UNIVERSITÀ
DEGLI STUDI
DI BRESCIA

DOTTORATO DI RICERCA IN
MODELLI E METODI PER L'ECONOMIA E IL MANAGEMENT
(ANALYTICS FOR ECONOMICS AND MANAGEMENT - AEM)

SECS-P/02 POLITICA ECONOMICA

XXXIII CICLO

TAX EFFECTS ON FIRM'S DECISIONS UNDER UNCERTAINTY: A NUMERICAL ANALYSIS

Candidato:
Nicola Comincioli
f.to digitalmente ex art. 24 D.Lgs. 82/05

Relatore:
Prof. Sergio Vergalli
f.to digitalmente ex art. 24 D.Lgs. 82/05

Secondo Relatore:
Prof. Paolo M. Panteghini
f.to digitalmente ex art. 24 D.Lgs. 82/05

ANNO ACCADEMICO 2019/2020

This page intentionally left blank.

Contents

Acknowledgements . . . . **iii**

Abstract . . . . **v**

Introduction . . . . **vii**

Chapter 1. Welfare effects of business taxation under default risk . . **1**

1.1. Introduction 1
1.2. The model 3
1.2.1. The value of debt and equity 4
1.2.2. Optimal default 5
1.2.3. Optimal coupon 6
1.2.4. Tax revenue and welfare 7
1.3. A numerical analysis 8
1.3.1. Sensitivity analysis on the cost of default α 10
1.3.2. Sensitivity analysis on the EBIT volatility σ 12
1.3.3. An insight on taxation effects 13
1.4. Conclusion 14
1.A. Appendix 16
1.A.1. The value of debt 16
1.A.2. The value of equity 17

Chapter 2. The start-up decision under default risk 19

2.1. Introduction 19
2.2. The model 20
2.2.1. Random process 20
2.2.2. The start-up decision 21
2.2.3. Tax revenue and welfare under investment decision 23
2.3. A numerical analysis 24

2.3.1. The relevant parameter values . . . 25
2.3.2. Effects on investment decision . . . 26
2.3.3. Sensitivity analysis . . . 28
2.3.4. Comparing start-ups, obliged and mature firms . . . 30
2.4. Conclusion . . . 33
2.A. Appendix . . . 33
2.A.1. The value of the firm . . . 33
2.A.2. Optimal default trigger and optimal coupon . . . 35
2.A.3. Optimal investment trigger . . . 36

Chapter 3. Debt shifting and transfer pricing in a volatile world . . . 37

3.1. Introduction . . . 37
3.2. The model . . . 38
3.2.1. EBIT dynamics . . . 39
3.2.2. Net profit of the multinational company . . . 40
3.2.3. The value of equity . . . 41
3.2.4. The value of debt . . . 43
3.2.5. The value of multinational company . . . 43
3.2.6. Some comparative statics . . . 44
3.3. A numerical analysis . . . 45
3.3.1. Purpose and parameters . . . 45
3.3.2. Effects of tax avoidance practices . . . 46
3.3.3. Sensitivity analysis . . . 48
3.4. Conclusion . . . 51
3.A. Appendix . . . 52
3.A.1. The value of equity . . . 52
3.A.2. The value of debt . . . 54
3.A.3. The value of the MNC . . . 55
3.A.4. Derivatives of optimal controls with respect to m and n . . . 56

Bibliography . . . 57

Acknowledgements

The successful completion of this doctoral program and the development of this thesis would not have been possible without the guidance and the assistance of several people who, in one way or another, contributed to it and extended their valuable help.

In the first place, I would like to express my sincere gratitude to my supervisors Prof. Sergio Vergalli and Prof. Paolo M. Panteghini, for the continuous support to my studies and my research, for their patience, motivation and immense knowledge. Their advisement and support enabled me to develop a great understanding of the subject. In addition, I am grateful to Prof. Sergio Vergalli also for his role as a tutor. During these years, I have been considered not only as a PhD student but also as a valuable member of his research team. Besides being a great honor, such an involvement has resulted in an remarkable academic growth.

A special thanks goes to all those with whom I had the pleasure of collaborating during these years: Verena Hagspiel, Peter M. Kort, Francesco Menoncin and Raffaele Miniaci, to name a few. They have always shown great interest and kindness and allowed me to learn a lot. In addition, I would like to extend my thanks to Chiara D'Alpaos and Jacco Thijssen for their valuable role as reviewers of this thesis.

Then I would like to heartily thank my family: my parents Elena and Sergio and my sister Susanna, for the support and motivation given me throughout these years and for those I am sure they will not leave me out in the future.

Last but not least, I am grateful towards Ilenia, who has accompanied me with patience and encouragement during this journey.

Brescia, March 2021 *Nicola Comincioli*

This page intentionally left blank.

Abstract

La presente tesi si compone di tre capitoli che, in modo diverso, si occupano di modellare il comportamento di aziende rappresentative in condizioni di incertezza, per studiarne gli effetti sul loro valore e sul welfare da esse generato.

Il primo capitolo è dedicato allo sviluppo di un modello stocastico, mediante il quale vengono studiati gli effetti della politica fiscale su un'azienda rappresentativa. Per avere un maggiore realismo, viene ipotizzato che tale azienda sia soggetta al rischio di default ed operi in un contesto economico caratterizzato da instabilità finanziaria. Questo modello mette in luce come il gettito fiscale ed il welfare siano profondamente influenzati dal rischio di default e dal relativo costo, a condizione che gli interessi passivi siano deducibili. Vengono inoltre mostrati gli effetti della politica fiscale e dell'instabilità finanziaria sul welfare di sistema, introducendo una misura di *deadweight loss*. Lo sviluppo del modello teorico è supportato da adeguate simulazioni numeriche.

Il secondo capitolo presenta un modello stocastico dedicato allo studio degli effetti della politica fiscale e dell'instabilità finanziaria sulla decisione di investimento di una nuova azienda e sul welfare corrispondente. In particolare, inizialmente l'attenzione è rivolta alla scelta ottimale della tempistica dell'investimento e sulla successiva probabilità di default. In seguito vengono invece presi in esame il valore attuale netto di un progetto di investimento, il gettito fiscale da questo generato ed il welfare conseguente. Il punto centrale di questo capitolo è rappresentato dal confronto degli effetti della politica fiscale sul welfare relativi al caso di una start-up, di un'azienda matura e di un'azienda obbligata ad investire al tempo zero. Questo confronto fornisce ai policy-maker uno strumento utile a calibrare i sistemi fiscali secondo la prevalenza delle aziende soggette ad esso. Queste analisi sono supportate da estese simulazioni numeriche, calibrate su dati reali, che mettono in luce come una politica fiscale aggressiva riduca il livello generale di welfare, dal momento che le nuove imprese possono ritardare l'inizio dell'operatività.

Il terzo capitolo infine introduce un modello stocastico per descrivere un'impresa multinazionale che si avvale di pratiche di elusione fiscale. In particolare, vengono presi in considerazione sia il *transfer pricing* che il *debt shifting* e viene mostrato come gli azionisti scelgano il relativo livello ottimale. Questo modello è oggetto di numerose simulazioni numeriche, calibrate su dati reali, volte a misurare l'impatto di questi comportamenti sul valore della multinazionale e a studiare come il beneficio derivante da essi sia influenzato da variabili esogene. I principali risultati di queste simulazioni sono i seguenti: un aumento della variabilità dei profitti riduce la leva finanziaria e, in modo meno rilevante, il valore dell'azienda; il costo del *transfer pricing* riduce nettamente il valore dell'impresa lasciando invariata la *capital structure*, mentre quello del *debt shifting* influenza entrambi in modo rilevante; il ricorso alle pratiche di elusione fiscale è strettamente legato al differenziale dei livelli di tassazione cui l'impresa è soggetta.

Introduction

The success or failure of any business is profoundly linked with the financial stability of the environment to which it belongs. The concept of financial stability has been widely discussed in economic literature, where many possible definitions have been proposed. Most of them refer to the same key aspects: efficient resource allocation and risk distribution, facilitation and enhancement of economic processes, resilience to stress episodes and absence of systemic failures. Financial stability is intrinsically related also to business taxation. It is widely discussed in literature that tax policy may contribute to insolvency, especially in the case of highly leveraged firms, with consequential impacts on financial stability. The importance of this relationship is highlighted by the fact that, despite the most recent financial crises, the use of debt over equity is encouraged by almost all tax systems.

This thesis aims to investigate how enterprises' decisions are affected by the interaction of business taxation and financial stability. More in detail, this dissertation applies mathematical models to analyze decisions regarding the optimal capital structure and the optimal investment timing. Moreover, it studies welfare effects of such decisions. The main novelty of this work is the generalized use of a stochastic approach, in discontinuity with most literature. In addition, the financial (in)stability is represented by means of two key variables: default cost and profit variability. Finally, tax avoidance practices are also taken into account, studying how they dovetail in previous discussion. In this regard, a further novelty is that the two most relevant practices of this kind, that is transfer pricing and debt shifting, are considered at the same time. All the models developed in this thesis are supported by extensive numerical simulations and, to increase the robustness of results, these have always been calibrated on the basis of empirical evidence proposed in the literature.

This thesis is structured as follows. Chapter 1 uses the benchmark model,[1] which is then generalized in the two following chapters. More specifically, this chapter studies the behavior of a representative firm in a stochastic context. In particular, for a given tax rate, tax revenue and welfare are crucially affected by default risk and its costs, as long as interest expenses are deductible. Thus, an evaluation made without accounting for default may be dramatically biased. Moreover, this Chapter shows that the "debt bias" due to the tax treatment of debt finance causes a quite relevant deadweight loss.

Chapter 2 introduces a real option model to investigate how fiscal policy affects a representative firm's investment decision and to measure its welfare effects. On the one hand, the effects of financial instability on the optimal investment timing and on the probability of default are studied. On the other hand, it is shown how the net present value of an investment project, the tax revenue generated and the welfare are influenced by financial instability. Then, a comparison of welfare effects of tax policy on start-ups, mature and obliged firms is provided. This comparison provides policy-makers a tool to shape their tax systems according to the characteristics of their firms. All presented analyses are supported by extensive numerical simulations, based on realistic data.

Chapter 3 finally uses a stochastic model with a multinational company that exploits tax avoidance practices is introduced.[2] This model focuses on transfer pricing and debt shifting activities and show how their optimal level is chosen by the shareholders. In addition, this chapter provides a numerical approach to measure the impact of tax avoidance practices on a multinational company's value. In particular, this chapter shows that: an increase in risk sharply reduces leverage and slightly decreases a multinational company's value; the cost of transfer pricing leads to a sharp reduction in the multinational company's value, whereas it does not affect leverage; the impact on multinational company's decisions is increasing in the tax rate differential; finally, the cost of debt shifting has always a relevant impact on both multinational company's value and leverage.

---

[1]This Chapter has been published as: Comincioli, N., Panteghini, P. M., and Vergalli, S. (2021). Welfare Effects of Business Taxation under Default Risk. *International Tax and Public Finance.* doi: 10.1007/s10797-020-09650-1. This article is here reprinted, with few minor modifications, under the license number 4992420133210, issued by Springer Nature on January 19, 2021.

[2]This Chapter has been released as a working paper: Comincioli, N., Panteghini, P. M., Vergalli, S. (2021). Debt Shifting and Transfer Pricing in a Volatile World, CESifo Working Papers, 8807.2020.

CHAPTER 1

Welfare effects of business taxation under default risk

1.1. Introduction

Business taxation and financial stability are intrinsically related.[1] This is mainly due to the fact that, despite recent financial crises, almost all tax systems encourage the use of debt over equity finance. Though this "debt bias" has been reduced by tax devices such as thin cap and earning stripping rules, it still persists (see, e.g., De Mooij and Hebous (2018) and the articles quoted therein, as well as Sinn (2010)).

This "debt bias" has welfare effects. For instance, Sørensen (2017) shows that the socially optimal debt-asset ratio is $2\% - 3\%$ below the current debt level. Using a different approach, Weichenrieder and Klautke (2008) calculate the efficiency costs due to the debt-equity distortion. However, both articles are based on a deterministic framework: the default risk premium is simply proxied by a quadratic cost function. Since this simplification may lead to biased results, we will study welfare implications in a really stochastic context. In particular, we model a framework where default risk is linked to the stochastic process driving profitability. Moreover, we evaluate the joint effect of default risk and its (sunk) cost. We will therefore show that the "debt bias" causes a deadweight loss which is much greater than that obtained in the existing literature.

Our main aim is to investigate the effects of default costs and risk on both welfare and tax revenue, as well as that on the ratio between welfare loss and tax revenue (which is a measure of deadweight loss).[2] In order to focus on tax rate effects, we

[1]For instance, Kocherlakota (2010) argues that bailouts are inevitable if the default of firms causes systemic failure. For this reason, he proposes a Pigouvian tax, aimed at offsetting negative externalities arising from financial instability.

[2]For simplicity, we assume symmetric information and full interest deductibility. For a detailed analysis on business taxation under asymmetric information, see Cohen et al. (2016) and the articles cited therein. Partial interest deductibility does not change the quality of our results.

will use a very simple framework, where the firm's capital structure will be initially chosen.[3]

It is worth noting that the cost of default is affected by both market conditions and default rules. Since a change in these rules is feasible, we can say that, to some extent, a policy-maker can affect default costs.[4] Similarly, volatility is affected by both systemic and firm-specific risk. If therefore a policy-maker can affect systemic risk, it is useful to study the effects of volatility on a firm's value as well as on welfare.[5]

Using realistic parameter values we will show that welfare and tax revenue crucially depend on tax rates as well as on the volatility of the Earning Before Interest and Taxes (EBIT) and the default cost. In particular, welfare is decreasing in the tax rate under default risk. Tax revenue is always increasing in the statutory tax rate and hence no Laffer curve is found. Moreover, an increase in the expected cost of default raises tax revenue and decreases welfare. Similarly, volatility raises (reduces) the expected value of tax revenue (welfare). Thus, a more stable financial system is beneficial from a social point of view, although it reduces tax revenue and vice versa. A similar effect is found for the default cost, which increases (reduces) tax revenue (welfare).

The structure of this paper is as follows. Section 2 introduces a simple trade-off model and calculates the value function of a representative firm, as well as tax revenue and the value of welfare. Section 3 provides a numerical analysis where the

[3]Assuming that the capital structure changes over time, due to the variability of market conditions, would make our model complex (see, e.g., Strebulaev (2007), De Marzo and Sannikov (2007) and Bolton et al. (2019)) and would lead to a quite difficult interpretation of tax effects. For this reason, we leave this point for further research.

[4]McGowan and Andrews (2018) provide a comprehensive analysis of insolvency procedures across OECD countries and find that they are quite heterogeneous. In particular, they state that "[a] comparison of the 2010 and 2016 values suggests that recent reform efforts have been largest for prevention and streamlining, with reforms observable in 11 countries, especially European countries (e.g. Portugal). This may reflect the fact that such measures have been recently endorsed by the European Commission and the IMF, in response to the crisis [...]. Barriers to restructuring have also declined in 10 countries, while reform activity affecting the personal costs to failed entrepreneurs has been less ambitious, with only Chile, Greece and Spain undertaking reforms since 2010". Of course, heterogeneous rules can lead to heterogeneous default costs.

[5]Here, we deal with one policy-maker who can implement both monetary and fiscal policies, although we are aware that separate entities deal with them. As pointed out by Sinn (2018) however, the separation of roles sometimes vanishes and a central bank can affect fiscal policy. The reverse may also be true when bank taxes are levied (De Mooij and Keen (2016)).

effects of changes in the tax rate, default risk and its expected cost are examined. Section 4 summarizes our findings and discusses their policy implications.

1.2. The model

In this Section we apply a continuous-time model in line with Goldstein et al. (2001). By assumption, a representative firm can borrow from a perfectly competitive credit sector, where the discount factor is the risk-free interest rate r.[6] The firm's EBIT, defined as Π, is assumed to be stochastic and follow a Geometric Brownian Motion (GBM):[7]

$$\frac{d\Pi}{\Pi} = \sigma dz, \tag{1.2.1}$$

where $\Pi_0 > 0$ is the initial EBIT, σ is the instantaneous standard deviation of $\frac{d\Pi}{\Pi}$, and dz is the increment of a Wiener process. According to this assumption, EBIT follows a random walk. To keep the model as simple as possible, we assume that Π depends on previous decisions (such as investment).[8] It is worth noting that such an assumption is quite common in the literature (see, e.g., Bolton et al. (2019) and the articles quoted therein). Moreover, in line with Leland (1994) and Panteghini (2007b), we introduce the following:

Assumption 1.1. *At time 0, the firm can borrow some resources thereby paying a coupon C, which cannot be renegotiated.*

Assumption 1.2. *If the firm does not meet its debt obligations, default occurs and hence, the firm is expropriated by the lender.*

Assumption 1.3. *The cost of default is equal to a percentage $\alpha \in [0, 1)$ of defaulted firm's value.*

Assumption 1.1 means that the firm sets a coupon and then computes the debt market value. Without arbitrage, this is equivalent to first setting the debt value and then calculating the effective interest rate. For simplicity, we assume that debt

[6]This framework is built as a risk neutral world, according to Lucchetta et al. (2019). Then the risk-free rate can be used to evaluate any contingent claim on an asset.
[7]The general form of the geometric Brownian motion is $d\Pi = \mu\Pi dt + \sigma\Pi dz$ where μ is the expected rate of growth. With no loss of generality, here we set $\mu = 0$.
[8]In this model, our firm chooses its capital structure for a given investment. In Panteghini (2007b), Panteghini (2007a) and Panteghini and Vergalli (2016), it is shown that the qualitative properties of the model do not change when an investment decision is also made.

cannot be renegotiated: this means that we apply a *static* trade-off approach where the firm's financial policy cannot be reviewed later.

Assumptions 1.2 and 1.3 introduce default risk and its cost, respectively. Given (1.2.1), if the firm's EBIT falls to a given threshold value, defined as $\overline{\Pi}$, the firm is expropriated by the lender (Assumption 1.2), who becomes shareholder. Default causes a sunk cost borne by the lender. By assumption, this cost is proportional to the value of the defaulted firm (Assumption 1.3).

Finally, we also introduce the following:

ASSUMPTION 1.4. *The threshold level $\overline{\Pi}$ is chosen by shareholders at time 0.*

Assumption 1.4 implies that shareholders behave as if they owned a put option, whose exercise leads to default.[9]

Given these assumptions, the firm's net profit is equal to $\Pi^N = (1-\tau)(\Pi - C)$, where τ is the relevant tax rate. It is worth noting that a tax saving due to debt-finance arises as long as the business tax rate is higher than the lender's rate. For simplicity, we let the lender's pre-default tax rate be nil. When however default takes place, the lender becomes shareholder and is therefore subject to corporate taxation.

1.2.1. The value of debt and equity. Let us calculate the mark-to-market value of debt, $D(\Pi)$, and equity, $E(\Pi)$. Given these results we will be able to compute a firm's value function:

$$V(\Pi) = D(\Pi) + E(\Pi). \quad (1.2.2)$$

Let us start with debt. According to Assumption 1.3, the (sunk) default cost is a percentage α of the defaulted firm. Hence, the lender will own $(1-\alpha)$ of the defaulted firm.[10] Using dynamic programming, we can therefore write the value of debt as follows:

[9]For further details on the characteristics of default conditions see, e.g., Leland (1994) and Panteghini (2007b).

[10]As pointed out by Estrin et al. (2017), economic agents are sensitive to different elements of the bankruptcy codes. Moreover, the authors show that some countries are more debt-friendly than others. All of these features are here summarized by our parameter cost α. If therefore countries are debtor-friendly (-unfriendly), α is expected to be lower (higher). For a detailed (and economic) analysis of default procedures see also Claesens et al. (2001).

(1.2.3)
$$D(\Pi) = \begin{cases} (1-\alpha)(1-\tau)\Pi dt + e^{-rdt}\mathbb{E}[D(\Pi + d\Pi)] & \text{a.d.} \\ Cdt + e^{-rdt}\mathbb{E}[D(\Pi + d\Pi)] & \text{b.d.} \end{cases},$$

where $\mathbb{E}$ is the expected value operator. Labels "a.d." and "b.d." stand for "after default" and "before default", respectively. As shown in Appendix 1.A.1, (1.2.3) can be rewritten as:

(1.2.4)
$$D(\Pi) = \begin{cases} \frac{(1-\alpha)(1-\tau)\overline{\Pi}}{r} & \text{a.d.} \\ \frac{C}{r} + \left[\frac{(1-\alpha)(1-\tau)\overline{\Pi}-C}{r}\right]\left(\frac{\Pi}{\overline{\Pi}}\right)^{\beta_2} & \text{b.d.} \end{cases},$$

where $\beta_2 = \frac{1}{2} - \sqrt{\left(\frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0$. As shown in (1.2.4), before default the debt value consists of two terms. The first one, C/r, is a perpetual rent which measures the debt value without default, while the second term accounts for the default effects. In particular, the term $\left(\frac{\Pi}{\overline{\Pi}}\right)^{\beta_2}$ measures the present value of 1 euro contingent on the default event. After default, the lender becomes shareholder and the value of his/her claim is equal to $\frac{(1-\alpha)(1-\tau)\overline{\Pi}}{r}$.

Let us next focus on equity. Applying dynamic programming the value of equity can be written as follows:

(1.2.5)
$$E(\Pi) = \begin{cases} 0 & \text{a.d.} \\ (1-\tau)(\Pi - C)dt + e^{-rdt}\mathbb{E}[E(\Pi + d\Pi)] & \text{b.d.} \end{cases}.$$

As can be seen, after default the equity value is nil (according to Assumption 1.2). As proven in Appendix 1.A.2, (1.2.5) can be written as:

(1.2.6)
$$E(\Pi) = \begin{cases} 0 & \text{a.d.} \\ \frac{(1-\tau)(\Pi-C)}{r} - \frac{(1-\tau)\left(\overline{\Pi}-C\right)}{r}\left(\frac{\Pi}{\overline{\Pi}}\right)^{\beta_2} & \text{b.d.} \end{cases}.$$

As shown in (1.2.6), before default the equity value is given by the summation between the perpetual rent $\frac{(1-\tau)(\Pi-C)}{r}$ and the loss contingent on the event of default, $-\frac{(1-\tau)\left(\overline{\Pi}-C\right)}{r}\left(\frac{\Pi}{\overline{\Pi}}\right)^{\beta_2}$.

