UNIVERSITÀ
DEGLI STUDI
DI BRESCIA

DOTTORATO DI RICERCA IN
Modelli e Metodi per l'Economia e il Management
(Analytics for Economics and Management - AEM)

Settore scientifico - disciplinare
SECS-P/02 Politica Economica

CICLO XXXIII

Photovoltaic Smart Grids: prosumers' investment, uncertainty and energy communities.

CANDIDATA:
Marta Castellini
f.to digitalmente ex art. 24 D. Lgs. 82/05

RELATORE:
Prof. Sergio Vergalli
f.to digitalmente ex art. 24 D. Lgs. 82/05

CO-RELATORE:
Prof. Francesco Menoncin
f.to digitalmente ex art. 24 D. Lgs. 82/05

ANNO ACCADEMICO 2019/2020

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Acknowledgments

The matter is not the path,
but the people with whom you have the opportunity to share it.

I want to thank those who have been in this journey with me,
and who have allowed me to fulfill this achievement.
They know their names and they know how much I appreciate them.
A thank you to my family and my friends,
to my supervisor and co-supervisor, as well as to the co-authors.

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Abstract

The purpose of this thesis is to investigate some key features of the Smart Grids (SG, hereafter) topic, focusing on the agents' (i.e. prosumers) investment decisions in the photovoltaic (PV, hereafter) technology, in a context characterized by uncertainty and where the investment decision is undertaken cooperatively. The effect of allowing prosumers to exchange of energy with each other (exchange P2P, hereafter) is analyzed with a special emphasis on the demand and supply matching in exchange P2P as well as the conditions assuring its economic optimality. Discussion related to Renewable Energy Communities (REC,hereafter) is provided with the aim to understand how the findings of our models, can boost their diffusion.

The thesis is organized in four Chapters. Chapter 1 provides an overview of the topic. Chapter 2 presents a model where the prosumers' decision to invest in PV power plants is analysed, assuming that they are integrated in a SG. The main goal is to study the optimal plant size and the optimal investment threshold, in a context where exchange P2P is possible and under perfect complementarity in demand and supply of exchanged energy. To do so, the model is developed first in presence of exchange, and then in the absence of it. The results of both models are then compared, in order to understand the effect of the exchange P2P introduction. In Chapter 3 the prosumers' investment decisions in PV plant are modeled, with a focus on the set up of the exchange P2P. The novelty, compared to the model presented in Chapter 2, lies in the following elements: by removing the assumption of perfect complementarity in demand and supply of exchanged energy, the optimal size of the PV plant is determined under four different exchange scenarios. Once optimal capacities have been identified for each scenario, the discussion of the results is centered around the conditions assuring the existence of the exchange P2P in terms of prosumers' self consumption behavior, where the prosumers' aim is to maximize their joint economic pay-off. In addition to that, the first model is also extended as follows: i) the increment of the price that the prosumers are paid for the energy sold to the national grid is modeled as a Geometric Brownian Motion (GBM, hereafter) and ii) the price of the exchanged energy is the weighted average between the buying and selling price of the energy exchanged with the national grid.

Chapter 4 presents a brief discussion about the two models, focusing on the modeling framework, considerations on numerical analysis, possible policy implications and draws some overall conclusions.

Abstract (*Italian language*)

