for allowances to the power sector was instituted at the onset of Phase III. After Phase III, power producers in most member states (except new member states who are permitted to obtain EUA free of charge) purchase allowances in accordance with the polluter pays principle (Hobbie et al., 2019). The reform aims to negate windfall profits being earned by producers in the power sector by forcing electricity producers to assume material costs for the permits obtained. Auctioning addresses both criticisms of the previous grandfathering allocations in one fell swoop. First, windfall profits are eliminated through the auctioning of allowances. Second, the possibility of competitive distortions within the single EU market is solved by the force of demand and supply. Since the auction-based methods are increasingly becoming the preferred allocation mechanism by policy-makers[7], we assume in our model that the allowances are auctioned instead of allocating for free (see for example Rocha et al. (2015) and Tang et al. (2017) for a discussion of this issue).

Another instrument to combat the GHG effect is the expansion of renewable energy sources. The deployment of intermittent capacities, such as wind and solar, are effective instruments that can be done at a lower cost in the electricity sector with less structural changes than in other sectors. However, there are debates that policy combinations of renewable deployment and devising EU ETS to combat GHG have negatively interacted as the EU ETS has a dampening effect on the CO_2 price (del Río, 2017). The existence of such different policy goals and market failures are not panaceas and bring problems on their own. Rather there are arguments that the economic theory supports the combination of ETS and renewable energy targets by mitigating policy trade-offs through appropriate coordination and/or instrument choice and design, as recommended by del Río (2017) and Munoz et al. (2017).

In this chapter, we analyze the interaction of electricity generators in an oligopolistic market with a game-theoretical model where the electricity market and the emissions market interact in a two-stage framework. For analytical simplicity, we assume a single futures electricity market where a quantity is committed at the futures price. Similarly, the emissions allowance is contracted in advance prior to a spot market where the electricity and allowances delivery takes place. We represent these market interactions with a game-theoretical equilibrium model. Both risk neutral and risk

[7] The auction markets for carbon allowances can effectively avoid the intrinsic shortcomings of the centralized allocation methods, political misallocation, and regulatory distortion (Tang et al., 2017).

averse generators are taken into consideration with different hedging strategies. In order to achieve that, a nonlinear programming problem is solved after reformulating the CVaR optimization problem for each agent via its equivalent Karush-Kuhn-Tucker (KKT) conditions. Finally, the model is tested numerically with calibrated data. Our findings show that both RES and CO_2 parameters are effective in plummeting GHG by suppressing conventional generators and helping the low-carbon transition without compromising social welfare.

The chapter is organized as follows. Section 3.2 reviews the literature on electricity, emission allowance, and GHG reduction. Section 3.3 presents the game-theoretical stochastic programming model formulation and the approaches to solve it with futures emission allowances and RES parameters. Section 3.4 tests the model with calibrated data and presents relevant numerical results. Finally, we provide concluding remarks in Section 3.5.

3.2 Literature Review and Contributions

3.2.1 Literature Review

EU ETS and Electricity Market

Since the 1990s, there has been a pouring of research efforts to assess and to combat the effects of GHG emissions through different strategies, like cap-and-trade programs in Europe. The European Union has created a single European electricity market, operating under ETS in the first half of the 2000s. That coincided with electricity market liberalization. Market liberalization and increasing transmission capacity between regional electricity grids have been served as the key means for the promotion of internal electricity market creation and competition (Chevallier, 2011;

Aatola et al., 2013). This further promotes the creation and enhancement of the regulation between European countries. One of the economic activities in the regulation is the carbon market that has got the attention of countries around the world. The carbon market is considered an economically efficient mechanism to reduce the cost of emission abatement by institutional innovation and trading in financial markets (Cui et al., 2014). Recent studies reveal that integrating GHG combating with emission trading is quite significant to achieve economic and environmental objectives. EU ETS is an incentive-based and market-driven instrument purported to achieve environmental objectives by allocating and auctioning GHG emission allowances. In other words, the EU ETS is taken as an important pathway of development for the EU to respond to the global climate change calls by achieving simultaneously green and low-carbon economic development (Naso et al., 2017; Wang et al., 2019). Because economic activities foster higher demand for industrial production, companies (such as electricity generators) falling under the regulation of the EU ETS, need to produce more in order to meet their customers' demand. This, in turn yields large demand for CO_2 allowances (as they emit more CO_2) to cover emissions shortage which ultimately leads to the CO_2 price increase (Ellerman and Buchner, 2008).

The CO_2 price is determined by the interplay between supply and demand. In response to low emission prices, recently, the EU is deciding to harness the emission allowance supply through backloading[8] and the Market Stability Reserve (MSR)[9] which

[8]Backloading is a decision to postpone auctions of a certain number of allowances. This measure aimed at re-balancing supply and demand of allowances in the short term to improve the overall functioning of the EU ETS.

[9]The MSR is a carbon market reform aimed at providing price stability for installations covered under the EU ETS scheme and spurring low-carbon investments by increasing resilience to demand-supply imbalances. The MSR reduces yearly allocations depending on the size of the surplus. Later, it injects stored allowances back into the market once a certain scarcity threshold is reached. Hence, it shifts the auction date of allowances into the future (Perino and Willner, 2017). The MSR is a

are effected in 2019 so that both measures are directed at increasing prices (Salant, 2016). Therefore, the EU electricity market is an area within which the EU ETS and energy efficiency instruments interact with (Thema et al., 2013). After the onset of Phase III of the EU ETS, the electricity generating sector and energy-intensive industries are obliged to buy/sell from the auction market (as a function of market outcomes) that replaces the fixed cap with their emission permit surplus/shortage.

Emission Allowances Models in Electricity Market

The linkage between carbon futures markets with the energy market and the effectiveness of the carbon futures markets to combat GHG are two strands of studies relevant to this topic. This dynamic development of financial markets and carbon futures has attracted numerous researchers and practitioners with quantitative models (Wu et al., 2011). More often, the focus on the interaction between power systems and carbon policies is considered from the power market and emission reduction perspectives. In this regard, Rocha et al. (2015) analyze the interactions between a cap-and-trade program, investment decisions, and electricity markets for an electricity network in a restructured market environment by considering the CO_2 emissions. Another simulation model is done by Ruth et al. (2008) to examine the energy and economic implications whereby they conclude modest emissions reductions, despite profit is reduced when conventional plants are retired. Wang et al. (2020) propose an agent-based approach with Q-learning algorithm to model[10] interactions between the carbon trading market where carbon emission is regulated from the electricity

mechanism established by the EU in 2015 to reduce the surplus of emission allowances in the carbon market and to improve the EU ETS's resilience to future shocks

[10] Q-learning algorithm is a model-free reinforcement learning algorithm to learn a policy telling an agent what action to take under what circumstances.

generators side. On the other hand, since the carbon market is risk prone, it attracts attention in the field of energy economics and finance. In the carbon market, the market risk comes not only from its internal volatility but also from multiple aspects (generators technologies, costs of production uncertainties, demand, and nonstorability of the energy generated) (Hintermann, 2010; Lin and Jia, 2019)

In general, given that allowances trading has primarily been applied in the EU ETS, there are quite a lot of studies that analyze price behavior of tradable emission allowances from different perspectives. For instances, Daskalakis et al. (2009); Seifert et al. (2008); Paolella and Taschini (2006) from econometrics and management perspectives, Böhringer and Lange (2005); Daskalakis et al. (2009); Samadi et al. (2018) with price simulations against changes in market design parameters, Galiana and Khatib (2010); El Khatib and Galiana (2019) with deterministic Cournot-Nash electricity companies gaming strategies and emission permit, Chevallier (2010); Botterud et al. (2010); Boersen and Scholtens (2014) with futures/forward contracts and EUAs allowances, and earlier Chuang et al. (2001) using the competition of generators and Cournot theory of oligopoly, among others. Methodologically, there are estimation and optimization models. They have contributed to the development of the allowance market. However, there are still gaps in the literature that combines CO_2 allowances allocation through carbon market auction and RES penetration with stochastic programming[11].

[11] Stochastic programming method associated with scenario generation approach which has been extensively deployed in problems which include uncertainties. In such programming approach, the uncertain terms are expressed as stochastic variables and represented with scenarios (Hajibandeh et al., 2017)

Risk Aversion Models in Electricity Market

Risk management is a crucial task that has been considered in decision science. There are numerous ways to include risk aversion in market equilibrium models. For example, in the economics literature, concave utility functions[12] are popular as they can be used to convert monetary costs/profits into utilities, whose expected value is then optimized instead of the original objective (Fishburn, 1970). In such cases, the non-linearity nature of the functions makes them challenging to be included in large-scale market equilibrium models. In addition, if the distribution of the possible outcomes is normal, the exponential function can be written as a linear combination of expected outcomes and the standard deviation of the outcome distribution. This further complicates the model as the normality assumption is not realistic in the case of considering an small number of scenarios, which is usually the case due to computational complexity. As a result, in economics and finance, there are sophisticated theoretical and quantitative tours-de-force risk measures such as variance, shortfall probability, expected shortage, VaR, CVaR and stochastic dominance, which can control the trade-off between expected profit and the variability of profit.

Specifically, in financial mathematics literature, risk aversion is modeled by including VaR (Duffie and Pan, 1997), or CVaR (Rockafellar and Uryasev, 2000) in the

[12] That is, the utility functions can be specified as exponential functions (exhibiting constant absolute risk aversion, CARA) and isoelastic functions (exhibiting constant relative risk aversion, CRRA) (Eeckhoudt et al., 2011).

decision maker's objective, or constraints. However, VaR has undesirable mathematical properties[13]. CVaR[14], on the contrary, gives the expected value over outcomes that are worse than the VaR.

Our model applies CVaR risk aversion on profit optimization with financial derivatives in the electricity market and emission auctioning. We formulate a stochastic model that maximizes a weighted average of expected profits and their CVaR. In particular, we use stochastic programming because: i) it provides a powerful framework to model and include parameters' uncertainty in an optimization problem, via a plausible set of scenarios; and ii) the CVaR can be easily incorporated to the model with a linear formulation ((Rockafellar and Uryasev, 2000)).

The CVaR at α confidence level (CVaR$_\alpha$) can be defined as the expected value of the profit smaller than the $(1-\alpha)$-quantile of the profit distribution. CVaR has been widely used in various electricity power problems for different entities for different purposes (Artzner, 1999; Zhang, 2006; Conejo et al., 2008; Shapiro, 2009; Morales et al., 2010; Philpott et al., 2013; del Río, 2017). For instance, Neuhoff and De Vries (2004) use for competitive markets where risks cannot be traded and coupled with risk averse generators and consumers. Ehrenmann and Smeers (2011) show the generation cost increasing and market failure by applying stochastic equilibrium models with

[13] According to Uryasev (2010), VaR has the following undesirable mathematical properties:

1. It does not control scenarios exceeding VaR
2. It is a non-convex and discontinuous function of the confidence level for discrete distributions
3. VaR is difficult to control/optimize for non-normal distributions.

Whereas CVaR is continuous with respect to the confidence level and convex in decision variables.

[14] CVaR is a coherent risk measure (i.e., it exhibits good properties such as translation invariance, subadditivity, positive homogeneity, and monotonicity) that was introduced by Rockafellar and Uryasev (2000) as a technique for portfolio optimization which calculates VaR and optimizes CVaR simultaneously.

CVaR maximizing investors' return. In their models, they feature uncertain fuel costs, emissions reduction targets, and numbers of carbon allowances where risk averse investors build more open cycle gas turbines and less coal-fired generation capacity.

The novelty in our model is the analysis of the interaction between different financial derivatives (electricity and emission permits) and markets (futures and spot) from a game-theoretical perspective. Furthermore, we try to characterize the equilibrium between risk averse generators (CVaR) under different levels of competition in the market. Hence, our model combines different market designs and interactions between energy markets and financial derivatives. To this end, we develop a two-stage stochastic programming model which has a closed-form solution in the second stage and allows to be reformulated as a single nonlinear optimization problem. Subsequently, we examine strategic players reactions in both electricity and emission allowance markets through realistic numerical simulations.

3.2.2 Contributions

In this chapter, we develop market equilibrium models that combine emissions trading and RES deployment with oligopolistic risk neutral/averse players. The models are examined with trading data calibrated from European Energy Exchange (EEX) and the Spanish electricity market which implicate the main entities (generators, marketers, consumers, regulators) involved in power markets and climate change. The main contributions of this chapter are five-fold:

1. Modeling ad-hoc electricity market that involves renewable penetration and emission reduction simultaneously with a single futures market and a subsequent spot market.

2. Developing and examining a two-stage stochastic programming model with risk neutral/averse generators by introducing coherent risk measurement (CVaR) and different levels of competition (Cournot and perfect competition).

3. Deriving analytical expressions to characterize the equilibrium in both the futures and spot markets with emission futures and electricity futures, considering RES and CO_2 as parameters.

4. Testing the equilibrium model with relevant and insightful data to highlight the impact of low-carbon energy expansion and combating emission via EUA auctioning instead of grandfathering (allocations in the previous stages of the regulations).

5. Examining how to achieve the targets of plummeting GHG emissions in an economically efficient manner using both RES penetration and CO_2 auctioning in the derivatives market.

3.3 Problem Formulation

In this section, we present the model formulation that entails two specific models. The benchmark is referred to as the general model (GM) which includes a futures contract for both emission trading and electricity market in a two-stage stochastic programming framework. The second model examines a single spot market model for comparison and completeness.

3.3.1 The General Model (GM)

In the general model, we consider a set of oligopolistic conventional generators (I) and a set of RES generators (J) that compete to supply a homogeneous product (electricity) in a two-stage electricity market. In the first stage, conventional and renewable generators simultaneously choose the quantities q_k^F where $k \in I \cup J$, to sell in the futures market with futures price P^F, together with allowance futures commitment ε_i^F (only for conventional generators), with futures CO_2 price $P^{F_{CO_2}}$. In the second stage, generators participate in a spot market where, for each scenario ω, the amount of energy $q_{k\omega}^S$, is delivered at the spot price P_ω^S. Additionally, we assume conventional generators auction their shortage/surplus EUAs, $\varepsilon_{i\omega}^S$, with spot CO_2 price, $P_\omega^{S_{CO_2}}$. Note that $P_\omega^{S_{CO_2}}$ and $P^{F_{CO_2}}$ are exogenous parameters of the model.

Considering the emissions market, the cost (income) of buying (selling) the shortage (surplus) of emission allowances is added to the total profit function of conventional generators. We assume no grandfathering allowance allocations so that generators can only auction their emission permits (Brown and Eckert, 2017; Samadi et al., 2018; El Khatib and Galiana, 2019).

In a two-stage stochastic programming approach where futures and spot markets are considered, the profit for conventional generator i, and scenario ω, is expressed as follows:

$$\Pi_{i\omega} = P^F q_i^F + P_\omega^S q_{i\omega}^S - C_{i\omega}(q_i^F, q_{i\omega}^S) - P^{F_{CO_2}} \varepsilon_i^F + P_\omega^{S_{CO_2}} (\varepsilon_i^F - \hat{\eta_{i\omega}} (q_i^F + q_{i\omega}^S)) \quad (3.3.1)$$

where $C_{i\omega}(q_i^F, q_{i\omega}^S) = a_{i\omega} + b_{i\omega}(q_i^F + q_{i\omega}^S) + \frac{1}{2} c_{i\omega}(q_i^F + q_{i\omega}^S)^2$ is a quadratic production cost function[15] (concave and non-decreasing in input prices). $a_{i\omega} \geq 0$, $b_{i\omega} \geq 0$ and $c_{i\omega} \geq 0$ are the cost coefficients of conventional power generation i, and scenario ω. $P^{F_{CO_2}}$ and $P_\omega^{S_{CO_2}}$ are emission allowance prices in €/ton CO_2 in the futures market and in the spot market, respectively. Note that allowance price in the fututres market

[15] Quadratic cost functions accurately model the actual response of conventional generators where fuel is oil, coal and gas etc (Theerthamalai and Maheswarapu, 2010). Renewable energy sources are without cost function because the fuel that drives its power generation is without price.

and spot market are exogenous to the model. The total amount of CO_2 emissions for generator i, at scenario ω, is:

$$\varepsilon_{i\omega}^{tot} = \hat{\eta_{i\omega}} \sum_{i \in I} (q_{i\omega}^S + q_i^F) \tag{3.3.2}$$

The remaining emissions allowance that can be traded in the spot market is expressed as:

$$\varepsilon_{i\omega}^{S} = \hat{\eta_{i\omega}} \sum_{i \in I} (q_{i\omega}^S + q_i^F) - \varepsilon_i^F \quad \forall i, \forall \omega \tag{3.3.3}$$

where $\varepsilon_{i\omega}^S$ is the emissions allowance of generator i in the spot market and $\hat{\eta_{i\omega}}$ is the emission intensity factor for generator i and scenario ω. If $\varepsilon_{i\omega}^S > 0$, then generator i has shortage of emissions and surplus if $\varepsilon_{i\omega}^S < 0$. The profit for RES generator j, per scenario ω, is expressed in (3.3.4), where the production cost is assumed to be zero.

$$\Pi_{j\omega} = (P^F - P_\omega^S) q_j^F + P_\omega^S Q_{j\omega} \tag{3.3.4}$$

where $Q_{j\omega} = q_j^F + q_{j\omega}^S$ is a stochastic parameter representing the total production for RES generator j and scenario ω which is sold both in the spot and futures markets. Indeed, the energy that can be traded in the spot market is $q_{j\omega}^S = Q_{j\omega} - q_j^F$. As the occurrence of renewable spillage is a rare situation in most large power systems (it only occurs a few hours per year), we assume that $Q_{j\omega}$ is always dispatched at no costs or emissions with no possibility of spillage.

The spot price-inverse demand curve per scenario ω, is endogenously defined as follows:

$$P_\omega^S = \gamma_\omega^S - \beta_\omega^S \left(\sum_{i \in I} (q_{i\omega}^S + q_i^F) + \sum_{j \in J} Q_{j\omega} \right) \tag{3.3.5}$$

with γ^S_ω and β^S_ω being positive for the price-demand function to be well behaved[16]. The spot market price can be simplified as follows:

$$P^S_\omega = \hat{\gamma}^S_\omega - \beta^S_\omega \sum_{i\in I}(q^S_{i\omega} + q^F_i) \quad \forall\omega \tag{3.3.6}$$

where

$$\hat{\gamma}^S_\omega = \gamma^S_\omega - \beta^S_\omega \sum_{j\in J} Q_{j\omega} \quad \forall\omega.$$

Similarly, the inverse demand curve in the futures market is expressed as:

$$P^F = \gamma^F - \beta^F \left(\sum_{i\in I} q^F_i + \sum_{j\in J} q^F_j\right). \tag{3.3.7}$$

Second Stage: Spot Market Equilibrium

At this stage we assume that the futures market has already been settled so that we seek to characterize the spot market equilibrium, per scenario ω. Hence, the futures (first-stage) decisions variables are considered fixed at this stage. This is specified in Assumption 3.

Assumption 3 *The futures market quantity q^F_k, for $k \in I \cup J$ has already been committed with the futures price P^F, and in the spot market all firms simultaneously maximize their profits. Moreover, each generator has an estimation (conjecture) of the impact that its production, $q^S_{k\omega}$ and q^F_k, may have in the spot market price and rival quantities.*

Proposition 4 *Given Assumption 3 and the futures market quantities $q^F_k, \forall k$, at each scenario ω, the equilibrium spot market price is expressed as:*

$$P^S_\omega \quad = \quad \varphi_\omega \left[\hat{\gamma}^S_\omega + \beta^S_\omega \sum_{i\in I} \tau_{i\omega}\left(b_{i\omega} + c_{i\omega} q^F_i + \hat{\eta_{i\omega}} P^{S_{CO_2}}_\omega\right) - \beta^S_\omega \sum_{i\in I} q^F_i\right] \tag{3.3.8}$$

[16] The demand for electricity is assumed to exhibit the fundamental law of demand, that is, keeping other things constant, the quantity demanded decreases when the price increases.

where $\tau_{i\omega} = \frac{1}{\beta^S_\omega(1+\delta_i)+c_{i\omega}}$ *and* $\varphi_\omega = \frac{1}{1+\beta^S_\omega \sum_{i\in I}\tau_{i\omega}}$. *The parameter* $\delta_i = \sum_{k\neq i}^{I} \frac{\partial q^S_{k\omega}}{\partial q^S_{i\omega}}$ *measures the level of competition of generator* i *in the spot market, as analyzed in Lindh (1992), i.e,* $\delta_i = -1$ *represents perfect competition and* $\delta_i = 0$ *characterizes Cournot competition.*

Proof 4 *In the spot market profit for generator* i *and scenario* ω *is expressed as:*

$$\Pi^S_{i\omega}(q^S_{i\omega}, q^S_{-i\omega}) = P^S_\omega q^S_{i\omega} - C_{i\omega}(q^F_i, q^S_{i\omega}) + P^{S_{CO_2}}_\omega(\varepsilon^F_i - \hat{\eta_{i\omega}}(q^F_i + q^S_{i\omega})) \tag{3.3.9}$$

The optimal quantity $q^S_{i\omega}$ *is obtained by maximizing profit* $\Pi_{i\omega}$. *The first order optimality condition (FOC) with respect to* $q^S_{i\omega}$ *are:*

$$\begin{aligned}\frac{\partial \Pi_{i\omega}}{\partial q^S_{i\omega}} &= \frac{\partial P^S_\omega}{\partial q^S_{i\omega}} q^S_{i\omega} + P^S_\omega - b_{i\omega} - c_{i\omega}(q^F_i + q^S_{i\omega}) + P^{S_{CO_2}}_\omega \frac{\partial(\varepsilon^F_i - \hat{\eta_{i\omega}}(q^F_i + q^S_{i\omega}))}{\partial q^S_{i\omega}} \\ &= -\beta^S_\omega(1+\delta_i)q^S_{i\omega} + P^S_\omega - b_{i\omega} - c_{i\omega}(q^F_i + q^S_{i\omega}) - \hat{\eta_{i\omega}} P^{S_{CO_2}}_\omega = 0 \end{aligned} \tag{3.3.10}$$

where

$$\frac{\partial P^S_\omega}{\partial q^S_{i\omega}} = -\beta^S_\omega \left(1 + \sum_{(k\in I\cup J)\neq i} \frac{\partial q^S_{k\omega}}{\partial q^S_{i\omega}}\right) = -\beta^S_\omega(1+\delta_i) \quad \textit{and} \tag{3.3.11}$$

$$\delta_i = \sum_{k\neq i} \frac{\partial q^S_{k\omega}}{\partial q^S_{i\omega}} \tag{3.3.12}$$

Substituting (3.3.11) into (3.3.10), we have:

$$-\beta^S_\omega(1+\delta_i)q^S_{i\omega} - c_{i\omega}q^S_{i\omega} = -P^S_\omega + b_{i\omega} + c_{i\omega}q^F_i + \hat{\eta_{i\omega}} P^{S_{CO_2}}_\omega \tag{3.3.13}$$

$$q^S_{i\omega}\left[\beta^S_\omega(1+\delta_i) + c_{i\omega}\right] = P^S_\omega - b_{i\omega} - c_{i\omega}q^F_i - \hat{\eta_{i\omega}} P^{S_{CO_2}}_\omega \tag{3.3.14}$$

and after some rearrangements, we see that (3.3.14) is equivalent to:

$$q^S_{i\omega} = \tau_{i\omega}\left[P^S_\omega - b_{i\omega} - c_{i\omega}q^F_i - \hat{\eta_{i\omega}} P^{S_{CO_2}}_\omega\right] \tag{3.3.15}$$

where

$$\tau_{i\omega} = \frac{1}{\beta_\omega^S(1+\delta_i)+c_{i\omega}}.$$

By substituting $q_{i\omega}^S$ *of* (3.3.15) *into* P_ω^S *formula in (3.3.6):*

$$P_\omega^S = \hat{\gamma}_\omega^S + \beta_\omega^S \sum_{i\in I} \tau_{i\omega}\left[-P_\omega^S + b_{i\omega} + c_{i\omega}q_i^F + \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}}\right] - \beta_\omega^S \sum_{i\in I} q_i^F \tag{3.3.16}$$

and then:

$$P_\omega^S + \beta_\omega^S \sum_{i\in I} \tau_{i\omega} P_\omega^S = \hat{\gamma_{i\omega}}^S + \beta_\omega^S \sum_{i\in I} \tau_{i\omega}\left[b_{i\omega} + c_{i\omega}q_i^F + \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}}\right] - \beta_\omega^S \sum_{i\in I} q_i^F \tag{3.3.17}$$

Thus, the equilibrium spot price is stated as follows:

$$P_\omega^S = \varphi_\omega \left[\hat{\gamma}_\omega^S + \beta_\omega^S \sum_{i\in I} \tau_{i\omega}\left(b_{i\omega} + c_{i\omega}q_i^F + \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}}\right) - \beta_\omega^S \sum_{i\in I} q_i^F\right] \tag{3.3.18}$$

where $\varphi_\omega = \frac{1}{1+\beta_\omega^S \sum_{i\in I}\tau_{i\omega}}$.