1.2.2. Optimal default. Given (1.2.6), we can now calculate the default threshold point under debt financing. Following Leland (1994) and Goldstein et al. (2001), shareholders are assumed to solve the following problem:

(1.2.7)
$$\max_{\overline{\Pi}} E(\Pi).$$

Using (1.2.6) and rearranging the FOC of (1.2.7) gives:

$$\overline{\Pi} = \frac{\beta_2}{\beta_2 - 1} C < C. \tag{1.2.8}$$

This means that, if the firm's net cash flow is negative, shareholders can decide whether to inject further resources to meet the firm's debt obligations or to default. As long as they pay the coupon they can exploit future recoveries in the value of their claim. In addition, we introduce the following:

Assumption 1.5. *Given the stochastic process (1.2.1), after default the optimal policy is to use only equity.*

According to Assumption 1.5, after default former lenders become equity holders and thus the firm is fully equity financed. This means that a second default never takes place. This result certainly holds as long as the stochastic process of EBIT is unchanged. To our knowledge, there are no articles estimating the change (if any) of stochastic processes after default. For this reason, we let (1.2.1) drive the stochastic process of EBIT also after default. Intuitively, this simple model allows us to deal with a fairly realistic context where, after default, a bad company and a good company may be founded. The former is α times the value of the defaulted firm and faces procedure costs. The latter goes on producing. Of course, after the birth of the good company, its probability to default is close to zero.[11]

1.2.3. Optimal coupon. Substituting (1.2.4) and (1.2.6) into (1.2.2) gives the pre-default value of the firm:

$$V(\Pi) = \frac{(1-\tau)\,\Pi}{r} + \tau\frac{C}{r} - \left[(1-\tau)\,\alpha\frac{\overline{\Pi}}{r} + \tau\frac{C}{r}\right]\left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2}. \tag{1.2.9}$$

To find the optimal coupon we maximize (1.2.9) with respect to C.[12]

[11] This model, characterized by an one-off default, has a widespread use since Goldstein et al. (2001). Of course, in a dynamic model where the stochastic process of EBIT may change over time, and the optimal coupon is both state- and time-dependent, a good company might borrow and have some probability of default. We leave this topic for future research.

[12] This maximization implies that there do not exist conflicts of interest between shareholders and lenders.

Differentiating the value function (1.2.9) with respect to C and rearranging gives:

$$C = \frac{\beta_2 - 1}{\beta_2} \left\{ \frac{\tau}{(1-\beta_2)\left[(1-\tau)\,\alpha \frac{\beta_2}{\beta_2 - 1} + \tau\right]} \right\}^{-\frac{1}{\beta_2}} \Pi. \tag{1.2.10}$$

Given (1.2.10), it is straightforward to see that $\frac{\partial C}{\partial \alpha} < 0$. Moreover it is easy to prove that $\frac{\partial C}{\partial \tau} > 0$.[13] Finally, $\frac{\partial C}{\partial \sigma^2}$ is expected to be negative for realistic parameter values. The intuition behind this result is straightforward: *coeteris paribus*, an increase in volatility is expected to anticipate default and its sunk cost. This induces firms to borrow less (and hence, pay a lower coupon).

1.2.4. Tax revenue and welfare. Using the value function (1.2.9), we can also measure the present value of tax revenue:

$$T(\Pi) = \frac{\tau}{r} \left[\Pi - C + \left(C - \alpha \overline{\Pi}\right) \left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2} \right]. \tag{1.2.11}$$

Of course, $T(\Pi)$ is increasing in Π and decreasing in C. Moreover, given the inequality in (1.2.8), the term $\frac{\tau}{r}\left(C - \alpha\overline{\Pi}\right)\left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2}$, which measures the present value of tax revenues contingent on default, is positive. So, we can say that default leads to a twofold effect: on the one hand, it causes a sunk cost and, on the other hand, it leads to an increase in tax revenue, equal to $\tau\left(C - \alpha\overline{\Pi}\right)$ for any short period dt.

We let the welfare function be the summation of $V(\Pi)$ and $T(\Pi)$:[14]

$$W(\Pi) = V(\Pi) + T(\Pi) = \frac{\Pi}{r} - \alpha \frac{\overline{\Pi}}{r} \left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2}. \tag{1.2.12}$$

As can be seen, without default $W(\Pi)$ is equal to the perpetual rent $\frac{\Pi}{r}$. With default however, it is lower.[15] Of course, the welfare loss of taxation is $\alpha\frac{\overline{\Pi}}{r}\left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2}$. To

[13] Taking the log of (1.2.10) and differentiating it with respect to τ gives:

$$\frac{\partial \log C}{\partial \tau} = -\frac{1}{\beta_2}\left[\frac{1}{\tau} - \frac{1-\alpha\frac{\beta_2}{\beta_2-1}}{(1-\tau)\,\alpha\frac{\beta_2}{\beta_2-1}+\tau}\right] = -\frac{1}{\beta_2}\frac{\alpha\frac{\beta_2}{\beta_2-1}}{\tau\left((1-\tau)\,\alpha\frac{\beta_2}{\beta_2-1}+\tau\right)} > 0.$$

[14] In doing so we rule out consumer surplus for simplicity.

[15] Of course, this result holds under symmetric information. If, otherwise, information was asymmetric and there were agency costs of monitoring corporate managers, debt finance could have beneficial effects. Indeed, before lending, financial institutions would implement a due diligence on the firm which could considerably reduce equity holders' monitoring costs. In this case, welfare effects might be different. See, e.g., Cohen et al. (2016).

better understand the welfare effects of debt biases we will also analyze the ratio:

$$R = \frac{\alpha \frac{\overline{\Pi}}{r} \left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2}}{T(\Pi)} = \frac{\alpha \frac{\overline{\Pi}}{r} \left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2}}{\frac{\tau}{r}\left[\Pi - C + \left(C - \alpha\overline{\Pi}\right)\left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2}\right]}. \tag{1.2.13}$$

It is easy to show that the welfare cost (numerator) is increasing in τ.[16] As regards the denominator, we know that an increase in τ raises tax revenue, for any given tax base. Given the complexity of the tax base $\left[\Pi - C + \left(C - \alpha\overline{\Pi}\right)\left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2}\right]$ however, we find no simple mathematical solution. For this reason, in the next Section, we will run a numerical simulation: as will be shown, the tax base is decreasing in τ.

1.3. A numerical analysis

In this Section we use a numerical simulation to study the effects of both the default cost and EBIT volatility on our representative firm's value, as well as on tax revenue and welfare. In line with Sørensen (2017), we set $r = 2.5\%$.[17] We also let the initial value of EBIT be equal to 2.5. This allows us to normalize all the effects, since the perpetual rent Π/r is equal to 100.[18] We focus on values of τ between 10% and 45%: most statutory tax rates range between 20% and 30% (e.g., Sørensen (2017) uses an average rate of about 27%), although lower rates are applied in several countries.

In line with Dixit and Pindyck (1994), we let the benchmark value of standard deviation σ be 20%. Moreover, σ is assumed to range from 10% to 40% in order to perform our sensitivity analysis. Moreover, we will add a fifth scenario where σ is almost irrelevant (equal to 1%). This case allows us to see what happens when risk is negligible. The standard deviation σ is affected by both systemic and firm-specific risk. If therefore the policy-maker wants to improve financial stability we expect that it reduces σ.

---

[16]The numerator can be rewritten as $\zeta h(\tau)^{\xi}$, where $\zeta \equiv \alpha \frac{\Pi}{r} > 0$, $\xi \equiv \frac{\beta_2 - 1}{\beta_2} > 0$ and:

$$h(\tau) = \tau \left\{ (1-\beta_2)\left[(1-\tau)\,\alpha \frac{\beta_2}{\beta_2 - 1} + \tau\right]\right\}^{-1}.$$

Since $\frac{\partial}{\partial \tau} h(\tau) = -\alpha\beta_2 \left[\tau\left(\alpha\beta_2 - \beta_2 + 1\right) - \alpha\beta_2\right]^{-2} > 0$, $\zeta > 0$, and $\xi > 0$, the welfare cost is increasing in τ.

[17]We have also run a robustness check with $r = 5\%$. These results are available upon request and their quality is unaffected.

[18]In a tax-free context, the welfare value is equal to 100.

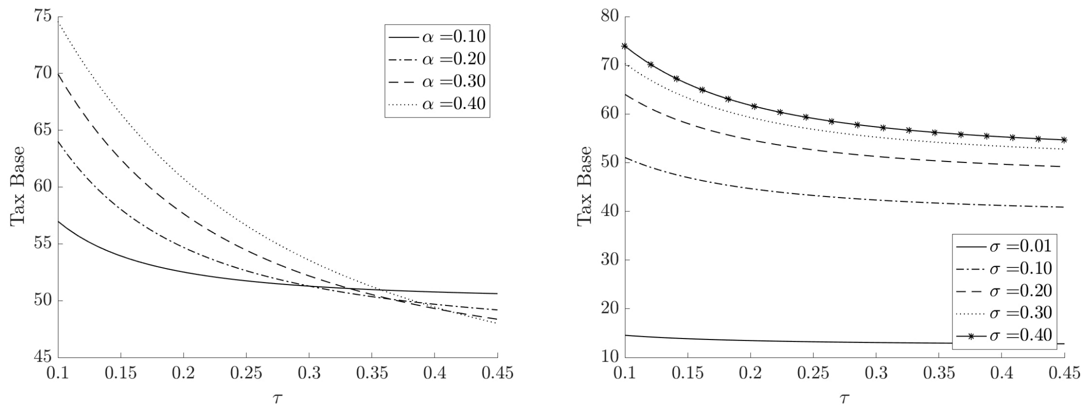

FIGURE 1.3.1. Sensitivity analysis about the tax base with different values of α (left panel) and σ (right panel).

As regards the value of α, the empirical evidence shows quite heterogeneous results. For instance, Andrade and Kaplan (1998) estimate distress costs of 10–23% of firm value for a sample of 31 highly leveraged transactions. Branch (2002) finds a total default-related cost that ranges between 12.7% and 20.5%. Davydenko et al. (2012) estimate the cost of default for an average defaulting firm to be 21.7% of assets' market value. These costs are shown to range from 14.7% for bond renegotiations to 30.5% for default. Interestingly, Glover (2016) finds that the average firm expects a default cost equal to 45% of its value under default. However, this cost is estimated to be less (25%) among defaulted firms. It is worth noting that the cost of default depends not only on market conditions but also on default rules.[19] This means that, to some extent, the government can affect the value of α by changing the insolvency regulation. Moreover, default costs may be only partially sunk, since a portion of them can be re-distributed among stakeholders. For these reasons we will use a wide range of values for α (from 10% to 40%).

We then provide a numerical analysis which enables us to analyze the effects of different values of α and σ, over the range of τ of our interest, on the value function (1.2.9), tax revenue (1.2.11) and welfare (1.2.12), respectively.[20] Since both baseline values of α and σ are 20%, all plots regarding the sensitivity analysis with respect to α (σ) are computed with the other parameter equal to its benchmark value. In

[19]For instance, time-consuming default procedures are expected to increase α and vice versa.

[20]Notice that leverage ratios implicit in the following simulations are in line with those collected by Aswath Damodaran. These data are available at: `http://pages.stern.nyu.edu/~adamodar/`.

each plot, the case with both parameters set equal to the baseline value are drawn with a dash-dot (dashed) line if the sensitivity analysis is performed with respect to α (σ). As pointed out, we will also provide a numerical simulation of the tax base. As shown in Figure 1.3.1, the tax base is decreasing in τ, irrespective of the values of α and on σ.

1.3.1. Sensitivity analysis on the cost of default α. In the four panels of Figure 1.3.2, we show the effects of α and τ on the firm's value, tax revenue, the welfare function and the welfare loss. As can be seen from the top-left panel, the firm's value is decreasing in τ, despite the tax benefit arising from the deductibility of C. Not surprisingly, the greater the (sunk) default cost the lower the value of $V(\Pi)$.[21]

As shown in the top-right panel, tax revenue is always increasing in τ. Of course, an increase in τ leads to a mechanical rise in $T(\Pi)$. It also increases C and the probability of default. On the one hand, after default, tax revenue increases as C is no longer deducted (thereby raising the tax base). On the other hand, the tax base is only a portion $(1-\alpha)$ of Π. Since the former effect dominates the latter one, we can therefore say that no Laffer curve exists. Moreover, if τ is low enough, α has a positive impact on $T(\Pi)$. This is due to the fact that default has a twofold effect. On the one hand, it causes a sunk cost; on the other hand, it increases tax revenue, due to the elimination of interest rate deductions: the value of such benefit is equal to $\tau\left(C-\alpha\overline{\Pi}\right)$ for any short period dt. If therefore τ is low enough, this latter effect dominates the previous one.

The bottom panels show the effects of α and τ on both welfare (left) and the welfare loss (right).[22] As can be seen, an increase in α leads to an increase in default cost and hence a rise in the welfare loss. Moreover, the overall impact of τ on $W(\Pi)$ is negative since the negative impact of τ on $V(\Pi)$ dominates the positive one on $T(\Pi)$. Furthermore, the welfare loss is about 1.5% irrespective of the value of α, if τ is around 10%. When however the tax rate is higher (and hence has a more realistic value), the effects of α on welfare are much more significant. For instance, if τ is 45%, the welfare loss grows from about 2 (with $\alpha = 10\%$) to about 7 (with $\alpha = 40\%$).

[21]Figures about equity and debt value (as a function of τ) are available upon request.

[22]The welfare loss is defined as the difference between the tax-free welfare function, i.e. 100, and its effective value with $\tau > 0$.

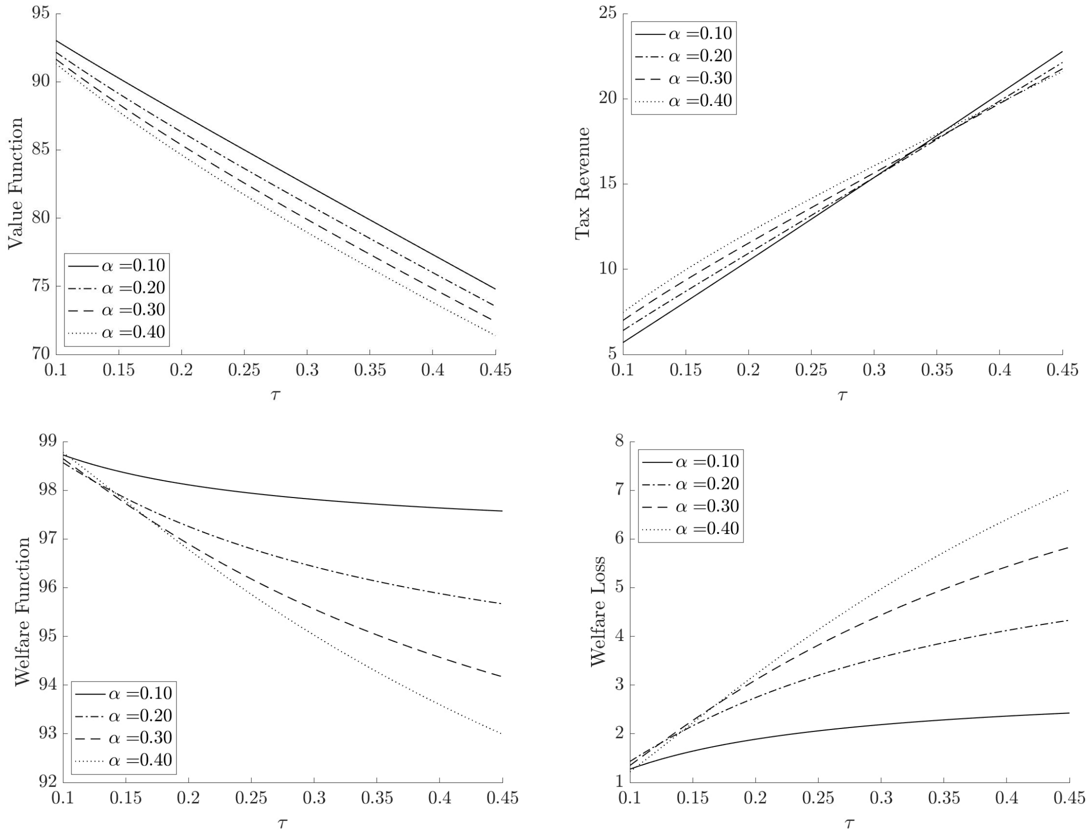

FIGURE 1.3.2. Sensitivity analysis about the effects of α (from top-left clockwise) on value function, tax revenue, welfare loss and welfare function.

In Figure 1.3.3 we focus on the ratio between the welfare loss and tax revenue as well as on its marginal value for different values of α and τ. As can be seen, the effects of τ crucially depend on the value of α. If the default cost is low enough (about 10%), the ratio between the welfare loss and tax revenue is characterized by an almost negligible change in the value of the numerator and by an increasing denominator, which dominates the tax effect on the welfare cost. Otherwise, i.e. with $\alpha \in [0.2 - 0.4]$, the ratio (13) is concave. In other words, the increase in the welfare loss dominates the tax revenue increase up to a certain value of τ. The converse is true when τ is high enough.

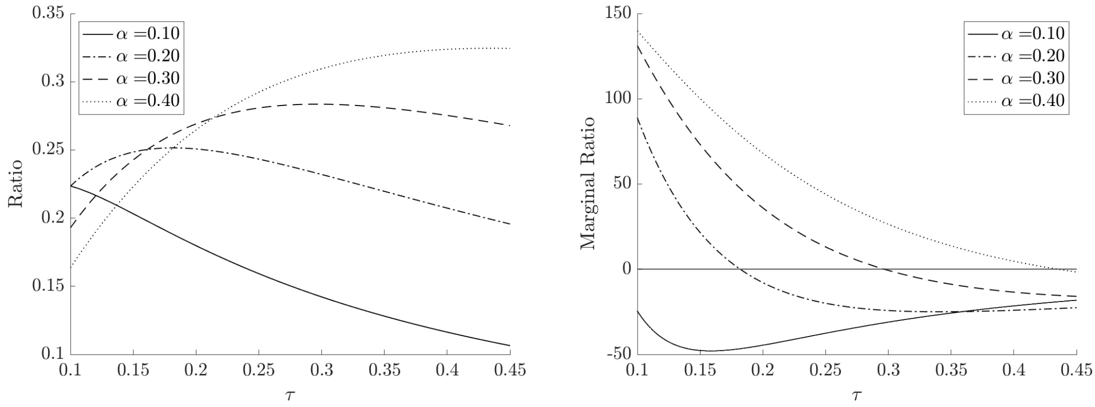

FIGURE 1.3.3. Sensitivity analysis about the impact of α on the ratio (1.2.13) (left panel) and its marginal value (right panel).

1.3.2. Sensitivity analysis on the EBIT volatility σ. Let us next focus on the four panels of Figure 1.3.4, where we show the effects of σ and τ on the firm's value, tax revenue, the welfare function and the welfare loss. A comparison with Figure 1.3.2 shows that the quality of results is somehow similar. Again, the value function (top-left panel) is decreasing in τ, while tax revenue (top-right panel) is always increasing. Moreover, the higher the volatility σ, the smaller the firm's value. This is due to the fact that an increase in volatility raises the contingent value of the default cost thereby reducing $V(\Pi)$.

As can be seen, the welfare function (bottom-left panel) is decreasing in σ. This is not surprising since a (costly) default has a negative impact on welfare.[23] Of course, if $\sigma = 0.01$ the welfare loss is almost negligible. Overall, our analysis shows that the negative effect of σ on the firm's value dominates the positive one on tax revenue.

Figure 1.3.5 shows both the ratio (1.2.13) and its marginal value for different values of σ and τ. Not surprisingly, the effects of τ crucially depend on the default risk σ. More precisely, this ratio (13) is concave in the range $\sigma \in [0.1, 0.4]$. In this case, up to a certain value of τ, an increase in the welfare loss dominates the rise of the denominator (i.e. tax revenue). The converse is true when τ is high enough. If however, σ is low enough (let us say 0.01 or so), the ratio is decreasing in τ and its marginal value is always negative. Again, in an almost deterministic context, the

[23]This relation is made explicit in equation (1.2.12).