La tesi è volta ad analizzare alcune delle caratteristiche delle Smart Grid (SG, a seguire), focalizzandosi sulle decisioni di investimento degli agenti (i.e prosumers) nella tecnologia fotovoltaica (PV, a seguire), in un contesto caratterizzato da incertezza e cooperazione nella decisione di investimento. L'effetto dell'introduzione dello scambio di energia tra prosumers (scambio P2P, a seguire) viene studiato ponendo particolare enfasi sull'incontro tra domanda ed offerta di energia in scambio P2P e sulle condizioni che ne determinano l'ottimalità in termini economici. Una specifica riflessione è dedicata alle Comunità Energetiche che utilizzano energia da fonti rinnovabili (REC, a seguire), al fine di comprendere la rilevanza dei risultati in relazione alla diffusione di quest'ultime. La tesi è organizzata in quattro capitoli. Nel Capitolo 1 viene descritto lo stato dell'arte. Il Capitolo 2 presenta un modello in cui si analizza la decisione d' investimento dei prosumer, assumendo che siano integrati in una SG. L'obiettivo è quello di studiare la capacità ottima dei rispettivi impianti ed il prezzo ottimale che induce i prosumer ad investire, in un contesto dove lo scambio P2P è possibile e assumendo perfetta complementarietà nella domanda ed offerta in scambio P2P. A tale scopo, il modello viene sviluppato in presenza ed in assenza di scambio P2P. Il risultati ottenuti vengono confrontati al fine di comprendere l'effetto dell'introduzione dello scambio P2P. Nel Capitolo 3, le decisioni d' investimento dei prosumer sono modellate ponendo enfasi sulle dinamiche che influenzano lo scambio P2P. Gli elementi di novità, rispetto al modello presentato nel Capitolo 2, sono i seguenti: rimuovendo l'assunzione di perfetta complementarietà in domanda ed offerta in scambio P2P, la dimensione ottima di ciascun impianto viene calcolata assumendo l'esistenza di quattro diversi scenari di scambio P2P. Sulla base dei risultati ottenuti, la discussione si focalizza sulle condizioni che determinano l'ottimalità dello scambio P2P, espresse in termini di autoconsumo da parte dei prosumers, considerando che il loro obiettivo è quello di massimizzare il loro beneficio economico congiunto. Inoltre, l'incremento nel tempo del prezzo che i prosumer ricevono per la vendita di energia alla rete viene modellato come un Moto Geometrico Browniano ed il prezzo dell'energia scambiata è una media ponderata tra il prezzo d'acquisto ed il prezzo di vendita dell'energia scambiata con la rete nazionale. Nel Capitolo 4 viene fatta una discussione dei due modelli e presentate alcune considerazioni relative alle possibili politiche a supporto dello scambio P2P e tratte alcune brevi conclusioni finali.

Contents

Abbreviations

ABM: Arithmetic Brownian Motion

CEC: Citizen Energy Communities

Exchange P2P: Exchange of energy between prosumers

GBM: Geometric Brownian Motion

GHG: greenhouse gas

ICT: Information and Communication Technologies

N: National grid

NEMO: National Energy Market Operator

PV: Photovoltaic

REC: Renewable energy communities

RO: Real Options

SG: Smart Grids

TSO: Transmission System Operators

Chapter 1

Introduction

Prosumers' investment decisions in clean energy plants will be able to affect the new shape of electricity networks, as well as play a central role in the transition path towards decarbonization. In this context, information and communication technologies (ICT) and new organizational models for the energy management, such as Smart Grids (SG, hereafter) and exchange of energy peer-to-peer (Exchange P2P, hereafter) [1], are among the most promising instruments for the deployment of renewable energies. [2]

The EU's *Clean energy for all Europeans package* [3] has set a new legal framework for the internal energy market and particular attention has been devoted to the benefits of consumers, from both an environmental and economic perspective. Directive 2018/2001 [4] formally introduces the *renewables consumers* and sets the elements needed to ensure the spread of such status as much as

[1] Exchange P2P: "peer-to-peer trading of renewable energy means the sale of renewable energy between market participants by means of a contract with pre-determined conditions governing the automated execution and settlement of the transaction, either directly between market participants or indirectly through a certified third-party market participant, such as an aggregator. The right to conduct peer-to-peer trading shall be without prejudice to the rights and obligations of the parties involved as final customers, producers, suppliers or aggregators"(EU, 2018).

[2] Zondag and Harmelink (2017) state that the diffusion of renewable energy will require the improvement of the physical network flexibility to accommodate the electricity produced with renewables and "SG technologies present a possible solution for this challenge". EU (2018) formally request Member States to ensure renewables self-consumers, individually or through aggregator, to be entitled, among other things, to sell the excess of their energy production via peer to peer trading agreements.