Proposition 5 *From (3.3.15) and (3.3.18), at each scenario* ω *the equilibrium quantity* $q_{i\omega}^S$ *in the spot market is expressed as:*

$$\begin{aligned} q_{i\omega}^S &= \tau_{i\omega}\varphi_\omega \left[\hat{\gamma}_\omega^S + \beta_\omega^S \sum_{i\in I} \tau_{i\omega}\left(b_{i\omega} + c_{i\omega}q_i^F + \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}}\right) - \beta_\omega^S \sum_{i\in I} q_i^F\right] + \\ &- \tau_{i\omega}\left[b_{i\omega} + c_{i\omega}q_i^F + \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}}\right] \quad \forall i \end{aligned} \tag{3.3.19}$$

Proof 5 *We can combine (3.3.15) and (3.3.18) to express the equilibrium quantity in terms of futures decision variables as follows:*

$$q_{i\omega}^S = \tau_{i\omega}\left[P_\omega^S - b_{i\omega} - c_{i\omega}q_i^F - \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}}\right] \quad \forall i, \forall\omega \tag{3.3.20}$$

Since we have the equilibrium spot market price in (3.3.18), substituting it into (3.3.20) gives:

$$\begin{aligned} q_{i\omega}^S &= \tau_{i\omega}\varphi_\omega \left[\hat{\gamma}_\omega^S + \beta_\omega^S \sum_{i\in I} \tau_{i\omega}\left(b_{i\omega} + c_{i\omega}q_i^F + \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}}\right) - \beta_\omega^S \sum_{i\in I} q_i^F\right] + \\ &- \tau_{i\omega}\left[b_{i\omega} + c_{i\omega}q_i^F + \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}}\right] \quad \forall i, \forall\omega \end{aligned}$$

Proposition 6 *Given the spot emissions in (3.3.3), the pre-existing allowance futures commitment, ε_i^F, and the knowledge of $q_{i\omega}^S$ from Proposition 5, the equilibrium amount of shortage/surplus of emission allowances for each generator at each scenario in the spot market is expressed as:*

$$\begin{aligned}\varepsilon_{i\omega}^S &= \hat{\eta_{i\omega}} \sum_{i\in I} q_i^F + \hat{\eta_{i\omega}} \sum_{i\in I} \tau_{i\omega}\varphi_\omega \left[\hat{\gamma}_\omega^S + \beta_\omega^S \sum_{i\in I} \tau_{i\omega}\left(b_{i\omega} + c_{i\omega}q_i^F + \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}}\right)\right] - \varepsilon_i^F + \\ &- \hat{\eta_{i\omega}} \sum_{i\in I} \tau_{i\omega}\varphi_\omega\beta_\omega^S \sum_{i\in I} q_i^F - \hat{\eta_{i\omega}} \sum_{i\in I} \tau_{i\omega}\left[b_{i\omega} + c_{i\omega}q_i^F + \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}}\right] \quad \forall i, \forall\omega \quad (3.3.21)\end{aligned}$$

Proof 6 *The spot market emissions are expressed as $\varepsilon_{i\omega}^S = \hat{\eta_{i\omega}} \sum_{i\in I}(q_{i\omega}^S + q_i^F) - \varepsilon_i^F$ in (3.3.3). Expression (3.3.21) is obtained by substituting the spot market equilibrium quantity from Proposition 5 into (3.3.3).*

Finally, as indicated above, for a given level of futures contracting, the optimal production at each scenario ω for RES generator j can be expressed as $q_{j\omega}^S = Q_{j\omega} - q_j^F$.

First Stage: Futures Market Analysis

Now we go one step backward in time to analyze generators' futures market contracts. We can substitute the spot market equilibrium outcomes obtained in the previous section (which are parameterized in the futures decisions variables) in the profit functions expressed in (3.3.1) and (3.3.4) for conventional and RES technologies, respectively.

Assumption 4 *Each generator k has an estimation (conjecture) of the impact that its production q_k^F may have in the futures price and competitors' production.*

Notice that this assumption is important to compute $\frac{\partial \Pi_{k\omega}}{\partial q_k^F}$ which in turn depends on $\psi_k = \frac{\partial q_{-k}^F}{\partial q_k^F}$, which is also a pre-specified parameter used to model different levels of competitions in the futures market. Let us first derive the partial derivatives of the profit function with respect to q_i^F, ε_i^F, and q_j^F, respectively, which play a key role in the following section.

$$\begin{aligned}\frac{\partial \Pi_{i\omega}}{\partial q_i^F} &= \frac{\partial P_\omega^S}{\partial q_i^F} q_{i\omega}^S + P_\omega^S \frac{\partial q_{i\omega}^S}{\partial q_i^F} - b_{i\omega}\left(1 + \frac{\partial q_{i\omega}^S}{\partial q_i^F}\right) - c_{i\omega}\left[q_i^F + q_{i\omega}^S\right]\left(1 + \frac{\partial q_{i\omega}^S}{\partial q_i^F}\right) \\ &\quad + \frac{\partial P^F}{\partial q_i^F} q_i^F - \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}}\left(1 + \frac{\partial q_{i\omega}^S}{\partial q_i^F}\right) + P^F \quad \forall i, \forall \omega \end{aligned} \tag{3.3.22a}$$

$$\frac{\partial \Pi_{i\omega}}{\partial \varepsilon_i^F} = P_\omega^{S_{CO_2}} - P^{F_{CO_2}} \quad \forall i, \forall \omega \tag{3.3.22b}$$

$$\frac{\partial \Pi_{j\omega}}{\partial q_j^F} = \left(\frac{\partial P^F}{\partial q_j^F} - \frac{\partial P_\omega^S}{\partial q_j^F}\right) q_j^F - P_\omega^S + \frac{\partial P_\omega^S}{\partial q_j^F} Q_{j\omega} + P^F \quad \forall j, \forall \omega \tag{3.3.22c}$$

Moreover, considering the spot market equilibrium outcomes derived in the aforementioned section, we can also compute the following partial derivatives:

$$\frac{\partial P^F}{\partial q_i^F} = -\beta^F\left(1 + \sum_{(k \in I \cup J) \neq i} \frac{\partial q_k^F}{\partial q_i^F} + \sum_{j \in J} \frac{\partial q_j^F}{\partial q_i^F}\right) = -\beta^F(1 + (I + J - 1)\psi_i) \quad \forall i \tag{3.3.23a}$$

$$\frac{\partial P_\omega^S}{\partial q_i^F} = \varphi_\omega \beta_\omega^S\left[-(1 + (I-1)\psi_i) + \varphi_\omega c_{i\omega}\tau_{i\omega} + \sum_{k \neq i} c_{k\omega}\psi_k\tau_{k\omega}\right] \quad \forall i, \forall \omega \tag{3.3.23b}$$

$$\frac{\partial q_{i\omega}^S}{\partial q_i^F} = \tau_{i\omega}\left(\frac{\partial P_\omega^S}{\partial q_i^F} - c_{i\omega}\right) \quad \forall i, \forall \omega \tag{3.3.23c}$$

$$\frac{\partial P^F}{\partial q_j^F} = -\beta^F(1 + (I + J - 1)\psi_j) \quad \forall j \tag{3.3.23d}$$

$$\frac{\partial P_\omega^S}{\partial q_j^F} = 0 \quad \forall j, \forall \omega \tag{3.3.23e}$$

$$\frac{\partial q_\omega^S}{\partial q_j^F} = -1 \quad \forall j, \forall \omega. \tag{3.3.23f}$$

Risk Averse Futures Market Equilibrium

As indicated, by replacing the equilibrium spot price (P_ω^S) and quantities ($q_{i\omega}^S$ and $q_{j\omega}^S$) in the profit functions $\Pi_{i\omega}$ and $\Pi_{j\omega}$, we can parameterize profits in the futures decision variables:

$$\Pi_{i\omega} = \Pi_{i\omega}(q_i^F, q_{-i}^F, \varepsilon_i^F, \varepsilon_{-i}^F, q_{j\in J}^F) \quad \forall i, \forall \omega \tag{3.3.24a}$$

$$\Pi_{j\omega} = \Pi_{j\omega}(q_j^F, q_{-j}^F, q_{i\in I}^F) \quad \forall j, \forall \omega \tag{3.3.24b}$$

The profit maximization problem solved by the risk averse generators is formulated in (3.3.25) where $\sigma_{k\omega}$ is the probability assigned by generator $k \in I \cup J$ to scenario ω. $1-\alpha$ represents the level of significance associated with the CVaR (linear formulation introduced by Rockafellar and Royset (2010)). Therefore, risk averse problem solved by each generator is:

$$\max_{\xi_k, \eta_{k\omega}, q_k^F, \varepsilon_i^F} (1-\phi) \sum_{\omega\in\Omega} \sigma_{k\omega}\Pi_{k\omega} + \phi \left[\xi_k - \frac{1}{1-\alpha} \sum_{\omega\in\Omega} \sigma_{k\omega}\eta_{k\omega} \right] \tag{3.3.25a}$$

s.t.

$$\eta_{k\omega} + \Pi_{k\omega} - \xi_k \geq 0 \colon \mu_{k\omega} \quad \forall k, \forall \omega \tag{3.3.25b}$$

$$\eta_{k\omega} \geq 0 \colon \theta_{k\omega} \quad \forall k, \forall \omega \tag{3.3.25c}$$

$$q_k^{F_{min}} \leq q_k^F \leq q_k^{F_{max}} \colon \nu_k^{min}, \nu_k^{max} \quad \forall k \tag{3.3.25d}$$

$$\varepsilon_k^{F_{min}} \leq \varepsilon_k^F \leq \varepsilon_k^{F_{max}} \colon \lambda_k^{min}, \lambda_k^{max} \quad \forall k \tag{3.3.25e}$$

The objective function is expressed as the expected profit (first term) and the CVaR (second term) multiplied by a risk aversion parameter $\phi \in [0,1]$. ϕ regulates the balance between expected profits and the CVaR for a given confidence level α. In other words, $\phi = 0$ represents maximization of the expected profits (risk neutral agent), and $\phi = 1$ corresponds to the maximization of the most risk averse setting in which all the weight in the objective function is placed on the CVaR. The optimal value of the auxiliary variable ξ_k corresponds to the VaR and $\eta_{k\omega}$ represents the positive discrepancy between VaR and each profit scenario. Constraints (3.3.25d) and (3.3.25e) enforce the lower and upper bounds for generation and emission, respectively, of generator k. $\theta_{k\omega}, \nu_k^{min}, \nu_k^{max}, \lambda_k^{min}$, and λ_k^{max} are the Lagrangian ($\mathcal{L}$) multipliers associated with the constraints of CVaR, minimum and maximum quantities of generation and emissions which are treated as variables in the equilibrium model. Note that $\Pi_{k\omega}$ has a quadratic expression (3.3.24) so that problem (3.3.25) is indeed a quadratic optimization problem with quadratic constraints.

Finally, the market equilibrium model is obtained by solving simultaneously (3.3.25) for all generators $k \in I \cup J$ after replacing (3.3.25) by its associated KKT system of optimality conditions. In particular, the KKT conditions associated with generator k is:

$$\frac{\partial \mathcal{L}}{\partial q_k^F} = -(1-\phi)\sum_{\omega\in\Omega}\sigma_{k\omega}\frac{\partial \Pi_{k\omega}}{\partial q_k^F} - \sum_{\omega\in\Omega}\mu_{k\omega}\frac{\partial \Pi_{k\omega}}{\partial q_k^F} - \nu_k^{min} + \nu_k^{max} = 0 \quad \forall\omega \tag{3.3.26a}$$

$$\frac{\partial \mathcal{L}}{\partial \varepsilon_k^F} = -(1-\phi)\sum_{\omega\in\Omega}\sigma_{k\omega}\frac{\partial \Pi_{k\omega}}{\partial \varepsilon_k^F} - \sum_{\omega\in\Omega}\mu_{k\omega}\frac{\partial \Pi_{k\omega}}{\partial \varepsilon_k^F} - \lambda_k^{min} + \lambda_k^{max} = 0 \quad \forall\omega \tag{3.3.26b}$$

$$\frac{\partial \mathcal{L}}{\partial \eta_{k\omega}} = \phi\frac{1}{1-\alpha}\sigma_{k\omega} - \mu_{k\omega} - \theta_{k\omega} = 0 \quad \forall\omega \tag{3.3.26c}$$

$$\frac{\partial \mathcal{L}}{\partial \xi_k} = -\phi + \sum_{\omega\in\Omega}\mu_{k\omega} = 0 \quad \forall\omega \tag{3.3.26d}$$

$$0 \le \eta_{k\omega} + \Pi_{k\omega} - \xi_k \perp \mu_{k\omega} \ge 0 \quad \forall\omega \tag{3.3.26e}$$

$$0 \le \eta_{k\omega} \perp \theta_{k\omega} \ge 0 \quad \forall\omega \tag{3.3.26f}$$

$$0 \le q_k^F - q_k^{F_{min}} \perp \nu_k^{min} \ge 0 \tag{3.3.26g}$$

$$0 \le q_k^{F_{max}} - q_k^F \perp \nu_k^{max} \ge 0 \tag{3.3.26h}$$

$$0 \le \varepsilon_k^F - \varepsilon_k^{F_{min}} \perp \lambda_k^{min} \ge 0 \tag{3.3.26i}$$

$$0 \le \varepsilon_k^{F_{max}} - \varepsilon_k^F \perp \lambda_k^{max} \ge 0. \tag{3.3.26j}$$

For computational purposes, it is convenient to reformulate the system of equations in (3.3.26) $\forall k$ as nonlinear optimization problem that minimizes sum of complementarity constraints (of a form $xy = 0$) in (3.3.26) subject to the remaining set of inequality and equality constraints in (3.3.26).

$$\begin{aligned}
&\text{Min} \sum_{i\in I}\sum_{\omega\in\Omega}\mu_{i\omega}(\eta_{i\omega}+\Pi_{i\omega}-\xi_i)+\sum_{j\in J}\sum_{\omega\in\Omega}\mu_{j\omega}(\eta_{j\omega}+\Pi_{j\omega}-\xi_j)+\sum_{i\in I}\sum_{\omega\in\Omega}\eta_{i\omega}\theta_{i\omega}\\
&+\sum_{j\in J}\sum_{\omega\in\Omega}\eta_{j\omega}\theta_{j\omega}+\sum_{i\in I}\left(q_i^F-q_i^{F_{min}}\right)\nu_i^{min}+\sum_{j\in J}\left(q_j^F-q_j^{F_{min}}\right)\nu_j^{min}\\
&+\sum_{i\in I}\left(q_i^{F_{max}}-q_i^F\right)\nu_i^{max}+\sum_{j\in J}\left(q_j^{F_{max}}-q_j^F\right)\nu_j^{max}+\sum_{i\in I}\left(\varepsilon_i^F-\varepsilon_i^{F_{min}}\right)\lambda_i^{min}\\
&+\sum_{i\in I}\left(\varepsilon_i^{F_{max}}-\varepsilon_i^F\right)\lambda_i^{max} \qquad (3.3.27)
\end{aligned}$$

subject to

$$\text{equalities} \quad (3.3.26a)-(3.3.26d) \qquad (3.3.28)$$

$$\text{inequalities} \quad (3.3.26e)-(3.3.26j) \qquad (3.3.29)$$

$$(3.3.1),(3.3.3),(3.3.8),(3.3.19),(3.3.23) \qquad (3.3.30)$$

where (3.3.1) and (3.3.3) are profit definitions for conventional and RES generators, respectively, whereas, (3.3.8) and (3.3.19) are equilibrium spot price and spot quantity, and (3.3.23) the partial derivatives completing the futures market.

Problem (3.3.27) can be tackled by using standard NLP solvers. Moreover, if a solution to (3.3.27) renders an objective function value equal to 0, then this is also the solution of system (3.3.26) (Leyffer and Munson, 2010).

3.3.2 Spot Market with no Futures

Finally, to complete the analysis and for comparison purposes, we derive the equilibrium market outcomes of a market with no futures trading (neither electricity nor allowances). The profit for conventional generator i with no futures trading, subject to uncertain emissions auctioning, is expressed as:

$$\Pi_{i\omega}(q_{i\omega}^S, q_{-i\omega}^S) = P_\omega^S q_{i\omega}^S - a_{i\omega} - b_{i\omega} q_{i\omega}^S - \frac{1}{2} c_{i\omega} (q_{i\omega}^S)^2 - \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}} q_{i\omega}^S \quad \forall \omega \tag{3.3.31}$$

and the profit for renewable generator j is expressed as:

$$\Pi_{j\omega} = P_\omega^S Q_{j\omega} \quad \forall \omega \tag{3.3.32}$$

In this case, the inverse demand function is expressed as:

$$P_\omega^S = \gamma_\omega^S - \beta_\omega^S \left(\sum_{i \in I} q_{i\omega}^S + \sum_{j \in J} Q_{j\omega} \right) \quad \forall \omega \tag{3.3.33}$$

Which can be simplified as:

$$P_\omega^S = \hat{\gamma}_\omega^S - \beta_\omega^S \sum_{i \in I} q_{i\omega}^S \quad \forall \omega \tag{3.3.34}$$

where

$$\hat{\gamma}_\omega^S = \gamma_\omega^S - \beta_\omega^S \sum_{j \in J} Q_{j\omega}. \quad \forall \omega$$

The equilibrium in the spot market, per scenario ω, is reached when all generators i maximize their profits simultaneously:

$$\frac{\partial \Pi_{i\omega}}{\partial q_{i\omega}^S} = \frac{\partial P_\omega^S}{\partial q_{i\omega}^S} q_{i\omega}^S + P_\omega^S - b_{i\omega} - c_{i\omega} q_{i\omega}^S - \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}} = 0 \tag{3.3.35}$$

Simplifying (3.3.35) gives:

$$q_{i\omega}^S = \tau_{i\omega} (P_\omega^S - b_{i\omega} - \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}}) \quad \forall i, \forall \omega \tag{3.3.36}$$

By substituting (3.3.34) into (3.3.36), we obtain:

$$P_\omega^S + \beta_\omega^S \sum_{i \in I} \tau_{i\omega} P_\omega^S = \hat{\gamma}_\omega^S + \beta_\omega^S \sum_{i \in I} \tau_{i\omega} (b_{i\omega} + \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}})$$

$$P_\omega^S \left(1 + \beta_\omega^S \sum_{i \in I} \tau_{i\omega} \right) = \hat{\gamma}_\omega^S + \beta_\omega^S \sum_{i \in I} \tau_{i\omega} (b_{i\omega} + \hat{\eta_{i\omega}} P_\omega^{S_{CO_2}}) \tag{3.3.37}$$

Finally, the equilibrium clearing price per scenario ω is:

$$P_{\omega}^{S} = \varphi_{\omega}\left[\hat{\gamma}_{\omega}^{S} + \beta_{\omega}^{S}\sum_{i\in I}\tau_{i\omega}(b_{i\omega} + \hat{\eta_{i\omega}}P_{\omega}^{S_{CO_2}})\right] \tag{3.3.38}$$

By substituting (3.3.38) into (3.3.36), we can get equilibrium quantity for conventional generators as:

$$q_{i\omega}^{S} = \tau_{i\omega}\left[\varphi_{\omega}\left(\hat{\gamma}_{\omega}^{S} + \beta_{\omega}^{S}\sum_{i\in I}\tau_{i\omega}(b_{i\omega} + \hat{\eta_{i\omega}}P_{\omega}^{S_{CO_2}})\right) - b_{i\omega} - \hat{\eta_{i\omega}}P_{\omega}^{S_{CO_2}}\right] \quad \forall i, \forall\omega \tag{3.3.39}$$

$$\varepsilon_{i\omega}^{S} = \hat{\eta_{i\omega}}\sum_{i\in I}q_{i\omega}^{S} \quad \forall i, \forall\omega \tag{3.3.40}$$

Finally, the emission in the spot market is expresses as:

$$\varepsilon_{i\omega}^{S} = \hat{\eta_{i\omega}}\sum_{i\in I}\tau_{i\omega}\left[\varphi_{\omega}\left(\hat{\gamma}_{\omega}^{S} + \beta_{\omega}^{S}\sum_{i\in I}\tau_{i\omega}(b_{i\omega} + \hat{\eta_{i\omega}}P_{\omega}^{S_{CO_2}})\right) - b_{i\omega} - \hat{\eta_{i\omega}}P_{\omega}^{S_{CO_2}}\right] \quad \forall i, \forall\omega \tag{3.3.41}$$

3.4 Numerical Analysis

We have analytically derived the game-theoretical equilibrium models (the general model and the spot only market model). It should be noted that modeling emissions auctioning in oligopolistic electricity market with financial derivatives is the key focus of this chapter. In order to evaluate the efficiency of these models, we carried out various numerical studies.

3.4.1 Data

This section provides numerical examples to show the performance of the proposed market equilibrium models. We have calibrated the data to get working parameters which approximate the behavior of a realistic electricity market with contracts and emissions auctioning. We compare the parameters with realistic data from the Spanish electricity market (OMIE, 2020) that combines futures contracts and pool market structures. We also use the EEX[17] (EEX, 2020) to compare the EUA futures contracting prices. Electricity generators in the 31 countries under EU ETS can sign futures contracts (quantity, price, and delivery date) for electricity and emission allowances.

Our model considers three conventional generators and one large RES generator participating in the electricity and emission trading market. To supply the aggregate demand, we are considering large generators whose total output is the sum of several units with different technologies. Hence, based on the Central Limit Theorem we can reasonably assume that the random parameters (renewable output, generation costs and demand curve) can be modeled by normal distribution. With that, we examine the impact of renewable penetration and CO_2 price variation on prices (futures and expected spot prices), quantities (futures and spot), and profits (for conventional and RES).

We randomly generate the quadratic cost parameters (which is consistent with the approach in Goudarzi et al. (2017); Ruiz et al. (2012); Oliveira et al. (2013)). For the emission intensity, our parameters are consistent with Farhat and Ugursal (2010). The mean values and coefficient of variations of cost parameters, emission intensity factors and the maximum generation quantities and maximum emissions auctioned for each conventional generator i are presented in Table 3.1. Note that the standard deviation is calculated with $\sigma = \mu \times CV$.