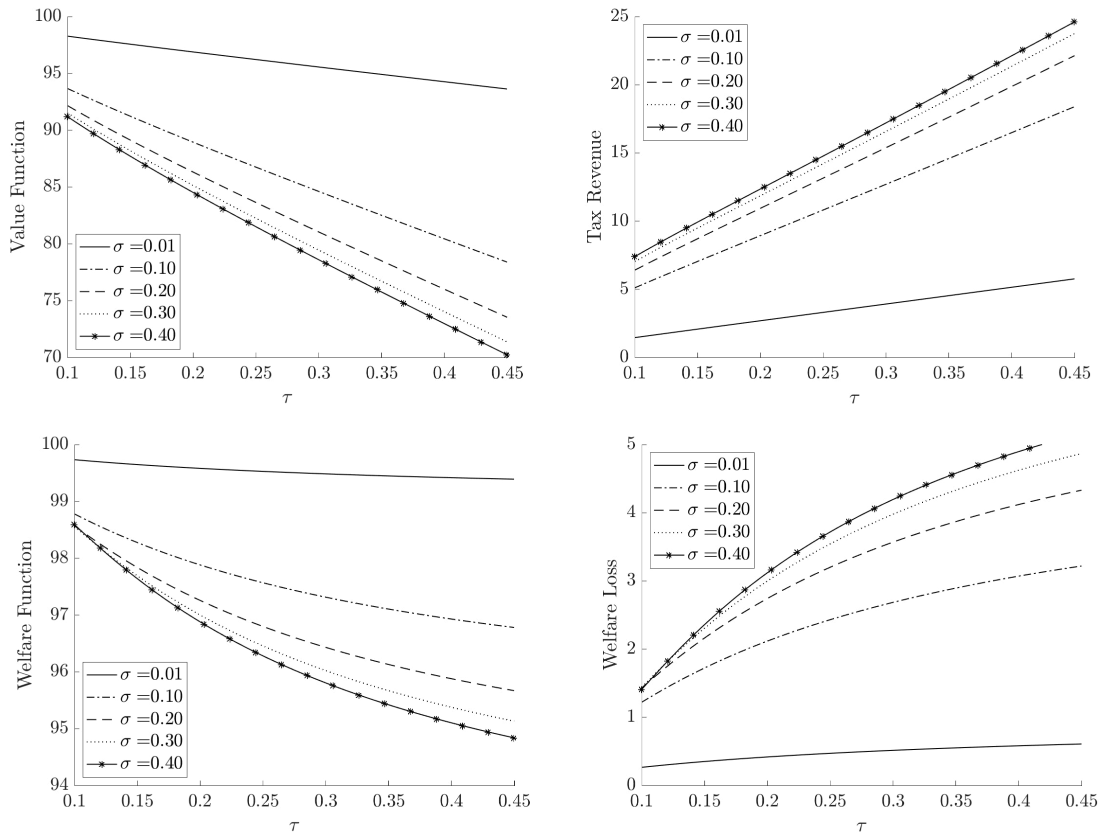

FIGURE 1.3.4. Sensitivity analysis about the effects of σ (from top-left clockwise) on value function, tax revenue, welfare loss and welfare function.

higher the tax rate, the lower the loss. Of course, these results help us to stress the importance of using realistic parameter values when dealing with the welfare effects of debt finance.

1.3.3. An insight on taxation effects. To provide a better understanding of tax effects, let us finally assume a given value of tax revenue, i.e. $T(\Pi) = 10$ (that is the10% of an unlevered firm's value) and then calculate both the relevant tax rate and the ratio (1.2.13). Figure 1.3.6 shows together the plots of both tax revenue (top) and the ratio (1.2.13) (bottom) for different values of α (left panels) and σ (right panels).

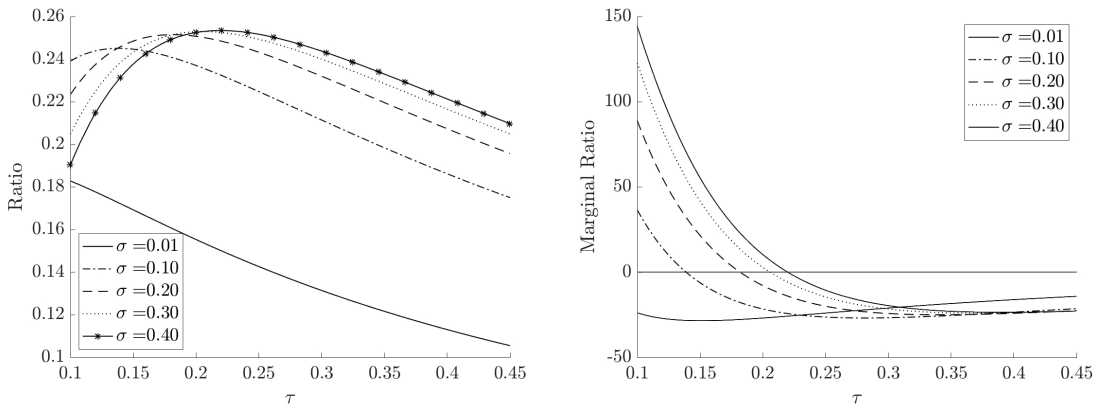

FIGURE 1.3.5. Sensitivity analysis about the impact of σ on the ratio (1.2.13) (left panel) and its marginal value (right panel).

As can be seen, the relevant tax rate crucially depends on the value of α: it ranges from about 15% to 18%. Moreover, the ratio between welfare loss and tax revenue is concave: this is due to the fact that α has a non-linear impact on $T(\Pi)$.

Furthermore, the relevant tax rate is increasing in σ and ranges from 15% to 23%. When however volatility is negligible (i.e. $\sigma = 0.01$) it is impossible to raise 10 even with a fairly high tax rate. Otherwise (i.e. when $\sigma \in [0.1, 0.4]$), the ratio (1.2.13) is around 25%. These results do highlight the importance of accounting for default.[24]

Let us finally compare our findings with those obtained in other articles. Using a calibration model, Sørensen (2017) finds a deadweight loss (i.e. a ratio between the welfare loss and tax revenue) around 5%. By applying a similar approach Weichenrieder and Klautke (2008) estimate a deadweight loss ranging from 5% to 15%. As we have shown, the deadweight loss is dramatically higher in a stochastic model.[25]

1.4. Conclusion

In this article we have shown that, under default, tax revenue and the ratio between the welfare loss and tax revenue depend not only on the relevant tax rate

[24]We have also run some robustness checks with different values of Π and r (which are available upon request). The quality of results does not change.

[25]As pointed out, our results have been obtained with a fairly simple model. Of course, the use of a more general framework, where, e.g., investment decisions, credit constraints, agency costs and a dynamic capital structure are considered, is left for further research.

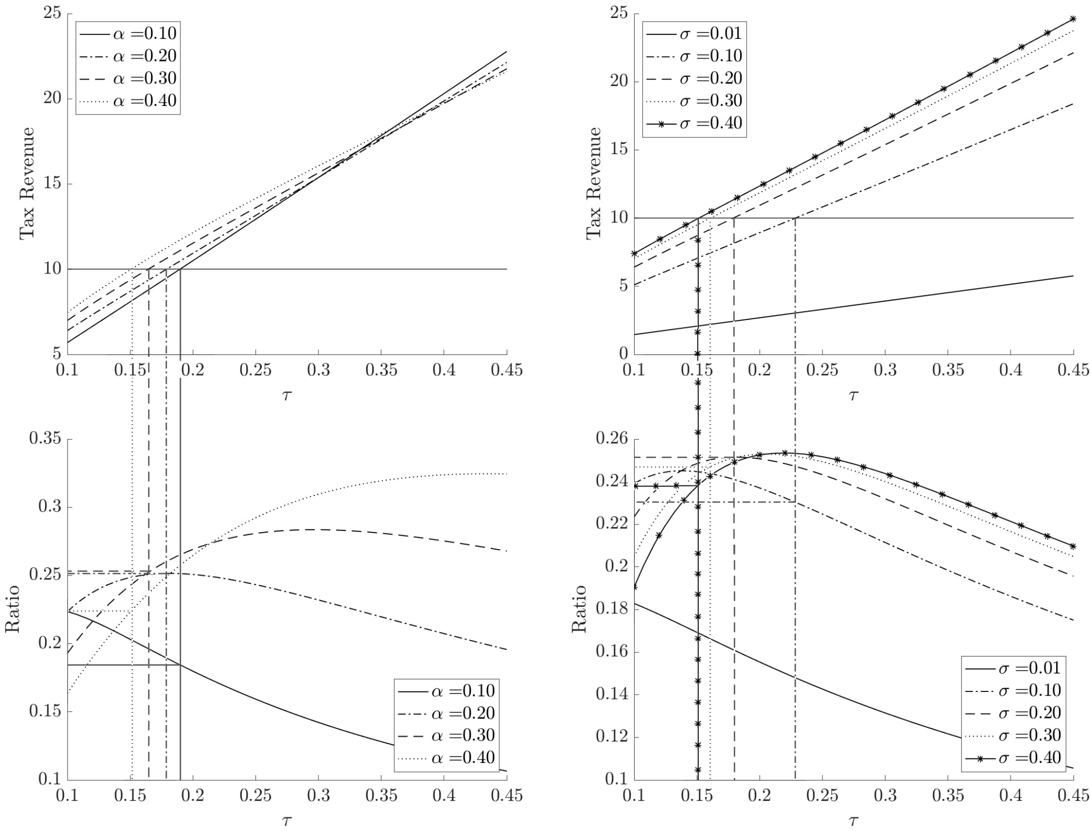

FIGURE 1.3.6. The relevant tax rate allowing the government to raise a revenue equal to 10 (top panels) for different values of α (left panels) and σ (right panels) and the ratio (1.2.13) (bottom panels).

but also on the default cost and the volatility of EBIT. If we disregard these factors we obtain a dramatically biased measure of tax effects. In particular, an increase in the expected cost of default raises tax revenue, although it is welfare deteriorating. Moreover, volatility raises (reduces) the expected value of tax revenue (welfare). In other words, a more stable financial system is beneficial from a social point of view, although it reduces tax revenue and vice versa. A similar effect is found for the default cost, which increases (reduces) tax revenue (welfare).

Since default leads to a dramatic welfare loss due to debt biases, policy-makers should change many existing systems. For this reason, over the last decades, scholars and tax experts have proposed different solutions aimed at eliminating the debt bias and ensure both financial and real tax neutrality (since the Meade report, (1978)). Some tax devices, such as the Allowance for Corporate Equity, have been implemented in some countries (e.g., Belgium and Italy) with interesting results.[26] Unfortunately, they have not yet had a widespread use. Others, such as CBIT, proposed by the U.S. Treasury (1992), which entails the absence of deductibility for both equity and debt, have never been applied. As a consequence, we are still dealing with such a debt bias.

A caveat is necessary: our results are obtained with a fairly simplified framework. In particular, we have assumed full deductibility of interest expenses as well as tax symmetry. In any case, the quality of results would not change if we assumed partial deductibility and asymmetric taxation: the tax benefit of debt finance would be simply reduced and welfare losses might be mitigated.

1.A. Appendix

1.A.1. The value of debt. Applying Itô's Lemma to (1.2.3) gives:

$$rD(\Pi) = L + \frac{\sigma^2}{2}\Pi^2 D_{\Pi\Pi}(\Pi), \tag{1.A.1}$$

where $L = (1-\alpha)(1-\tau)\Pi$, C and $D_{\Pi\Pi}(\Pi) \equiv \frac{\partial^2 D(\Pi)}{\partial \Pi^2}$. The general closed-form solution of function (1.A.1) is therefore equal to:

$$D(\Pi) = \begin{cases} \frac{(1-\alpha)(1-\tau)\overline{\Pi}}{r} + \sum_{i=1}^{2} B_i \Pi^{\beta_i} & \text{after default,} \\ \frac{C}{r} + \sum_{i=1}^{2} D_i \Pi^{\beta_i} & \text{before default,} \end{cases} \tag{1.A.2}$$

where $\beta_1 = \frac{1}{2} + \sqrt{\left(\frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1$, and $\beta_2 = \frac{1}{2} - \sqrt{\left(\frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0$ are the two roots of the characteristic equation $\Psi(\beta) \equiv \frac{1}{2}\sigma^2\beta(\beta-1) - r = 0$. To calculate B_i and D_i for $i = 1, 2$, we need three boundary conditions. Firstly, we assume that, whenever Π goes to zero, the lender's claim is nil, namely condition $D(0) = 0$ holds: this implies that $B_2 = 0$. Secondly, we assume that financial bubbles do not exist: this means that $B_1 = D_1 = 0$. Thirdly, we must consider that at point $\Pi = \overline{\Pi}$, the pre-default value of debt must be equal to the post-default one, net of the default cost. Using the two branches of (1.A.2) we thus obtain:

$$(1-\alpha)\frac{(1-\tau)\overline{\Pi}}{r} = \frac{C}{r} + D_2\overline{\Pi}^{\beta_2}.$$

[26]For a formal analysis of ACE taxation under uncertainty, see, e.g., Bond and Devereux (2003) and Panteghini (2006).

Rearranging gives $D_2 = \left[\frac{(1-\alpha)(1-\tau)\overline{\Pi}-C}{r}\right]\overline{\Pi}^{-\beta_2}$. Hence, the value of debt is (1.2.4).

1.A.2. The value of equity. Using (1.2.5) and Itô's Lemma, we obtain the following non-arbitrage condition:

$$rE(\Pi) = (1-\tau)(\Pi - C) + \frac{\sigma^2}{2}\Pi^2 E_{\Pi\Pi}(\Pi), \tag{1.A.3}$$

before default. Since, after default, the general-form solution of (1.A.3) is:

$$E(\Pi) = \begin{cases} 0 & \text{a.d.} \\ (1-\tau)\left(\frac{\Pi-C}{r}\right) + \sum_{i=1}^{2} A_i \Pi^{\beta_i} & \text{b.d.} \end{cases}, \tag{1.A.4}$$

In the absence of financial bubbles, A_1 is nil. To calculate A_2, we recall that default occurs when $\Pi = \overline{\overline{\Pi}}$. In this case, the value of equity falls to zero, namely:

$$E\left(\overline{\overline{\Pi}}\right) = 0. \tag{1.A.5}$$

Substituting (1.A.4) into (1.A.5), and solving for A_2 gives $A_2 = -\frac{(1-\tau)\left(\overline{\overline{\Pi}}-C\right)}{r}\overline{\overline{\Pi}}^{-\beta_2}$. Hence, the pre-default value of equity is equal to:

$$E(\Pi) = \frac{(1-\tau)(\Pi - C)}{r} - \frac{(1-\tau)\left(\overline{\overline{\Pi}} - C\right)}{r}\left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2} \tag{1.A.6}$$

and zero otherwise. These results give (1.2.6).

This page intentionally left blank.

CHAPTER 2

The start-up decision under default risk

2.1. Introduction

The relationship between business taxation and financial stability has been extensively studied in the scientific literature. In Chapter 1 we have investigated how this interaction can affect a representative firm's value, its capital structure, as well as the welfare generated. In our companion paper we have analyzed the behavior of a mature firm (which no longer makes an investment and just faces the risk of default). Here we focus instead on start-up firms, since financial stability can also affect their behavior. In this study we therefore introduce a real-option model aimed at investigating how a start-up option is affected by taxation and, at the same time, measuring expected tax revenue and welfare loss.

More specifically, we study how the value and the capital structure of a representative firm, together with the expected timing of exercise of the real option, are affected by: the relevant tax rate; profit volatility; the cost of default; and the start-up investment cost. Moreover, we compare these results to those obtained in Chapter 1, dealing with a mature firm. This comparison is aimed at highlighting how the investment decision affects the firm's key indicators.

The remaining part of this article is structured as follows. Section 2.2 presents the start-up decision of a new entrepreneurial business activity and measures the net present value (NPV) of the investment project, the corresponding tax revenue, welfare, together with the welfare loss and the deadweight loss. To gain more insights, Section 2.3 provides a numerical example, with the aim of comparing three companies at different stages of their existence, i.e. a start-up firm, an enterprise investing at time 0 and a mature one, in line with Chapter 1, to highlight the different effects of tax policy and financial instability. Section 2.4 summarizes our findings and discusses their policy implications.

2.2. The model

2.2.1. Random process. Let us consider a representative economic agent with an option to start a firm. By assumption, investment entails a sunk cost. After the investment is made, the new firm starts earning a cash flow. For simplicity, we assume that the economic agent is not subject to personal taxation and chooses her/his investment timing. The attractiveness of this investment opportunity depends on future earnings, that is on Earning Before Interest and Taxes (EBIT).

In line with Goldstein et al. (2001), we let the EBIT be driven by the following Geometric Brownian Motion (GBM):[1]

$$d\Pi = \mu \Pi dt + \sigma \Pi dz, \tag{2.2.1}$$

where $\Pi_0 > 0$ is its initial level, μ and σ are the drift and diffusion coefficients, respectively, and dz is the increment of a Wiener process. In line with Dixit and Pindyck (1994), we assume that $\delta \equiv r - \mu > 0$.[2] Moreover, in line with Chapter 1, we introduce the following:

Assumption 2.1. *While starting the business, the firm can borrow resources, thereby paying a non-renegotiable coupon C.*

Assumption 2.2. *If EBIT decreases to a certain trigger $\overline{\Pi}$, default occurs. If so, the firm is expropriated by the lender and loses access to credit market, but continues to operate.*

Assumption 2.3. *After default, the default cost is borne by lenders and is proportional to EBIT. We let former lenders become shareholders, earning a portion $1 - \alpha$, with $\alpha \in (0, 1)$, of the before-default EBIT.*

Assumption 2.1 means that the firm sets a coupon and then computes market value of debt.[3] For simplicity, we assume that debt cannot be renegotiated: this

[1]This choice rules out negative EBIT. However, this is not a relevant problem since the model is such that default occurs before EBIT falls to zero.

[2]As the expected growth rate is set equal to $\delta - r$, we refer to this framework as a risk neutral world. As a consequence, according to Lucchetta et al. (2019), by replacing the actual growth rate of cash flows with a certainty-equivalent growth rate, it is possible to evaluate any contingent claim on an asset. In addition, according to Shackleton and Sødal (2005), this condition is needed to allow the early exercise of the start-up option.

[3]Without arbitrage, this is equivalent to first setting the book value of debt and then calculating the effective interest rate.

means that we apply a *static* trade-off approach, where the firm's financial policy cannot be reviewed later.[4]

Assumptions 2.2 and 2.3 introduce default risk and its cost, respectively. More in detail, if the firm's EBIT falls to the threshold value $\overline{\Pi}$, the firm is expropriated by the lender, who becomes the new shareholder. The cost of default, whose impact is driven by the parameter α, is borne by the lender. For further details on these assumptions see, e.g., Goldstein et al. (2001) and Panteghini (2007b).

2.2.2. The start-up decision. As pointed out, our economic agent maximizes the expected discounted NPV of the investment project:

$$\max_{T\geq 0, C\geq 0} = \mathbb{E}\left[e^{-rT} NPV\left(\Pi\right)\right], \tag{2.2.2}$$

where $\mathbb{E}$ is the expected value operator. The control variables are investment timing T and the fixed non-renegotiable coupon C, respectively. Recall that $NPV\left(\Pi\right) = V\left(\Pi\right) - I$, that is the value function $V\left(\Pi\right)$ less cost I.[5] Following Panteghini (2007a), $V\left(\Pi\right)$ denotes the firm's value function at the unknown establishing time T, i.e. the discounted present value of all future after-tax cash flows generated from T onwards:

$$V\left(\Pi\right) = \mathbb{E}\left[\int_T^{\infty} \left[\left(1-\tau\right)\left(\Pi - C\right)\right] e^{-rdt} dt\right]. \tag{2.2.3}$$

As shown in Appendix 2.A.1, using dynamic programming, $V\left(\Pi\right)$ can be rewritten as:

$$V\left(\Pi\right) = \frac{\left(1-\tau\right)\Pi}{\delta} + \tau\frac{C}{r} - \left[\left(1-\tau\right)\alpha\frac{\overline{\Pi}}{\delta} + \tau\frac{C}{r}\right]\left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2}, \tag{2.2.4}$$

where $\overline{\Pi}$ is the level of EBIT below which default occurs, derived in Appendix 2.A.2. According to Panteghini (2007a), we assume $\overline{\Pi} = C$, namely, EBIT is such that default occurs when it hits C.[6]

Notice that choosing an optimal investment timing is equivalent to setting the optimal level of EBIT, Π^*, above which the investment is optimal. As shown by Harrison (1985), the relationship between T and Π^* is such that the equation:

$$\mathbb{E}\left[e^{-rT}\right] = \left(\frac{\Pi}{\Pi^*}\right)^{\beta_1} \tag{2.2.5}$$

[4]We leave this point for future research.
[5]For simplicity, we assume that no tax credit is ensured to cost I.
[6]The relaxation of this assumption is left for future research.

holds. This means that finding the optimal control of the former is equivalent to finding that of the latter. Hence, using equations (2.2.4) and (2.2.5), problem (2.2.2) can then be rewritten as:

$$\max_{\Pi^* \geq 0, C \geq 0} \left(\frac{\Pi}{\Pi^*} \right)^{\beta_1} \left[\frac{(1-\tau)\,\Pi^*}{\delta} + \tau \frac{C}{r} - \kappa C \left(\frac{\Pi^*}{C} \right)^{\beta_2} - I \right], \tag{2.2.6}$$

where, $\kappa \equiv (1-\tau)\,\alpha\delta^{-1} + \tau r^{-1}$. As shown in Appendix 2.A.3, solving problem (2.2.6) leads to the optimal values of Π^* and C, defined as:

$$\Pi^* = \frac{r}{1+m(\tau,\kappa)} \frac{\beta_1}{\beta_1 - 1} \frac{1}{1-\tau} I \tag{2.2.7}$$

and:

$$C = \frac{r}{1+m(\tau,\kappa)} \frac{\beta_1}{\beta_1 - 1} \frac{1}{1-\tau} I \left(\frac{\tau}{r\kappa\,(1-\beta_2)} \right)^{-\frac{1}{\beta_2}}, \tag{2.2.8}$$

respectively, with $m(\tau,\kappa) \equiv \frac{\tau}{1-\tau} \frac{\beta_2}{\beta_2 - 1} \frac{\delta}{r} \left(\frac{1}{1-\beta_2} \frac{\tau}{r\kappa} \right)^{-\frac{1}{\beta_2}}$, $\beta_1 > 1$ and $\beta_2 < 0$. From equation (2.2.7) we easily notice that Π^* is increasing in τ, as $\frac{\partial \Pi^*}{\partial \tau} > 0$.