[3] This policy framework sets the new energy union strategy with eight legislative acts, where the main pillars are: energy performance in buildings, renewable energy, energy efficiency, governance regulation, electricity market design. The recast of EU Directive 2018/2001 aims "at keeping the EU a global leader in renewables" and sets new binding targets on renewable energy. Directive 2019/944 (EU, 2019) focuses on the new common rules for the internal market for electricity, where the "consumer is put at the center of the clean energy transition" and new rules are defined with the aim to enable their active participation in this process.

[4] In the bibliography as EU (2018). Source: `https://eur-lex.europa.eu/legal-content/en/TXT/?uri=CELEX%3A32018L2001`.

possible.
Even though the figure of prosumers (as agents that produce and consume energy) was widespread in the last years, the most promising instruments in this context to achieve the EU decarbonization goals, such as SG, together with exchange P2P, as well new prosumers' organizational structures like Renewable Energy Communities (REC, hereafter)[5] are still far away to be deployed. In the last decades, researchers have focused their attention on these topics, deepening several aspects and with different methodologies and approaches, but still many facets of it need further analysis. Understanding the interaction and the inter dependencies between green energy technologies' diffusion, SG, exchange P2P and REC may provide important insights on key elements to boost decarbonization process.

1 The main framework

The SG has been identified by researchers and policymakers as one of the best instruments to face the new challenges of the electricity network in a *decarbonization oriented era*.[6] The recurring imbalances [7] of the electricity network "combined with the needs to reduce GHG emissions, increase the share of renewable energy sources in the power generation mix as well as energy efficiency (Moretti et al., 2017), make the deployment of this new energy network's structure a key goal for the regulators. Apart from the technical benefits that the deployment of the SG might imply for physical energy infrastructure [8], the economic ones are as important as the former.[9] The growing number of small sized renewable energy plants owned by the consumers, combined with the introduction of ICT, are expected to deeply change the role of these agents in the energy market. Indeed, SG represents the trigger for truly active participation of the energy consumers, in a framework where, only few years ago, they were perceived as fairly passive actors. With particular reference to the PV tech-

[5]Hunkin and Krell (2018) defines renewable energy communities as an organizational framework that involves "generation of energy from renewable resources and technologies, which are partly or wholly owned by local communities".

[6]See, among other, Zondag and Harmelink (2017).

[7]i.e. schedule or energy volume deviations. In particular, the Commission Regulation (EU) No 543/2013 (EU, 2013) states that "even after careful planning producers, suppliers and traders may find themselves out of balance and be exposed to transmission system operators (TSO) balancing and settlement regime. In order to optimally mitigate imbalance risk market participants need accurate, clear and timely information about balancing markets. TSOs should provide such information in a comparable format across borders including details about the reserves they have contracted, prices paid and volumes activated for balancing purposes".

[8]improvement of reliability and security of energy distribution, shift of the peak load, reduction of contamination and losses inherent to the transmission process as well as lowering of the probability of network congestion (Moretti et al. (2017); Cardenas et al. (2014)).

[9]SG technical aspects and implications are always central elements in the discussion related to electricity network innovation process. However, shifting to an SG framework will also have economic implications, such as the arising of the new behaviors and opportunities for the energy consumers as well as the need of a well defined set of economic policies to manage the investments required for its set up.

nology, in the last years, consumers are no longer considered in this way but also as active producers, i.e *prosumers*. As underlined by Cardenas et al. (2014) [10], among others, prosumers' investments decisions in renewables, as well as their self-consumption behavior, are now key drivers that regulators must take into account when designing specific policy measures aimed at boosting green technologies' diffusion. On the other hand, to do that, "policymakers and planners need knowledge about how prosumers could be integrated effectively and efficiently into competitive electricity markets" (Parag and Sovacool, 2016). Indeed, as Parag and Sovacool (2016) clearly stated in their work, it must be also acknowledged the importance of a "proper market design for the prosumer era". [11]