[17]EEX is a leading exchange platforms in the European power market that offers trading in power derivatives for €-denominated cash-settled futures contracts for 20 European power markets across Europe.

	Generators			
Parameter	$i=1$	$i=2$	$i=3$	CV
$a_{i\omega}$	35.00	45.00	50.00	0.10
$b_{i\omega}$	27.00	35.00	43.00	0.13
$c_{i\omega}$	0.015	0.008	0.013	0.15
$\hat{\eta_{i\omega}}$	0.67	0.50	0.49	0.05
$\varepsilon_i^{F_{max}}$	20,000	23,000	19,000	
$q_k^{F_{max}}$	21,000	21,000	25,000	

Table 3.1: Mean values and coefficient of variations used to calibrate the data applied in our simulations for conventional generators.

The cost parameter $a_{i\omega}$ is the cost intercept and load cost for generator i, and scenario ω, generated randomly with mean values equal to [35,45,50][€/MWh] and CV of 10%. Similarly, $b_{i\omega}$ and $c_{i\omega}$ are generated randomly for each scenario. That means, each realization of $b_{i\omega}$ is generated from a multivariate normal distribution with mean $[27, 35, 43]$ [€/MWh] and a CV of 13%, and $c_{i\omega}$ is randomly generated from a normal distribution with mean $[0.015, 0.008, 0.013]$ [€/MWh^2] and CV of 0.015. The intensity factor for generator i, $\hat{\eta_{i\omega}}$ is randomly generated from a normal distribution with mean $[0.67, 0.50, 0.492]$ [CO_2/MWh] and CV of 0.05.

The demand curve parameters for electricity market are generated by approximating the aggregated step-wise demand curve in the spot market, using the futures market intercept $\mu_{\gamma^F} = 180$ and CV of 0.15. The slope is $\mu_{\beta^F} = 0.005$ with CV of 0.057. The expected value of the parameters of the inverse spot market demand curve equal those of the futures demand curve, i.e., $\gamma^F = E[\gamma_\omega^S]$ and $\beta^F = E[\beta_\omega^S]$. From this, γ_ω^S for each scenario is randomly generated with a normal distribution as $\gamma_\omega^S \sim N(\gamma^F, \sigma_{\gamma^F})$ and $\beta_\omega^S \sim N(\beta^F, \sigma_{\beta^F})$, where CV for γ^F and β^F are equal to

0.15. and 0.005, respectively. For simplicity, no correlation among cost, or demand parameters are considered.

For the CO_2 price in the spot market, we take the mean futures CO_2 price to be 25€/MWh with a CV of 0.16 and randomly generated with ω scenario. The parameters for the futures CO_2 prices are chosen based on the EUA futures prices presented by EEX futures market. The data covers daily trading prices of 215 working days from January 7, 2019, to December 16, 2019 (EEX, 2020). Thus, for the CO_2 price analysis, we use the CO_2 price vector in the range of $[0, 50]$ which corresponds with EEX EUA futures price.

The average renewable energy total production for generator j is in the range $[0, 10000]$MWh with a CV of 0.057.

These data are generated to test the models and study the behavior of the equilibrium market outcomes, with increasing penetration of RES capacity and CO_2 prices. Each market equilibrium is first characterized by the optimal values of the first-stage decision variables (P^F and q_k^F), and by the second-stage variables (P_ω^S and $q_{k\omega}^S$), that determine the spot equilibrium market outcomes with scenario ω. The models have been implemented in JuMP version 0.21.1 (Dunning et al., 2017) under the open source Julia programming Language version 5.2.1 (Bezanson et al., 2017). We use Artelys Knitro solver version 12.2 (Byrd et al., 2006) on an CPU E5-1650v2@3.50GHz and 64.00 GB of RAM running workstation. We analyze multiple cases with different number of equiprobable scenarios which are selected based on the overall computational performance. For instance, the risk aversion simulations using RES as a parameter for the Cournot model with $|\Omega| = 320$ scenario is solved in 675.094 seconds of elapsed CPU time (120.38 M allocations: 4.165 GiB, 0.24% gc time).

$\lvert\Omega\rvert$	Risk Neutral Model				Risk Averse Model			
	Cournot Model		Competitive Model		Cournot Model		Competitive Model	
	RES	CO_2	RES	CO_2	RES	CO_2	RES	CO_2
100		✓						
125	✓		✓					
200				✓				
320					✓	✓	✓	✓

Table 3.2: Scenarios for each case.

3.4.2 Result Presentation and Discussion

Based on the risk aversion problems specified in (3.3.1), two risk profiles are demonstrated and discussed. The parameter $\phi = 0$ is used for the risk neutral generator, whereas $\phi = 1$ is applied for the risk averse generators (by changing the parameter ϕ between zero to one, the tolerable risk level of generators is adjusted). The threshold of the value at risk α is fixed at 0.90.

As we have multiple cases, we simulate them differently and the results are reported based on different working scenarios[18]. Table 3.2 presents the number of working scenarios for all the cases into consideration. In Table 3.3, the Simulation run for Cournot competition with RES parameter for different number of scenarios. The sensitivity results (market outcomes) reported are the mean values for the RES parameters, from 0 to 10,000 in 1000 ranges (11 parameters). As the results show,

[18]Most stochastic models use a limited number of scenarios for computational reasons. However, to investigate the effect of risk aversion using CVaR, this is not enough. In the real world, we need to consider low-probability/high-consequence scenarios that can have a significant effect on expected profits or welfare if they occur. Moreover, the smallest meaningful CVaR threshold level of α in a model with Ω scenarios is $1/\Omega$, while typical threshold values are closer to 5% (Munoz et al., 2017). Hence, we simulate as large as 320 scenarios for the risk aversion (models with CVaR) to characterize the model realistically.

Ω	EX.P^F	EX.P^S_ω	Con. Profit (Expected)	RES. Profit (Expected)
150	97.90	86.29	70,546	432,652
200	96.28	86.06	71,056	432,851
320	98.08	87.01	73,291	434,447
350	98.53	86.65	72,256	433,660
400	98.23	86.22	70,003	431,351
500	97.67	85.80	69,254	428,312

Table 3.3: Simulation for Cournot competition with RES parameter for different number of scenarios.

there is no significant difference in outcomes due to the increase/decrease in the number of scenarios. This attests the number of scenarios used in our reporting works properly and change neither the concluding results nor the objective of the models.

For the CVaR simulations, our model works even for a larger scenario (for instance, 500 and more). However, if we further increase the number of scenarios for some parameter values, we will have non-zero objective function results, which show the solution is not optimal. To that end, we report the case studies with $\omega = 320$ (well above the threshold (≥ 30) for CVaR at $\alpha = 0.90$), where the number of scenarios is sufficiently large to ensure the stability of the CVaR while keeping the problem tractable.

For the sake of convenience, we present our simulation results in two parts. Since we intend to characterize the risk neutral and risk averse generators by CVaR formulation, we present the result as:

1. *Risk Neutral Generators:* with the Cournot market and the perfect competitive market structure, we discuss the general model and the spot only market. The focus is on the equilibrium prices, profits and quantities (total trading) that

are presented and discussed with respect to RES penetration and CO_2 price parameters.

2. *Risk Averse Generators:* we use a similar approach than the risk neutral case, except, we impose the CVaR parameter to be $\phi = 1$.

3.4.3 Risk Neutral Generators Numerical Results

This section discusses the equilibrium market for risk neutral generators using RES penetration and CO_2 price as parameters of interest.

Sensitivity on Renewable Penetration

For the RES penetration sensitivity analysis, CO_2 price is fixed at its mean value of 25€/MWh and a CV of 0.16. Let us first examine the Cournot competition equilibrium results from Figure 3.1. The futures price of electricity is negatively correlated to the RES penetration (diminishing from around 126 to 103€/MWh). Similar behavior is observed for the expected spot price (with a range of variation of approximately $[84, 113]$ €/MWh). This can be explained by the substitution of conventional generations for RES ones-exhibiting null production costs and curb the level of electricity prices in both markets. Expected spot electricity prices (with the presence of futures market and without the presence of futures) are overlapped for RES less than 5,000MWh. For RES $> 5,000$MWh, the expected spot price in the presence of the futures market decreases faster than the expected spot price in the GM. The futures price is higher than the expected spot one.

The overall RES generators' profit increases with an increasing rate with respect to RES penetration. However, conventional generators' profit decreases with RES penetration. Despite conventional generators spot market trading consistently decreases with RES penetration, their trading quantities in the futures market increases, which in turn helps them to balance their profit. Since RES penetration increases the total quantities traded by RES generators, their profit is always increasing. Conventional generators trade emissions only in the spot market. Therefore, futures market emission is almost zero with RES penetration. Note that total-vis-à-vis spot CO_2 emissions consistently decreases with respect to RES penetration.

With the perfect competition, we observe different behaviors. Futures and expected spot prices without the presence of futures market overlapped and vary from 85 to 106€/MWh (refer Table 3.4 and Figure 3.2). Since RES penetration decreases electricity prices, there is no incentive for conventional generators to compete in the futures market where their futures market trading decreases. This is the opposite of the Cournot competition. Comparing the two competitions, spot trading consistently decreases in the Cournot competition and slightly increases/unchanges in the perfect competition corresponding to the RES penetration. Conventional generators earn higher profit with Cournot competition than with perfect competition where futures price ranges from [103,126] to, [85,106]€/MWh, respectively. In the perfect competition, the reduced conventional quantities are substituted by the increase in RES generators quantities. The overall impact of RES penetration for conventional generators is higher in the Cournot competition than in the perfect competition.

Sensitivity on Carbon Price Increase

With the sensitivity analysis on CO_2 price, we try to examine if a EUA allowance auctioning, or charging a higher price of CO_2 can contribute to decarbonizing the electricity system. Therefore, we try to explore its effects on prices, profits, and quantities for RES and conventional generators. While simulating the model by

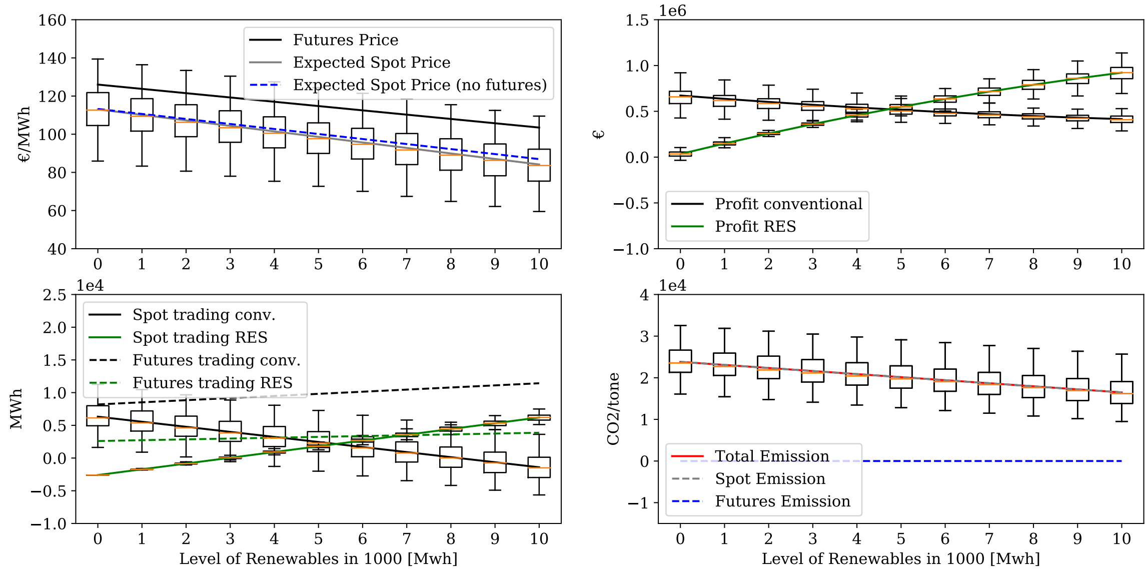


Figure 3.1: Risk neutral Cournot model simulation with results sensitivity analysis on RES penetration.

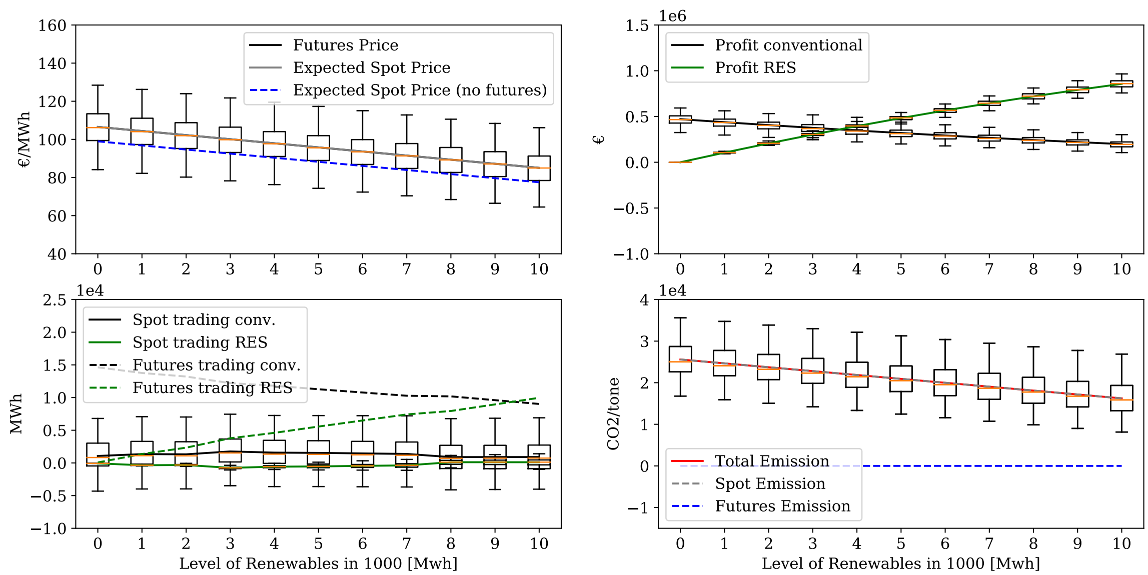


Figure 3.2: Risk neutral perfect competitive model with sensitivity analysis on RES penetration

varying the CO_2 price, the RES parameter is randomly generated/fixed with a mean of 5,000MWh and a CV of 0.057. To examine the results, let us start from the Cournot competition.

The increase in CO_2 price has an increasing impact on the overall electricity price. Futures price increases within the range [108,120]€/MWh, while the expected spot price varies within the range [91,105]€/MWh. The expected spot market electricity price increases faster with the presence of futures market than it does with no futures market. In other words, there is no significant variation regarding the spot only electricity price (refer Table 3.4 for comparison). The insight is that increase in electricity price is partially to cover the increase in the marginal cost of conventional generators.

Despite the large gap between the minimum and the maximum profits based on the whiskers at CO_2 price 30 and 40€/MWh), the overall impact of CO_2 price increase on market outcomes is limited.

The amount of CO_2 emitted by conventional generators generally decreases as their production decreases.[19] The generation of sustainable energy sources contributes to reducing CO_2 emissions, which corroborates with total emissions reduction targets by EU ETS.

Looking at the perfect competition model, the spot and futures market prices are overlapped within the interval [85,101]€/MWh. Prices are lower in the perfect competition than in the Cournot competition (for instance $[85, 101] < [108, 120]$ for futures price in Table 3.4). This explains why conventional generators earn a lower

[19]Note that due to the risk neutrality nature of generators with the CO_2 price, emissions trading in the futures market is highly volatile. Thus, to manage the inherent volatility, we plot the total emissions than the futures and spot emissions in the other cases.

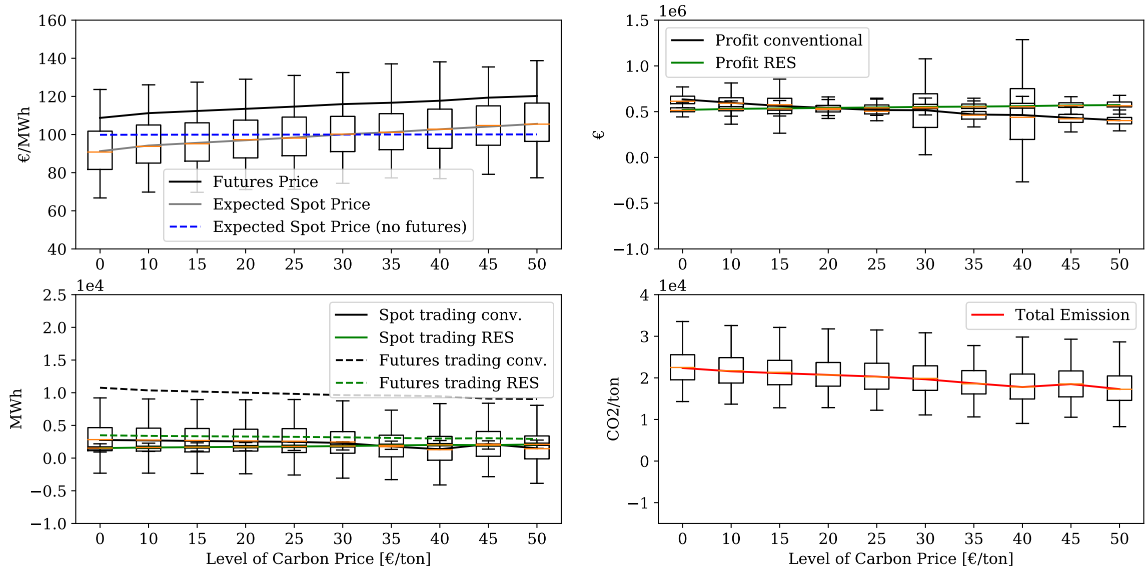


Figure 3.3: Risk neutral Cournot model with sensitivity on CO_2 price

profit than the Cournot competition, albeit trading a larger quantity in perfect competition. Part of the increase of CO_2 price passed into the expected spot and futures prices so that conventional generators still maintain a constant profit with the CO_2 price increase. Similar to the Cournot competition, in the perfect competition the profit whiskers are large.

The negative trading quantities by conventional generators in the spot market are short selling as they trade more electricity in the futures market. Total emissions trading decreases with respect to CO_2 price.

Levels of Competition and Policy Implication

RES penetration and CO_2 price increase affect electricity prices in both levels of competition. RES penetration and CO_2 price increase have an opposite impact on prices. RES penetration reduces prices in both stages/levels of competition. Whereas

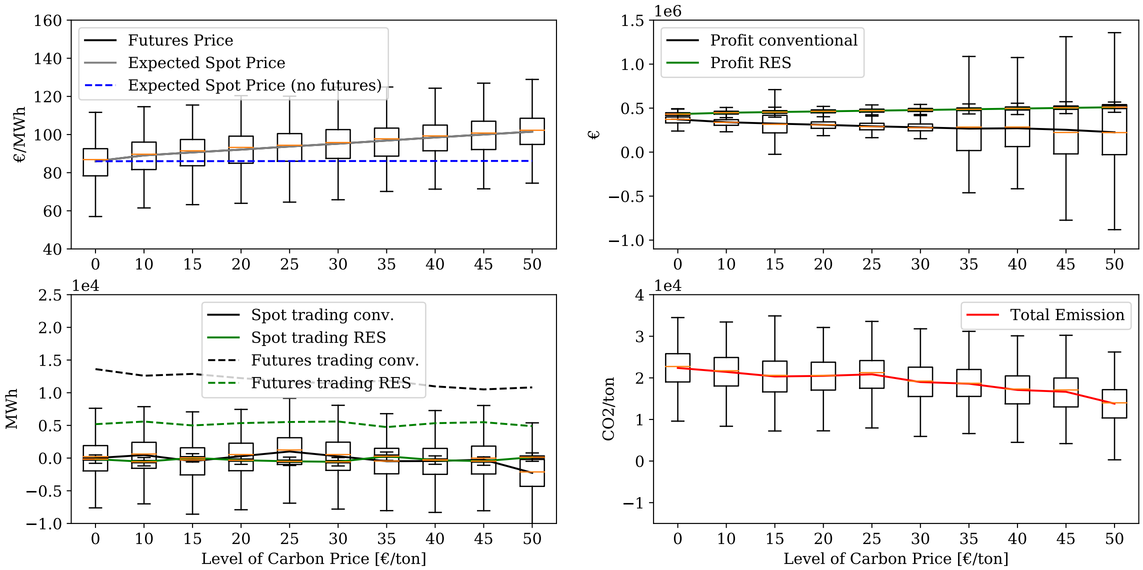


Figure 3.4: Risk neutral perfect competitive model with sensitivity on CO_2 price

the surge in CO_2 price increases the overall price. In the Cournot competition, prices are relatively higher than the perfect competition. Both parameters reduce total emissions trading. With RES penetration, generators trade emissions only in the spot market with a clear diminishing trend. Likewise, total emissions trading decreases with CO_2 price increase.

From the social welfare point of view, the auction-based allowance trading transfers part of generators' profit into social welfare, hence avoiding windfall profit. This helps to reduce GHG emissions with both policies. From the generators' point of view, RES generators penetrate with low-carbon and sustainable generation. These generators obtain higher profits as they do not have production and allowance permits related costs. With both policies, the conventional generators face challenges to stay in the market with competition.

Level of competition with parameter	P^F		P^S_ω the GM		P^S_ω the spot only	
	Risk					
	Neutral	Averse	Neutral	Averse	Neutral	Averse
Cournot RES	103-126	103-122	84-113	83-107	86-113	84-110
Competitive RES	85-106	87-108	85-106	83-105	77-98	76-97
Cournot CO_2	108-120	103-116	91-105	88-103	99-100	97-98
Competitive CO_2	85-101	90-106	85-101	86-102	85-86	86-87

Table 3.4: The minimum and the maximum futures price and expected spot prices, for the GM and spot only market, by competitions with RES penetration and CO_2 price increase.

3.4.4 Risk Averse Generators' Numerical Results

This section discusses the risk averse generators simulation results where generators maximize the CVaR with the risk parameter $\phi = 1$. Computationally, the risk averse model works with a large number of scenarios ($|\Omega| = 320$). In this section, we present the risk averse generators' results based on sensitivities on the two parameters.