Given these results, we can introduce the following:

LEMMA 2.1. *The expected time to exercise the option to invest, which depends not only on the GMB parameters but also on its initial value and on investment trigger, under the condition that* $\mu > \frac{\sigma^2}{2}$, *is:*

$$\mathbb{E}[T] = \ln \frac{\Pi^*}{\Pi_0} \left(\mu - \frac{\sigma^2}{2} \right)^{-1}. \tag{2.2.9}$$

PROOF. See: Wong (2007). □

It is worth noting that the wider the gap between Π^* and Π_0, the farther the first passage time is. Finally, to better understand the effects of taxation on a firm's decisions, we calculate the probability of default as follows:

LEMMA 2.2. *The probability that* Π *hits the default trigger* $\overline{\Pi} = C$ *within* θ *periods after the start-up decision, that is over the* $[T, T+\theta]$ *interval, is equal to:*

$$PD_\theta = \left(\frac{C}{\Pi_0} \right)^{\frac{2\zeta}{\sigma^2}} \Phi \left[\frac{\ln \frac{C}{\Pi_0} + \zeta\theta}{\sigma\sqrt{\theta}} \right] + \Phi \left[\frac{\ln \frac{C}{\Pi_0} - \zeta\theta}{\sigma\sqrt{\theta}} \right], \tag{2.2.10}$$

where $\zeta \equiv \mu - \frac{\sigma^2}{2} > 0$.

Proof. See: Carini et al. (2020). □

Note that, as the probability of default before the start-up decision is nil, the probability (2.2.10) is equal to the probability of default in the $[0, T+\theta]$ interval. The optimal investment trigger (2.2.7) and the expected time to exercise (2.2.9), together with the probability of default (2.2.10), will be further analyzed in Section 2.3.2.

2.2.3. Tax revenue and welfare under investment decision. As shown in Chapter 1, given the value function (2.2.4) we can calculate the tax revenue $R(\Pi)$. Then, the welfare function $W(\Pi)$, given by the sum of value function and tax revenue, immediately follows.[7] Thereafter, the difference between the maximum possible value of $W(\Pi)$, reached with $\tau = 0$, and $W(\Pi)$ measures the welfare loss $WL(\Pi)$. Finally, following Sørensen (2017), we use the ratio between $WL(\Pi)$ and $R(\Pi)$ to measure the deadweight loss. In this Section we analyze these functions in a real option setting. Accordingly, given (2.2.4) and (2.2.5), the expected NPV is:

$$\overline{NPV}(\Pi) = \left(\frac{\Pi}{\Pi^*}\right)^{\beta_1}\left[\frac{(1-\tau)\,\Pi^*}{\delta} + \tau\frac{C}{r} - \kappa C\left(\frac{\Pi^*}{C}\right)^{\beta_2} - I\right]. \tag{2.2.11}$$

where $\left(\frac{\Pi}{\Pi^*}\right)^{\beta_1}$measures the value of 1 Euro contingent on the future investment project. Using this definition,[8] we can also measure the present value of tax revenue:

$$\overline{R}(\Pi) = \left(\frac{\Pi}{\Pi^*}\right)^{\beta_1}\tau\left[\frac{\Pi^*}{\delta} - \frac{C}{r} + \left(\frac{1}{r} - \frac{\alpha}{\delta}\right)C\left(\frac{\Pi^*}{C}\right)^{\beta_2}\right]. \tag{2.2.12}$$

Then, using (2.3.3) and (2.2.12) we obtain:

$$\overline{W}(\Pi) = \overline{NPV}(\Pi) + \overline{R}(\Pi) = \left(\frac{\Pi}{\Pi^*}\right)^{\beta_1}\left[\frac{\Pi^*}{\delta} - \frac{\alpha}{\delta}C\left(\frac{\Pi^*}{C}\right)^{\beta_2} - I\right]. \tag{2.2.13}$$

Finally, the calculation of welfare loss is straightforward:

$$\overline{WL}(\Pi) = \overline{W}(\Pi)\,|_{\tau=0} - \overline{W}(\Pi)\,. \tag{2.2.14}$$

Dividing (2.2.14) by (2.2.12) gives the deadweight loss:

[7]In doing so we rule out consumer surplus for simplicity.

[8]Notice that the part in square brackets of (2.2.11), except for I, corresponds with the value function of a mature firm studied in Chapter 1.

$$\overline{DWL}\,(\Pi) = \frac{\overline{WL}\,(\Pi)}{\overline{R}\,(\Pi)}. \tag{2.2.15}$$

Since taxation has a nonlinear impact on equations (2.2.11) to (2.2.15), we therefore need a numerical analysis based on realistic parameter values.

2.3. A numerical analysis

The goal of our numerical analysis is twofold. Firstly, we aim at investigating the role of exogenous variables representative of financial instability, that is the cost of default α and EBIT volatility σ. More in detail, we study the influence of these variables on both (i) the investment decision process and (ii) the NPV of the investment project and the consequent welfare. In the first case, we analyze how the investment trigger Π^* and the probability of default within θ periods after the start-up decision PD_θ, are influenced by financial instability. In the second case, we focus on the effects that changes in α and σ induce on the welfare loss $\overline{WL}\,(\Pi)$, on tax revenue $\overline{R}\,(\Pi)$ and the deadweight loss $\overline{DWL}\,(\Pi)$. These results are collected in Sections 2.3.2 and 2.3.3 respectively.[9]

Secondly, to highlight the effects of the possibility of postponing the investment, we compare a start-up firm with another firm, which cannot decide when to invest and rather is obliged to invest at time 0. Moreover, we add a third mature firm, which is assumed to be indefinitely active and to have fully amortized its investment cost. The comparison between these cases is helpful to understand the heterogeneous impact of taxation on different firms, in line with Sinn (1991). To perform this analysis we focus on the NPV of the three firms, the corresponding tax revenue, and welfare, together with the welfare loss and the deadweight loss. This is dealt with in Section 2.3.4.

Figure 2.3.1 shows the status of three firms object of analysis. The mature firm (top) is assumed to be active indefinitely and to have amortized its investment cost. The obliged firm (amid) is forced to enter the market at $t = 0$ by paying the investment cost. This allows us to better understand the effect of investment

[9]In addition to the results shown below, the following analyses were also executed. On the one hand, we studied how the investment cost I impacts the investment decision and the consequent welfare, noting an effect quite similar to that of σ. On the other hand, we evaluated the effects of financial instability also on the expected investment timing $\mathbb{E}\,[T]$ which, not surprisingly, which reacts in the same way as Π^*. Since these analyses produced results entirely comparable to previous ones, we preferred to omit them for brevity. They are however available upon request.

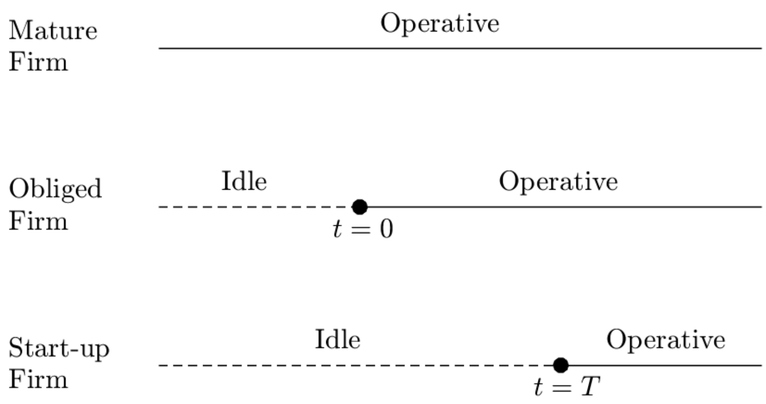

FIGURE 2.3.1. Diagram showing when the mature (top), the obliged (amid) and the start-up (bottom) firms enter the market.

	Variable	Value
r	Risk-free interest rate	0.025
Π_0	EBIT initial value	2.5
μ	EBIT drift	0
I	Investment cost	25
θ	Periods before default	10
α	Cost of default	0.20
σ	EBIT diffusion	0.20

TABLE 1. Benchmark values of parameters used in the numerical simulations.

flexibility, which is the main feature of start-up firms. The start-up (bottom) can choose optimal investment timing $t = T$, defined in equation (2.2.9). It is worth noting that, depending on exogenous parameters, the optimal investment timing may be equal to 0: in this case the scenarios of the obliged and start-up firm coincide.

2.3.1. The relevant parameter values. In our analysis, we let the statutory tax rate range from 0 to 0.30. This range is in line with the empirical evidence.

Table 1 contains the benchmark values of the other parameters used in our study. Firstly, we set the risk-free interest rate r equal to 0.025, in line with Sørensen (2017). Secondly, we arbitrarily set EBIT initial value Π_0 equal to 2.5, so as to normalize the ratio Π_0/r to 100. Thirdly, in line with Chapter 1, we set $\mu = 0$. This allows us to compare the effects of both mature and start-up firms. Then, we set $I = 25$ which is calibrated with respect to the magnitude of Π_0.[10] Finally, in line with Carini et al. (2020), we study the probability that the default occurs within $\theta = 10$ periods.[11]

To run our sensitivity analysis, we first set a benchmark value of both α and σ. The benchmark value chosen for the cost of default is $\alpha = 0.20$, that is a good average of those proposed in the relevant literature.[12] Considering the two additional levels, the values of α used for our sensitivity analysis are $\{0.10, 0.20, 0.30\}$. Moreover, we set $\sigma = 0.20$ as benchmark value, which is again in line with the literature (see, e.g., Dixit and Pindyck (1994)). We also consider two additional scenarios, so the values of σ used are $\{0.15, 0.20, 0.25\}$.

2.3.2. Effects on investment decision. Let us next focus on the investment trigger level and the probability of default (PD). Of course, in order to have an immediate indication as to wether the option can be immediately exercised, we compare Π^* with EBIT initial value Π_0.

As shown in Figure 2.3.2, Π^* is always increasing in τ. This is due to the fact that, *coeteris paribus*, a rise in τ reduces net profit. Since a higher EBIT is needed to make investment profitable, the investment project is delayed. When however τ is sufficiently low, an increase in taxation dramatically increases the PD. However, beyond a certain level of τ depending on other parameters, we notice that an increase of τ has a slightly negative effect on the PD. As pointed out by Carini et al. (2020), this happens because an increase in τ reduces the default trigger and hence PD_θ.

The left panels focus on the effects of α and show that both Π^* and $\mathbb{E}[T]$ are slightly increasing in α: the more costly the default, the lower the expected profitability for any EBIT level. To offset the increase in α, a higher Π^* is needed. The intuition is as follows: the higher the default cost, the higher the loss contingent to this event. This latter effect has a negative impact on the value of a feasible investment project. Moreover, we see that α has a remarkable negative effect on the PD: this is due to the fact that the coupon is also decreasing in α as, the higher the cost of default, the lower the optimal level of debt. Therefore, the lower the coupon, the lower the default trigger and thus the probability that EBIT will reach it. For

[10]It implies that, without taxation, investment can be paid back on average in 10 periods.

[11]We also ran a robustness check with $r = 5\%$ and with $\mu = 0.01$, in line with Dixit and Pindyck (1994). The quality of results, available upon request, is unaffected.

[12]For example, Andrade and Kaplan (1998) estimates distress costs from 0.10 to 0.23 of firm value, Davydenko et al. (2012) proposes 0.22, while Glover (2016) expects 0.45.

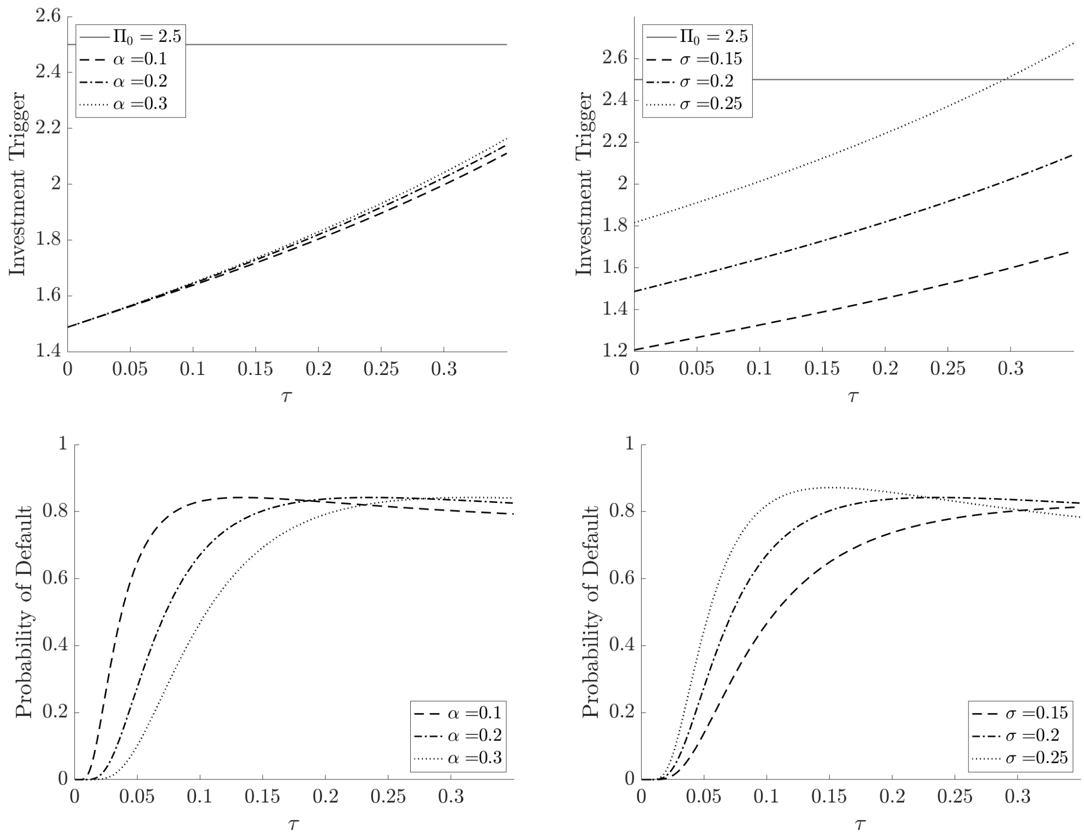

FIGURE 2.3.2. Effects on investment trigger (top panels) and probability of default within 10 periods after the exercise of the start-up option (bottom panels), expressed as functions of effective tax rate τ, of different values of default's cost α (left panels) and of EBIT diffusion σ (right panels).

example, given $\tau = 0.15$, a rise in α from 0.20 to 0.30 leads to an increase of Π^* of 0.34% and reduces PD_θ by -13.62%. Instead a drop of α from 0.20 to 0.10 reduces Π^* by -0.64% and rises PD_θ by 4.78%.

The right panels show the effects of σ. This parameter has a positive effect on both Π^* and PD_θ. The rationale behind this effect is straightforward: the higher the volatility, the further the expected investment time. In addition, the higher the volatility, the higher the probability that Π hits the default trigger and thus the PD. For example, given $\tau = 0.15$, a rise of σ from 0.20 to 0.25 leads to an increase of both Π^* and PD_θ by 22.82% and 8.65% respectively. Instead a drop from 0.20 to 0.15 reduces Π^* by -19.63% and PD_θ by -19.10%.

2.3.3. Sensitivity analysis. Let us next focus on the welfare loss (2.2.14), the tax revenue (2.2.12) and the deadweight loss (2.2.15). The purpose of this analysis is to isolate the effect of changes in α or σ in a real options context, i.e. relative to the start-up. This allows us to complement the analysis focused on mature firms shown in Chapter 1. Results are shown in Figure 2.3.3.

The top panels focus on $\overline{WL}(\Pi)$. First of all, we notice that, except for small values of τ where the effect is negligible, $\overline{WL}(\Pi)$ is positively influenced by changes in α. This happens because the higher the loss contingent on the case of default, the lower the value function (2.2.4) on which $\overline{NPV}(\Pi)$ is based and then, as a consequence, $\overline{R}(\Pi)$. These two joint effects reduce overall welfare and then increase $\overline{WL}(\Pi)$. For example, given $\tau = 0.15$, a rise of α from 0.20 to 0.30 increases $\overline{WL}(\Pi)$ by 4.22%, while a drop from 0.20 to 0.10 reduces $\overline{WL}(\Pi)$ by -22.17%. Moreover, we see that $\overline{WL}(\Pi)$ is negatively correlated with σ. This is due to the combined dynamics of $\overline{NPV}(\Pi)$ and $\overline{R}(\Pi)$, which is increasing in σ. The former takes benefit from an increase of σ, as, under the real option framework that allows to postpone the market entry, a higher σ may lead to hit the investment trigger earlier. The latter behaves oppositely, as it is decreasing in σ, however failing to offset the rise of the NPV. For example, given $\tau = 0.15$, a rise of σ from 0.20 to 0.25 $\overline{WL}(\Pi)$ by -32.84%, while a drop from 0.20 to 0.15 increases $\overline{WL}(\Pi)$ by 108.20%.

The middle panels focus on $\overline{R}(\Pi)$ and show its obvious mechanical rise following an increase in τ. In addition, when financial instability, i.e. α or σ, increases, we observe a reduction in tax revenue. As anticipated in Section 2.3.2, this happens because financial instability causes a postponement of the start-up decision, thus the firm's operations and eventually the generation of taxable profit. In other words, the expected value of tax base decreases. For example, given $\tau = 0.15$, a rise of α from 0.20 to 0.30 reduces $\overline{R}(\Pi)$ by -13.72%, while a drop from 0.20 to 0.10 increases $\overline{R}(\Pi)$ by 34.34%. Similarly, a rise of σ from 0.20 to 0.25 reduces $\overline{R}(\Pi)$ by -17.75%, while a drop from 0.20 to 0.15 increases $\overline{R}(\Pi)$ by 68.29%. Looking at these data we can argue that the maximum impact on $\overline{R}(\Pi)$ occurs when passing from low values of σ to average levels, as the marginal effect of an increase of σ is decreasing.

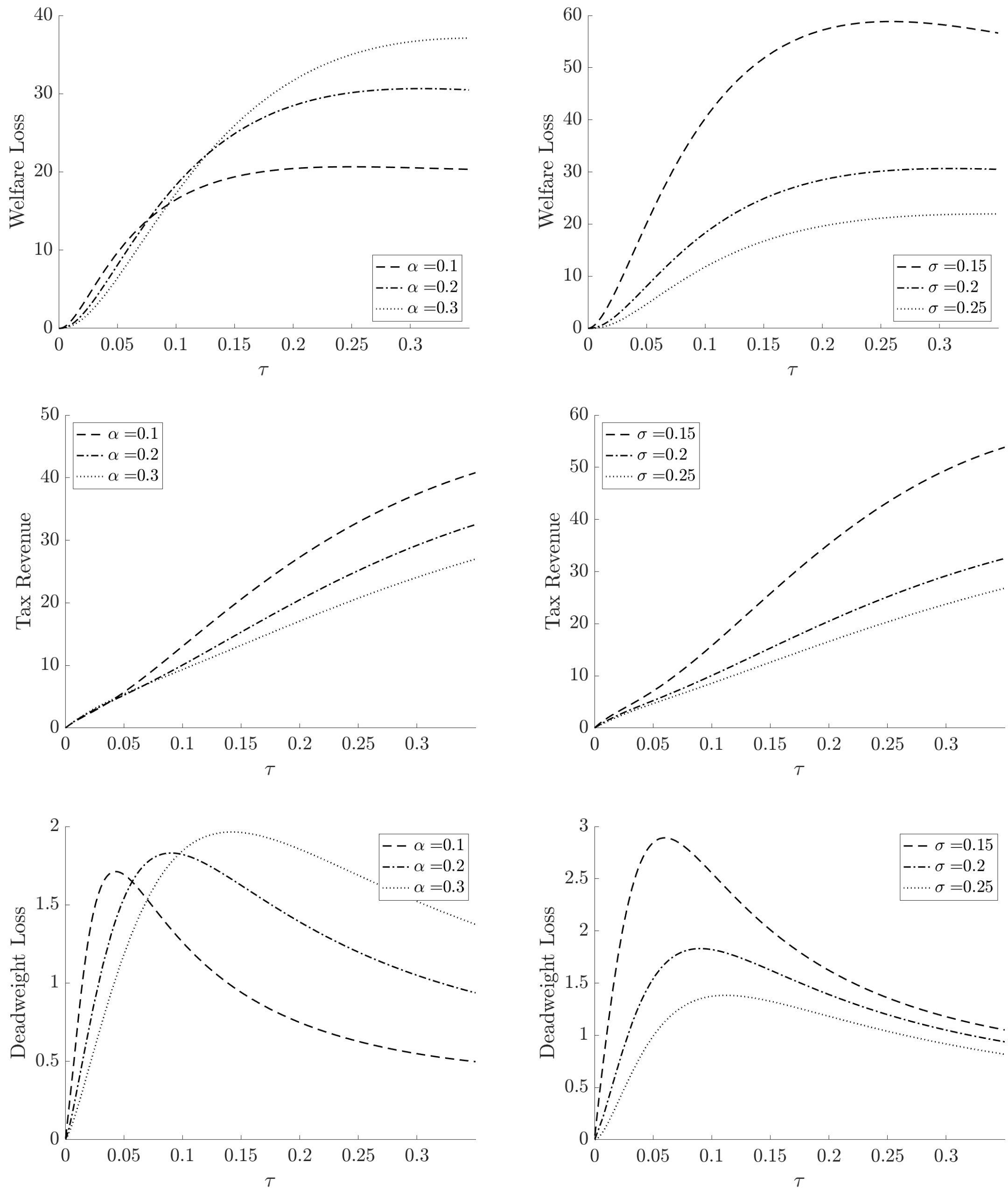

FIGURE 2.3.3. Effects on welfare loss (top panels), tax revenue (middle panels) and deadweight loss(bottom panels), expressed as functions of effective tax rate τ, of different values of default's cost α (left panels) and of EBIT diffusion σ (right panels).