From the side of the prosumers' perspective, one of the most relevant features of the SG is the possibility of interacting instantaneously with the electricity grid. Thanks to the introduction of ICT in the overall energy framework, "the grid can send signals (through prices) to the agents, which can respond to those signals and obtain monetary gain as a counterpart" (Bertolini et al., 2018). The prosumers' involvement in the energy market increases and, in a certain way, induces those that are yet to own a PV plant to invest, attracted by future potential net gains, thanks to the chance of interacting directly with the energy market as energy suppliers. [12]

So far, prosumers that sell energy to the national grid are basically still price takers, thus their revenues are strictly related to the price agreed with it[13]. It is widely recognized by scientific literature (especially the one in the energy branch studying uncertainty), that the volatility of the energy price that the prosumers receive for the energy sold to the national grid, is among the main elements affecting the prosumers' investment decisions. With reference in particular to the Real Options (RO) field, it is well known that the higher the uncertainty the higher the value of the option to postpone the investment.

Prosumers' investments decisions in renewable energy have been analyzed by researchers from different perspectives and with multidisciplinary approaches.[14] The same path is now undertaken by the research on the exchange P2P topic, as fundamental part of the SG landscape (Hernández-Callejo (2019)) as well as in the path towards REC diffusion.

In its recent directives, EU stated that "a well-functioning electricity market designs is the key factor enabling the uptake of renewable energy" EU (2019). As clearly already stressed in EU (2018), national governments of the EU "shall ensure the renewables self-consumers, individually of thought aggregators, are

[10]"consumer participation is going to play a key role in the near future as it requires developing a new business model with the inclusion of self-generation and selling-back of excess capacity to the utility company" (Cardenas et al., 2014).

[11]see also Caramizaru and Uihlein (2020), Zafar et al. (2018), Bellekom et al. (2016) and Espe et al. (2018).

[12]see also Caramizaru and Uihlein (2020).

[13]It is important to remark as well the relevance in the last 15 years of national policies aimed at boosting investments in renewables, such as feed-in tariffs and support schemes.

[14]Among other see Mondol et al. (2009), Pillai et al. (2014), Cucchiella et al. (2016), Ioannou et al. (2017), Hartner et al. (2017), Guerrero-Liquet et al. (2018)

entitled to generate renewable energy, including for their own consumption, store and sell their excess production of renewable electricity, including through renewables power purchase agreements, electricity suppliers and peer-to-peer trading arrangements". The main reasons of the attention on the exchange P2P topic is due to its potential to boost investments in renewable energy, the consumption of green energy as well as to provide alternative solutions for the management of electricity network. In addition to that, understanding exchange P2P dynamics represents a key point to in REC context.[15]
Exchange P2P possibility has been deepened combining engineering, mathematical, financial and economic knowledge to assess the impact of P2P exchange possibility introduction on i) the prosumers' behavior in energy consumption and supply, ii) to define the required features of the potentials P2P energy markets, and iii) to understand possible structures and dynamics of the prosumers' REC.[16] Empirical evidence provided by pilot projects suggests some additional insights on these topics.[17] In particular, Zhang et al. (2017) underline that in many pilot projects much attention is dedicated to the development of business models while the possibility of introducing those models to smaller-scale local energy market is ignored. Hahnel et al. (2020) provide some important considerations on by analyzing homeowners' trading preferences in simulated P2P electricity trading scenarios. Insights on the community electricity prices and state of charge of private energy storage were identified on the basis of a sample of 301 homeowners.

[15]Among other, one of the most interesting work is the one of Hahnel et al. (2020). The authors try to address this research gap analyzing homeowners? trading decisions in simulated P2P electricity trading scenarios, where agents were assumed to be organized in peer-to-peer energy communities.

[16]for the sake of brevity, see literature on exchange P2P in following Chapters 2 and 3.