Sensitivity on Renewable Penetration

Figure 3.5 and Figure 3.6 illustrate the Cournot competition and perfect competition results for the RES penetration, respectively. In the Cournot competition with risk aversion, the expected spot and futures price behavior is similar to the risk neutral case. The expected spot prices and the futures price decrease from 107 to 83€/MWh, and from 126 to 103€/MWh, respectively. Unlike the risk neutral case with Cournot competition, conventional generators' futures trading slightly decreases. Spot trading is slightly affected by RES penetration especially when the level of RES penetration is greater than 7000MWh. Both spot and futures tradings with respect to RES generators increase. The loss in terms of quantities for conventional generators

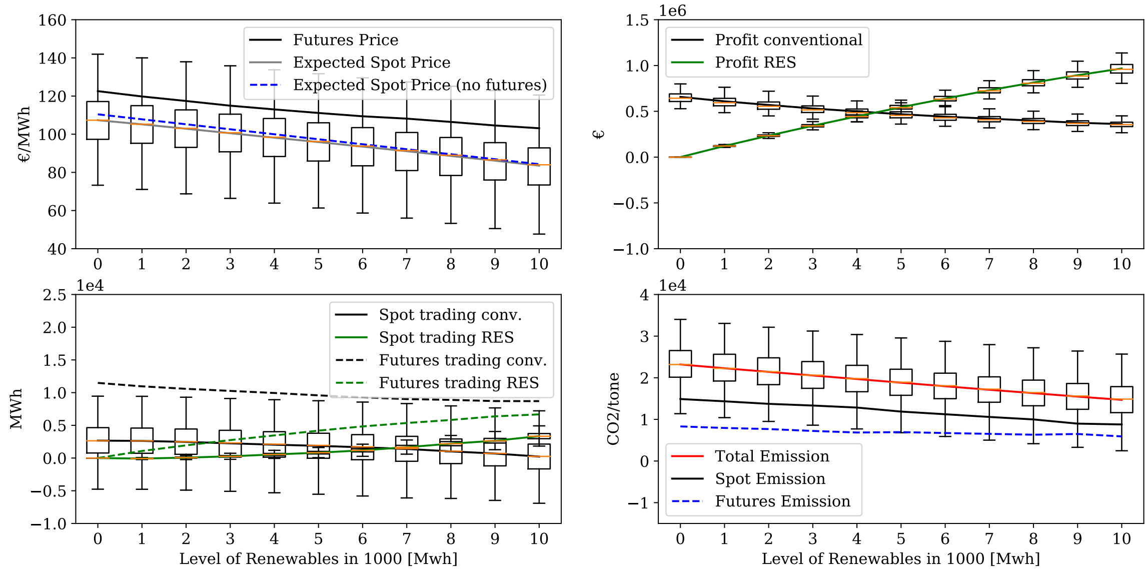


Figure 3.5: Risk Averse Cournot model simulation results with sensitivity analysis on RES penetration

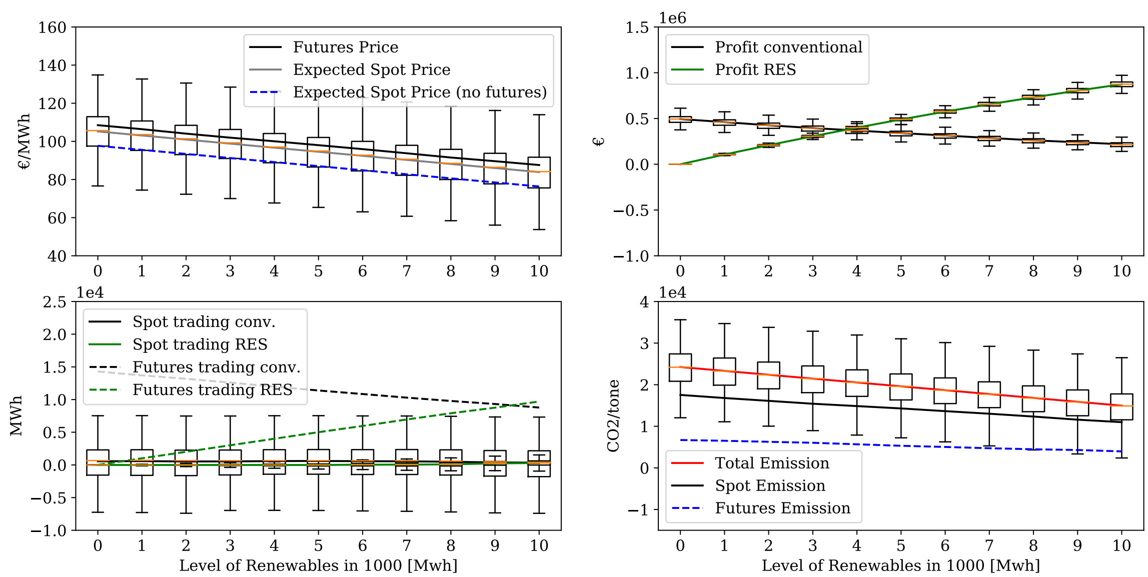


Figure 3.6: Risk Averse perfect competition model with sensitivity analysis on RES penetration

is compensated by the increase in the RES generators' quantities in futures and spot markets. We can then conclude that in the risk aversion with Cournot competition RES penetration is an effective policy, which is a counter-intuitive result of our model.

In the perfect competition, we can observe that the futures and expected spot prices are no more overlapped. P^F and P^S_ω in the GM and P^S_ω in the spot only market are within the range of [87,108],[83,105] and [76,97], respectively (see Table 3.4). For trading quantities, RES penetration increases the traded quantities for RES generators in the futures market and decreases futures trading for conventional generators. Trading quantities in the spot market remain unchanged for both sets of generators. Since the decrease in electricity generation for conventional generators is substituted by the RES, the consumers' welfare is unaffected as large quantities are traded with relatively lower competition prices than the Cournot competition.

Sensitivity on Carbon Price Increase

Figure 3.7 and Figure 3.8 show the results of the increase in CO_2 price for Cournot and perfect competition levels, respectively. In the Cournot competition, the effect of an increase in the CO_2 price on profit is slightly different compared to the risk neutral case. Risk averse conventional generators' profit is decreased more than the risk neutral ones. Similar to the risk neutral case, futures conventional generators trading slightly decreases and spot market tradings are not significantly affected.

For the perfect competition, the expected spot and futures market prices are not overlapped. The higher CO_2 price is partially recovered by higher futures market electricity price that ranges from 90 to 106€/MWh. The increase in CO_2 price increases the overall prices in both stages of the market. This electricity price increase corresponds with the decrease in the futures trading. Spot trading for conventional

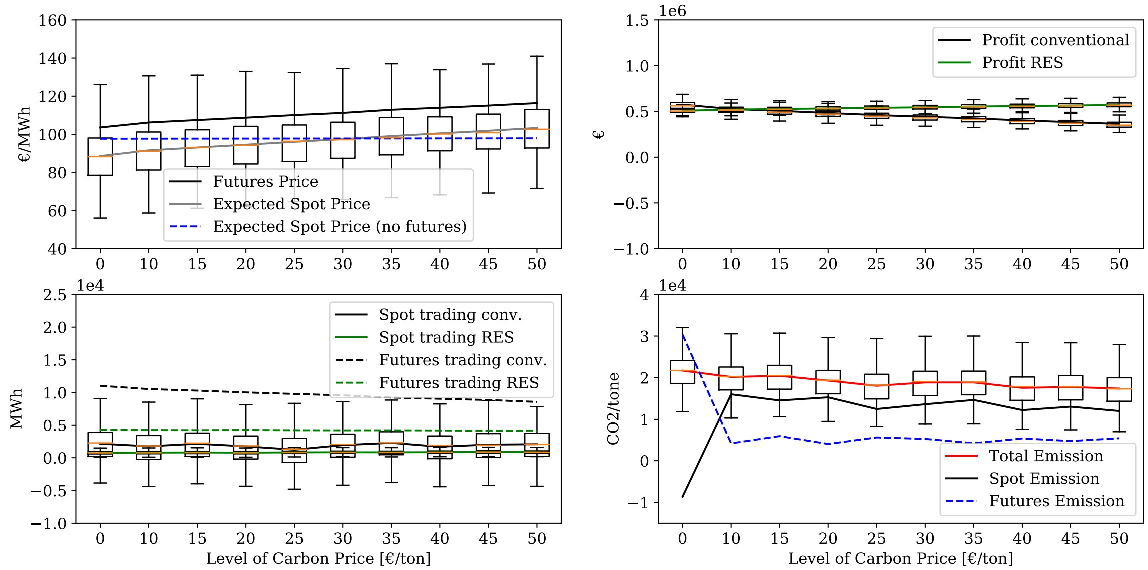


Figure 3.7: Risk Averse Cournot model with sensitivity on CO_2 price.

generators is not affected by the increase in CO_2 price. However, the expected spot price slightly increases for CO_2 price increase. This is induced by the cost increase for conventional generators so that is a cost pass-through to the consumers.

Differently from the risk neutral case in the Cournot competition (as examined above) conventional generators react to the higher CO_2 prices by slightly increasing their spot trading and reducing futures trading. This reduces the impact on the total profit. RES generators benefit from this policy where their profit increases with respect to CO_2 price increase, though no significant increase in their generation. It is possible to conclude that the risk averse setting in both competitions CO_2 price increase is less effective policy than RES penetration.

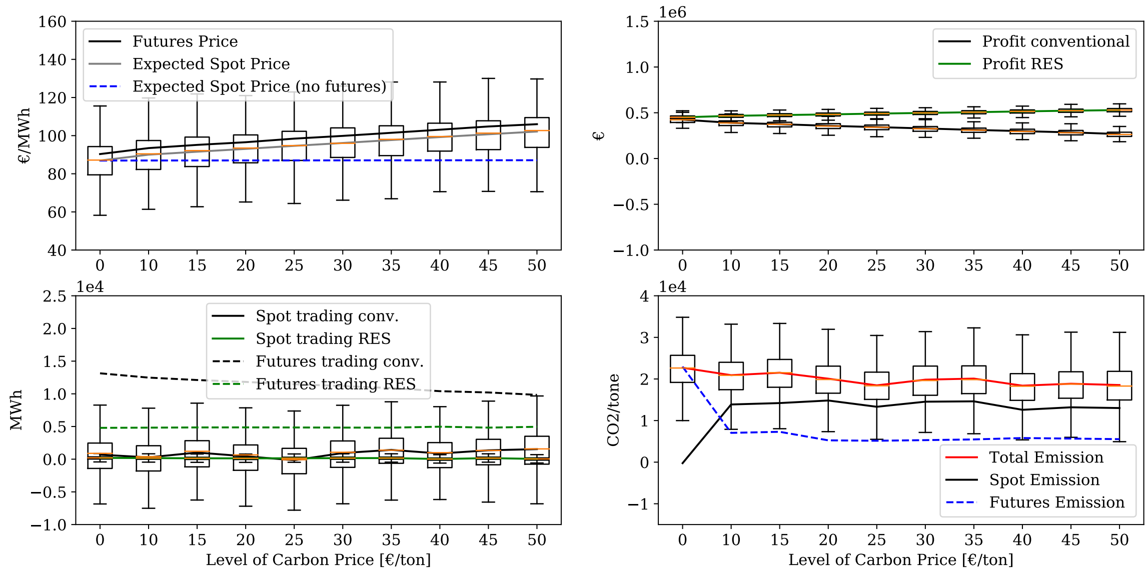


Figure 3.8: Risk Averse perfect competitive model with sensitivity on CO_2 price

Variables	Parameters			
	RES Penetration ([5,000-10,000]MWh)		CO_2 price increase ([25-50]€/MWh)	
	Cournot Model	Competitive model	Cournot Model	Competitive model
P^S_ω	92.70	86.54	94.47	95.96
P^F	107.14	92.74	113.89	103.09
total $\sum_i^I(\varepsilon^S_{i\omega} + \varepsilon^F_i)$	**16,730.78**	**17,289.33**	**18,088.58**	**19,142.94**
$\sum_i^I q^F_i$	9,028.78	10,069.13	9,064.43	10,514.06
$\sum_i^I q^S_{i\omega}$	1,138.59	491.94	1,988.28	1,218.53
total $\sum_i^I(q^F_i + q^S_{i\omega})$	**10,167.37**	**10,561.07**	**11,052.71**	**11,732.59**
$\sum_j^J q^F_j$	5,542.88	7,382.86	4,157.07	4,866.1
$\sum_j^J q^S_{j\omega}$	1,975.83	115.64	849.28	128.74
total $\sum_j^J(q^F_j + q^S_{j\omega})$	**7,518.71**	**7,498.50**	**5,006.35**	**4,994.84**
$\sum_i^I \Pi_{i\omega}$	410,644.03	277,124.65	402,861.70	299,116.87
$\sum_j^J \Pi_{j\omega}$	765,046.94	688,389.88	558,535.16	514,268.11
$\sum_i^I CVaR_i$	317,690.49	190,997.50	305,364.36	207,757.98
$\sum_j^J CVaR_j$	688,191.82	624,651.80	509,343.34	464,237.22

Table 3.5: Expected values of the market outcomes with the CVaR formulation.

Levels of Competition and Policy Implication

Table 3.4 and 3.5 are useful to compare the Cournot and the perfect competitions based on the two sensitivities. With RES penetration, the expected spot electricity price is lower in the perfect competition that range from 83 to 105 €/MWh, and futures price varies from 87 to 108€/MWh. Conventional generators earn higher profit in Cournot competition with RES penetration. This higher profit is induced by the higher futures price ([103,122]€/MWh), and higher expected total tradings (11,480.01MWh). Similarly, with CO_2 price increase, the Cournot competition generates higher profit as a result of a higher P^F ([103,116]€/MWh). RES generators also trade lower total quantities in the Cournot competition but manage to obtain a higher profit with respect to CO_2 price increase. Measured with profit, Cournot competition is better for both sets of generators. However, the perfect competition prevails lower electricity prices with respect to both policies.

Table 3.5 summarizes the two policies' impact on market outcomes when the sensitivities are considered above the values of their respective average. Accordingly, RES penetration decreases total emissions as the conventional generation decreases. However, it increases RES generators' trading share in the market. Note that despite the relatively higher total tradings, both RES penetration and CO_2 price increase in the perfect competition reduces the quantities sold in the spot market for both sets of generators. In other words, higher total quantities are traded in perfect competition than the Cournot competition with respect to both parameters. Electricity prices are lower since there is no cost pass through. More interestingly, RES generators expected profit is higher than their conventional counterparts, which shows both policies are effective in achieving emission reduction targets and green economic path. However, in the grand scheme of things, RES penetration seems to be more effective than CO_2 price increase since it results in lower emissions, lower electricity prices, and higher trading quantities.

3.5 Summary and Conclusion

This chapter proposes a model based on game theory to assess the effects of CO_2 auctioning with futures market contract designs in a stochastic programming approach, with high renewable penetration in oligopolistic electricity markets. The game is based on electricity derivatives and EUA allowances, applying for different levels of competition (Cournot and perfect competition) in a two-stage stochastic model.

We examine risk neutral/averse stochastic and oligopolistic generators, with the option of primarily trading their production and EUA allowances in a futures market (stage-one) and later in a spot market (stage-two) anticipating the inherent uncertainties. The sources of uncertainties considered are electricity, EUA allowance prices, generation cost for conventional generators, and level of nondispatchable RES generation.

A coherent risk measure (CVaR) is used in the model to characterize the behavior of risk averse generators. The futures contracts are introduced for both electricity and allowances (to trade surplus/shortage of allowances in the auction market) to hedge the risk of both prices and RES generation capacity in the equilibrium market model.

Analytically, we have closed-form solutions in the second stage, where we start to derive and move backward to characterize the first stage variables in terms of the second stage equilibrium market outcomes. The global equilibrium of the market is computed from the joint solution of all the generators' profit maximization problems through maximizing the sum of expected profits for risk neutral generators and maximizing the CVaR for risk averse generators. This is done by solving an equivalent system of optimality (KKT) conditions.

The analytical results are checked with a wide computational test to analyze different market configurations, Cournot, and perfect competitive generators together with risk neutrality, or risk averse strategies.

We conclude that RES penetration is an effective and economically efficient parameter in plummeting GHG emissions. The increase of CO_2 emission prices, on the other hand, encourages RES generators by strategically penalizing CO_2 emitters from the market so that it decreases futures and spot emissions. The result corroborates with other findings such as RES deployment and an increase of CO_2 prices lead to higher energy-intensive sectors to leave the industry and lower energy-intensive sectors to be attracted to enter the economy. Future research may include to study the financial and environmental impact of emerging trends in power systems, such as distributed generation, large-scale electricity storage, demand response programs, etc.

Chapter 4: Retailer-Customer Model in Electricity Market under Demand Response

4.1 Introduction

In order to meet emission reduction targets, power systems are evolving towards a generation mix that is more decentralized, less predictable and less flexible to operate due to the massive integration of renewable and distributed energy sources. To enable the large-scale integration of these renewables and to enhance the decarbonizing of electricity systems without endangering the security of supply, additional flexibility is needed to be provided in the form of demand-side management (DSM), and in particular, via demand response programs (He et al., 2013). DSM will be a key component of the future power system that can help reduce peak load and adapt elastic demand to fluctuating and non-dispatchable power generation. In this paper, we consider households (consumers) that operate under demand response (DR) program, explicitly modeled via their utility functions, and a retailer that tries to maximize its profit in a dynamic pricing environment under uncertainty. Determining an optimal operation strategy in electricity markets where traditionally supply must follow demand in real-time is challenging, particularly in the face of high penetration of renewable energy resources (RES) and demand uncertainties (He et al., 2013; Zugno et al., 2013;

Niromandfam et al., 2020). Technically, power generators should submit hourly offer curves the day before energy delivery. Similarly, retailers need to anticipate their total demand load and bid accordingly in the day-ahead market. Otherwise, the market imposes penalties if the power is not delivered/consumed in real-time as planned. The new approaches to cope with this challenge is via techno-economic solutions including DR, RES generation, storage units, and smart grid technologies to exploit end-users flexibility.

With the introduction of tailor-made tariffs, DR programs can motivate end-consumers to change their normal consumption patterns in response to changes in electricity prices over time. These incentive payments can be designed to induce lower electricity use at times of high wholesales market prices or when system reliability is jeopardized. Thus, consumers are expected to be able to manage and adjust their electricity consumption in response to real-time information and changing price signals so that a higher efficiency can be achieved in the power system (Meliopoulos et al., 2011; Blumsack and Fernandez, 2012). In particular, DR models are based on the assumption that consumer demand is elastic and, thus, that consumers will respond to higher prices by reducing demand (Muratori et al., 2014). With DR, consumers can have three options or a combination of therein. These are: reducing consumption during peak demand, shifting to low demand periods, or offsetting power imbalances using on-site/distributive generation sources during high demand periods. As a result, DR in the power system has been recognized as an effective tool to facilitate the integration of intermittent and stochastic energy sources such as wind or solar energy into the power system (Nguyen and Le, 2014b). However, the uncertainties on electricity price, production cost, and electricity demand in the day-ahead (DA) wholesale electricity market are the main challenges that have been dealt with DR programs (Lau et al., 2015; Morales et al., 2013; Hu et al., 2016).

Mainly, the electricity sector deregulation and restructuring, the increasing demand to integrate RES into the power system (Ramchurn et al., 2012; Bonneuil and Boucekkine, 2016), and the increasing application of novel technologies (smart meters and intelligence appliances) have played an important role to integrate DR programs in the electricity system. One of the advantages of DR programs is that it allows higher penetration of intermittent RES in the electric power system. In balancing generation and demand, DR programs help power system to overcome difficulties arising from uncertain nature of intermittent RES. Hence, DR is considered as a resource to access the electricity market so as to reduce system costs, secure supply during peak-demand hours, manage price volatility in real-time and reduce emissions. To exploit these benefits in the fundamental electricity market structure, bidirectional communication systems have been in place to send appropriate signals to responsive consumers nearly in real-time while monitoring system reliability. As shown in many DR programs, consumers tend to change their consumption time and magnitude in response to dynamic tariffs or financial incentives when smart metering and bidirectional communication are available.

DR programs can be facilitated by measures such as load shedding programs or peak-saving, time-of-use, or real-time based consumer tariffs (Zugno et al., 2013; Setlhaolo et al., 2014; Aussel et al., 2020; Brahman et al., 2015; Alipour et al., 2019). As such, they have been categorized as price-based and incentive-based programs. Price-based DR program offers time-differentiated rates, which may depend on the time of the day, week, or season and is expected to lead consumers to adapt the timing of load operation to make the most of the lower price periods (Morales et al., 2013; Hu et al., 2016; Antunes et al., 2020). On the other hand, incentive-based DR program intends to change the consumption patterns via direct load control, interruptible load contracts, peak time rebate, demand bidding, emergency programs, capacity, and ancillary service markets (Albadi and El-Saadany, 2008; Antunes et al., 2020), which provide financial compensation to consumers to shift their consumption. In response to the programs, consumers may weigh their benefits to reduce/shift consumption at critical times while keeping other periods of consumption patterns. Some of these problems have been dealt with price elasticity and consumer utility/welfare functions under DR programs (Morales et al., 2013).

Studying the behavior of consumers using utility functions is an important concept in microeconomics where preferences are measured with mathematical models. Accordingly, a consumer can be considered as flexible (elastic) and inflexible (inelastic) in its electricity demand. Consumers participating in DR can expect savings either through incentive payments, or market-based performance in electricity bills if they reduce their electricity consumption during peak periods. This brings efficient resource utilization that reduces the overall expected electricity prices which eventually increases utility for end-consumers. Similarly, the role of retailers fundamentally lies in acquiring energy from the power markets to resell it to customers by optimizing their benefits/costs. On top of this comes also the need to consider and account for uncertainties which are of great importance for obtaining meaningful solutions for reliability and security. Consequently, there is a need for novel approaches which guide the design of DR programs that integrate RES into the power system wherein consumers' behavior towards the market has to be accounted for explicitly via utility functions under uncertainty. Our paper addresses this research gap: we model and solve an energy retailer's problem whose objective is to maximize its profit, and consumers' problem whose objective is to maximize their utility and reduce their electricity bills, given the inherent uncertainties in the market. The problem is solved by two market frameworks, first by modeling retailer's market power through a bi-level program which can be solved as a nonlinear mathematical program with equilibrium constraints (MPEC), and second by considering an equilibrium model with perfect competition solved as a mixed-integer linear programming (MILP) problem. The required techniques to test and compare the performance of the models are undertaken with realistic data. The paper finds the following main results: The retailer is able to maximize its expected profit with market power, and the consumers are able to minimize their procurement cost and disutility with the equilibrium model. The proposed models are adaptable to any group of consumers with flexible demand and with different market configurations. There is also a significant cost-saving potential while adopting the DR program as the social welfare is optimal for both players. Moreover, modeling the consumers' behavior explicitly with their utility function under a dynamic pricing environment reveals more information towards the integration of the much-needed renewable sources into the power system using smart technologies.

The remainder of this paper is organized as follows. Section 4.2 reviews the current literature on the topic and underlines our contribution. Section 4.3 demonstrates the model formulation which entails the retailer's and consumers' problems. The formulation of the problems as an MPEC, linear reformulation of the problem as MILP in the equilibrium, and the NLP reformulation of the equilibrium are presented. Computational and simulation results are reported and discussed in Section 4.4. Finally, Section 4.5 closes by drawing some concluding remarks from the provided discussions and results.