Variable	**Firm**	$\tau = 0.10$	$\tau = 0.20$	$\tau = 0.30$
NPV	Mature	92.17	86.32	81.05
	Obliged	61.22	48.02	36.25
	Start-up	56.11	35.52	24.64
R	Mature	6.40	10.94	15.38
	Obliged	7.40	14.75	22.82
	Start-up	10.05	20.48	29.19
W	Mature	98.57	97.26	96.43
	Obliged	68.62	62.77	59.08
	Start-up	66.16	56.00	53.83
WL	Mature	1.43	2.74	3.57
	Obliged	6.38	12.23	15.92
	Start-up	18.31	28.48	30.64
DWL	Mature	0.22	0.25	0.23
	Obliged	0.86	0.83	0.70
	Start-up	1.82	1.39	1.05

TABLE 2. NPV, tax revenue, welfare function, welfare loss and deadweight loss relative to the case of a mature, obliged and start-up firm, for different values of effective tax rate τ. All other parameters are set to their benchmark value.

The bottom panels finally focus on the deadweight loss $\overline{DWL}(\Pi)$, which is increasing in α (except for small values of τ) and decreasing in σ. Given the dynamics described above, on the one hand the positive effect of α is easily explained by changes in $\overline{WL}(\Pi)$ and $\overline{R}(\Pi)$, which are increasing and decreasing in α respectively. On the other hand, the negative effect of σ is due to the fact that $\overline{WL}(\Pi)$ decreases faster that $\overline{R}(\Pi)$. For example, given $\tau = 0.15$, a rise of α to its highest level increases $\overline{WLR}(\Pi)$ by 20.79%, while a drop to its lowest level reduces $\overline{WLR}(\Pi)$ by -42.06%. Conversely, a rise of σ from 0.20 to 0.25 reduces $\overline{WLR}(\Pi)$ by -18.35%, while a drop from 0.20 to 0.15 increases $\overline{WLR}(\Pi)$ by 23.72%.

2.3.4. Comparing start-ups, obliged and mature firms. Let us now focus on the comparison between a start-up, an obliged firm and a mature one, studying their NPV and their corresponding welfare indicators.

Table 2 shows the numerical results obtained with the benchmark parameter values, regarding the three firms object of investigation. In addition, Figure 2.3.4 provides a graphical representation of the NPV (2.2.11), welfare loss (2.2.14), tax

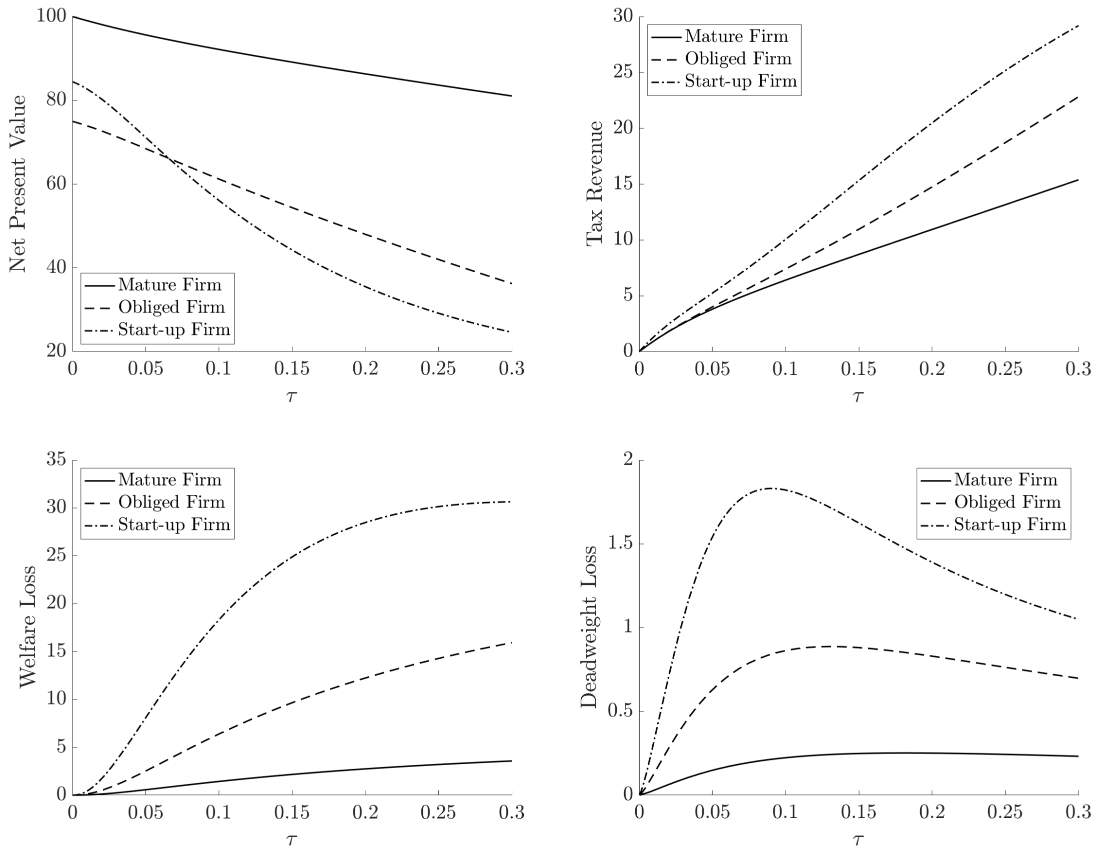

FIGURE 2.3.4. Net present value (top left), welfare loss (bottom left), tax revenue (top right) and deadweight loss (bottom right) of a mature, obliged and start-up firm. All parameters are set to their benchmark value.

revenue (2.2.12) and deadweight loss (2.2.15) of the three firms. Notice that the case of the mature firm refers to the scenario outlined in Chapter 1.

As can be seen, the NPV of all three firms is obviously always decreasing in τ. More specifically, in the case of mature and obliged firm, the decrease is almost linear, due to the reduction of net profit, with a difference mainly attributable to the investment cost paid by the obliged firm. The start-up case shows a different trend: the decrease appears more (less) marked for low (high) values of τ. This is due to the fact that an increase in the tax rate both reduces the after-tax profit and postpones investment. This postponement is greater if τ is low enough. For example, given $\tau = 0.15$, the NPV of a start-up is 50.33% and 18.60% lower than those of a mature and an obliged firm respectively.

Let us first focus on the tax revenue which is always increasing in τ. Moreover we see that no Laffer curve is therefore found. In addition, we notice that the highest level of tax revenue is generated by the start-up firm. This happens because of the effect of both an increase in future cash flow and investment delay, the former of which dominates the latter. This happens because, the higher τ, the higher the investment trigger (see: Figure 2.3.2) and thus the more delayed the market entry. However, although postponing the investment decision delays the generation of tax revenue, this negative effect is offset by the higher level of taxable profit after operations start. For example, given $\tau = 0.15$, a start-up firm generates 75.95% and 39.77% more tax revenue than a mature and an obliged firm respectively.

Let us now focus on the welfare loss. As can be seen, it is always increasing in in τ. As outlined in Chapter 1, this happens because the negative effect on the NPV offsets the positive one on tax revenue, thereby causing a deadweight loss. When the real option is available, however, the increase in the welfare loss is first smoothed and then interrupted. This happens as the slowed down decrease of the NPV, due to the possibility of postponing the investment decision, is finally offset by the benefit of tax revenue. More precisely, the growth of the welfare loss stops at around $\tau = 0.3$: above this level welfare starts to grow, thanks to tax revenue increase. Because of the sharp decrease in the start-up's NPV there are huge differences between the three welfare losses of the three firms. For example, given $\tau = 0.15$, the welfare loss of a start-up case is more than ten times (four times) larger than faced with a mature (obliged) firm.

Finally, we deal with the deadweight loss, defined by the ratio between welfare loss and tax revenue. As can be seen, its maximum value is reached when $\tau = 0.09$. Below (above) this level the increase in welfare loss offsets (is offset by) the tax revenue rise. Given current business taxation, policy-makers are aware of the present deadweight loss (2.2.15). It is then possible to calibrate fiscal policy in order to reduce the deadweight loss. In addition, policy-makers are able to know how much this benefit derives from a greater NPV or tax revenues. Also in this case, the differences among the three firms are significant. For example, given $\tau = 0.15$, the deadweight loss caused by a start-up firm is more than six times larger (twice as big) than the one caused by a mature (obliged) firm.

2.4. Conclusion

This study represents the natural development of the model described in Chapter 1. The assumption of a start-up option is motivated by the fact that financial stability and business taxation influence not only the behavior of existing firms, but also the decisions of new entrepreneurs. For this reason, we have studied how the economic environment affects investment timing and all the indicators of benefit arising from a firm's operations, for this purpose redefined to be consistent with this extended framework.

As we have shown, the default cost is a relevant burden that is almost always disregarded by the tax literature. In fact, together with profit volatility, it affects both investment decision and welfare measures. More specifically, they both postpone market entry and reduce tax revenue, while the cost of default (profit volatility) increases (reduces) welfare loss and then the deadweight loss.

Moreover, we have shown that an aggressive tax policy, despite the greater tax revenue, increases the welfare loss. This damage is particularly relevant in the case of start-ups, as a tightening of taxation forces them to postpone their market entry. The effect on deadweight loss is however twofold: it is increasing (decreasing) in business taxation when it is sufficiently low (high), as welfare loss offsets (is offset by) tax revenue. The most relevant deadweight loss is faced when start-up firms are considered: this is due to the fact that they must postpone investment.

These results, regarding financial instability, welfare effects and investment decisions, are a useful tool to better shape fiscal policy.

2.A. Appendix

2.A.1. The value of the firm. Since the firm's value function is given by the sum of the net present value of equity and debt, they have to be firstly computed.

Following Chapter 1, at any time the value of equity before default (b.d.) is equal to the sum of its immediately preceding value, the instantaneous net profit and the expected capital gain, while after default (a.d.) its value falls to zero. The value of equity can then be defined as:

$$E(\Pi) = \begin{cases} (1-\tau)(\Pi - C)\,dt + e^{-rdt}\mathbb{E}\left[D(\Pi + d\Pi)\right] & \text{b.d.} \\ 0 & \text{a.d.} \end{cases} . \tag{2.A.1}$$

Recalling that $e^{-rdt} = (1 - rdt)$ when $dt \to 0$ and after having applied Itô's lemma to define the increment $dE(\Pi)$ and some rearrangements, we obtain the following second order differential equation:[13]

$$\frac{\sigma^2}{2}\Pi^2 E'' + \mu\Pi E' - rE + (1-\tau)\Pi - (1-\tau)C = 0, \tag{2.A.2}$$

whose solution can be guessed to be in the form $E = H_0 + H_1\Pi + A\Pi^{\beta}$. Then, by substituting into equation (2.A.2) the guessed solution and its primes, we find that:

$$E(\Pi) = \frac{(1-\tau)}{\delta}\Pi - \frac{(1-\tau)}{r}C + \sum_{i=1}^{2} A_i \Pi^{\beta_i}, \tag{2.A.3}$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are defined as:

$$\beta_{1,2} = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}.$$

By letting $A_1 = 0$ to avoid the presence of financial bubbles and setting $E\left(\overline{\Pi}\right) = 0$ to derive A_2, we finally find the net present value of equity:

$$E(\Pi) = \frac{(1-\tau)}{\delta}\Pi - \frac{(1-\tau)}{r}C - \left[\frac{(1-\tau)}{\delta}\overline{\Pi} - \frac{(1-\tau)}{r}C\right]\left(\frac{\Pi}{\overline{\Pi}}\right)^{\beta_2}. \tag{2.A.4}$$

Again following Chapter 1, at any time the value of debt b.d. is equal to the sum of its immediately preceding value, the instantaneous coupon and its expected increment. It is worth noting that, a.d., the value of $D(\Pi)$ does not fall to zero: as from Assumption 2.2, the firm continues to produce. In this case, the lender will benefit from the future net profit flow, reduced proportionally to α, according to Assumption 2.3. The value of debt can then be defined as:

$$D(\Pi) = \begin{cases} Cdt + e^{-rdt}\mathbb{E}\left[D(\Pi + d\Pi)\right] & \text{b.d.} \\ (1-\alpha)(1-\tau)\Pi dt + e^{-rdt}\mathbb{E}\left[D(\Pi + d\Pi)\right] & \text{a.d.} \end{cases}. \tag{2.A.5}$$

By executing the same procedure described for the case of equity, we obtain the two following second order differential equations:[14]

$$\begin{cases} \frac{\sigma^2}{2}\Pi^2 D'' + \mu\Pi D' - rD + C = 0 & \text{b.d.} \\ \frac{\sigma^2}{2}\Pi^2 D'' + \mu\Pi D' - rD + (1-\alpha)(1-\tau)\Pi = 0 & \text{a.d.} \end{cases}, \tag{2.A.6}$$

[13] The dependency of E on Π is henceforward omitted to lighten the notation. Moreover, we denote the two first derivatives of E with respect to Π as E' and E'' respectively.

[14] The dependency of D on Π is henceforward omitted to lighten the notation. Moreover, we denote the two first derivatives of D with respect to Π as D' and D'' respectively.

whose solutions can be guessed to be in the form $D = H_0 + B\Pi^{\beta}$ and $D = H_1\Pi + F\Pi^{\beta}$ respectively. Then, by substituting into equation (2.A.6) the guessed solutions and their primes, we find that:

$$D(\Pi) = \begin{cases} \frac{C}{r} + \sum_{i=1}^{2} B_i \Pi^{\beta_i} & \text{b.d.} \\ \frac{(1-\alpha)(1-\tau)}{\delta}\Pi + \sum_{i=1}^{2} F_i \Pi^{\beta_i} & \text{a.d.} \end{cases}, \tag{2.A.7}$$

where $\beta_{1,2}$ are the same as the case of equity. To calculate B_i and F_i, three boundary conditions are needed. Firstly, we set $B_1 = F_1 = 0$ to avoid the presence of financial bubbles. In addition, we assume that if the profit falls to zero, so does the lender's claim a.d., namely $D(0) = 0$. For this reason we can set $F_1 = 0$. To derive the value of the only not null constant D_2 we must equate the value of debt b.d. and a.d., in correspondence of $\overline{\overline{\Pi}}$. After that, we find the net present value of debt:

$$D(\Pi) = \begin{cases} \frac{C}{r} + \left[\frac{(1-\alpha)(1-\tau)}{\delta}\overline{\overline{\Pi}} - \frac{C}{r}\right]\left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2} & \text{b.d.} \\ \frac{(1-\alpha)(1-\tau)}{\delta}\Pi & \text{a.d.} \end{cases}. \tag{2.A.8}$$

Finally, we can compute the firm's value function, shown in equation (2.2.4). This result is obtained by simply adding up the net present value of equity and debt, as shown in equations (2.A.4) and (2.A.8) respectively.

2.A.2. Optimal default trigger and optimal coupon. The problems solved by shareholders and lenders, to find the optimal controls for default trigger and coupon, are defined as $\max_{\overline{\overline{\Pi}}} E(\Pi)$ and $\max_C V(\Pi)$ respectively, where the value of equity and the firm's value function are those defined in equations (2.A.4) and (2.2.4). The FOC of the first problem is:

$$\frac{\partial E(\Pi)}{\partial \overline{\overline{\Pi}}} = -\left\{\frac{(1-\tau)}{\delta}\left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2} - \left[\frac{(1-\tau)}{\delta}\overline{\overline{\Pi}} - \frac{(1-\tau)}{r}C\right]\frac{\beta_2}{\overline{\overline{\Pi}}}\left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2}\right\} = 0,$$

from which it easily follows that:

$$\overline{\overline{\Pi}}(\Pi) = \frac{\delta}{r}\frac{\beta_2}{\beta_2 - 1}C.$$

Then, exploiting this result, we find the FOC of the second problem:

$$\frac{\partial V(\Pi)}{\partial C} = \frac{\tau}{r} - \left[(1-\tau)\alpha\frac{\beta_2}{\beta_2 - 1} + \tau\right]\left[\frac{1}{r}\left(\frac{\Pi}{C}\frac{r}{\delta}\frac{\beta_2 - 1}{\beta_2}\right)^{\beta_2} - \frac{\beta_2}{C}\frac{C}{r}\left(\frac{\Pi}{C}\frac{r}{\delta}\frac{\beta_2 - 1}{\beta_2}\right)^{\beta_2}\right] = 0,$$

which finally leads to the optimal coupon:

$$C(\Pi) = \frac{r}{\delta}\frac{\beta_2 - 1}{\beta_2}\left\{\frac{\tau}{(1-\beta_2)\left[(1-\tau)\,\alpha\frac{\beta_2}{\beta_2-1} + \tau\right]}\right\}^{-\frac{1}{\beta_2}}\Pi.$$

2.A.3. Optimal investment trigger. The objective function of problem (2.2.6) is the NPV defined in equation (2.2.11). The FOCs of this problem are then defined as:

$$\frac{\partial\overline{NPV}(\Pi)}{\partial C} = \left(\frac{\Pi}{\Pi^*}\right)^{\beta_1}\left[\frac{\tau}{r} - \kappa(1-\beta_2)\left(\frac{\Pi^*}{C}\right)^{\beta_2}\right] = 0 \tag{2.A.9}$$

and:

$$\frac{\partial\overline{NPV}(\Pi)}{\partial\Pi^*} = \left(\frac{\Pi}{\Pi^*}\right)^{\beta_1}\left[\frac{(1-\tau)}{\delta} - \kappa\beta_2\left(\frac{\Pi^*}{C}\right)^{\beta_2-1}\right] - \frac{\beta_1}{\Pi^*}\overline{N}(\Pi, C) = 0 \tag{2.A.10}$$

respectively. Rearranging equation (2.A.9), it easily follows that:

$$\frac{C}{\Pi^*} = \left(\frac{\tau}{r\kappa(1-\beta_2)}\right)^{-\frac{1}{\beta_2}}. \tag{2.A.11}$$

Then, substituting this result into equation (2.A.10) and rearranging thus yields equation (2.2.7) that, using (2.A.11) and after some rearrangements, finally leads to (2.2.8).

CHAPTER 3

Debt shifting and transfer pricing in a volatile world

3.1. Introduction

Debt shifting (DS) and transfer pricing (TP) activities, allowing multinational companies (MNCs) to shift debt and profit towards countries with a favorable tax treatment, are worldwide phenomena and hence have been studied for a long time. The debate about these practices has become increasingly important, as worldwide governments have experienced substantial losses on corporate tax revenue, due to erosion of the taxable base.[1]

To our knowledge, despite the existence of several empirical articles, only Schenkelberg (2020) studies both TP and DS and finds that TP is, on average, 85% of the increase in pre-tax earnings, while less than 15% is attributable to DS. Again, only Schindler and Schjelderup (2016) provide a theoretical model where both TP and DS are studied together. They assume that a higher leverage may reduce marginal

[1]The empirical literature on both debt shifting (see, for example, the pioneering articles by Collins and Shackelford (1992) and Froot and Hines (1995)) and transfer pricing (see, for example, Grubert and Slemrod (1998)) began in America. Subsequently, it spread around the world. In Europe, for instance, Ramb and Weichenrieder (2005) showed that the tax rates of parent companies have no statistically significant effect on their subsidiaries' leverage, whereas Overesch and Wamser (2014) studied the effects of parent companies' tax rates on their own capital structure. Moreover, using the effective cross-border tax rates, Huizinga et al. (2008) estimated a negative impact of parent company taxation. As shown by Miniaci et al. (2014) however, the effects of a change in parent company tax rate are much more complex, because taxes affect both a MNC's borrowing decision and the distribution of debt among its entities. Accordingly, the meta-analysis of the empirical literature on corporate capital structure provided by Feld et al. (2013) emphasizes the complexity of tax effects at a multinational level. Based on 48 studies, they estimate a marginal tax effect on the debt ratio of about 0.27, that is, the debt-to-assets ratio rises by 2.7% if the marginal tax rate increases by 10%. When, however, they focus on the capital structure of foreign subsidiaries, taxation has a more complex impact, as the tax sensitivity of inter-company debt financing is particularly strong. Overall, their meta-analysis does not support the idea that the international tax system affects the financing decisions of multinational firms. These results show that there is room for further research aimed at focusing on firms' heterogeneity. As regards TP, many authors show that MNCs shift income to low-tax subsidiaries in order to minimize their overall tax burden around the world (see, for example, Dischinger et al. (2014); Dischinger (2010); Devereux and Maffini (2007); Hines and Rice (1994)).

concealment costs of transfer pricing (and vice versa). Moreover, they assume that the concealment costs related to TP may rise when debt shifting increases (and vice versa). In principle, this cross effect is interesting. However, there is no empirical evidence that supports such an hypothesis. For this reason, we disentangle the concealment costs of DS and TP by simply assuming standard quadratic cost functions. Moreover, Schindler and Schjelderup (2016) apply a deterministic model. Since uncertainty can dramatically affect firms' decisions, we start from their assumptions and introduce a representative MNC, which operates in a stochastic environment. Firstly, it makes a MNC's profitability volatile, thereby affecting the amount of shifted profit. Secondly, it may lead to default: the probability of this event can affect both financial choices and DS activities.

Our aim is therefore to show that, assuming a stochastic Earning Before Interest and Taxes (EBIT), our MNC's choices are crucially affected by that. In particular, we show that: (i) an increase in uncertainty leads to a dramatic drop in leverage and a slight decrease in the MNC's value, (ii) the cost of TP leads to a sharp reduction in the MNC's value, whereas it does not affect leverage, (iii): the impact on the MNC's decisions is increasing in the tax rate differential and (iv) the cost of DS always has a relevant impact on both the MNC's value and leverage.

Using realistic parameter values, properly calibrated on available empirical literature, we show results in line with Schenkelberg (2020): namely, the tax saving arising from TP is more relevant than that from DS.

The remaining part of this article is as follows. Section 3.2 introduces the stochastic model describing the behavior of a representative MNC. Section 3.3 provides a numerical analysis. A set of sensitivity analyses is also added to show the robustness of our results. Section 3.4 summarizes our findings and discusses their policy implications.