[17]With reference in particular to exchange P2P, among others, see Zhang et al. (2017),Ecker et al. (2018), Morstyn et al. (2018) and Zhang et al. (2018). While, for the Energy communities (REC included), see van Summeren et al. (2020), Caramizaru and Uihlein (2020), Vernay and Sebi (2020), Ruggiero et al. (2021).

2 Research challenges

On the economics' perspective side, the most relevant challenges that the researchers may face in the SG field could be related to the topics of: i) uncertainty, ii) cooperation in investment's decisions, iii) demand and supply matching in exchange P2P, as well as its iv) technical and economical optimality, v) the market design for new actors' diffusion like energy communities, which in turn also comprehends vi) the study of the mechanism and dynamics of the price paid/received for the energy exchanged P2P, and finally vii) the development of specific policies aimed at regulating and stimulating the SG, exchange P2P and the energy communities, with particular attention to the REC ones.[18]

The effect of uncertainty in SG and prosumers' investment decision in renewable energy has been widely studied by the RO literature.[19] However, there are no scientific publications investigating its influence in the contexts of exchange P2P as well as the REC. Indeed, both may be characterized by different types of uncertainty, from the regulatory one[20], to the one related to the renewable energy's production or the prices' dynamics, for instance.

With reference instead to cooperation in investment decision, game theory, together with behavioral economics, have been widely applied with the aim to understand exchange P2P dynamics as well as energy communities' networks interactions.[21] However, much of the research effort focuses on the study of networks and coalitions dynamics.

On the micro level side, two challenging matters are: the demand and supply matching in exchange P2P and the discussion on the price of the exchanged energy. Starting from the first, the features of the prosumers' loads curves have been studied especially by energy engineers. The most relevant topics identified by the researches, with respect to the thesis' context, are the issues related to the energy networks' management, the development of specific algorithms aimed to allow and assess the energy exchange as well as the ICT platform to facilitate the virtual exchange between members of the energy communities. It is important also to underline that specific attention has been dedicated to the energy storage, as key element of this new framework. So far, most of the literature[22], has recognized such device as an essential part of the SG, as well a exchange P2P and REC, structure. It is true that the prosumers' behavior in exchange P2P may be deeply affected by its presence or absence. [23]

[18] See among others also D'Alpaos and Andreolli (2020) and Caramizaru and Uihlein (2020).

[19] see Kumbaroğlu et al. (2008), Boomsma et al. (2012), Feng et al. (2016), Schachter and Mancarella (2016), Schachter et al. (2016), Bertolini et al. (2018)

[20] see also Frieden et al. (2019),Inês et al. (2020)

[21] Among others see Brosch et al. (2014),Zhang et al. (2014), Zhang et al. (2015), Celik et al. (2017),Motalleb and Ghorbani (2017), Mei et al. (2019), Tarditi et al. (2020)

[22] Among others, see Alam et al. (2013),Mandelli et al. (2016), Bakke et al. (2016),Gonzalez-Romera et al. (2019),Hahnel et al. (2020)

[23] With reference in particular to the two models presented in Chapters 2 and 3, in both no storage possibility assumption is set, since both focuses on a context where prosumers are conceived as households and so far, storages are still far way to be adopted by this kind of agents. Of course, a possible extension of this current work could be the development of a model in a similar framework but with storage possibility.

With reference instead to the economic analysis, several optimization techniques have been applied with the aim to model this topic.[24] However, to the best of the current knowledge, the traditional economic concept of demand and supply matching, as well as the identification of a price for the energy exchanged on the basis of institutional economic market rules, are yet to be discussed by researchers under this framework. Another interesting point of discussion is about the prosumers' preferences with respect to the exchange P2P and the purchase of energy from traditional energy providers. Several works[25] analyze such topic exploiting algorithm aimed at simulating possible exchange P2P scenarios, however there is still a lack of studies on the preferences of prosumers under an economic perspective. [26] Surprisingly few models address such "prosumers' problem" starting from the traditional utility function.[27] Finally, with reference to the price of the energy exchanged P2P, this issue remains still a topic addressed only marginally by the research.[28]. Further discussion on this is presented in Chapter 4.