4.2 Literature review

DR enhances reliability by reducing outages, increasing customers participation and diversifying resources, which, in turn, can benefit retailers by reducing price volatility and increasing market capacity. The objective functions in most DR models encompass economical and technical conditions: quality of service, peak power demand, utility associated with electricity consumption, and discomfort associated with existing consumption patterns (Afşar et al., 2016; Soares et al., 2019; Aussel et al., 2020; Soares et al., 2020; Luo et al., 2020, 1996; Ruiz et al., 2018). For instance, an agent-based model that incorporates both balancing and spot power markets is presented in Kühnlenz et al. (2018). A stochastic multi-objective unit commitment real-time DR models with robust optimization and scenario-based stochastic optimization is proposed in Conejo et al. (2010); Alipour et al. (2019). Corradi et al. (2012) show that price-response dynamics can be used to control electricity consumption by applying a one-way price signal based on data measurable at the grid level. Afşar et al. (2016) present a bi-level optimization model where the supplier establishes time-differentiated electricity prices, and the consumers minimize their electricity consumption costs to enhance grid efficiency. Soares et al. (2019) present a model considering a single leader (retailer) whose aim is to maximize profit and multi-followers (consumers) whose objective is to minimize their electricity cost. Aussel et al. (2020) propose the interactions among electricity suppliers, local agents, aggregators, and consumers with a trilevel multi-leader-multi-follower game model for load shifting induced by the time of use pricing. They solve this presumably hard problem by assuming that the decision variables of all electricity suppliers but one, is known and optimize the decisions of the remaining suppliers as a single-leader-multi-follower game. A novel price-based optimization framework for a DR aggregator was introduced in Brahman et al. (2015) to maximize the profit of the DR aggregator. Soares et al. (2020) develop a comprehensive model in which a retailer maximizes its profit and a cluster of consumers react to the decision of the retailer based on electricity prices by determining the operation of the controllable loads to minimize the electricity bill and a monetized discomfort factor associated with the indoor temperature deviations. Moreover, unit commitment model to analyze the cost of day-ahead unit commitment errors (Sioshansi, 2009), energy scheduling problem for a load-serving entity using flexible and inflexible loads (Nguyen et al., 2016), integrating economic dispatch and efficient coordination of wind power (Ilic et al., 2011), and forecasting conditional load response to varying prices (Corradi et al., 2012), are other examples of effective applications of DR programs. However, there is scarcity of DR models which explicitly incorporate customers' utility function in their lower level problems. Due to its desirable properties, the utility function is a suitable tool to consider consumers' preferences regarding their decision making under uncertainty. In this vein, Ma et al. (2014) and Cicek and Delic (2015) incorporate welfare from electricity consumption using customers' utility function and generation cost with constrained optimization. In additions, Ruiz et al. (2018) analyze the interactions between asymmetric retailers that compete in prices to increase their profits and consumers whose aim is to maximize their utility functions in electricity market. Their simulations show how retail price tariffs can soften the spot price variations if retailers can purchase part of their electricity at fixed tariffs.

4.2.1 Approach and contributions

The objective of this work is to define meaningful decision-making tools and to analyze potential market interactions between electricity retailers and responsive consumers participating in the day-ahead electricity market. We consider a retailer who wants to optimize its expected profit, and $|J|$ consumers who want to minimize the difference between the cost of purchasing electricity and the utility brought by its consumption. That is, the retailer determines the selling prices to maximize the expected profit, which depend on the consumers' purchases and the consumers determine their loads, which depend on the retail prices so as to minimize the sum of the purchasing and the disutility cost. For comparison purposes, the problem is solved with two market configurations, the first one assuming retailers' market power, and the second one assuming a competitive-based equilibrium between the retailer and the consumers.

The market power is dealt with a hierarchical modeling MPEC where there is one retailer, and $|J|$ consumers (followers) expected to react to its price tariffs decisions in the market. This stochastic MPEC (includes uncertainties on consumers' reaction and spot market) is solved by recasting the consumers' problem as its KKT optimality conditions, which enter as constraints in the retailer's problem. Moreover, we explicitly introduce a quadratic utility function in the consumers' problem to provide a mathematical expression for the consumers' preferences for different electricity consumption levels, a concept adapted from consumer theory[20] in microeconomics (Rubinstein et al., 2006). The utility function allows consumers to decide dynamically whether to buy or not at a particular hour. Due to the assumption that the model provides no possibility for consumers to switch to a different retailer (short-term model), the utility function prevents the retailer from increasing the price tariffs possibly up to infinity to maximize its profit. To handle such challenges, for example, Zugno et al. (2013) include minimum, maximum, and average prices into the consumers' problem as constraints. In our model, we rather explicitly model the consumers' problem using their consumption preferences where the utility function operates based on the law of demand. Since we use a quadratic utility in the objective function of the consumers' problem, it is not possible to linearize the resulting MPEC problem. However, we are able to solve the MPEC efficiently as a nonlinear problem by using nonlinear solvers.

For the equilibrium model with perfect competition, we formulate a MILP problem by linearizing the complementarity conditions of the corresponding retailer and consumers KKT optimality conditions. In addition, a NLP reformulation of the equilibrium problem is derived using the methods proposed by Leyffer and Munson (2010), which is equivalent to the MILP formulation.

The model has first-stage and second-stage decisions. Prior to the day-ahead market, the retailer decides the price tariff for hour t (first-stage). However, the spot market price, the quantity purchased by the retailer from the electricity market, the

[20] This theory concerns how consumers spend their money given their preferences and budget constraints. The two primary tools of this theory are utility functions and budget constraints, which allows the consumer to make a decision in regard to their consumption level (Sharifi et al., 2017).

quantity bought by consumers, and the electricity quantity imbalance decisions are postponed (second-stage and scenario dependent) to the time of energy delivery.

The main contributions of this paper can be summarized as follows: (1) to incorporate the DR program in the decision making problem that retailers' and consumers' face when participating in the electricity market and formulating it as nonlinear MPEC under uncertainty. (2) to model the potential market interactions with two frameworks: perfect competition and a leader-follower game. (3) to transform the stochastic complementarity model between retailers and consumers into an MPEC and the equilibrium under perfect competition into an equivalent MILP. (4) to account for a quadratic utility function in the consumers' problem so that the marginal utility corresponds to the consumers' demand function faced by the retailer. This renders an incentive for the retailer to offer adequate tariffs to the elastic consumers. (5) to better characterize the DR program of consumers by allowing to displace consumption within hours to better adapt their utility to price variations. (6) to analyze and compare the two considered market settings with a variety of realistic case studies and to explore the impact of DR programs in the electricity market welfare and efficiency, under different types of competition.

4.3 Formulation of Retailer and Consumer Optimization Problems

Nomenclature

The main notation used in this chapter is stated below for quick reference. Other symbols are defined as needed throughout the text.

Sets:

$\omega \in \Omega$ set of scenarios ranging from $\omega = 1, ..., \mid \Omega \mid$.

$j \in J$ set of consumers ranging from $j = 1, ..., \mid J \mid$.

$t \in T$ hours considered ranging from $t = 1, ..., \mid T \mid$.

Parameters:

$\Delta_j^{C_{max}}$ level of flexibility of consumer j.

$a_{jt\omega}$ linear coefficient of utility function for consumer j, hour t, scenario ω.

$B_{jt\omega}$ quadratic coefficient of utility function for consumer j, hour t, scenario ω.

C penalization cost for power imbalances.

$P_{t\omega}^S$ spot price at hour t, scenario ω.

Variables:

$\Delta_{jt\omega}^C$ variation of demand consumer j, hour t, scenario ω.

$\delta_{t\omega}$ electricity imbalance by retailer at hour t, scenario ω.

P_t retailer price tariff at hour t.

$q_{jt\omega}$ electricity purchased by consumer j, hour t, scenario ω.

$q^S_{t\omega}$ electricity purchased by retailer in the spot market, hour t, scenario ω.

Dual Variables:

The following dual variables are associated with constraints:

$\alpha^+_{t\omega}$ upper limit for electricity imbalance for retailer hour t, scenario ω.

$\alpha^-_{t\omega}$ lower limit for electricity imbalance for retailer hour t, scenario ω.

$\beta_{t\omega}$ nonnegative electricity imbalance at hour t, scenario ω.

γ_t nonnegative tariff decided by retailer at hour t.

$\lambda_{j\omega}$ total quantity variation of demand for consumer j at scenario ω.

$\mu_{t\omega}$ electricity imbalance hour t, scenario ω.

$\nu^{C_{max}}_{jt\omega}$ maximum flexibility for consumer j at hour t, scenario ω.

$\nu^{C_{min}}_{jt\omega}$ minimum flexibility for consumer j at hour t, scenario ω.

$\theta_{t\omega}$ nonnegative spot market electricity at hour t, scenario ω.

We consider an economic optimization problem of an electricity retailer, which acts as an intermediary between the wholesalers and end-consumers. The retailer purchases the electricity at the wholesale market and sells it to the consumers with the aim of maximizing its profit (income minus purchasing cost). The consumers aim is to minimize their energy consumption cost (welfare maximization) given their preferences via the utility function. The retailer and the consumers interact in the real-time electricity market where they require to settle these conflicting objectives under uncertainty.

4.3.1 Retailer's Problem

The retailer maximizes its expected profit, subject to technical and economical constraints, and accounting for uncertainty, which is modeled by a discrete number of scenarios. The retailer's problem is stated mathematically as follows:

$$\max_{\mathbf{X}} \sum_{\omega \in \Omega} \sigma_\omega \left(\sum_{j \in J, t \in T} P_t q_{jt\omega} - \sum_{t \in T} (P^S_{t\omega} q^S_{t\omega} + C y_{t\omega}) \right) \tag{4.3.1a}$$

subject to

$$\sum_{j \in J} q_{jt\omega} - q^S_{t\omega} = \delta_{t\omega} : \mu_{t\omega} \quad \forall t, \forall \omega \tag{4.3.1b}$$

$$\delta_{t\omega} \leq y_{t\omega} : \alpha^+_{t\omega} \quad \forall t, \forall \omega \tag{4.3.1c}$$

$$-\delta_{t\omega} \leq y_{t\omega} : \alpha^-_{t\omega} \quad \forall t, \forall \omega \tag{4.3.1d}$$

$$q^S_{t\omega} \geq 0 : \theta_{t\omega} \quad \forall t, \forall \omega \tag{4.3.1e}$$

$$P_t \geq 0 : \gamma_t \quad \forall t \tag{4.3.1f}$$

$$y_{t\omega} \geq 0 : \beta_{t\omega} \quad \forall t, \forall \omega \tag{4.3.1g}$$

where $\mathbf{X}$=$\{P_t, q^S_{t\omega}, \delta_{t\omega}, y_{t\omega}\}$ is the retailer's set of decision variables.

The first term in the objective function is the revenue for the retailer from the quantity bought by consumer j, at hour t, with scenario ω and price P_t, which is the retail price of electricity (tariff), a first-stage decision determined by the retailer for each hour t. The second term is the cost of purchasing the electricity $q^S_{t\omega}$ at the spot market price $P^S_{t\omega}$ at hour t and with scenario ω. The last term is a cost (penalization of electricity imbalances), which can be positive or negative (variable $y_{t\omega}$ represents the absolute value of deviations of the electricity imbalances), and σ_ω is the probability associated with scenario ω. The summation of the product over index $\omega \in \Omega$ represents the expected profit, which is what the retailer wants to maximize over the time horizon.

Constraint (4.3.1b) is the electricity balancing constraint that guarantees the total demand by all consumers at hour t with scenario ω minus retailer's supply is the imbalance at that particular time. The linear set of constraints (4.3.1c), (4.3.1d) and (4.3.1g) enforce that $y_{t\omega} = |\delta_{t\omega}|$. These imbalances are penalized in the objective function with weight C. Constraints (4.3.1e) and (4.3.1f) are non-negativity constraints on the spot market quantity and price decided by the retailer for tariff at hour t, respectively. P_t is a first-stage decision, which needs to be settled before the uncertainty realization, while $q_{jt\omega}, q^S_{t\omega}, \delta_{t\omega}$, $P^S_{t\omega}$ and $y_{t\omega}$ are second stage decisions. Dual variables are indicated at their corresponding constraint with semicolons.

4.3.2 Consumers' problem

In modeling the consumers' problem with DR, it is common to express explicitly the type of consumers (residential, industrial, or commercial) so that part of their consumption is considered flexible, and part of the consumption is inelastic (demand that must be met strictly at all times). For instance, Halvgaard et al. (2012) model heat pumps for heating residential buildings, where the heating system of the house becomes a flexible consumption in the Smart Grid, and Zugno et al. (2013) extend the above model by considering the heating dynamics of a building, indoor temperature, or temperature inside a water tank where flexibility (the output of interest) is characterized through the indoor temperature. They also consider the inflexible part of the consumption in their model.

Electricity consumption is not only related to its costs but also to the consumers' behavior towards energy consumption, as it is an essential service that brings additional benefits/costs. For that reason, it is crucial to explore electricity consumption behaviors to cope with the main market challenges. In microeconomics, consumers' behavior is studied using the utility function, which is a suitable tool to consider customers' preferences. The utility function shows how a rational consumer would make consumption decisions under uncertainty.

By explicitly considering the consumers' utility function in the objective function, we explore a residential consumers' model with flexibility towards the dynamic price reported by the retailer. Thus, the consumers face an economic problem where it is crucial to balance the trade-off between the cost of electricity procurement and the discomfort of deviating from the utility of consumption given a flexible price and the possibility to shift the consumption to low price periods. In particular, the consumers' problem can be represented within a game-theoretical setting as it is parametrized by the decision of the retailer (hourly tariff P_t). We study two particular version of this setting: (a) a leader-follower problem, or (b) a competitive market setting. Therefore, the problem for consumer j and scenario ω can be expressed as:

$$\min_{q_{jt\omega}} \sum_{t \in T} \left(P_t q_{jt\omega} - a_{jt\omega}(q_{jt\omega} + \Delta^C_{jt\omega}) + \frac{1}{2} B_{jt\omega}(q_{jt\omega} + \Delta^C_{jt\omega})^2 \right) \tag{4.3.2a}$$

subject to

$$q_{jt\omega} + \Delta^C_{jt\omega} \geq 0 : \varepsilon_{jt\omega} \quad \forall j, \forall t, \forall \omega \tag{4.3.2b}$$

$$- \Delta^{C_{max}}_j \leq \Delta^C_{jt\omega} \leq +\Delta^{C_{max}}_j : \nu^{C_{min}}_{jt\omega}, \nu^{C_{max}}_{jt\omega} \quad \forall j, \forall t, \forall \omega \tag{4.3.2c}$$

$$\sum_{t \in T} \Delta^C_{jt\omega} = 0 \quad : \lambda_{j\omega} \quad \forall j, \forall \omega. \tag{4.3.2d}$$

where the objective function is the minus social welfare (cost minus utility) and $\{q_{jt\omega}, \Delta^C_{jt\omega}\}$ are consumer's decision variables. In the objective function $P_t q_{jt\omega}$ is the cost of purchasing $q_{jt\omega}$ electricity and $a_{jt\omega}(q_{jt\omega} + \Delta^C_{jt\omega}) - \frac{1}{2} B_{jt\omega}(q_{jt\omega} + \Delta^C_{jt\omega})^2$ is the quadratic utility function that measures the benefit the consumer achieves by consuming the amount of energy $(q_{jt\omega} + \Delta^C_{jt\omega})$ during hour t, scenario ω.

Quadratic (Mohsenian-Rad et al., 2010; Samadi et al., 2010; Chen et al., 2010) and logarithmic (Fan, 2012) utility functions are frequently used in DR programs, because they are non-decreasing and their marginal benefits are non-decreasing. In this paper, we adopt the quadratic utility function. Considering the consumers' objective function without any further constraints, consumption would only take place in periods when the real-time price is lower than the marginal benefit. The marginal utility is a demand function $P_t = a_{jt\omega} - B_{jt\omega}(q_{jt\omega} + \Delta^C_{jt\omega})$ where electricity quantity demanded decreases as electricity price increases. In other words, the consumers preferences are specified by considering an smooth utility function $U_{jt\omega}(q_{jt\omega} + \Delta^C_{jt\omega})$, that can depict the levels of satisfaction obtained by the demand as a function of the total power consumption.

Parameters $a_{jt\omega}$ and $B_{jt\omega}$ are the coefficients of the linear and the quadratic utility function, respectively. They are the intercept and slope of the demand function, which are important parameters to characterize the behavior of consumers towards market outcomes of the model. Clearly, different values for these parameters can capture the dynamics of consumers' demand. Constraint (4.3.2c) sets the lower and upper limits for flexible consumption, where $\Delta_j^{C_{max}}$ is the level of flexibility of consumer j. The inequalities guarantee that the flexible part of consumption at hour t falls in the range between $-\Delta_j^{C_{max}}$ and $+\Delta_j^{C_{max}}$, which bound consumption increase and decrease, respectively. If $\Delta_j^{C_{max}} = 0$, there is no flexibility in demand, while $\Delta_j^{C_{max}} > 0$, there is flexible demand response. In the latter case, the consumer considers a dynamic price tariff that ensures cost savings and better welfare. Besides, constraint (4.3.2d) ensures the net total flexible demand during the planning time horizon is zero. In the consumers' problem, $q_{jt\omega}$ and $\Delta_{jt\omega}^{C}$ are second-stage decisions. Note that since the dynamic electricity price P_t enters the consumers' problem as a parameter (it is only a variable in the retailer problem), the lower level problem is a linear and hence convex optimization problem. Therefore, we can replace each consumer's problem with its corresponding KKT optimality conditions (Haghighat and Kennedy, 2012) which are sufficient for optimality.

4.3.3 KKT formulation of the consumer problem

To formulate both the leader-follower (MPEC), and the equilibrium settings, the consumer's problem is replaced by the KKT optimality conditions. The Lagrangian function for consumer j and scenario ω problem (4.3.2) can be expressed as:

$$\mathcal{L}_{j\omega}(q_{jt\omega}, \Delta_j^{C_{max}}) = \sum_{t \in T} \Big(P_t q_{jt\omega} - a_{jt\omega}(q_{jt\omega} + \Delta_{jt\omega}^C) + \frac{1}{2} B_{jt\omega}(q_{jt\omega} + \Delta_{jt\omega}^C)^2 - \lambda_{j\omega}\Delta_{jt\omega}^C +$$
$$+ \nu_{jt\omega}^{C_{min}}(-\Delta_{jt\omega}^C - \Delta_j^{C_{max}}) + \nu_{jt\omega}^{C_{max}}(-\Delta_j^{C_{max}} + \Delta_{jt\omega}^C) - \varepsilon_{jt\omega}(q_{jt\omega} + \Delta_{jt\omega}^C)\Big) \tag{3}$$

The first-order KKT optimality conditions for all consumers and scenarios are:

$$\frac{\partial \mathcal{L}_{j\omega}}{\partial q_{jt\omega}} = P_t - a_{jt\omega} + B_{jt\omega}(q_{jt\omega} + \Delta_{jt\omega}^C) - \varepsilon_{jt\omega} = 0 \quad \forall j, \forall t, \forall \omega \tag{4.3.4a}$$

$$\frac{\partial \mathcal{L}_{j\omega}}{\partial \Delta_{jt\omega}^C} = -a_{jt\omega} + B_{jt\omega}(q_{jt\omega} + \Delta_{jt\omega}^C) - \nu_{jt\omega}^{C_{min}} + \nu_{jt\omega}^{C_{max}} - \lambda_{j\omega} - \varepsilon_{jt\omega} = 0 \quad \forall j, \forall t, \forall \omega \tag{4.3.4b}$$

$$\sum_{t \in T} \Delta_{jt\omega}^C = 0 \quad \forall j, \forall \omega \tag{4.3.4c}$$

$$0 \leq q_{jt\omega} + \Delta_{jt\omega}^C \perp \varepsilon_{jt\omega} \geq 0 \quad \forall j, \forall t, \forall \omega \tag{4.3.4d}$$

$$0 \leq \Delta_{jt\omega}^C + \Delta_j^{C_{max}} \perp \nu_{jt\omega}^{C_{min}} \geq 0 \quad \forall j, \forall t, \forall \omega \tag{4.3.4e}$$

$$0 \leq +\Delta_j^{C_{max}} - \Delta_{jt\omega}^C \perp \nu_{jt\omega}^{C_{max}} \geq 0 \quad \forall j, \forall t, \forall \omega \tag{4.3.4f}$$

$$\nu_{jt\omega}^{C_{min}}, \nu_{jt\omega}^{C_{max}}, \varepsilon_{jt\omega} \geq 0 \quad \forall j, \forall t, \forall \omega \tag{4.3.4g}$$

$$\lambda_{j\omega} \quad \text{free} \quad \forall j, \forall \omega. \tag{4.3.4h}$$

By convention, the symbol $\perp$ indicates complementarity so that any of the two inequalities is satisfied as an equality, i.e., the product of each expression and the corresponding dual variable must be zero. Note that the system of KKT optimality conditions is linear, with the exception of the complementarity conditions (4.3.4e)-(4.3.4g).

4.3.4 The Retailer's MPEC problem

In game theory, hierarchical optimization problems of this type can be formulated mathematically as MPECs (we refer the interested reader to Luo et al. (1996)). Thus,

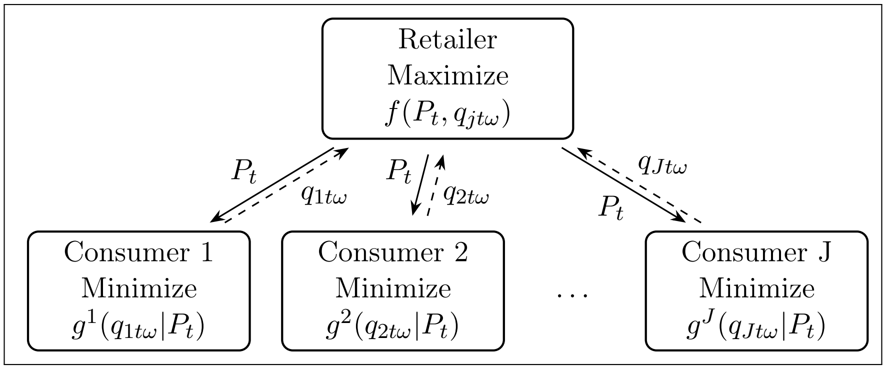


Figure 4.1: Game-theoretical framework for modeling demand response as a single leader-followers problem. Solid arrows indicate information signals from the retailer, while dashed arrows represent inferences on the consumers behavior.

the MPEC is expressed as follows:

$$\min_{\mathbf{X}^{\mathbf{MPEC}}} -\sum_{\omega\in\Omega}\sigma_\omega\left(\sum_{t\in T, j\in J} P_t q_{jt\omega} - \sum_{t\in T}(P^S_{t\omega}q^S_{t\omega} + Cy_{t\omega})\right) \tag{4.3.5a}$$

subjec to

$$\sum_{j\in J} q_{jt\omega} - q^S_{t\omega} = \delta_{t\omega} \quad \forall t, \forall\omega \tag{4.3.5b}$$

$$\delta_{t\omega} \le y_{t\omega} \quad \forall t, \forall\omega \tag{4.3.5c}$$

$$-\delta_{t\omega} \le y_{t\omega} \quad \forall t, \forall\omega \tag{4.3.5d}$$

$$q^S_{t\omega} \ge 0 \quad \forall t, \forall\omega \tag{4.3.5e}$$

$$P_t \ge 0 : \gamma_t \quad \forall t \tag{4.3.5f}$$

$$y_{t\omega} \ge 0 \quad \forall t, \forall\omega \tag{4.3.5g}$$

$$P_t - a_{jt\omega} + B_{jt\omega}(q_{jt\omega} + \Delta^C_{jt\omega}) - \varepsilon_{jt\omega} = 0 \quad \forall j, \forall t, \forall\omega \tag{4.3.5h}$$

$$-a_{jt\omega} + B_{jt\omega}(q_{jt\omega} + \Delta^C_{jt\omega}) - \nu^{C_{min}}_{jt\omega} + \nu^{C_{max}}_{jt\omega} - \lambda_{j\omega} - \varepsilon_{jt\omega} = 0 \quad \forall j, \forall t, \forall\omega \tag{4.3.5i}$$

$$\sum_{t\in T}\Delta^C_{jt\omega} = 0 \quad \forall j, \forall\omega \tag{4.3.5j}$$

$$0 \le q_{jt\omega} + \Delta^C_{jt\omega} \perp \varepsilon_{jt\omega} \ge 0 \quad \forall j, \forall t, \forall\omega \tag{4.3.5k}$$

$$0 \le \Delta^C_{jt\omega} + \Delta^{C_{max}}_j \perp \nu^{C_{min}}_{jt\omega} \ge 0 \quad \forall j, \forall t, \forall\omega \tag{4.3.5l}$$

$$0 \le +\Delta^{C_{max}}_j - \Delta^C_{jt\omega} \perp \nu^{C_{max}}_{jt\omega} \ge 0 \quad \forall j, \forall t, \forall\omega \tag{4.3.5m}$$

$$\nu^{C_{min}}_{jt\omega}, \nu^{C_{max}}_{jt\omega}, \varepsilon_{jt\omega} \ge 0 \quad \forall j, \forall t, \forall\omega \tag{4.3.5n}$$

where $\mathbf{X^{MPEC}}$=$\{P_t, q_{jt\omega}, q^S_{t\omega}, \Delta^C_{jt\omega}, \nu^{C_{min}}_{jt\omega}, \nu^{C_{max}}_{jt\omega}, \varepsilon_{jt\omega}, \lambda_{j\omega}\}$ is the set of retailer's decision variables. The MPEC optimizes the retailer's profit subject to its constraints and the KKT optimality conditions of the consumers' problem. However, the MPEC model in (4.3.5a)-(4.3.5o) has two sources of nonlinearities: (1) the bilinear product $P_t q_{jt\omega}$ in the objective function, and (2) the complementarity conditions (4.3.5k)-(4.3.5m) in the consumers' KKT optimality conditions.