3.2. The model

Savings arising from tax avoiding activities crucially depend on the characteristics of concealment costs. Since there is no evidence about the functional form of these costs, we let them be separate and convex. Moreover, we let shareholders choose the optimal value of both TP and DS, as well as the optimal threshold level of EBIT, below which default takes place (this assumption is, for example, in line with Leland (1994) and Goldstein et al. (2001)). Moreover, the optimal debt level is obtained by maximizing the levered value of a representative MNC. In doing so, we allow lenders and shareholders to decide the leverage ratio together, in order to avoid conflicts.[2]

3.2.1. EBIT dynamics. In this Section, we use a continuous-time model based on Goldstein et al. (2001), where a representative MNC's EBIT is stochastic and, hence, may lead to default. Accordingly, EBIT, defined as Π,[3] is assumed to follow a Geometric Brownian Motion (GBM):

$$\frac{d\Pi_t}{\Pi_t} = \mu dt + \sigma dz_t, \tag{3.2.1}$$

where $\Pi_0 > 0$ is its initial value, μ and σ are the drift and the instantaneous standard deviation, respectively. Moreover, dz_t is the increment of a Weiner process. In line with Dixit and Pindyck (1994), we let $\delta \equiv r - \mu$ be positive.[4] Moreover, we also assume that the firm can borrow from a perfectly competitive credit sector, where the discount factor is the risk-free interest rate r. In addition, we introduce the following:

ASSUMPTION 3.1. *At time* 0, *shareholders maximize the value of equity with respect to the threshold* $\overline{\Pi}$ *below which the default occurs, as well as with respect to the optimal transfer pricing and debt shifting strategies.*

ASSUMPTION 3.2. *At time* 0, *the MNC can borrow resources thereby paying a non-renegotiable coupon* C. *The optimal value of* C *is such that the levered value of the MNC is maximized.*

ASSUMPTION 3.3. *If the MNC does not meet its obligations, default occurs and hence the firm is expropriated by the lender.*

ASSUMPTION 3.4. *After default, the lender becomes shareholder and can exploit transfer price activities to reduce its tax bills.*

---

[2]This assumption entails a lack of informational asymmetries and lenders and creditors deciding the optimal leverage ratio together. We are aware that bargaining processes might exist, however this point is beyond the scope of this article. Of course, this simplifying assumption allows us to find a closed-form solution. Asymmetric information is left for further research.

[3]According to Panteghini and Vergalli (2016), Π can be considered as the result of previous investment decisions.

[4]As the expected growth rate is set equal to $\delta - r$, we refer to this framework as a risk neutral world, thus, according to Lucchetta et al. (2019), any contingent claim on an asset can be evaluated discounting the expected cash flows at the risk-less rate.

Assumption 3.1 implies that shareholders behave as if they owned a put option, whose exercise leads to default.[5] Moreover, it entails the MNC being able to reduce its tax burden by means of DS and TP activities. Assumption 3.2 means that the firm sets a coupon and then computes the debt market value. Without arbitrage, this is equivalent to first setting the book value of debt and then calculating the effective interest rate. For simplicity, we assume that debt cannot be renegotiated: this means that we apply a *static* trade-off approach where the firm's financial policy cannot be reviewed later.[6]

Assumption 3.3 introduces default risk. Such an event occurs if the firm's EBIT falls below a given threshold value $\overline{\Pi}$. In this case, the MNC is expropriated by the lender who bears the cost of default and then becomes shareholder. Notice that, after default, the firm is, by assumption, still producing EBIT. In this case the former lender, who has become shareholder, can exploit TP activities to reduce its tax liabilities.[7] As usual, we let the MNC's operations continue after default.

It is worth noting that tax savings due to debt finance arises as long as the business tax rate is higher than the lender's rate (see, for example, Panteghini (2007b)). For simplicity and without loss of generality, we let the lender's pre-default tax rate be nil. When, however, default takes place, the lender becomes shareholder and he/she is therefore subject to corporate taxation.

3.2.2. Net profit of the multinational company. Let us assume, for simplicity, that our representative MNC owns two subsidiaries: A and B, which are located in two different countries, whose relevant tax rates are τ_A and τ_B, respectively. Both subsidiaries are operating and their joint EBIT is Π. Accordingly, we assume that a portion $\theta \in (0,1)$ of EBIT is produced by the subsidiary located in A. The remaining portion $(1-\theta)$ is produced in country B.

In line with the empirical literature, we let the MNC shift a share $\alpha \in [0,1]$ of Π from the high-tax country to the low-tax one. Likewise, a share $\gamma \in [0,1]$ of C (if any) can be shifted from the low-tax country to the high-tax one, under the assumption that interest expenses are fully deductible.[8] It is worth noting that

[5]For further details on the characteristics of default conditions see, for example, Leland (1994) and Panteghini (2007a).

[6]The analysis of a dynamic trade-off model, where firms can subsequently adjust their capital structure, is again left for future research.

[7]After default the value of debt is nil.

[8]The quality of results does not change under partial deductibility of interest expenses.

shifting both EBIT and debt is costly. For this reason, we introduce ad hoc cost functions, i.e. the TP cost function denoted as $\phi(\alpha)$, and the DS cost function, i.e. $\nu(\gamma)$. For simplicity, we assume that both the cost functions are quadratic, namely:[9]

$$\phi(\alpha) = \frac{m}{2}\alpha^2 \quad \text{and} \quad \nu(\gamma) = \frac{n}{2}\gamma^2, \tag{3.2.2}$$

where m and n are scale parameters. Given these assumptions, our MNC's profit and coupon are, due to TP and DS, equal to $(\theta+\alpha)\Pi$ and $(\theta+\gamma)C$, respectively. Accordingly, the shares of subsidiary B are $(1-\theta-\alpha)\Pi$ and $(1-\theta-\gamma)C$. Since the cost of these operations, as from equation (3.2.2), are respectively $\phi(\alpha)\Pi$ and $\nu(\gamma)C$, the net profit Π^N is given by:

$$\begin{aligned}\Pi^N = &(1-\widetilde{\tau})(\Pi - C) + [(\tau_B - \tau_A)\alpha - \phi(\alpha)]\Pi + \\ &+ [(\tau_A - \tau_B)\gamma - \nu(\gamma)]C.\end{aligned} \tag{3.2.3}$$

where $\widetilde{\tau} \equiv \tau_A\theta + (1-\theta)\tau_B$ is the effective tax rate without tax avoidance.[10]

3.2.3. The value of equity. As later shown in Section 3.2.5, the MNC's value is given by the sum of its equity and debt. If the debt is nil, the MNC's value coincides with the value of equity, $E(\Pi)$. When, however, the MNC is debt financed and default occurs, $E(\Pi)$ goes to zero. Using the notation of Dixit and Pindyck (1994), we can therefore write:

$$E(\Pi) = \begin{cases} 0 & \text{a.d.} \\ \Pi^N dt + e^{-rdt}\mathbb{E}[E(\Pi + d\Pi)] & \text{b.d.} \end{cases}, \tag{3.2.4}$$

where $\mathbb{E}$ is the expected value operator. Labels "a.d." and "b.d." stand for "after default" and "before default", respectively. As proven in Appendix 3.A.1.1, equation (3.2.4) can be rewritten as:

[9]As pointed out, this choice is motivated by the lack of empirical evidence about the (hidden) cost of such operations. However, despite its simplicity, the functional form we propose introduces a penalty which is more than proportional to the shifted share α or γ, thereby setting a limit to the exploitation of these techniques. The study of a more realistic unique cost function, accounting for both TP and DS, is left for future research.

[10]The effective tax-rate without TP and DS $\tilde{\tau}$ is a function of τ_A, τ_B and θ, and it is obtained by solving the following equation:

$$1 - \widetilde{\tau} \equiv (1-\tau_A)\theta + (1-\tau_B)(1-\theta).$$

$$E(\Pi) = \begin{cases} 0 & \text{a.d.} \\ (1-\widetilde{\tau})\left(\frac{\Pi}{\delta}-\frac{C}{r}\right) + \left[(\tau_B-\tau_A)\alpha - \phi(\alpha)\right]\frac{\Pi}{\delta} + \\ + \left[(\tau_A-\tau_B)\gamma - \nu(\gamma)\right]\frac{C}{r} + \sum_{i=1}^{2} A_i \Pi^{\beta_i} & \text{b.d.} \end{cases} \tag{3.2.5}$$

As shown in Appendix 3.A.1.2, in the absence of financial bubbles, we have $A_1 = 0$. Solving the equation for A_2 at point $\Pi = \overline{\Pi}$, corresponding to the default trigger introduced by Assumption 3.1, we therefore obtain:

$$\begin{aligned} E(\Pi) = {} & (1-\widetilde{\tau})\left(\frac{\Pi}{\delta}-\frac{C}{r}\right) + (\tau_B-\tau_A)\left(\alpha\frac{\Pi}{\delta}-\gamma\frac{C}{r}\right) - \phi(\alpha)\frac{\Pi}{\delta} - \nu(\gamma)\frac{C}{r} + \\ & - \left[(1-\widetilde{\tau})\left(\frac{\overline{\Pi}}{\delta}-\frac{C}{r}\right) + (\tau_B-\tau_A)\left(\alpha\frac{\overline{\Pi}}{\delta}-\gamma\frac{C}{r}\right) - \phi(\alpha)\frac{\overline{\Pi}}{\delta} - \nu(\gamma)\frac{C}{r}\right]\left(\frac{\Pi}{\overline{\Pi}}\right)^{\beta_2}. \end{aligned} \tag{3.2.6}$$

According to Goldstein et al. (2001), shareholders choose the optimal default timing. Moreover, we also let them choose the optimal tax avoidance strategy. Their problem is therefore the following:

$$\max_{\overline{\Pi},\alpha,\gamma} E(\Pi). \tag{3.2.7}$$

As shown in Appendix 3.A.1.3, the solution of this problem leads to the optimal controls for α and γ:

$$\alpha^* = \frac{\tau_B-\tau_A}{m} \quad \text{and} \quad \gamma^* = \frac{\tau_A-\tau_B}{n}. \tag{3.2.8}$$

Hence, an increase in m and n reduces the absolute value of α^* and γ^*, respectively. Moreover, the trigger point, below which default takes place, is:

$$\overline{\Pi}^* = \frac{\beta_2}{\beta_2-1}\frac{1-\widetilde{\tau}-\frac{(\tau_A-\tau_B)^2}{2n}}{1-\widetilde{\tau}+\frac{(\tau_B-\tau_A)^2}{2m}}\frac{\delta}{r}C \equiv \Delta C, \tag{3.2.9}$$

where $\Delta < 1$. It is worth noting that, *coeteris paribus*, m and n affect not only the absolute value of α^* and γ^* but also the optimal threshold $\overline{\Pi}^*$ for a given coupon. In particular, an increase (decrease) in either m or n raises (reduces) $\overline{\Pi}^*$, thereby increasing (decreasing) the probability that Π hits $\overline{\Pi}^*$. This means that an increase (decrease) in either m or n raises (reduces) the default risk. Given these results, we can rewrite (3.2.6) as:

$$\begin{aligned} E(\Pi) = {} & \left(\frac{\Pi}{\delta}-\frac{C}{r}\right) + \frac{(\tau_B-\tau_A)^2}{2m}\frac{\Pi}{\delta} + \frac{(\tau_A-\tau_B)^2}{2n}\frac{C}{r} + \\ & - \left[(1-\widetilde{\tau})\left(\frac{\overline{\Pi}}{\delta}-\frac{C}{r}\right) + \frac{(\tau_B-\tau_A)^2}{2m}\frac{\overline{\Pi}}{\delta} + \frac{(\tau_A-\tau_B)^2}{2n}\frac{C}{r}\right]\left(\frac{\Pi}{\overline{\Pi}}\right)^{\beta_2}. \end{aligned} \tag{3.2.10}$$

3.2.4. The value of debt. In order to calculate the value of debt, $D(\Pi)$, we account for the fact that, before default, debt is equal to the sum between the coupon C over the short period dt and is expected to change in the future. It is worth noting that, after default, the value of $D(\Pi)$ does not fall to zero. As pointed out by Assumption 3.3, the MNC is still producing: in this case, the lender benefits from the future net profit flow.[11] Thus, the value of debt after default is equal to a portion $\Omega \in (0,1)$ of the discounted perpetual rent of future net profit:

$$D(\Pi) = \begin{cases} \Omega \frac{[(1-\tau_A)(\theta+\alpha)+(1-\tau_B)(1-\theta-\alpha)-\phi(\alpha)]\Pi}{\delta} & \text{a.d.} \\ Cdt + e^{-rdt}\mathbb{E}\left[D(\Pi + d\Pi)\right] & \text{b.d.} \end{cases} . \tag{3.2.11}$$

As proven in Appendix 3.A.2.1, the equation (3.2.11) can be rewritten as:

$$D(\Pi) = \begin{cases} \Omega \frac{[(1-\tau_A)(\theta+\alpha)+(1-\tau_B)(1-\theta-\alpha)-\phi(\alpha)]\Pi}{\delta} & \text{a.d.} \\ \frac{C}{r} + \sum_{i=1}^{2} B_i \Pi^{\beta_i} & \text{b.d.} \end{cases} . \tag{3.2.12}$$

Moreover, as shown in Appendix 3.A.2.2, assuming the absence of financial bubbles (i.e. $B_1 = 0$) and solving for B_2 at point $\Pi = \overline{\Pi}$ gives:

$$D(\Pi) = \begin{cases} \Omega \frac{[1-\widetilde{\tau}+(\tau_B-\tau_A)\alpha-\phi(\alpha)]\Pi}{\delta} & \text{a.d.} \\ \frac{C}{r} + \left[\Omega \frac{[1-\widetilde{\tau}+(\tau_B-\tau_A)\alpha-\phi(\alpha)]\overline{\Pi}}{\delta} - \frac{C}{r}\right]\left(\frac{\Pi}{\overline{\Pi}}\right)^{\beta_2} & \text{b.d.} \end{cases} , \tag{3.2.13}$$

After default, the lender chooses the optimal level of transfer pricing (see Appendix 3.A.2.3):

$$\max_{\alpha} D(\Pi) \quad \text{a.d.}, \tag{3.2.14}$$

which coincides with the result shown in equation (3.2.8), i.e. $\alpha^* = \frac{\tau_B - \tau_A}{m}$. Hence, solving (3.2.14) gives:

$$D(\Pi) = \begin{cases} \Omega \frac{1-\widetilde{\tau}+\frac{(\tau_B-\tau_A)^2}{2m}}{\delta}\Pi & \text{a.d.} \\ \frac{C}{r} + \left[\Omega \frac{1-\widetilde{\tau}+\frac{(\tau_B-\tau_A)^2}{2m}}{\delta}\overline{\Pi} - \frac{C}{r}\right]\left(\frac{\Pi}{\overline{\Pi}}\right)^{\beta_2} & \text{b.d.} \end{cases} , \tag{3.2.15}$$

As can be seen, the optimal TP choice is unaffected by default. Moreover, DS is no longer exploited.

3.2.5. The value of multinational company. The overall value of the MNC is given by the summation between equity and debt:

[11] In line with Chapter 1, we let the MNC not to borrow after default.

$$V(\Pi) = E(\Pi) + D(\Pi). \tag{3.2.16}$$

Substituting (3.2.10) and (3.2.15) into (3.2.16) gives:

$$\begin{aligned} V(\Pi) =& (1-\widetilde{\tau})\left(\tfrac{\Pi}{\delta}-\tfrac{C}{r}\right)+\tfrac{(\tau_B-\tau_A)^2}{2m}\tfrac{\Pi}{\delta}+\tfrac{(\tau_A-\tau_B)^2}{2n}\tfrac{C}{r}+ \\ & -\left\{(1-\widetilde{\tau})\left(\tfrac{\Delta C}{\delta}-\tfrac{C}{r}\right)+\tfrac{(\tau_B-\tau_A)^2}{2m}\tfrac{\Delta C}{\delta}+\tfrac{(\tau_A-\tau_B)^2}{2n}\tfrac{C}{r}\right\}\left(\tfrac{\Pi}{\Delta C}\right)^{\beta_2}+ \\ & +\tfrac{C}{r}+\left\{\Omega\left[1-\widetilde{\tau}+\tfrac{(\tau_B-\tau_A)^2}{2m}\right]\tfrac{\Delta C}{\delta}-\tfrac{C}{r}\right\}\left(\tfrac{\Pi}{\Delta C}\right)^{\beta_2}. \end{aligned} \tag{3.2.17}$$

As shown in Appendix 3.A.3, maximizing (3.2.17) with respect to C gives:

$$C^* = \left\{\frac{\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2n}}{(1-\beta_2)\left[(1-\Omega)\left[1-\widetilde{\tau}+\frac{(\tau_B-\tau_A)^2}{2m}\right]\frac{\Delta}{\delta}+\left[\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2n}\right]\frac{1}{r}\right]}\frac{1}{r}\right\}^{-\frac{1}{\beta_2}}\frac{\Pi}{\Delta}. \tag{3.2.18}$$

As can be seen, the relevant parameter values have a non-linear impact on endogenous variables. As shown in Appendix 3.A.3, substituting (3.2.18) into (3.2.9) gives the optimal default threshold point as a function of C^*:

$$\overline{\Pi}^*(C^*) = \frac{\delta}{r}\left\{\frac{\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2n}}{(1-\beta_2)\left[(1-\Omega)\frac{\beta_2}{\beta_2-1}+\left[1-(1-\Omega)\frac{\beta_2}{\beta_2-1}\right]\left[\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2n}\right]\right]}\right\}^{-\frac{1}{\beta_2}}\Pi. \tag{3.2.19}$$

A comparison between (3.2.9) and (3.2.19) leads to an interesting result: if C is given (as in (3.2.9)) both TP and DS affect the threshold level $\overline{\Pi}^*(C^*)$. If however C is optimally chosen (as in (3.2.19)), only DS affects $\overline{\Pi}^*(C^*)$ and hence it is unaffected by m. This result is useful to explain the asymmetric effects of TP and DS on the optimal leverage ratio.

3.2.6. Some comparative statics. Before introducing the numerical simulation of Section 3.3, let us provide some useful comparative statics. Given our two-stage approach, we use equations (3.2.8), (3.2.9) and (3.2.18), and analyze the effects of TP's and DS's concealment costs. In doing so, we focus on the effects of a change in either m or n on α^*, γ^*, $\overline{\Pi}^*(C^*)$ and C^*.

It is straightforward to find: $\partial\alpha^*/\partial m < 0$ when $\alpha \in (0,1)$, $\partial\alpha^*/\partial n = 0$, $\partial\gamma^*/\partial m = 0$ and $\partial\gamma^*/\partial n < 0$ if $\gamma \in (0,1)$. These results depend on the fact that, contrary to Schindler and Schjelderup (2016), we use additive concealment cost functions, regarding TP and DS, respectively.

As pointed out, we find that $\partial\overline{\Pi}^*(C^*)/\partial m = 0$. Given (3.2.19) this result is not surprising. The reasoning is as follows: TP activities are, by assumption, made both before and after default. As shown in (3.2.2) and (3.2.14), TP activities do not depend on the default event. Since, given m, TP strategies are the same irrespective of default, they do not affect default timing. In addition, we find that $\partial\overline{\Pi}\left(C^{*}\right)/\partial n < 0$. This result shows the negative effect of the DS cost on the optimal default trigger. Finally, it is easy to show that $\partial C^{*}/\partial m < 0$ and that $\partial C^{*}/\partial n < 0$. This is due to the fact that the more costly the tax avoidance, the lower the coupon and, hence, the debt value. In other words, an increase in concealment costs discourages borrowing because it reduces tax savings.

3.3. A numerical analysis

The effects of TP and DS on the capital structure of the MNC choices are next investigated. To do so, we use a numerical approach and focus on: the value of equity E, the value of debt D, the overall value V and the leverage ratio L, i.e. the ratio between D and V. The behavior of these indicators is studied with respect to both the relevant tax rate in country B, τ_B, and the drift coefficient μ, which determines the expected growth of EBIT.

The purpose of this exercise is twofold. Firstly, we evaluate if and how much the exploitation of TP and DP affects the MNC's value (Section 3.3.2 contains our main results). Secondly, in Section 3.3.3 we perform a sensitivity analysis aimed at evaluating the impact of changes in exogenous parameters. More in detail, we study the effects of EBIT drift and diffusion coefficients, namely μ and σ, the relevant tax rate τ_B (which, given a constant τ_A, allows control of the tax differential between countries) as well as the costs of transfer pricing and debt shifting, represented by m and n respectively.[12]

3.3.1. Purpose and parameters. The benchmark values of both parameters m and n, as well as those regarding the other variables, are shown in Table 1. The starting values of relevant tax rates in country A and B are respectively 0.15 and 0.25: this differential would make TP and DS feasible. The drift μ and the diffusion σ of the GMB are equal to 0.02 and 0.2 respectively: these values are in line with Dixit and Pindyck (1994). In order to normalize our results, the current values of Π (2.5) and r (0.025) are such that perpetual rent Π/r is equal to 100. As pointed out, the empirical evidence on concealment costs is quite poor. For this reason, we arbitrarily set m and n equal to 0.05 and 0.1 respectively (although we also

[12]For simplicity we omit the plots of equity and debt. Rather, we focus on the MNC's value. Such plots are of course available upon request.

	Variable	Value		Variable	Value
τ_A	Tax rate in country A	0.15	r	Risk-free interest rate	0.025
τ_B	Tax rate in country B	0.25	m	Scale parameter of TP cost	0.05
μ	GBM drift	0.02	n	Scale parameter of DS cost	0.1
σ	GMB diffusion	0.2	θ	Relative weight of firm A	0.5
Π	Current profitability	2.5	$1-\Omega$	Cost of default	0.2

TABLE 1. Benchmark values of parameters and variables used in the numerical simulations.

run some robustness checks). Finally, with no loss of generality, we set $\theta = 0.5$ and $1 - \Omega = 0.2$.[13] The other parameter values are not relevant for our sensitivity analysis: we showed that their change does not affect the quality of results.