3 Contribution of the work and relevance of the findings in the field of the energy communities

The two models presented in the following Chapters 2 and 3 aim at complementing the research strand of SG and exchange P2P. Both extends the work of Bertolini et al. (2018). Figure 1 shows the context and the overall framework of the two models.

In Chapter 2, the model's aim is to study the prosumers' investment decision in PV power plants, assuming that they are integrated in a SG. The main goal is to identify the optimal plant size (PV plant) and the optimal investment threshold, in a context where exchange P2P is possible and under perfect complementarity in demand and supply in exchange P2P. To do so, the model is developed first in presence of exchange, and then in the absence of it. The results of both models are then compared, in order to understand the effect of the introduction of exchange P2P . This first approach allows to understand the impact of the exchange of energy among prosumers in terms of PV plant sizing, which increases with its introduction, while investments' decision in PV plants are boosted as prosumers increase the energy self consumed and exchanged P2P.

[24]Zafar et al. (2018) provide a brief overview on most common optimization techniques, starting from integer linear, mixed linear integer and non linear programming, particle swarm optimization and genetic algorithm. Angelidakis and Chalkiadakis (2015) use Factored Markov Decision Processes (FMDP).

[25]Among others Hahnel et al. (2020), Bellekom et al. (2016), Zafar et al. (2018) and Zhou et al. (2018).

[26]Especially in the model presented in Chapter 3, we try to provide also some insights on the basis of numerical analysis' outcomes.

[27]Sun et al. (2013) investigate, under an economic perspective, the prosumers' problem taking into account their the utility function. Zhang et al. (2014) developed an algorithm for demand-side management based on social welfare maximization and using a dynamic game.

[28]see Luo et al. (2014), Ghosh et al. (2018)

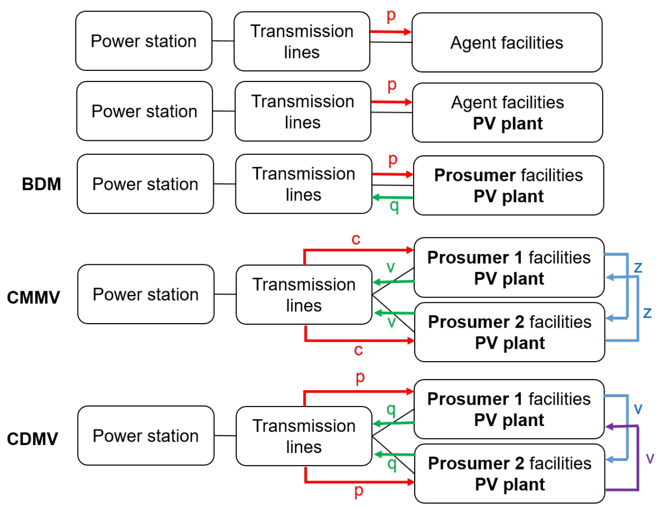

Figure 1: *Context and overall framework*, where the acronym BDM refers to Bertolini et al. (2018), CMMV to the model presented in Chapter 2 and CDMV to the one of Chapter 3.

The model presented in Chapter 3 focuses instead on the set up of the exchange P2P. The novelty, compared to the previous one, lies in the following elements: by removing the assumption of perfect complementarity in demand and supply of exchanged energy, the optimal size of the PV plant is determined under four different exchange scenarios (excess demand, excess supply and two non complementarity specific cases). Once optimal capacities have been identified for all of them, the discussion of the results is centered around the conditions assuring the existence of the exchange P2P in terms of prosumers' self consumption behavior, where the prosumers' aim is to maximize their joint economic pay-off. In addition to that, the first model is also extended as follows: i) the increment of the price that the prosumers are paid for the energy sold to the national grid is modeled as a Geometric Brownian Motion (GBM, hereafter) and ii) the price of the exchanged energy is the weighted average between the buying and selling price of the energy exchanged with the national grid. The outcomes of the numerical part allow to compare the optimal prosumers' profiles identified with the model with real load curves. This perspective allows to provide some insights on the real feasibility of the exchange P2P framework.