The second type of nonlinearities can be linearized using the strategy proposed in Fortuny-Amat and McCarl (1981), which is by applying the big-M technique. However, it is not possible to find an exact linear expression of the bilinear product $P_t q_{jt\omega}$. However, we can efficiently solve the resulting MPEC is solved with off-the-shelf nonlinear solvers (we refer the interested reader to Ruiz and Conejo (2009); Sadat and Fan (2017)).

4.3.5 The equilibrium model

Considering perfect competition, we tackle the equilibrium model by the joint solution of retailers and consumers KKT optimality conditions. The resulting system of equations can be linearized and cast as a MILP problem which can be solved efficiently (Fortuny-Amat and McCarl, 1981). In addition, this problem can be addressed directly as a NLP with the same resulting market outcomes (this formulation is included in A.1).

4.3.6 KKT formulation of the retailer problem

The Lagrangian function for the retailer's problem (4.3.1) is:

$$\mathcal{L}(q_{t\omega}, q_{jt\omega}) = -\sum_{\omega\in\Omega,t\in T,j\in J} \sigma_\omega P_t q_{jt\omega} + \sum_{\omega\in\Omega,t\in T} \Big(\sigma_\omega (P^S_{t\omega} q^S_{t\omega} + C y_{t\omega}) + \mu_{t\omega}(\delta_{t\omega} - \\ + \sum_{t\in T} q_{jt\omega} + q^S_{t\omega}) + \alpha^+_{t\omega}(-y_{t\omega} + \delta_{t\omega}) + \alpha^-_{t\omega}(-y_{t\omega} - \delta_{t\omega}) - \theta_{t\omega} q^S_{t\omega} - \gamma_t P_t - \beta_{t\omega} y_{t\omega} \Big) \tag{4.3.6}$$

To derive the retailer's KKT optimality conditions and to deal with the equilibrium model, we assume that the market is in perfect competition, where players compete with quantity. Thus, $q_{jt\omega}$ also becomes a decision variable for the retailer. This implies that tariff P_t is considered fixed for both retailers and consumers problems, and its equilibrium value will naturally come out as a market clearing price i.e., a unique price that satisfies all the players' (retailer's and consumers') optimality conditions.

The KKT optimality conditions derived from (4.3.1) and (4.3.6) are:

$$\frac{\partial \mathcal{L}}{\partial q_{jt\omega}} = \sigma_\omega P_t + \mu_{t\omega} = 0 \quad \forall j, \forall t, \forall \omega \tag{4.3.7a}$$

$$\frac{\partial \mathcal{L}}{\partial q^S_{t\omega}} = \mu_{t\omega} + \sigma_\omega P^S_{t\omega} - \theta_{t\omega} = 0 \quad \forall t, \forall \omega \tag{4.3.7b}$$

$$\frac{\partial \mathcal{L}}{\partial \delta_{t\omega}} = \mu_{t\omega} + \alpha^+_{t\omega} - \alpha^-_{t\omega} = 0 \quad \forall t, \forall \omega \tag{4.3.7c}$$

$$\frac{\partial \mathcal{L}}{\partial y_{t\omega}} = \sigma_\omega C - \alpha^+_{t\omega} - \alpha^-_{t\omega} - \beta_{t\omega} = 0 \quad \forall t, \forall \omega \tag{4.3.7d}$$

$$\sum_{j\in J} q_{jt\omega} - q^S_{t\omega} = \delta_{t\omega} \quad \forall t, \forall \omega \tag{4.3.7e}$$

$$0 \le y_{t\omega} - \delta_{t\omega} \perp \alpha^+_{t\omega} \ge 0 \quad \forall t, \forall \omega \tag{4.3.7f}$$

$$0 \le y_{t\omega} + \delta_{t\omega} \perp \alpha^-_{t\omega} \ge 0 \quad \forall t, \forall \omega \tag{4.3.7g}$$

$$0 \le q^S_{t\omega} \perp \theta_{t\omega} \ge 0 \quad \forall t, \forall \omega \tag{4.3.7h}$$

$$0 \le y_{t\omega} \perp \beta_{t\omega} \ge 0 \quad \forall t, \forall \omega \tag{4.3.7i}$$

$$\lambda_{j\omega}, \mu_{t\omega} \quad \text{free.} \tag{4.3.7j}$$

4.3.7 MILP formulation

We solve the equilibrium by concatenating the KKT optimality conditions. The complementarity slackness conditions (4.3.4d)-(4.3.4f) and (4.3.7f)-(4.3.7i) can be linearized problem by introducing a binary variable for each condition.

Then to exploit the efficient performance of MILP solvers, the resulting system of MIL constraints can be reformulated as a MILP by including an arbitrary constant as the objective function:

$$\min_{\mathbf{X^{Comp}}} 1 \tag{4.3.8a}$$

subject to

$$\sigma_\omega P_t + \mu_{t\omega} = 0 \quad \forall t, \forall \omega \tag{4.3.8b}$$

$$\mu_{t\omega} + \sigma_\omega P^S_{t\omega} - \theta_{t\omega} = 0 \quad \forall t, \forall \omega \tag{4.3.8c}$$

$$\mu_{t\omega} + \alpha^+_{t\omega} - \alpha^-_{t\omega} = 0 \quad \forall t, \forall \omega \tag{4.3.8d}$$

$$\sigma_\omega C - \alpha^+_{t\omega} - \alpha^-_{t\omega} - \beta_{t\omega} = 0 \quad \forall t, \forall \omega \tag{4.3.8e}$$

$$\sum_{j \in J} q_{jt\omega} - q^S_{t\omega} = \delta_{t\omega} \quad \forall t, \forall \omega \tag{4.3.8f}$$

$$P_t - a_{jt\omega} + B_{jt\omega}(q_{jt\omega} + \Delta^C_{jt\omega}) - \varepsilon_{jt\omega} = 0 \quad \forall j, \forall t, \forall \omega \tag{4.3.8g}$$

$$-a_{jt\omega} + B_{jt\omega}(q_{jt\omega} + \Delta^C_{jt\omega}) - \nu^{C_{min}}_{jt\omega} + \nu^{C_{max}}_{jt\omega} - \lambda_{j\omega} - \varepsilon_{jt\omega} = 0 \quad \forall j, \forall t, \forall \omega \tag{4.3.8h}$$

$$\sum_{t \in T} \Delta^C_{jt\omega} = 0 \quad \forall j, \forall \omega \tag{4.3.8i}$$

$$(4.3.9a) - (4.3.9m), (4.3.10a) - (4.3.10i) \quad \text{and} \quad (4.3.11a) - (4.3.11i) \tag{4.3.8j}$$

$$\lambda_{j\omega}, \mu_{t\omega} \quad \text{free} \quad \forall j, \forall t, \forall \omega. \tag{4.3.8k}$$

where $\mathbf{X^{Comp}} = \{q_{jt\omega}, q^S_{t\omega}, \Delta^C_{jt\omega}, \nu^{C_{min}}_{jt\omega}, \nu^{C_{max}}_{jt\omega}, \varepsilon_{jt\omega}, \lambda_{j\omega}, \mu_{t\omega}, \alpha^+_{t\omega}, \alpha^-_{t\omega}, \theta_{t\omega}, \beta_{t\omega}\}$ is the set of the decision variables. In particular, the complementarity conditions from the consumer's KKT conditions can be linearized as follows:

$$\Delta_{jt\omega}^{C} + \Delta_{j}^{C_{max}} \geq 0, \quad \forall j, \forall t, \forall \omega \tag{4.3.9a}$$

$$\Delta_{j}^{C_{max}} - \Delta_{jt\omega}^{C} \geq 0, \quad \forall j, \forall t, \forall \omega \tag{4.3.9b}$$

$$q_{jt\omega} + \Delta_{jt\omega}^{C} \geq 0 \quad \forall j, \forall t, \forall \omega \tag{4.3.9c}$$

$$\nu_{jt\omega}^{C_{min}} \geq 0 \quad \forall j, \forall t, \forall \omega \tag{4.3.9d}$$

$$\nu_{jt\omega}^{C_{max}} \geq 0 \quad \forall j, \forall t, \forall \omega \tag{4.3.9e}$$

$$\varepsilon_{jt\omega} \geq 0 \quad \forall j, \forall t, \forall \omega \tag{4.3.9f}$$

$$\Delta_{jt\omega}^{C} + \Delta_{j}^{C_{max}} \leq (1 - \psi_{jt\omega}^{C_{min}}) M^{c}, \quad \forall j, \forall t, \forall \omega \tag{4.3.9g}$$

$$\Delta_{j}^{C_{max}} - \Delta_{jt\omega}^{C} \leq (1 - \psi_{jt\omega}^{C_{max}}) M^{c}, \quad \forall j, \forall t, \forall \omega \tag{4.3.9h}$$

$$q_{jt\omega} + \Delta_{jt\omega}^{C} \leq (1 - \psi_{jt\omega}) M^{c} \quad \forall j, \forall t, \forall \omega \tag{4.3.9i}$$

$$\nu_{jt\omega}^{C_{min}} \leq \psi_{jt\omega}^{C_{min}} M^{P}, \quad \forall j, \forall t, \forall \omega \tag{4.3.9j}$$

$$\nu_{jt\omega}^{C_{max}} \leq \psi_{jt\omega}^{C_{max}} M^{P}, \quad \forall j, \forall t, \forall \omega \tag{4.3.9k}$$

$$\varepsilon_{jt\omega} \leq \psi_{jt\omega} M^{P}, \quad \forall j, \forall t, \forall \omega \tag{4.3.9l}$$

$$\psi_{jt\omega}, \psi_{jt\omega}^{C_{min}}, \psi_{jt\omega}^{C_{max}} \in \{0, 1\} \quad \forall j, \forall t, \forall \omega \tag{4.3.9m}$$

where M^c and M^P are large enough constants.

(4.3.7f) and (4.3.7g) are linearized as follows:

$$y_{t\omega} - \delta_{t\omega} \geq 0, \quad \forall t, \forall \omega \tag{4.3.10a}$$

$$y_{t\omega} + \delta_{t\omega} \geq 0, \quad \forall t, \forall \omega \tag{4.3.10b}$$

$$\alpha^{+}_{t\omega} \geq 0, \quad \forall t, \forall \omega \tag{4.3.10c}$$

$$\alpha^{-}_{t\omega} \geq 0, \quad \forall t, \forall \omega \tag{4.3.10d}$$

$$y_{t\omega} - \delta_{t\omega} \leq (1 - \tau^{+}_{t\omega}) M^S \quad \forall t, \forall \omega \tag{4.3.10e}$$

$$y_{t\omega} + \delta_{t\omega} \leq (1 - \tau^{-}_{t\omega}) M^S \quad \forall t, \forall \omega \tag{4.3.10f}$$

$$\alpha^{+}_{t\omega} \leq \tau^{+}_{t\omega} M^T, \quad \forall t, \forall \omega \tag{4.3.10g}$$

$$\alpha^{-}_{t\omega} \leq \tau^{-}_{t\omega} M^T, \quad \forall t, \forall \omega \tag{4.3.10h}$$

$$\tau^{+}_{t\omega}, \tau^{-}_{t\omega} \in \{0, 1\} \tag{4.3.10i}$$

where M^S and M^T are large enough constants.

Finally, the linearization of (3.3.26d)-(3.3.26e) renders:

$$q^S_{t\omega} \geq 0 \quad \forall t, \forall \omega \tag{4.3.11a}$$

$$y_{t\omega} \geq 0 \quad \forall t, \forall \omega \tag{4.3.11b}$$

$$\theta_{t\omega} \geq 0 \quad \forall t, \forall \omega \tag{4.3.11c}$$

$$\beta_{t\omega} \geq 0 \quad \forall t, \forall \omega \tag{4.3.11d}$$

$$q^S_{t\omega} \leq (1 - \tau^q_{t\omega})M^S, \quad \forall t, \forall \omega \tag{4.3.11e}$$

$$y_{t\omega} \leq (1 - \tau^y_{t\omega})M^S, \quad \forall t, \forall \omega \tag{4.3.11f}$$

$$\theta_{t\omega} \leq \tau^q_{t\omega}M^T, \quad \forall t, \forall \omega \tag{4.3.11g}$$

$$\beta_{t\omega} \leq \tau^y_{t\omega}M^T, \quad \forall t, \forall \omega \tag{4.3.11h}$$

$$\tau^q_{t\omega}, \tau^y_{t\omega}, \tau^P_t \in \{0, 1\} \tag{4.3.11i}$$

$$\tag{4.3.11j}$$

where M^S and M^T are large enough constants.

4.4 Numerical results and discussion

In this section we provide numerical examples to complement the analytical results in the previous sections.

4.4.1 Data and scenario generation

We simulate scenarios using day-ahead electricity spot market prices $P^S_{t\omega}$ from EEX, the European Energy Exchange market. These electricity prices are available in one-hour intervals, and we arbitrarily consider the prices on the 20/12/2020, which are available at EEX (2020) along with other market data.

Therefore, Figure 4.2 depicts hourly spot market electricity prices and quantities observed on that day on EEX where the left axis is the spot price, and the right axis is the spot quantity sold for the corresponding hours. As we discuss in the model formulation part, the marginal utility is the price-demand curve where $a_{jt\omega}$ and $B_{jt\omega}$ are price-demand curve intercept and slope, respectively. Given the data in Figure 4.2, it is possible to estimate appropriate expected values for the utility parameters ($a_{jt\omega}$ and $B_{jt\omega}$) and include some perturbations afterwards to consider different levels of residential consumers' flexibility.

In spite of the fact that there is no theoretical limit on the number of consumers, for simplicity and illustrative purposes, we consider three residential consumers. We extend the cases to accommodate different preferences (by varying the demand parameters and demand flexibility) of consumers with respect to consumption shifts. The consumers considered in our model can be regarded as types of consumers with different behaviors in the day-ahead market. Hence, the expected value of their utility parameters i.e., a_j and B_j, are assumed to be slightly different so that they have different reactions towards the real-time electricity price. Then, the spot prices for the simulation are randomly generated taking the observed spot market prices as mean values and a coefficient of variation (CV) of 0.015 in a Gaussian process based on the Central Limit Theorem (CLT). The standard deviation is calculated as $CV \times E[P_t^S]$, which used to generate $P_{t\omega}^S$ with a multivariate normal distribution.

Table 4.1 shows parameter mean values considered to represent consumers behavior for different case studies, where a_j, B_j and $\Delta_j^{C_{max}}$ are in €/kWh, €/kWh^2 and kWh, respectively. The simulations are done by moving the intercepts (a_j), the slopes

Cases	Parameters								
	a_1	a_2	a_3	B_1	B_2	B_3	$\Delta_1^{C_{max}}$	$\Delta_2^{C_{max}}$	$\Delta_1^{C_{max}}$
Case 1	0.0291	0.0302	0.0271	0.0013	0.0015	0.0014	2.50	1.40	2.00
Case 2	0.035	0.0375	0.0341	0.0017	0.020	0.0019	5.00	2.40	3.50

Table 4.1: Parameter mean values considered to represent consumers behavior for different case studies.

B_j and the demand flexibility parameter $\Delta_j^{C_{max}}$. That is the demand parameters arbitrarily increased by 25% and the demand flexibility by 75%, given Case 1 as the benchmark (BM).

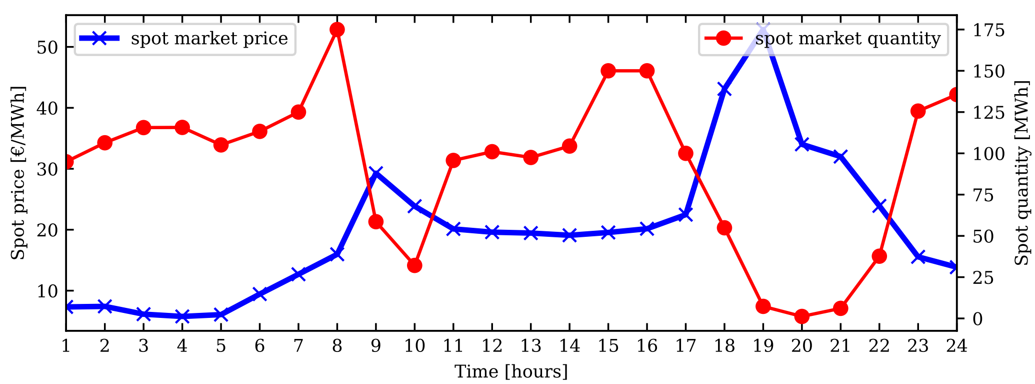


Figure 4.2: Observed spot price (left axis) and quantity (right axis) sold on EEX with the price on 20 December 2020.

Table 4.1 presents the mean parameter values used for data calibration for the different cases considered in the simulation. The corresponding CVs for a_j and B_j are 0.013 and 0.0013 so that $a_{jt} \sim N(a_{jt}, \sigma_{a_{jt}})$ and $B_{jt} \sim N(B_{jt}, \sigma_{B_{jt}})$. Note that we use the same CV (0.013 for a_j and 0.0013 for B_j) for the data calibration in all the cases. The demand flexibility parameter $\Delta_j^{C_{max}}$ is arbitrarily allowed to vary from 0 to 5.0kWh, which are reasonable values for individual consumer within DR programs. Given Case 1 is as a benchmark, we consider the other cases by varying (increasing) the parameter values from the benchmark for all the other cases. In other words, the simulation is done by allowing the demand parameters a_j to increase while fixing B_j and the demand flexibility $\Delta_j^{C_{max}}$ at the benchmark and vice versa.

The models have been solved using JuMP version 0.21.1 (Dunning et al., 2017) under the open-source Julia programming Language version 5.2.1 (Bezanson et al., 2017). We use Artelys Knitro solver version 12.2 (Artelys Knitro, 2021) for the MPEC and Gurobi version 0.9.12 (Gurobi Optimization, 2021) for the MILP on a CPU E5-1650v2@3.50GHz and 64.00 GB of RAM running workstation. It is worth noting that increasing the number of scenarios further increases the computational complexity of the nonlinear MPEC problem, particularly for $\Omega > 30$. That is, the number of variables and the number of constraints exponentially increase (Table 4.2), which may make the problem computationally intractable. In fact, possible alternatives to tackle this computational challenge may include utilizing a supercomputer with parallel computing techniques (we refer the interested reader to Ahmadi-Khatir et al. (2013); Nasri et al. (2015)). In our case, we have employed the option: *Knitro multistart in parallel with 12 threads*. Moreover, a sensitivity analysis on the number of scenarios shows that, increasing the number of scenarios barely affects the reported market outcomes (Table 4.5). Note that for the MILP (in the equilibrium), computational burden is not an issue as it could solve for larger sets of scenarios, for example $\Omega = 300$.

Model	Number of variables			Number of constraints				Number of nonzeros in Jacobian	Number of nonzeros in Hessian	CPU time
	bounded below only	free	otal	linear equalities	quadratic equalities	linear one-sided inequlities	Total			
MPEC	10,848	5,850	**16,698**	8,640	6,480	15,864	**30,984**	118,824	12,984	**106.922**
NLP	10,848	5,850	**16,698**	11,520	0	9,360	**20,881**	120,240	31,680	**18.172**

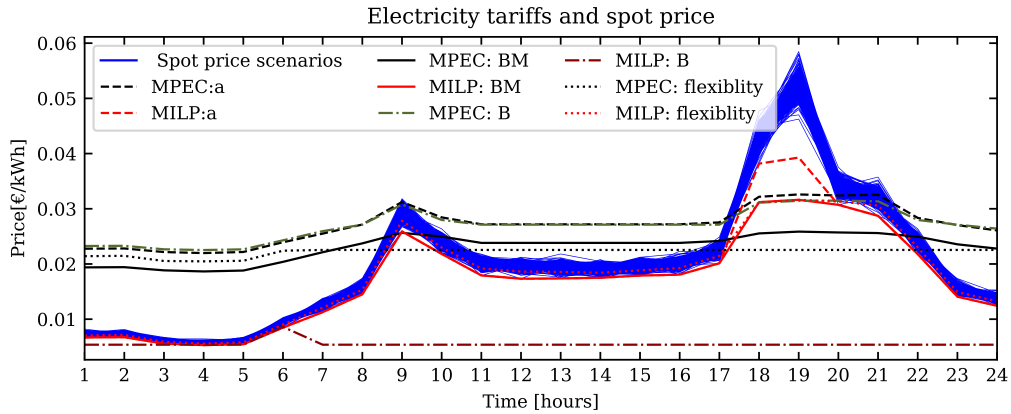
Table 4.2: Computational considerations under the two proposed models.

4.4.2 Results of the experiments

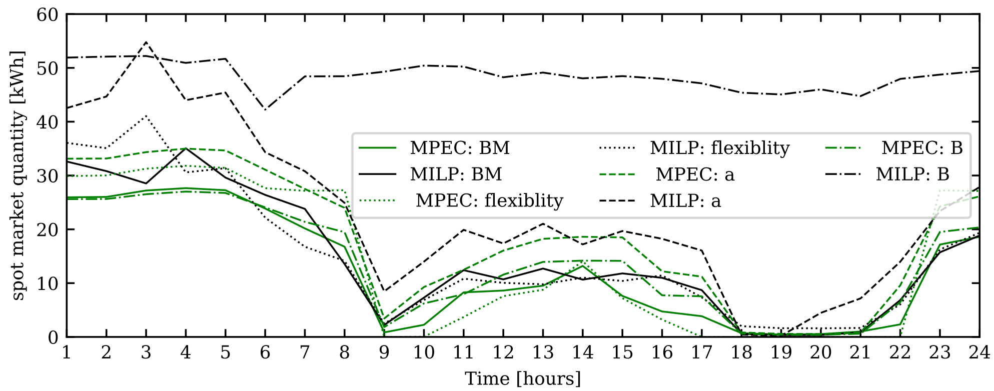
In the following we present simulation results to analyze the market implications of the level of competition and consumers' flexibility.