3.3.2. Effects of tax avoidance practices. As pointed out, our numerical simulation is based on the parameter values of Table 1. The only exception regards the value of m and n. They have been properly set to define the following scenarios: (i) both transfer pricing and debt shifting are exploited, (ii) only debt shifting is feasible, (iii) only transfer pricing is allowed and (iv) tax avoidance is impossible (this happens if both m and n are high enough). In what follows we focus on both V and L as functions of μ and τ_B respectively.

In the top-left panel of Figure 3.3.1, V is shown to be increasing in μ: namely, the higher the drift, the higher the expected future profitability and the higher the MNC's value. Moreover, V is increasing in the tax avoidance opportunities (see the blue line): when both TP and DS are feasible, V is higher. Of course, when only one of these tax avoiding practices is available, V is lower for any μ. Without tax avoidance (purple line), not surprisingly, V has the lowest value. Interestingly, we also see that the effect of TP always dominates the DS one. This result is in line with Schenkelberg (2020), who estimated that about 85% of the tax avoidance benefit is due to TP, whereas the remaining 15% is due to DS. Here we find similar values: with $\mu = 0.01$, the portion of benefit arising from TP (DS) is 78.1% (21.9%).

The top-right panel focuses on L. As can be seen, the leverage ratio is increasing in μ: this relationship is in line with static trade-off models. This behavior is also in line with Dwenger and Steiner (2014), who show that the marginal tax rate has

[13]For further details about default cost, see Chapter 1.

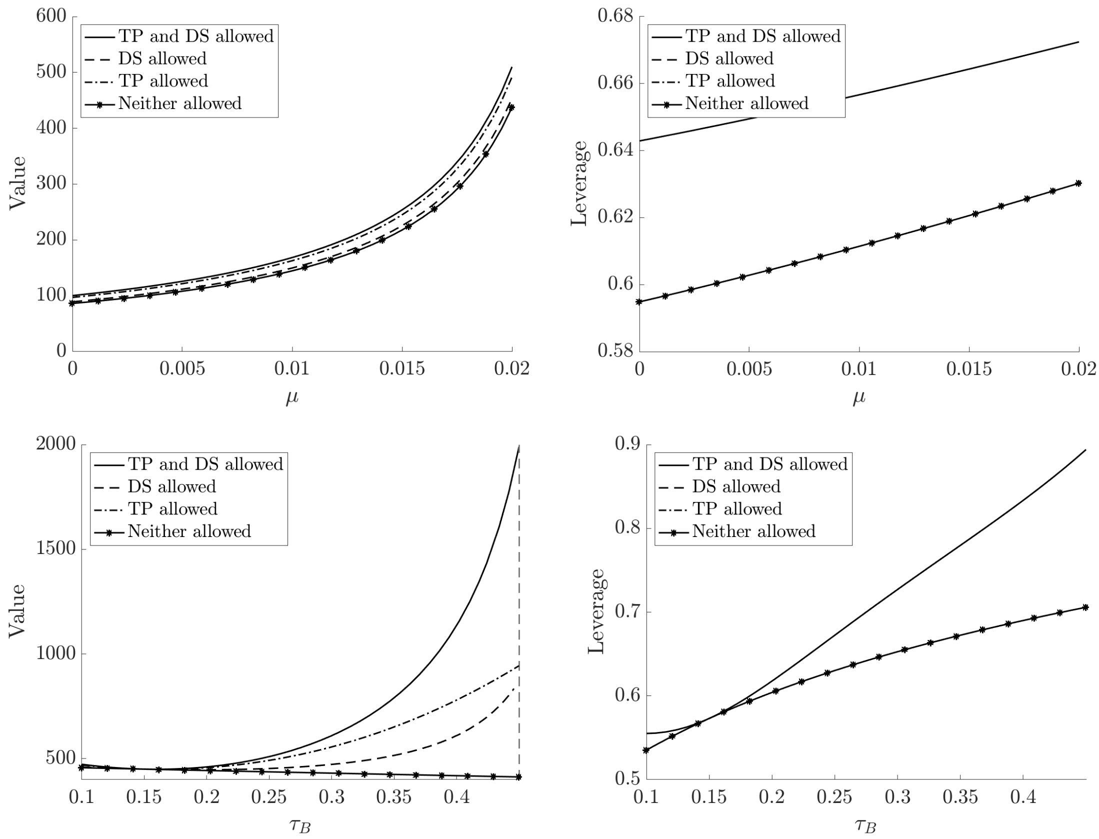

FIGURE 3.3.1. Effects on V (left panels) and L (right panels), expressed as functions of EBIT drift μ (top panels) and of effective tax rate τ_B (bottom panels), of different availability of tax avoidance practices.

a statistically significant and relatively large positive effect on corporate leverage.[14] It is worth pointing out that that cases (i) and (iii) coincide with cases (ii) and (iv). This means that the use (absence) of only DS leads L to be higher (lower) and that TP does not matter. This is due to the dynamics highlighted in Section 3.2.6: as $\overline{\Pi}^*(C^*)$ is unaffected by m, TP does not affect $L = D/V$.

In the bottom-left panel, we focus on the effects of τ_B, given $\tau_A = 0.15$. Of course, a change in τ_B affects the tax rate differential. Obviously, the higher the tax rate differential, the greater the tax benefit. If however the equality $\tau_A = \tau_B$

[14]It is worth noting that, when a dynamic trade-off model is applied, leverage is not necessarily increasing in μ (see, for example, Strebulaev (2007)).

holds, no benefit is ensured. As can be seen, V is higher when both TP and DS are feasible, for any tax rate differential. Not surprisingly, tax benefits vanish when $\tau_A = \tau_B$: in this case, all the lines are meeting at rate $\tau_B = 0.15$.

Finally, in the bottom-right panel we show the leverage as a function of τ_B. Accordingly, TP has no effect on leverage and all the lines meet at point $\tau_B = 0.15$. Finally, we can see that the higher the tax differential, the higher the leverage ratio.

We believe that policy-makers can draw useful insights from these results for two reasons. Firstly, knowing the benefits arising from DS and TP – and thus the corresponding losses in terms of tax revenue – helps to better fight the use of these practices. In addition, as almost all tax systems encourage the use of debt over equity finance, it is possible to address the MNC's capital structure towards a desirable level (for example, a higher L), by making it harder to exploit the TP.

3.3.3. Sensitivity analysis. To better understand the MNC's choices we run additional simulations investigating the effects of μ and τ_B, with the aim of quantifying the effect of a variation in μ, for a given τ_B and vice versa. In these simulations, all other parameters are set to their benchmark level.

In Figure 3.3.2 both top panels focus on the sensitivity analysis on μ. Not surprisingly, we see that V is increasing in both the tax differential and μ.[15] For example, given $\tau_B = 0.25$, an increase in μ from its benchmark value (0.02) to 0.021 leads to a dramatic increase in V (by 25.1%). In all cases, the minimum value is obtained when $\tau_A = \tau_B = 0.15$. Not surprisingly, L is also increasing in tax differential, although its sensitivity to changes in μ is almost negligible.

The bottom panels deal with the sensitivity analysis of τ_B. The left plot shows that, in line with our previous results, V is increasing in τ_B for any value of μ. Moreover, the higher the rate τ_B, the higher the MNC's value, given $\tau_A = 0.15$. For example, if $\mu = 0.01$, an increase in τ_B from its benchmark value (0.25) to 0.30 raises V by 19.3%. In addition, the right panel shows that L is also increasing in τ_B. In other words, an increase in the tax differential reduces the tax burden and allows the MNC to retain more resources. Since the marginal benefit arising from tax savings rises, the MNC borrows more, thereby leading to an higher marginal cost, due to higher default risk. This intuition is also supported by the fact that, if $\tau_B > \tau_A$, the default trigger $\overline{\Pi}$ is increasing with the tax differential as, notwithstanding the

[15]Note that when the tax differential is low enough, i.e. below 5%, the tax avoidance benefit is close to zero.

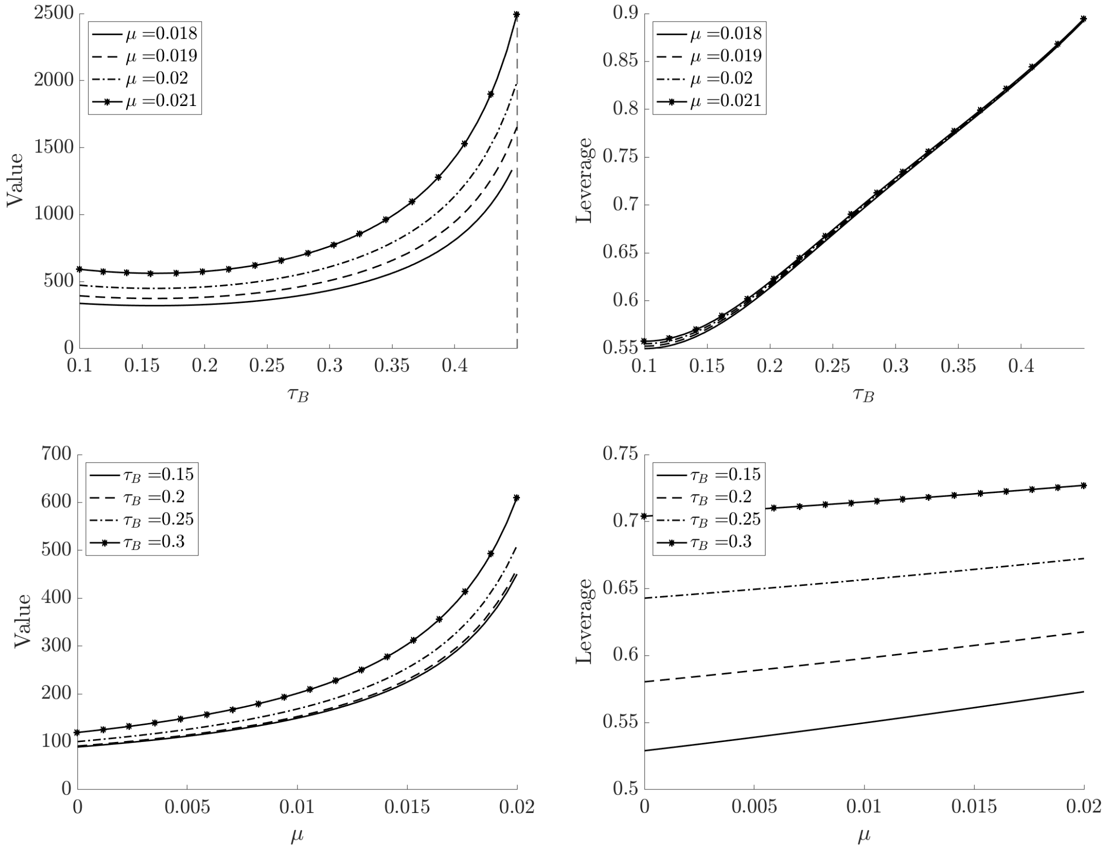

FIGURE 3.3.2. Effects on V (left panels) and L (right panels), expressed as functions of effective tax rate τ_B (top panels) and of EBIT drift μ (bottom panels) of different values of the same variables.

benefit arising from tax avoidance, just one increasing effective tax rate leads to a higher probability of default.

Figure 3.3.3 shows the effect of σ on V and L. Again, we set μ (upper panels) and τ_B (lower panels) on the horizontal axis. As can be seen, V is slightly decreasing in σ, since the higher the profit volatility, the lower the value of E. This effect dominates the negative one on D, explaining the low sensitivity of V to changes in σ. This however does not happen in L by construction: in this case the dynamics of E and D do not offset each other. For example, for $\mu = 0.01$, an increase in σ from its benchmark value (0.2) to 0.25 leads to a decrease in V and in L by -1.3% and -3.6% respectively. Similarly, when τ_B is on the horizontal axis, the decrease in both V and L is respectively -1.6% and -4.3% respectively. Again, the low

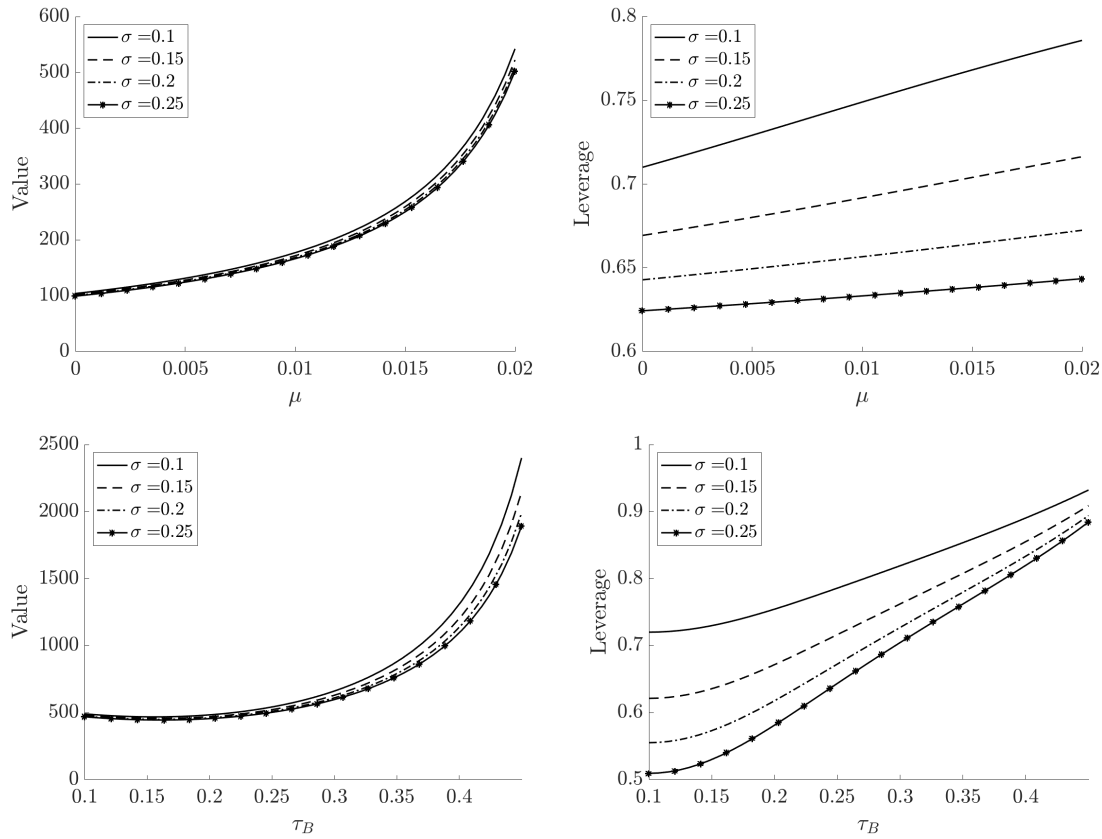

FIGURE 3.3.3. Effects on V (left panels) and L (right panels), expressed as functions of EBIT drift μ (top panels) and of effective tax rate τ_B (bottom panels), of different values of EBIT diffusion σ.

sensitivity of V is explained by the fact that the effect on E dominates the one on D, while L varies more significantly.

Figure 3.3.4 finally shows the effects of both m and n on V and L, for any given value of τ_B.[16] As can be seen, V is decreasing in both m and n. In other words, the more costly the TP and DS activities, the lower the MNC's value. For example, increasing m from its benchmark value (0.05) to 0.1 lowers V by -5.6%. Similarly, the reduction of V due to an increase in n from its benchmark value (0.1) to 0.25 is equal to -2.2%.

[16]We show only plots with τ_B set on the horizontal axis as it is more useful to see the combined effect of τ_B and one between m and n, since they are all determinants of the optimal shares of TP and DS, as from equation (3.2.8).

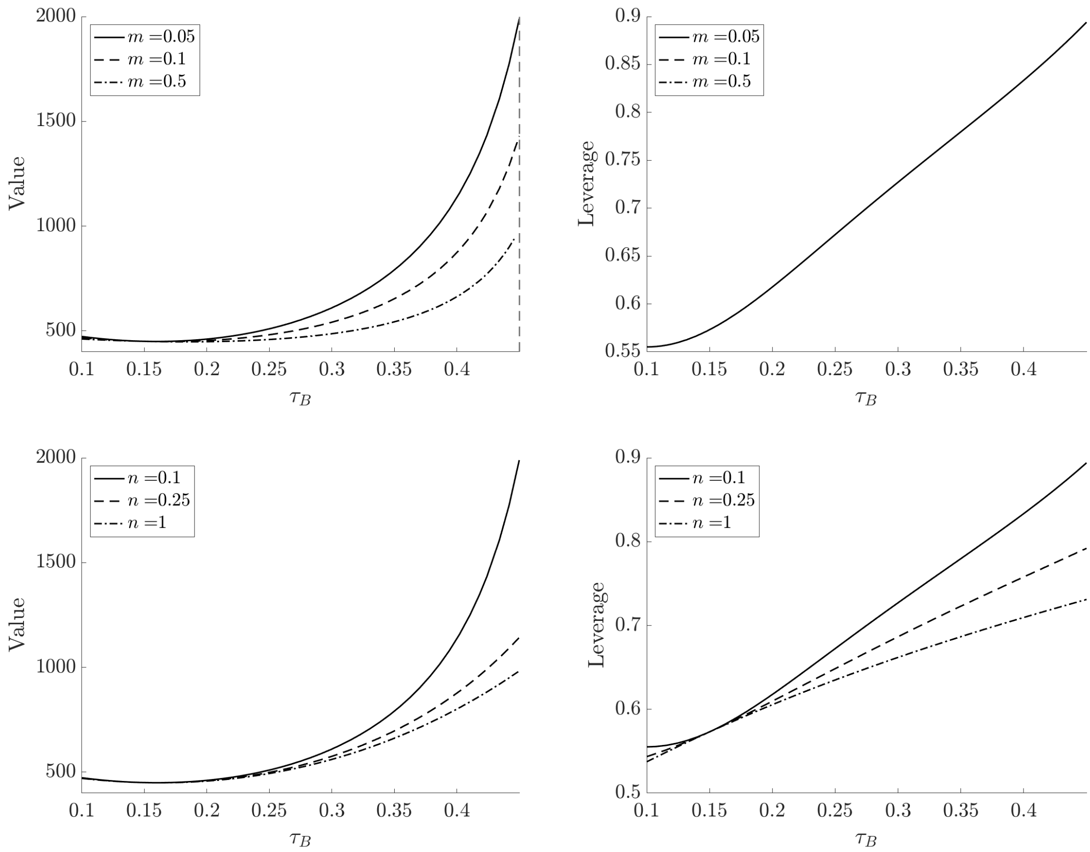

FIGURE 3.3.4. Effects on V (left panels) and L (right panels), expressed as functions of effective tax rate τ_B, of different values of scale parameter cost of TP m (upper panels) and of DS n (bottom panels).

It is worth noting that L is unaffected by m since, given (3.2.19), the changes in both E and D are such that leverage remains unchanged. Moreover, L is decreasing in n: this is due to the fact that an increase in n raises E and reduces D: this latter effect dominates the former one.

3.4. Conclusion

In this paper we have described the behavior of a representative MNC under default risk. In particular, we have focused on tax avoidance strategies in an uncertain context, due to the EBIT stochasticity. In addition, unlike most literature we have jointly analyzed TP and DS practices. In order to study the effects of volatility, we have run numerical simulations, where we show that results dramatically differ from deterministic ones.

More in detail, we have found a strongly positive effect of tax avoidance on the MNC's value. We have also shown that TP has a larger effect than DS on MNC decisions. In doing so, we can provide a theoretical rationale for the empirical findings of Schenkelberg (2020). Furthermore, we have studied how default risk impacts on our MNC's capital structure. We have shown that EBIT variability, despite a minimal negative effect on the value function, dramatically reduces the leverage ratio. Finally, we have run some robustness check regarding the costs of tax avoidance. In doing so, we have studied the effects on the MNC's value function and leverage ratio, thereby showing that tax avoidance has an asymmetric effect on the latter. Even if we use fairly simple concealment cost functions, we can say that default risk must be considered by policy-makers when designing their tax system.

3.A. Appendix

3.A.1. The value of equity.