Impact on price tariffs

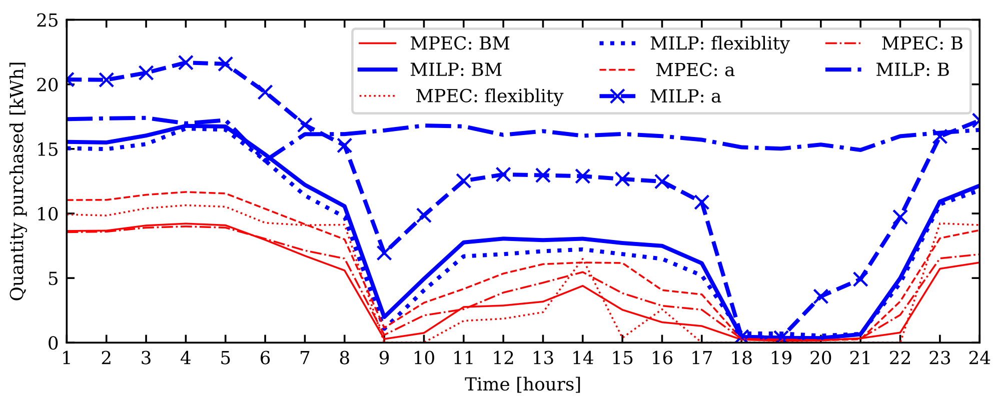
The retailer's price tariffs in both the MPEC and in the equilibrium models are depicted along with the generated spot market prices scenarios for the sake of comparison in Figure 4.3(a). Moreover, there are four cases for the simulation considering the two models. The first one is called the benchmark (BM), the second and the third considered by increasing demand intercept (a) and demand slope (B) by more than 25% from the BM. Finally, the fourth case explores a higher (75% higher from the BM) demand flexibility $\Delta_j^{C_{max}}$. These cases give us the chance to see the impacts of the utility parameters movement and demand flexibility on market outcomes under DR program.



(a) Electricity prices in both models with different cases



(b) Spot market quantity in both models with different cases



(c) Purchased quantity in both models with different cases

Figure 4.3: Electricity price, quantity purchased by the retailer from the spot market and quantity purchased by consumers in the retail market with the two models and with the different cases.

As expected, in the competitive equilibrium model (MILP), price tariffs are apparently lower than the retailer's tariffs in the MPEC where there is market power. The overall retailer's price tariffs P_t communicated to consumers at time t with the MPEC are proportional and higher than the spot market prices $P_{t\omega}^S$, which is economically intuitive. In other words, the retailer tries to decrease the procurement of power at periods $t = 9, 10, 17, 18, 19, 20, 21$ where the spot market prices are relatively high, and to increase at periods $t = 1 - 8, 10 - 17$ where there are lower spot market prices. These price tariff results corroborate with the retailer's quantities bought in these peak-periods, which are very low, as can be seen in the spot quantity purchased by retailers (more detail comparison in Figure 4.9(a), (b), (c), (d) for these four cases), and higher quantities in the periods where there are lower prices. Despite the early hours where there are already lower electricity price tariffs, the increase in demand flexibility decreases electricity price tariffs in the MPEC model. However, the increase in both demand parameters (a and B) increases price tariffs. Conversely, for the equilibrium model (MILP), only the increase in consumer's slope B can significantly decrease price tariffs, particularly in peak periods, which is normally expected from consumers' theory. The overall, price tariff is always below the spot price except the peak periods in the equilibrium model.

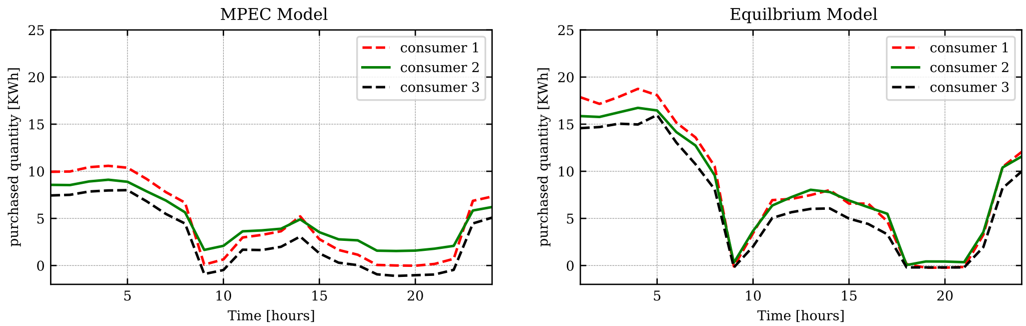
The increase in consumers' flexibility decreases the retailer price tariffs in the MPEC, particularly after 7:00. That means that, if consumers are flexible in their consumption, it is economically meaningful for price to increase and vice versa. However, in the equilibrium, demand flexibility has slightly an opposite impact. Conversely, with the MPEC, the increase in demand flexibility decreases optimal electricity price tariffs.

Figure 4.3(b) shows electricity procurement in the wholesales market with different cases and models. The retailer buys higher quantities in the perfect competition where there is a lower price, particularly with higher B. The retailer's purchasing trend resembles the observed market trend presented in Figure 4.2 for all the cases. With the demand flexibility and the BM where there are lower values for a and $\Delta_j^{C_{max}}$, the retailer purchases a lower quantity in the equilibrium model. Figure 4.3(c) shows the aggregate consumers purchase in the retail market with different cases. Since the quantities purchased from the spot market are higher with lower price tariffs, consumers purchase higher quantities in the MILP model irrespective of the case. In the equilibrium model, the increase in B creates higher and stable electricity purchases in the retail market for consumers, which corresponds to the steadily low price-tariffs in this case. With the increase in a, for both the MPEC and MILP, consumers buy higher electricity during off-peak periods, otherwise they face higher price tariffs. Because prices in these periods are already low, higher quantities are purchased from the spot market and delivered to the retail market. In general, comparing the price tariffs, spot quantity purchased by the retailer, and consumers quantity purchased (Figure 4.3), which gives insightful results regarding the dynamic price DR model that explicitly models consumers' preferences.

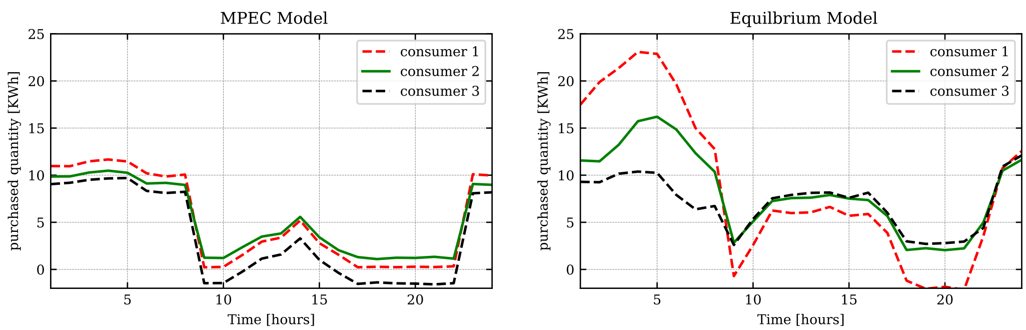
Impact on retailer's procurement and consumers' quantity

As discussed above, Figure 4.3(b) shows electricity procurement in the wholesales market with different cases and with respect to the two models. The retailer buys higher quantities in the early and late hours of the day-ahead market. With the MILP, particularly with higher B, the retailer purchases higher quantities in the spot market, which resulted to a lower selling price in the retail market. The higher the competition, the lower is the impact the retailer could have on the market outcomes, which, in turn, lowers the selling price back to the consumers. That is why consumers benefit from purchasing their electricity in a competitive equilibrium setting. On the other hand, Figure 4.3(c) shows the aggregate consumers purchase in the retail market with different cases. As can be observed, consumers procure much of their electricity consumption in the equilibrium model where there are more quantities and lower price tariffs. Despite the aggregate purchase trend is similar to the individuals purchase, comparing consumers' purchases in the BM reveals that consumer 1 buys more when electricity price tariffs are low (early morning and late evening). Because in the BM, consumer 1 has the minimum slope (0.0013) and the highest demand flexibility (2.50).

In addition, since loads are shifted from expensive periods to cheap periods, it can be seen that new peaks and valley periods arise. In other words, if there is a peak period, then the next will be a valley period.



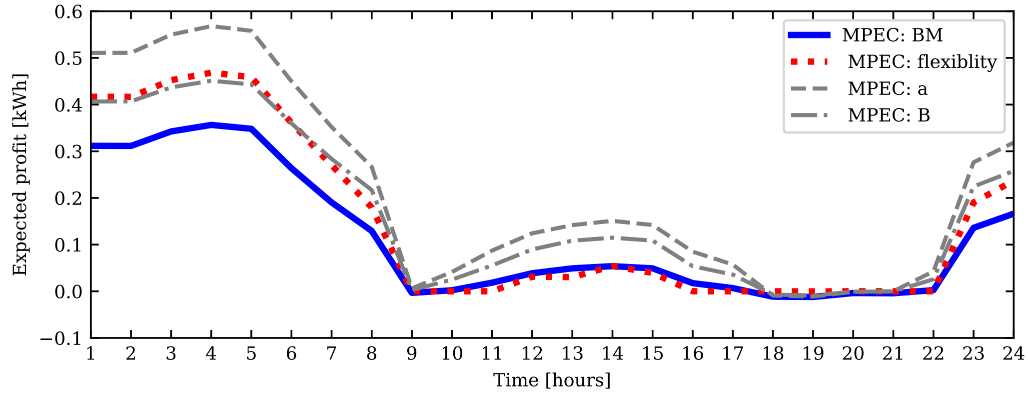
(a) Consumers' purchase: BM



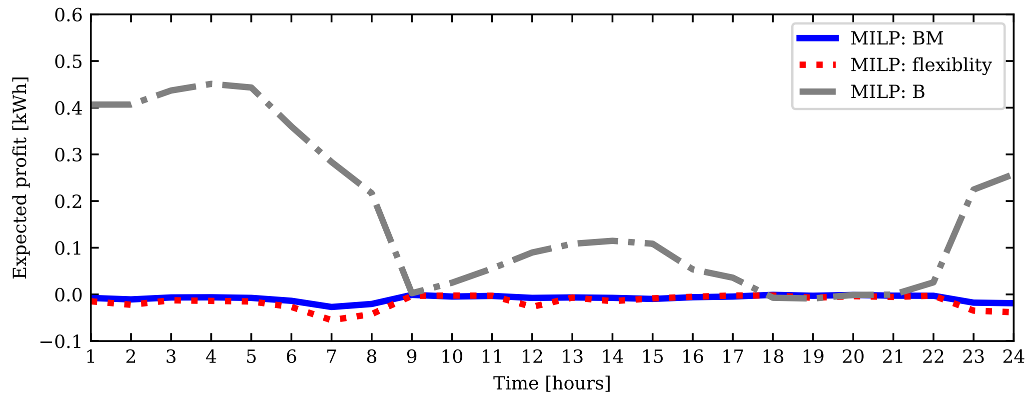
(b) Consumers' purchase: high flexibility

Figure 4.4: Consumers' purchased quantities in both models with the BM case.

Impact on expected profit



(a) Expected profit for different cases: MPEC



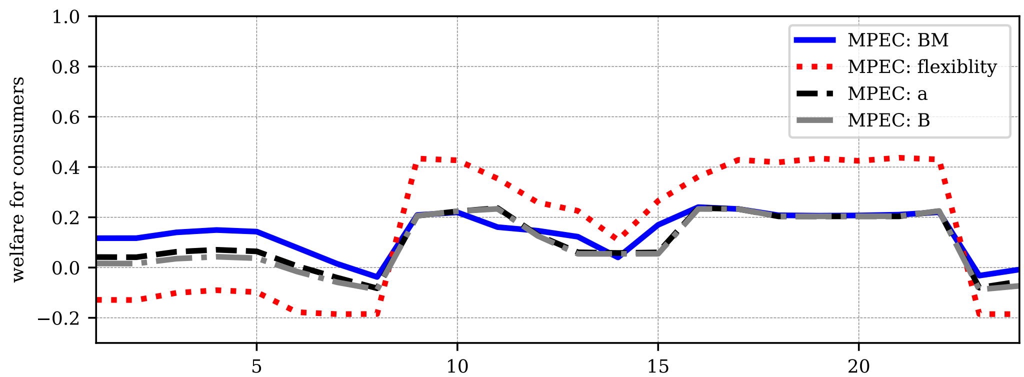
(b) Expected profit for different cases: MILP

Figure 4.5: Expected profit comparison between the two models with different cases.

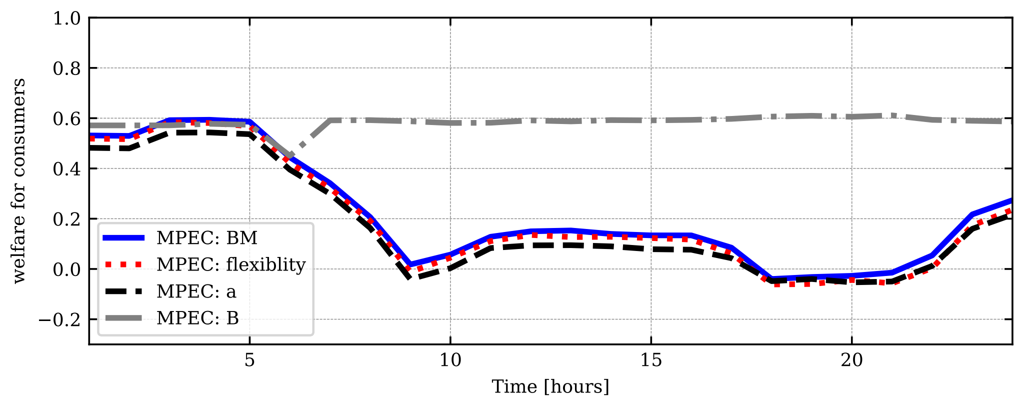
Looking at the results depicted in Figure 4.5(a), and (b), which show the expected profit for the retailer in both models, we can observe that the results are directly related to price tariffs and quantities. Since the expected profit for the retailer in the equilibrium model using the increase in a is entirely negative in all periods (most unfavorable case for the retailer), we do not include in the current analysis (A.2). The results show that the retailer obtains a higher expected profit in the MPEC model than in the equilibrium model. Note that even the price tariffs P_t at some periods are not necessarily cheaper than the spot market prices $P^S_{t\omega}$, because the retailer maximizes the expected value of total profit over the entire time horizon (24 hours), hence it may face negative profits in some periods. However, the retailer faces lower average payoffs with the equilibrium model as the equilibrium price is always below the wholesales market price in all the cases. As opposed to an increase in a where there are money losses for the retailer, the increase in parameter B i.e., more inelastic consumers, can significantly increase the retailer's profit even with the equilibrium model.

Even though the retailer loses money in the equilibrium model, Figure 4.6 reveals that the model is better off for consumers. They have higher utility during the off-peak demand periods, since there are lower electricity prices, which, in turn, results in higher welfare for consumers. Thus, in off-peak periods, consumption is higher with lower electricity prices and can increase consumers' comfort. This helps to achieve the goal of supply security and reducing price unpredictability, which is the main goal of demand response programs.

Consumers' welfare analysis



(a) Consumers' welfare for different cases: MPEC



(b) Consumers' welfare for different cases: MILP

Figure 4.6: Consumers' welfare comparison in the two models with different cases.

Furthermore, the welfare results corroborate with other results as consumers obtain higher utility and better welfare in the equilibrium model than the MPEC.

Since welfare is a direct implication of utility, it increases as demand flexibility increases. For consumers, the equilibrium model provides better welfare as competition decreases price and increases demand. Figure 4.6 depicts the aggregate consumers' welfare in both models with different cases. As can be seen, consumers are better off with the equilibrium model where they enjoy lower price tariffs and higher quantity purchases. Therefore, consumers maximize their welfare by shifting their consumption to off-peak periods, which helps to ensure reliable electricity supply during peak periods. Hence, with higher demand flexibility, consumers can have better welfare even during peak periods. This is a very insightful result since consumers can increase their welfare by increasing demand flexibility even with the MPEC model where the retailer can obtain higher expected profits. The implication is that the benefits obtained from DR program offsets the cost incurred due the market power exercised by the retailer.

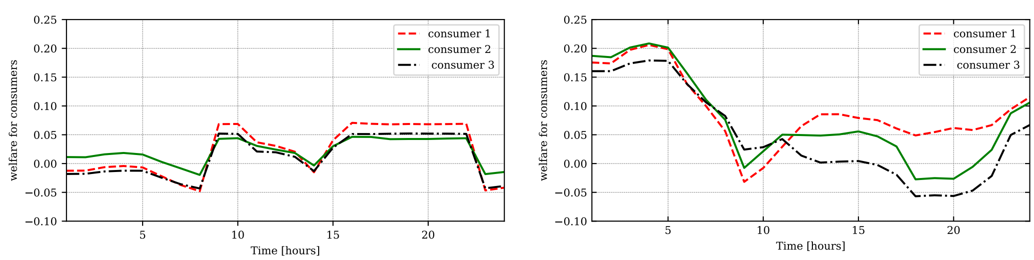


Figure 4.7: Consumer welfare in both models with respect to the three consumers.

Figure 4.7 shows individual consumers' welfare comparison between the two models applying the BM case. All consumers maximize their welfare during off-peak periods in the MILP. With respect to both models, consumer 1 is relatively better off during peak periods. Because consumer 1 has higher demand flexibility. On the other hand, consumer 3 has lower welfare in both models due to its lower flexibility.

Therefore, it is possible to conclude that welfare is significantly affected by consumers demand flexibility $\Delta_j^{C_{max}}$.

In addition, Figure 4.8 shows the pattern of the three consumers' utility during the 24-hours for the two models. Like their welfare, consumers obtain higher satisfaction during the low-demand periods using the BM case where they incur the least costs. For that reason, consumers shift consumption to off-peak periods because they have lower utility during the peak periods. Overall, the equilibrium provides higher utility, albeit that reverses during peak periods.

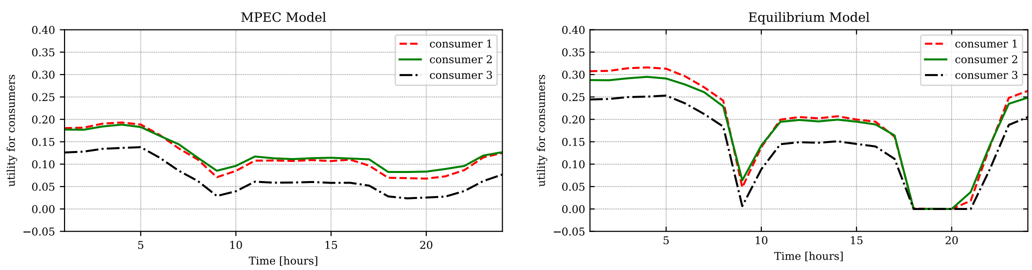
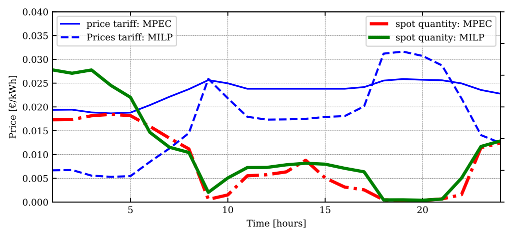
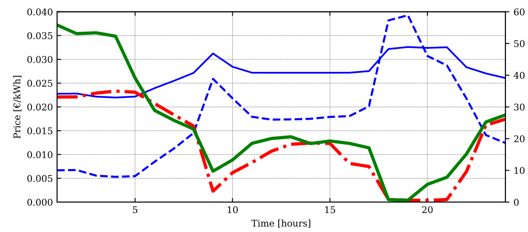


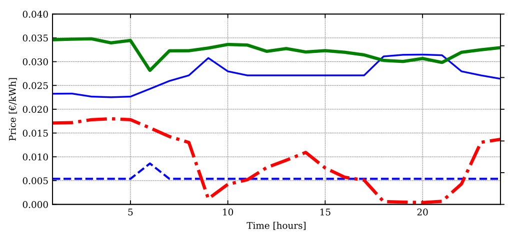
Figure 4.8: Consumers' utility in both models: BM.



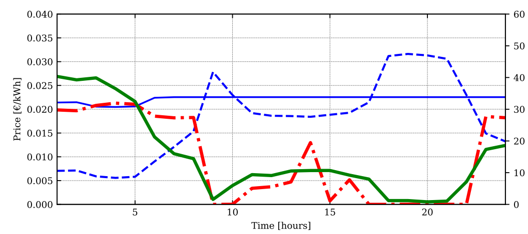
(a) Price-quantity comparison: BM



(b) Price-quantity comparison: a



(c) Price-quantity comparison: B



(d) Price-quantity comparison: flexibility

Figure 4.9: Price (left axis) and spot quantity (right axis) comparison in both models and for the four cases.

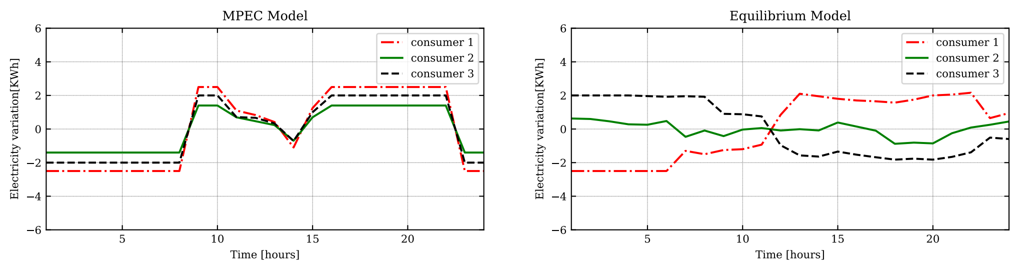
Figure 4.9 represent price tariffs and quantities purchased by the retailer from the spot market in both models. For the sake of comparison, we plot price tariffs on the left axis and spot market quantity on the right axis so that possible to analyze the impact of parameters change on the spot market purchase and price tariffs. As discussed before, price tariffs, particularly with the MILP, decrease when demand flexibility increases. However, the increase in a increases price tariffs, whereas the increase in B decreases price, which is in line with the economic theory of demand. For that reason, the retailer buys a higher spot quantity from the wholesale market if he expects that B increases.

In general, electricity price tariffs set by the retailer and spot market quantity follow a similar trend to the observed real market trend and the consumers' procurement, which we discussed before. Thus, the models fully characterize the market participants' (the retailer's and consumers') behaviors so that policy implications can be drawn from the DR models proposed. Consumers are better off purchasing their electricity consumption with the equilibrium model as it has a lower price than the MPEC model. However, the retailer benefits from the MPEC model as it gives the retailer a higher expected profit, which is its primary objective.

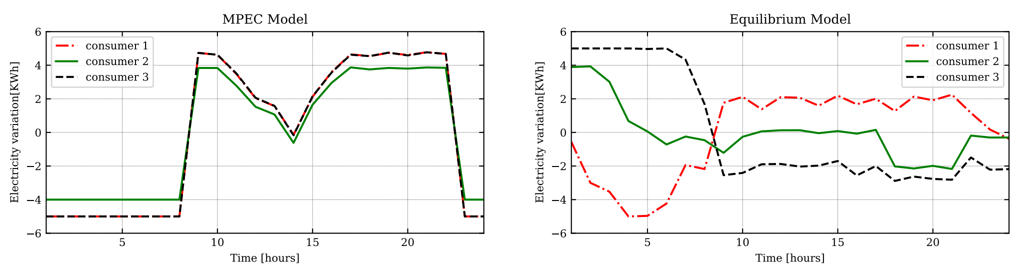
Figure 4.10 represents results regarding consumers' flexibility (DR) towards their consumption for the two models with the BM and with higher flexibility. The overall trend for flexible demand is almost similar in both models, though differences in magnitudes. Consumers are more flexible from 9:00-11:00 and from 17:00-22:00 as they shift their consumption to the off-peak periods. In relation to this, the negative values (lower blocks) show consumers shift part of their consumption in those hours as they consume higher energy in the off-peak periods. The result also guarantees the constraint $\sum_{t\in T} \Delta^{C}_{jt\omega} = 0$, and the flexibility boundary values given for each consumers in Table 4.1. For instance, consumer 2 is the most flexible in consumption up to 5kWh. For the case where we have more flexibility, the overall electricity price is decreased, and the quantity purchased is increased. As a result, the results on the retailer's profit and consumers' welfare are robust. Thus, the DR models effectively characterize the market in both market configurations.

Retailer	Model		Consumers	Model	
Performance index	MPEC	Equilibrium	performance index	MPEC	Equilibrium
Expected profit	0.1149	-0.0076	Cost	0.2602	0.2894
Revenue	0.2602	0.2894	Price	0.0230	0.0177
Cost	0.1454	0.2970	Welfare	0.0177	0.0784

Table 4.3: Market performance of retailer and consumers in the simulations (with the benchmark case).



(a) Level of flexibility for consumers: BM



(b) Level of flexibility for consumers: high flexibility 3

Figure 4.10: Demand variation in both models with respect to the two cases.

Market performance and sensitivity analysis

Table 4.3 presents the performance of the two models for the retailer and consumers where values are measured on average under the BM case. Note all the values but price (€/kWh) are averages for the considered scenarios and expressed in €. As

Spot price uncertainty	Exp. profit	Max. profit	Min. profit	$q_{t\omega}$	$q_{jt\omega}$	Tariff	Utility	Welfare
0.015	0.1143	0.3555	-0.0128	12.29	4.198	0.0230	0.1046	0.0177
0.030	0.1156	0.3551	-0.0140	12.30	4.101	0.0230	0.1046	0.0177
0.035	0.1156	0.3547	-0.0141	12.31	4.105	0.0230	0.1047	0.0178

Table 4.4: Sensitivity analysis on spot price uncertainty under the MPEC model for the BM case.

	Market outcomes					Market outcomes			
Ω	Expected profit	P_t	$q^S_{t\omega}$	$q_{jt\omega}$	Ω	Expected profit	P_t	$q^S_{t\omega}$	$q_{jt\omega}$
10	0.1857	0.0268	14.047	4.682	20	0.1839	0.0267	14.211	4.737
15	0.1850	0.0268	14.074	4.691	30	0.1844	0.02267	14.201	4.733

Table 4.5: Sensitivity on number of scenarios. All values are averages for the considered scenarios.

the expected profit reveals, the retailer loses money on average in the equilibrium despite consumers enjoy better welfare from their consumption with lower prices. Note that cost for the retailer comes of two sources: the spot market purchasing costs if the retailer had perfect information from future consumption and the imbalance penalties, which are the costs of imperfect information. Furthermore, note that the retailer's revenue is a cost for the consumers. In Table 4.4, we present the market outcomes regarding the uncertainty of the spot market prices $P^S_{t\omega}$ under the BM case and the MPEC model, where profits, quantities, and tariffs, utilities, welfare are in €, kWh and €/kWh, respectively. We consider the coefficient of variations for this analysis to be only 0.015, 0.0.03, and 0.035 for the sake of simplicity. The results show that the uncertainty on the spot market price has no significant impact on market outcomes for both market participants. Instead, as proved through the simulation, the level of demand flexibility affects the market outcomes significantly. Other than increasing a computational burden, the increase in the number of scenarios does not have a significant impact on optimal market outcomes using the BM case

4.5 Conclusions

This paper presents two game-theoretical models for a retailer and consumers participating in the electricity market under demand response. Our research complements prior works on demand response by considering the flexibility of consumers under dynamic price tariffs and characterizing their behavior using a utility function.

We consider welfare-maximizing consumers that interact in the retail market in a leader-follower or in a perfect competition market configuration. The first case is modeled with a bilevel optimization problem. In the lower level, a dynamic model with responsive demand based on realistic consumers' preferences is explicitly formulated using a quadratic utility function. This leader-followers game is solved as a nonlinear MPEC where a retailer with market power selects an optimal tariff taking into account the behavior of the consumers, which can react to these selling prices by modifying their consumption profiles. We also explore the equilibrium model with perfect competition which is solved as a MILP problem.

Dynamic pricing DR program has been shown to effectively shift electricity demand if there are incentives for both retailers and consumers to participate in the program. The results allow the retailer to optimize its pricing strategy with market power (MPEC), and the consumers to maximize their welfare in a competitive equilibrium setting. The numerical results further demonstrate that the proposed models are adaptable to any group of consumers with participation in DR programs and can be adjusted for any period according to the preferences given by consumers. For both models, consumers are flexible enough to exercise the DR program. We discuss the impact of demand parameters and demand-side flexibility on market outcomes in detail with different case studies. The results attest that including consumers' utility function in a DR program explicitly, reveals important features of consumers' behavior. Finally, through a sensitivity analysis and under spot price uncertainty, it is shown that there is an economic incentive for consumers to increase their flexibility, which, in turn, benefits the retailer as well.

The proposed game-theoretical optimization model can be further extended in different directions. First, we can model generating companies and market operators' problems with a DR program by applying a similar methodology employed in this paper. Second, within the same model and players, we can include the RES generation schedule and main grid trading constraints by explicitly modeling consumers' problems with intelligent appliances so that they can trade their energy instead of being net consumers. Third, since only a few works address on environmental effects of DR programs, we can consider pollution in the objective function of DR optimization problem which could give better policy direction to the integration of technology, market, and innovation in the power system to achieve emission reduction targets.

Chapter 5: Summary, Conclusions, and Suggestions for Future Research Work

This chapter provides a summary, some conclusions of the thesis work and suggests future works.

5.1 Summary

In this dissertation, models and methods in the electricity market are developed by introducing financial derivatives with a game-theoretical framework to help agents in the market to make optimal decisions under uncertainty. The first two chapters focus on RES integration and emissions trading in the electricity system with risk aversion under the CVaR methodology. The final chapter explores the demand response program for system reliability and efficiency under different market configurations. Stochastic programming is employed in all the models where we have a powerful framework to model and to include parameters' uncertainty in an optimization problem, via a plausible set of scenarios to incorporate the CVaR with linear formulation (Rockafellar and Uryasev, 2000; Leyffer and Munson, 2010), and to deal the demand response with market power and perfect competition setups. Much stochastic modelling on the electricity market and RES integration, emission allowance trading and demand response in the electricity market have preceded this research (Conejo et al., 2010; Zugno et al., 2013; Hu et al., 2016; Luo et al., 2020; Vehviläinen and Pyykkönen, 2005; Galiana and Khatib, 2010; Aïd et al., 2011; Du et al., 2018; Vehviläinen and Pyykkönen, 2005; Galiana and Khatib, 2010,?; El Khatib and Galiana, 2019; Morales et al., 2013; Hu et al., 2016; Antunes et al., 2020). For instance, Du et al. (2018) propose a three-stage structure stochastic model for power systems operation under high renewable energy penetration. Vehviläinen and Pyykkönen (2005) present a stochastic factor-based approach to mid-term modeling of spot market prices in deregulated electricity markets. Galiana and Khatib (2010) for emissions trading under stochastic programming, Galiana and Khatib (2010); El Khatib and Galiana (2019) for emissions trading with Nash game models, and Morales et al. (2013); Hu et al. (2016); Antunes et al. (2020) for demand response programs, among others.

None of these, however, (1) combine the integration of RES in the electricity market considering participants' risk aversion and the relationships between futures and spot markets. (2) explore the EUA allowances trading and its role in decarbonizing the energy system with different market competition levels and a coherent risk measurement simultaneously, and (3) model the demand response program with different market configurations and consumers' behavior using utility function under uncertainty. Thus, in this thesis, we have tried to exploit those gaps so that we can contribute to the literature.

In the following, we briefly summarize both the theoretical properties and the characteristics of the solution approaches of the three models that we have developed in the thesis to tackle the above challenges.

5.1.1 Integrating RES into the Electricity Market

Integrating RES is a new challenge in the electricity sector following the deregulation and restructuring of the electricity sector, which in turn promotes great regional and inter-regional competition. In Chapter 2, we propose an equilibrium model that integrates RES and conventional generators with financial derivatives in a two-stage electricity market. First, we solve the game-theoretical model in a single spot market where the dispatchable and non-dispatchable generations are delivered with a pre-existing futures market contract. We endogenously define both spot market and futures market demand functions by relaxing and generalizing the standard non-arbitrage condition. After we obtain a closed-form solution of the second stage equilibrium, we move one stage backward in time to characterize the futures market decision variables in terms of the spot market equilibrium outcomes. The global optimization solution is solved after the CVaR is formulated and the KKT optimality conditions are concatenated to reformulate them as an equivalent NLP. The results show that the level of risk aversion and market competition have a strong impact on electricity market outcomes with respect to RES penetration where RES deployment decreases overall electricity prices. Though both physical and financial contracts help to mitigate market risks pertaining to the uncertainties, physical contracts benefit generators and CFD for consumers.

5.1.2 Introducing EUA allowance Contracts in the Electricity Market

In Chapter 3, we introduce the EU ETS in the equilibrium model to derive an ad-hoc model for plummeting GHG caused by the electricity sector, where the scheme covers around 40% of the EU's greenhouse gas emissions according to European Commission (2020). The conventional and RES generators are integrated with financial contracts in which a stochastic programming setting is applied to manage uncertainty. Despite electricity prices in both stages of the markets are defined endogenously, emission prices are taken as exogenous parameters for simplicity, in both futures and spot markets. Regarding emission trading, market-based auctioning is considered where each generator decides to buy the emission permits required for their operation instead of grandfathering allowance allocation, which may result in windfall profits. The risk neutral and risk averse generators models are developed with a coherent risk measurement and the sensitivity analyses regarding the RES parameter and the CO_2 price increase are illustrated with multiple case studies.

5.1.3 Retailer-Consumer model in Electricity Market under Demand Response

The participation of an electricity retailer with flexible demand under real-time consumer prices is dealt with a game-theoretical model. The problem is considered as a leader-followers game and solved as a nonlinear MPEC where the retailer exercises market power, and the consumers react to his decision based on their welfare-maximizing objective. We also explore the equilibrium model with perfect competition, which is solved as MILP/NLP problem. A quadratic consumer's utility function is used to model consumers' behavior that forces the retailer to impose adequate price tariffs. Illustrative examples with realistic data are used to analyze the retailer's decisions and consumer's reactions under market power or perfect competition.

5.2 Conclusions

The main conclusions of this dissertation are summarized below:

(1) Financial contracts (both physical and financial) facilitate the integration of RES deployment into the electricity system. A coherent risk measurement properly manages risk averse generators participation in the electricity market under uncertainty and enhance RES integration into the power system by boosting generators confidence towards RES deployment.

(2) Auction-based emissions trading, coupled with financial contracts, and RES penetration effectively plummet GHG emissions without compromising the social welfare. The EUAs allowances trading also encourages the low energy-intensive sectors to enter the economy as the market based mechanism negates the windfall profits caused by grandfathered allowance allocations.

(3) Demand-side management of electricity with DR programs is effectively measured by consumer's welfare (via consumers' utility function) and retailer's profit optimization so that system reliability and participants objectives are achieved simultaneously.

5.3 Suggestions for Future Research Work

Some suggestions for future research work are listed below:

1. By relaxing conventional generators' production cost function to be linear, the model could be extended to study the interaction between market power, and risk aversion considering different contracts over time. Moreover, the model can be applied for problems that incorporate financial contracts, and risk aversion to study equilibrium solutions.

2. Another future research may be extended from Chapter 3 to study the financial and environmental impact of emerging trends in power systems, such as distributed generation and large-scale electricity storage systems.

3. In relation to the demand response problem, it can be applied to include generators' and the independent system operator's problem under uncertainty where the respective participants objectives are met simultaneously. In addition, within a similar modelling framework to us, the RES generation schedule and main grid trading constraints could be included by explicitly modeling intelligent prosumers so that they can trade their energy instead of being only net consumers.

Appendix A: Appendix One

A.1 NLP formulation

The NLP can also be formulated by concatenating and minimizing the complementarity conditions as objective function of the problem and entering all the remaining equality and inequality KKT optimality conditions as constraints of the problem. This is done by leading all the complementarities to zero while enforcing both equalities and inequalities in the KKT optimality conditions as system constraints as proposed by Leyffer and Munson (2010).

$$\min \sum_{\omega\in\Omega,t\in T,j\in J} \{\nu_{jt\omega}^{C_{min}}(\Delta_{jt\omega}^{C}+\Delta_{j}^{C_{max}})+\nu_{jt\omega}^{C_{max}}(\Delta_{j}^{C_{max}}-\Delta_{jt\omega}^{C})+\varepsilon_{jt\omega}(q_{jt\omega}+\Delta_{jt\omega}^{C})\}+$$
$$+\sum_{\omega\in\Omega,t\in T}\{\alpha_{t\omega}^{+}(y_{t\omega}-\delta_{t\omega})+\alpha_{t\omega}^{-}(y_{t\omega}+\delta_{t\omega})+y_{t\omega}\beta_{t\omega}+q_{t\omega}^{S}\theta_{t\omega}\} \tag{A.1.1a}$$

subject to

$$\sigma_\omega P_t+\mu_{t\omega}=0 \quad \forall j,\forall t,\forall\omega \tag{A.1.1b}$$

$$\mu_{t\omega}+\sigma_\omega P_{t\omega}^{S}-\theta_{t\omega}=0 \quad \forall t,\forall\omega \tag{A.1.1c}$$

$$\mu_{t\omega}+\alpha_{t\omega}^{+}-\alpha_{t\omega}^{-}=0 \quad \forall t,\forall\omega \tag{A.1.1d}$$

$$\sigma_\omega C-\alpha_{t\omega}^{+}-\alpha_{t\omega}^{-}-\beta_{t\omega}=0 \quad \forall t,\forall\omega \tag{A.1.1e}$$

$$\sum_{j\in J} q_{jt\omega}-q_{t\omega}^{S}=\delta_{t\omega} \quad \forall t,\forall\omega \tag{A.1.1f}$$

$$P_t-a_{jt\omega}+B_{jt\omega}(q_{jt\omega}+\Delta_{jt\omega}^{C})-\varepsilon_{jt\omega}=0 \quad \forall j,\forall t,\forall\omega \tag{A.1.1g}$$

$$-a_{jt\omega}+B_{jt\omega}(q_{jt\omega}+\Delta_{jt\omega}^{C})-\nu_{jt\omega}^{C_{min}}+\nu_{jt\omega}^{C_{max}}-\lambda_{j\omega}-\varepsilon_{jt\omega}=0 \quad \forall j,\forall t,\forall\omega \tag{A.1.1h}$$

$$\sum_{t\in T}\Delta_{jt\omega}^{C}=0 \quad \forall j,\forall\omega \tag{A.1.1i}$$

$$\Delta_{jt\omega}^{C}+\Delta_{j}^{C_{max}}\geq 0 \quad \forall j,\forall t,\forall\omega \tag{A.1.1j}$$

$$q_{jt\omega}+\Delta_{jt\omega}^{C}\geq 0 \quad \forall j,\forall t,\forall\omega \tag{A.1.1k}$$

$$\Delta_{j}^{C_{max}}-\Delta_{jt\omega}^{C}\geq 0 \quad \forall j,\forall t,\forall\omega \tag{A.1.1l}$$

$$y_{t\omega}-\delta_{t\omega}\geq 0 \quad \forall t,\forall\omega \tag{A.1.1m}$$

$$y_{t\omega}+\delta_{t\omega}\geq 0 \quad \forall t,\forall\omega \tag{A.1.1n}$$

$$q_{t\omega}^{S}\geq 0 \quad \forall t,\forall\omega \tag{A.1.1o}$$

$$y_{t\omega}\geq 0 \quad \forall t,\forall\omega \tag{A.1.1p}$$

$$\alpha_{t\omega}^{+},\alpha_{t\omega}^{-},\beta_{t\omega},\varepsilon_{jt\omega},\theta_{t\omega},\nu_{jt\omega}^{C_{min}},\nu_{jt\omega}^{C_{max}}\geq 0 \tag{A.1.1q}$$

$$\lambda_{j\omega},\mu_{t\omega} \quad \forall j,\forall t,\forall\omega \quad \text{free.} \tag{A.1.1r}$$

If the objective function (A.1.1a), is 0 at the optimal solution, then this is a solution of the retailer-consumer equilibrium.

A.2 Some Results

Since the shape of the plot is not convenient to be shown with the other cases, in Figure A.1, we depict expected profit results using the increase in parameter B for the MILP model. As we can see, the expected profit is entirely negative. That means the retailer may participate in the DR program to avoid a higher penalty in this case. On the other hand, as the increase in parameter B decreases price tariffs, it is unavoidable to have expect negative profits.

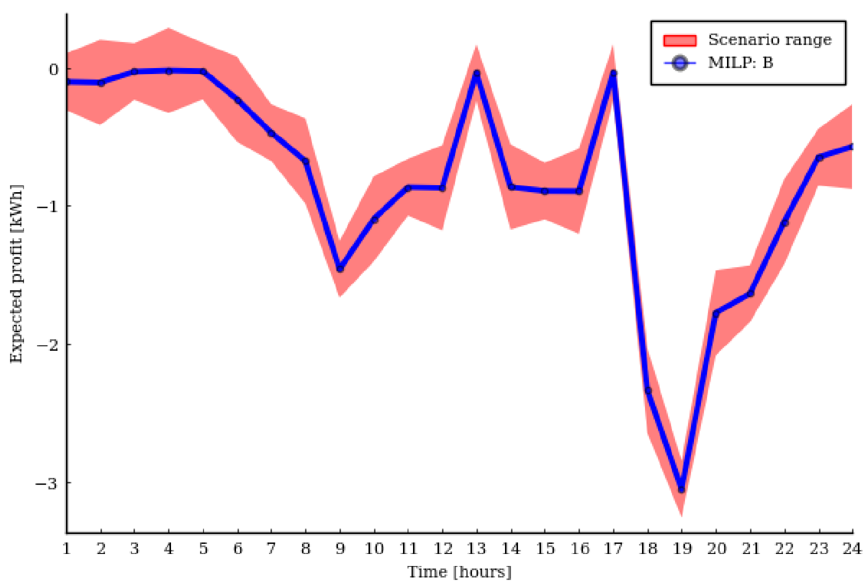


Figure A.1: Expected profit in the equilibrium model. increasing parameters a.

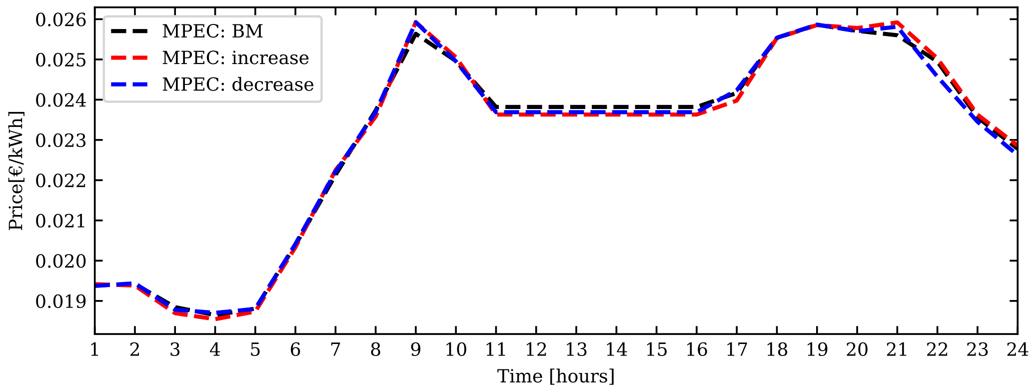


Figure A.2: Price tariffs with the MPEC model using the BM for different values of penalty (500, 1000, 3000).

Though it affects the power imbalance at a specific time, penalizing the retailer's objective function with weight C has not significantly affected market outcomes. For instance, Figure A.2 shows price tariffs using the BM($C = 1000, C = 500$ and $C = 3000$). As can be seen, the three price tariff curves seem to be indifferent throughout the period. However, a higher imbalance penalty may result in negative expected profits.

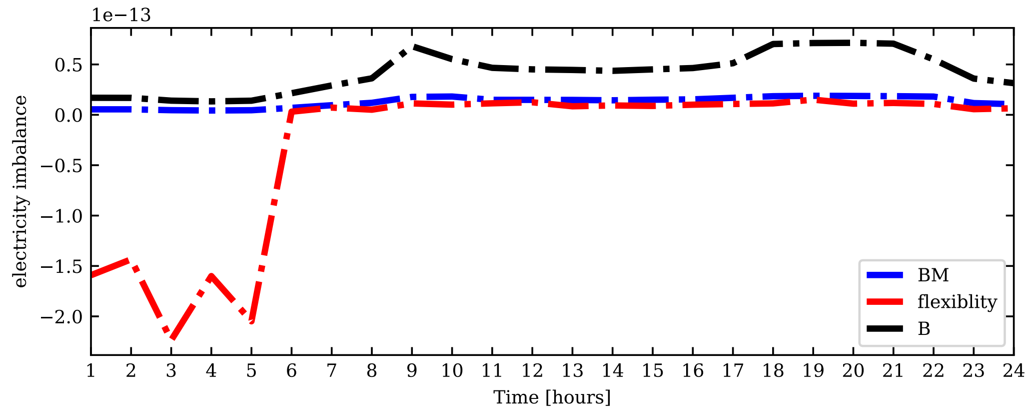


Figure A.3: Electricity imbalances for different cases in the MPEC model.

Figure A.3 shows electricity imbalance $\delta_{t\omega}$ with the BM, and by increasing B and $\Delta_j^{C_{max}}$. When demand flexibility increases, $\delta_{t\omega}$ becomes negative in the off-peak periods, which economically makes sense. The opposite can happen when $B_{jt\omega}$ increases.

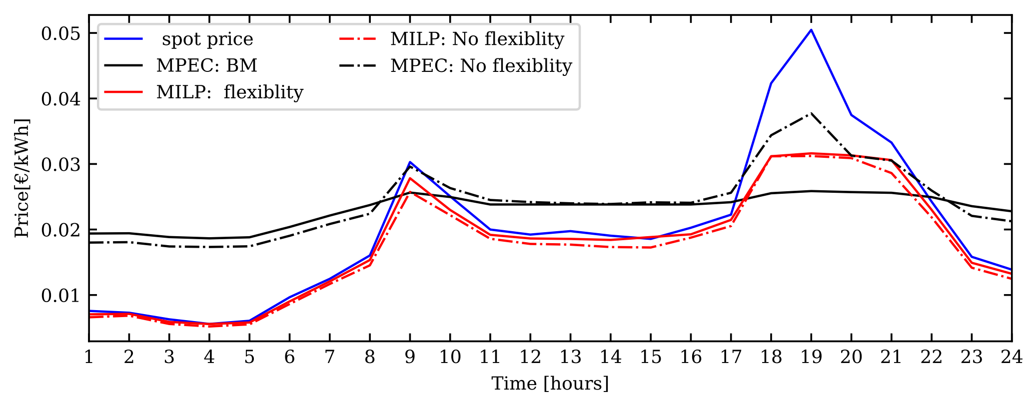


Figure A.4: Electricity prices comparison in both models considering non flexible demand and the BM.

We also compare prices with and without demand flexibility. Figure A.4 depicts all prices (spot market price, price tariffs with the MPEC and the MILP) so that possible to compare price tariffs without demand flexibility to price tariffs with flexibility and spot market price. Without demand flexibility, price tariffs are higher in the BM under the MPEC model. However, they are the opposite in the equilibrium model. Since prices are not significantly different using the BM case, we compare them with higher flexibility, which resulted in higher prices. This is a counterintuitive result that indicates price tariffs increase if consumer demand flexibility increases, which may be due to the nature of the competition since prices are already below the spot market prices under the BM case.

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