3.A.1.1. *The derivation of* (3.2.5). In order to derive the value of equity, it is first necessary to rearrange the net profit defined in equation (3.2.3) as:

$$\Pi^N = \left[-1 + \tilde{\tau} + (\tau_B - \tau_A)\,\gamma - \nu\,(\gamma)\right] C + \left[1 - \tilde{\tau} + (\tau_B - \tau_A)\,\alpha - \phi\,(\alpha)\right] \Pi,$$

that is as the sum of a term constant in Π, $a \equiv \left[-1 + \tilde{\tau} + (\tau_B - \tau_A)\,\gamma - \nu\,(\gamma)\right] C$, and one depending on Π, namely $b\Pi$, with $b \equiv \left[1 - \tilde{\tau} + (\tau_B - \tau_A)\,\alpha - \phi\,(\alpha)\right]$. Applying Itô's lemma to equation (3.2.4) the following second order differential equation is:[17]

$$\frac{\sigma^2}{2}\Pi^2 E_{\Pi\Pi} + \mu\Pi E_{\Pi} - rE = -a - b\Pi. \tag{3.A.1}$$

The general solution of equation (3.A.1) is:

$$E = H_0 + H_1\Pi + A\Pi^{\beta}.$$

Substituting it into (3.A.1) thus leads to:

$$\frac{\sigma^2}{2}\Pi^2\beta\,(\beta - 1)\,A\Pi^{\beta-2} + \mu\Pi\left(H_1 + \beta A\Pi^{\beta-1}\right) - r\left(H_0 + H_1\Pi + A\Pi^{\beta}\right) + a + b\Pi = 0,$$

which is satisfied if:

[17]The dependency of E on Π is omitted to lighten the notation. Moreover, we denote the two first derivatives of E with respect to Π as E' and E'' respectively.

$$\begin{cases} \frac{\sigma^2}{2}\beta\left(\beta-1\right)+\mu\beta-r=0 \\ \mu H_1 - rH_1 + b = 0 \\ -rH_0 + a = 0 \end{cases}.$$

From the second and the third equations it easily follows that $H_0 = ar^{-1}$ and $H_1 = b\left(r-\mu\right)^{-1}$, respectively. Moreover, the solution of the first equation leads to:

$$\beta_{1,2} = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{3.A.2}$$

where $\beta_1 > 1$ and $\beta_2 < 0$. It follows that the general solution of equation (3.A.1) is:

$$E(\Pi) = \left[-1+\tilde{\tau}+\left(\tau_B-\tau_A\right)\gamma-\nu\left(\gamma\right)\right]\tfrac{C}{r} + \left[1-\tilde{\tau}+\left(\tau_B-\tau_A\right)\alpha-\phi\left(\alpha\right)\right]\tfrac{\Pi}{r-\mu} + \textstyle\sum_{i=1}^{2} A_i\Pi^{\beta_i}.$$

After some rearrangements, we therefore obtain (3.2.5).

3.A.1.2. *The derivation of* (3.2.6)*.* Under the assumption that financial bubbles do not exist, we set $A_1 = 0$. Since, given $\overline{\Pi}$, the value of equity b.d. and a.d. must be equal, it holds that:

$$E\left(\overline{\Pi}\right) = \left(1-\widetilde{\tau}\right)\left(\tfrac{\overline{\Pi}}{\delta}-\tfrac{C}{r}\right)+\left(\tau_B-\tau_A\right)\left[\alpha\tfrac{\overline{\Pi}}{\delta}-\gamma\tfrac{C}{r}\right]-\phi\left(\alpha\right)\tfrac{\overline{\Pi}}{\delta}-\nu\left(\gamma\right)\tfrac{C}{r}+A_2\overline{\Pi}^{\beta_2}=0,$$

Solving for A_2 gives:

$$A_2 = -\left[\left(1-\widetilde{\tau}\right)\left(\tfrac{\overline{\Pi}}{\delta}-\tfrac{C}{r}\right)+\left(\tau_B-\tau_A\right)\left(\alpha\tfrac{\overline{\Pi}}{\delta}-\gamma\tfrac{C}{r}\right)-\phi\left(\alpha\right)\tfrac{\overline{\Pi}}{\delta}-\nu\left(\gamma\right)\tfrac{C}{r}\right]\overline{\Pi}^{-\beta_2}, \tag{3.A.3}$$

Using (3.2.5) and (3.A.3) allows us to obtain (3.2.6).

3.A.1.3. *The derivation of* (3.2.8)*,* (3.2.9) *and* (3.2.10)*.* To find the optimal controls of $\overline{\Pi}$, α and γ that solve problem (3.2.7) maximizing the value of equity shown in equation (3.2.6), it is necessary to set all its partials equal to zero. With regard to the optimal default $\overline{\Pi}$,we find that:

$$\begin{aligned} \tfrac{\partial E(\Pi)}{\partial\overline{\Pi}} &= -\tfrac{1}{\delta}\left[1-\widetilde{\tau}+\left(\tau_B-\tau_A\right)\alpha-\phi\left(\alpha\right)\right]\left(\tfrac{\Pi}{\overline{\Pi}}\right)^{\beta_2}+ \\ &+\left[\left(1-\widetilde{\tau}\right)\left(\tfrac{\overline{\Pi}}{\delta}-\tfrac{C}{r}\right)+\left(\tau_B-\tau_A\right)\left(\alpha\tfrac{\overline{\Pi}}{\delta}-\gamma\tfrac{C}{r}\right)-\phi\left(\alpha\right)\tfrac{\overline{\Pi}}{\delta}-\nu\left(\gamma\right)\tfrac{C}{r}\right]\tfrac{\beta_2}{\overline{\Pi}}\left(\tfrac{\Pi}{\overline{\Pi}}\right)^{\beta_2}=0, \end{aligned}$$

from which it easily follows that:

$$\overline{\Pi}^* = \frac{\beta_2}{\beta_2-1}\frac{\left[1-\widetilde{\tau}+\left(\tau_B-\tau_A\right)\gamma+\nu\left(\gamma\right)\right]}{\left[1-\widetilde{\tau}+\left(\tau_B-\tau_A\right)\alpha-\phi\left(\alpha\right)\right]}\frac{\delta}{r}C. \tag{3.A.4}$$

Let us next focus on the optimal tax avoidance choices. Differentiating $E\left(\Pi\right)$ with respect to α and γ gives:

$$\frac{\partial E(\Pi)}{\partial \alpha} = [(\tau_B - \tau_A) - m\alpha]\left[\frac{\Pi}{\delta} - \frac{\overline{\Pi}}{\delta}\left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2}\right] = 0$$

and:

$$\frac{\partial E(\Pi)}{\partial \gamma} = -[(\tau_B - \tau_A) + n\gamma]\left[1 + \left(\frac{\Pi}{\overline{\overline{\Pi}}}\right)^{\beta_2}\right] = 0,$$

respectively. Their solutions easily lead to equation (3.2.8). Substituting into equation (3.A.4) the optimal controls for α and γ, we obtain (3.2.9). Finally, by substituting the values of α^* and γ^* into equation (3.2.6) the value of $E(\Pi)$, shown in equation (3.2.10), finally follows.

3.A.2. The value of debt.

3.A.2.1. *The derivation of* (3.2.12). Applying Ito's lemma to the increment $dD(\Pi)$, the value of debt before default in equation (3.2.11) can be rewritten as:[18]

$$\frac{\sigma^2}{2}\Pi^2 D_{\Pi\Pi} + \mu\Pi D_{\Pi} - rD = -C. \tag{3.A.5}$$

The general solution of (3.A.5) is:

$$D = K + B\Pi^{\beta}.$$

Rearranging therefore gives:

$$\left[\frac{\sigma^2}{2}\beta(\beta-1) + \mu\beta - r\right]B\Pi^{\beta} - rK + C = 0,$$

which holds if:

$$\begin{cases} \frac{\sigma^2}{2}\beta(\beta-1) + \mu\beta - r = 0 \\ -rK + C = 0 \end{cases}.$$

From the second equation, it easily follows that $K = Cr^{-1}$, while the first one is equal to the one in the case of equity and then leads to the same β_1 and β_2. It finally follows the general solution of equation (3.A.5) , that immediately leads to the value of debt before default in equation (3.2.12).

[18]The dependency of D on Π is omitted to lighten the notation. Moreover, we denote the two first derivatives of D with respect to Π as D' and D'' respectively.

3.A.2.2. *The derivation of* (3.2.13). For the same reason detailed in Section 3.A.1.2, B_1 must be set equal to 0, leaving only the constant B_2 to be computed. Since in correspondence of default trigger $\overline{\Pi}$ the value of debt before and after default must be equal and set to zero, it holds that:

$$D\left(\overline{\Pi}\right) = \frac{C}{r} + B_2 \overline{\Pi}^{\beta_2} = \Omega \frac{\left[(1-\tau_A)(\theta+\alpha) + (1-\tau_B)(1-\theta-\alpha) - \phi(\alpha)\right]\overline{\Pi}}{\delta},$$

from which, also recalling the definition of effective tax rate $\widetilde{\tau}$ shown in Section (3.2.2), the value of B_2 easily follows:

$$B_2 = \left[\Omega \frac{\left[1-\widetilde{\tau} + (\tau_B - \tau_A)\alpha - \phi(\alpha)\right]\overline{\Pi}}{\delta} - \frac{C}{r}\right]\overline{\Pi}^{-\beta_2},$$

which once substituted in the equation above leads to equation (3.2.13). The value of debt after default can be simplified, in the same way, by the definition of $\widetilde{\tau}$.

3.A.2.3. *The derivation of* (3.2.15). The derivative with respect to α of the value of debt after default, defined in equation (3.2.13), is:

$$\frac{\partial D(\Pi)}{\partial \alpha} = \Omega \frac{\left[(1-\tau_A) - (1-\tau_B) - m\alpha\right]\Pi}{\delta},$$

which, once set equal to zero, leads to the same solution of shareholders' problem before default, shown in equation 3.2.8. By substituting this result into equation (3.2.13) and after some rearrangements, equation (3.2.15) easily follows.

3.A.3. The value of the MNC. The value of the MNC defined in equation (3.2.17) can be rearranged as:

$$\begin{aligned} V(\Pi) =& \left[1-\widetilde{\tau}+\tfrac{(\tau_B-\tau_A)^2}{2m}\right]\tfrac{\Pi}{\delta} + \left[\widetilde{\tau}+\tfrac{(\tau_A-\tau_B)^2}{2n}\right]\tfrac{C}{r} \\ & - \left\{(1-\Omega)\left(1-\widetilde{\tau}+\tfrac{(\tau_B-\tau_A)^2}{2m}\right)\tfrac{\Delta C}{\delta} + \left[\widetilde{\tau}+\tfrac{(\tau_A-\tau_B)^2}{2n}\right]\tfrac{C}{r}\right\}\left(\tfrac{\Pi}{\Delta C}\right)^{\beta_2}, \end{aligned}$$

whose derivative with respect to C is:

$$\tfrac{\partial V(\Pi)}{\partial C} = \left[\widetilde{\tau}+\tfrac{(\tau_A-\tau_B)^2}{2n}\right]\tfrac{1}{r} - (1-\beta_2)\left\{(1-\Omega)\left[1-\widetilde{\tau}+\tfrac{(\tau_B-\tau_A)^2}{2m}\right]\tfrac{\Delta}{\delta} + \left[\widetilde{\tau}+\tfrac{(\tau_A-\tau_B)^2}{2n}\right]\tfrac{1}{r}\right\}\left(\tfrac{\Pi}{\Delta C}\right)^{\beta_2}.$$

By setting it equal to zero and rearranging, equation (3.2.18) follows. Then, we can rewrite the optimal default trigger defined in equation (3.2.9) as:

$$\overline{\Pi}^*(C^*) = \tfrac{\delta}{r}\left[\frac{\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2n}}{(1-\beta_2)\left[(1-\Omega)\frac{\beta_2}{\beta_2-1}\left[1-\widetilde{\tau}-\frac{(\tau_A-\tau_B)^2}{2n}\right]+\left[\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2n}\right]\right]}\right]^{-\frac{1}{\beta_2}}\Pi,$$

from which, after some rearrangements, equation (3.2.19) easily follows.

3.A.4. Derivatives of optimal controls with respect to m and n. As regards $\overline{\Pi}^*(C^*)$, we notice that it is unaffected by m. Thus, we immediately conclude that $\partial\overline{\Pi}^*(C^*)/\partial m = 0$. Moreover, differentiating $\overline{\Pi}^*(C^*)$ with respect to n gives:

$$\frac{\partial\overline{\Pi}^*(C^*)}{\partial n} = \Pi\left[r(1-\beta_2)\right]^{\frac{1}{\beta_2}}\frac{1}{\beta_2}\left\{\frac{\left[(1-\Omega)\frac{\beta_2}{\beta_2-1}(1-\widetilde{\tau})+\widetilde{\tau}\right]+\left[1-(1-\Omega)\frac{\beta_2}{\beta_2-1}\right]\frac{(\tau_A-\tau_B)^2}{2}n^{-1}}{\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2}n^{-1}}\right\}^{\frac{1}{\beta_2}}\frac{\frac{(\tau_A-\tau_B)^2}{2}\left(\left[(1-\Omega)\frac{\beta_2}{\beta_2-1}(1-\widetilde{\tau})+\widetilde{\tau}\right]-\left[1-(1-\Omega)\frac{\beta_2}{\beta_2-1}\right]\widetilde{\tau}\right)}{\left(\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2}n^{-1}\right)^2}n^{-2}.$$

Notice that all factors are positive, with the only exception of β_2^{-1}. This implies that, given (3.A.2), the derivative $\partial\overline{\Pi}^*(C^*)/\partial n$ is negative.

As regards C^*, we find that:

$$\frac{\partial C^*}{\partial m} = -\left\{\frac{\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2n}}{(1-\beta_2)\left[(1-\Omega)\frac{\beta_2}{\beta_2-1}\left[1-\widetilde{\tau}-\frac{(\tau_A-\tau_B)^2}{2n}\right]+\left[\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2n}\right]\right]}\right\}^{-\frac{1}{\beta_2}}\frac{\beta_2-1}{\beta_2}\frac{\Pi}{1-\widetilde{\tau}-\frac{(\tau_A-\tau_B)^2}{2n}}\frac{(\tau_B-\tau_A)^2}{2}m^{-2},$$

whose factors are all always positive. Given the initial minus, we conclude that $\partial C^*/\partial m < 0$. Differentiating with respect to n gives:

$$\frac{\partial C^*}{\partial n} = \frac{\beta_2-1}{\beta_2}\left[1-\widetilde{\tau}+\frac{(\tau_B-\tau_A)^2}{2m}\right]\frac{r}{\delta}\Pi\left\{\frac{(1-\beta_2)\left[(1-\Omega)\frac{\beta_2}{\beta_2-1}-\widetilde{\tau}(1-\Omega)\frac{\beta_2}{\beta_2-1}+\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2}\left[1-(1-\Omega)\frac{\beta_2}{\beta_2-1}\right]n^{-1}\right]}{\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2}n^{-1}}\right\}^{\frac{1}{\beta_2}}$$

$$\left\{\frac{1}{\beta_2}\left[\frac{(1-\beta_2)\left[(1-\Omega)\frac{\beta_2}{\beta_2-1}-\widetilde{\tau}(1-\Omega)\frac{\beta_2}{\beta_2-1}+\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2}\left[1-(1-\Omega)\frac{\beta_2}{\beta_2-1}\right]n^{-1}\right]}{\widetilde{\tau}+\frac{(\tau_A-\tau_B)^2}{2}n^{-1}}\right]^{-1}\left(1-\widetilde{\tau}-\frac{(\tau_A-\tau_B)^2}{2}n^{-1}\right)^{-1}-\frac{\frac{(\tau_A-\tau_B)^2}{2}}{\left((1-\widetilde{\tau})n-\frac{(\tau_A-\tau_B)^2}{2}\right)^2}\right\},$$

whose factors in the first row are all positive. In the second row, we notice that both terms are negative (given $\beta_2^{-1} < 0$). It is straightforward to see that $\partial C^*/\partial n < 0$.

Bibliography

Andrade, G. and Kaplan, S. N. (1998). How costly is financial (not economic) distress? Evidence from highly leveraged transactions that became distressed. *Journal of Finance*, 53(5):1443–1493.

Bolton, P., Wang, N., and Yang, J. (2019). Investment under uncertainty with financial constraints. *Journal of Economic Theory*, 184(104912).

Bond, S. and Devereux, M. B. (2003). Generalised R-based and S-based taxes under uncertainty. *Journal of Public Economics*, 87(5–6):1291–1311.

Branch, B. (2002). The costs of bankruptcy: A review. *International Review of Financial Analysis*, 11(1):39–57.

Carini, C., Moretto, M., Panteghini, P. M., and Vergalli, S. (2020). Deferred taxation under default risk. *Journal of Economics*, 129:33–48.

Claesens, S., Djankov, S., and Mody, A. (2001). *Resolution of Financial Distress: An International Perspective on the Design of Bankruptcy Laws*. World Bank.

Cohen, F., Fedele, A., and Panteghini, P. M. (2016). Corporate taxation and financial strategies under asymmetric information. *Economia Politica*, 33(1):9–34.

Collins, J. H. and Shackelford, D. A. (1992). Foreign tax credit limitations and preferred stock issuances. *Journal of Accounting Research*, 30:103–124.

Davydenko, S., Strebulaev, I. A., and Zhao, X. (2012). A market-based study of the costs of default. *Review of Financial Studies*, 25(10):2959–2999.

De Marzo, P. M. and Sannikov, Y. (2007). Optimal security design and dynamic capital structure in a continuous-time agency model. *Journal of Finance*, 61(6):2681–2724.

De Mooij, R. and Hebous, S. (2018). Curbing corporate debt bias: Do limitations to interest deductibility work? *Journal of Banking and Finance*, 96:368–378.

De Mooij, R. A. and Keen, M. (2016). Debt, taxes, and banks. *Journal of Money, Credit and Banking*, 48(1):5–33.

Devereux, M. P. and Maffini, G. (2007). The impact of taxation on the location of capital, firms and profit: A survey of empirical evidence. *Oxford University Center for Business Taxation Working Paper*.

Dischinger, M. (2010). Profit shifting by multinationals: Indirect evidence from european micro data. *Ludwig-Maximilians-University Discussion Paper*.

Dischinger, M., Knoll, B., and Riedel, N. (2014). The role of headquarters in multinational transfer pricing strategies. *International Tax and Public Finance*, 21(2):248–271.

Dixit, A. and Pindyck, R. (1994). *Investment under Uncertainty*. Princeton University Press.

Dwenger, N. and Steiner, V. (2014). Financial leverage and corporate taxation: Evidence from German corporate tax return data. *International Tax and Public Finance*, 21:1–28.

Estrin, S., Mickiewicz, T., and Rebmann, A. (2017). Prospect theory and the effects of bankruptcy laws on entrepreneurial aspirations. *Small Business Economics*, 48:977–997.

Feld, L. P., Heckemeyer, J. H., and Overesch, M. (2013). Capital structure choice and company taxation: A meta-study. *Journal of Banking & Finance*, 37(8):2850–2866.

Froot, K. A. and Hines, J. R. (1995). Interest allocation rules, financing patterns, and the operation of US multinationals'. *National Bureau of Economic Research*.

Glover, B. (2016). The expected cost of default. *Journal of Financial Economics*, 119(2):284–299.

Goldstein, R., Jun, N., and Leland, H. E. (2001). An EBIT-based model of dynamic capital structure. *Journal of Business*, 74(4):483–512.

Grubert, H. and Slemrod, J. (1998). The effect of taxes on investment and income shifting to Puerto Rico. *Review of Economics and Statistics*, 80(3):365–373.

Harrison, J. M. (1985). *Brownian Motion and Stochastic Flow Systems*. John Wiley & Sons Inc.

Hines, J. R. and Rice, E. (1994). Fiscal paradise: Foreign tax havens and american business. *Quarterly Journal of Economics*, 109(1):149–182.

Huizinga, H., Laeven, L., and Nicodeme, G. (2008). Capital structure and international debt shifting. *Journal of Financial Economics*, 88(1):80–118.

Kocherlakota, N. (2010). Taxing risk and the optimal regulation of financial institutions. *Economic Policy Paper, Federal Reserve Bank of Minneapolis*, #10–3.

Leland, H. E. (1994). Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance*, 49(4):1213–1252.

Lucchetta, M., Moretto, M., and Parigi, B. M. (2019). Optimal bailouts, bank's incentive and risk. *Annals of Finance*, 15:369–399.

McGowan, M. A. and Andrews, D. (2018). Design of insolvency regimes across countries. *OECD Working Paper*, 52.

Meade, J. (1978). The structure and reform of direct taxation. *London: Allen and Unwin.*

Miniaci, R., Parisi, M. L., and Panteghini, P. M. (2014). Debt shifting in Europe. *International Tax and Public Finance*, 21(3):397–435.

Overesch, M. and Wamser, G. (2014). Bilateral internal debt financing and tax planning of multinational firms. *Review of Quantitative Finance and Accounting*, 42:191–209.

Panteghini, P. M. (2006). S-based taxation under default risk. *Journal of Public Economics*, 90(10):1923–1937.

Panteghini, P. M. (2007a). *Corporate Taxation in a Dynamic World*. Springer.

Panteghini, P. M. (2007b). Interest deductibility under default risk and the unfavorable tax treatment of investment costs: A simple explanation. *Economics Letters*, 96(1):1–7.

Panteghini, P. M. and Vergalli, S. (2016). Accelerated depreciation, default risk and investment decisions. *Journal of Economics*, 119:113–130.

Ramb, F. and Weichenrieder, A. J. (2005). Taxes and the financial structure of German inward FDI. *Review of World Economics*, 141:670–692.

Schenkelberg, S. (2020). The Cadbury Schweppes judgment and its implications on transfer pricing activities within Europe. *International Tax and Public Finance*, 27:1–31.

Schindler, D. and Schjelderup, G. (2016). Multinationals and income shifting by debt. *International Journal of the Economics of Business*, 23(3):1–24.

Shackleton, M. B. and Sødal, S. (2005). Smooth pasting as rate of return equalization. *Economics Letters*, 89:200–206.

Sinn, H. W. (1991). The vanishing Harberger triangle. *Journal of Public Economics*, 45:271–300.

Sinn, H. W. (2010). *Casino Capitalism.* Oxford: Oxford University Press.

Sinn, H. W. (2018). The ECB's fiscal policy. *International Tax and Public Finance*, 25(6):1404–1433.

Sørensen, P. B. (2017). Taxation and the optimal constraint on corporate debt finance: Why a comprehensive business income tax is suboptimal. *International Tax and Public Finance*, 24:731–753.

Strebulaev, I. A. (2007). Do tests of capital structure theory mean what they say? *The Journal of Finance*, 62(4):1747–1787.

U.S. Department of the Treasury (1992). Integration of the individual and corporate tax system: Taxing business income once.

Weichenrieder, A. and Klautke, T. (2008). Taxes and the efficiency costs of capital distortions. *CESifo W.P. 2431.*

Wong, K. P. (2007). The effect of uncertainty on investment timing in a real options model. *Journal of Economic Dynamics and Control*, 31(7):2152–2167.