3.1 Smart grids, exchange P2P and renewable energy communities.

Understanding the inter-linkages and interdependencies between SG and exchange P2P can provide relevant insights in the field of energy communities, and specifically in the case of REC. Indeed, "the recent growth of decentralized renewable energy technology has made direct participation in energy production and management more accessible" CEER (2019), leading to the arising of energy communities under different forms.
For the sake of clarity, it is important to underline that there are several definitions of energy communities and different types as well. Among others, Caramizaru and Uihlein (2020) provide a general definition on the basis of Roberts et al. (2014). Such new structures, expected to be active players in the energy markets of the future, are intended as "a way to organize collective energy actions around open and, democratic participation and governance and the provision of benefits for the members of the local community". Furthermore, EU Directives 2018/2001 and 2019/944 [29] identify two main types of energy communities: the first is *citizen energy communities* (CEC) [30], while the second is *renewable energy communities* (REC). [31]
These two categories are similar but have also some relevant differences.[32] The most important differences, with reference to the context of this work, are:

- Technology: CEC can be renewable and fossil fuel based (i.e technology neutral), while REC are limited to renewable energy technology.
- Membership: participation to REC is limited those private undertakings whose participation does not constitute primary commercial or professional activity.
- Geography: CEC have no bound in geographical location, while for REC the members "must be located in the proximity of the renewable energy projects that are owned and developed by the REC" (CEER, 2019).

[29] see EU (2018) and EU (2019) respectively.

[30] Citizen energy communities are formally defined by EU (2019) as entities: i) "based on voluntary and open participation and effectively controlled by members or shareholders that are natural persons, local authorities, including municipalities, or small enterprises"; ii) the "primary purpose is to provide environmental, economic or social community benefits to its members or shareholders or to the local areas where it operates rather than to generate financial profits"; iii) they "may engage in generation, including from renewable sources, distribution, supply, consumption, aggregation, energy storage, energy efficiency services or charging services for electric vehicles or provide other energy services to its members or shareholders".

[31] The definition of renewable energy communities (REC) is provided instead by EU (2018). According to it, such entities are entitled to: i) "produce, consume, store and sell renewable energy, including through renewables power purchase agreements"; ii) "share, within the renewable energy community, renewable energy that is produced by the production units owned by that renewable energy community, and maintaining the rights and obligations of the renewable energy community members as customers"; iii) "access all suitable energy markets both directly or through aggregation in a non-discriminatory manner".

[32] CEER (2019), Caramizaru and Uihlein (2020) and Frieden et al. (2020) list their common elements and main differences in details.

It is important to remark that the classification provided by these two EU Directives does not exclude, so far, the use of the term "energy community" for those collective frameworks that do not comply exactly with it. Indeed, many pilot projects do not correspond entirely to one of the two categories.[33] Such entities could face a more complex path towards their legal recognition according to EU rules in the future, or may open a further debate on additional recognizable types of energy communities in the EU law.[34]
The case of REC represents the one more in line with the framework described by the models presented in following Chapters 2 and 3. In particular, EU (2018) stresses that the participation of the final customers in the REC, such as households ones, must be assured by the Members States. To achieve such goal, policymakers must be aware of the main dynamics, drivers and determinants related to the participation of such specific category to the REC.
The study of "household prosumers" has been widely addressed by the researchers in the context of photovoltaic energy. However, further research effort must be done to understand the effect of allowing such type of actors to organize themselves in REC, or to take part in existing ones, which in turns imply the exchange of energy within it, in a framework characterized by uncertainty.
Even though the prosumers' participation to a REC is not the same of allowing only two of them to exchange energy one with each other, the two models presented in following chapters are an attempt to understand the effect of the exchange P2P introduction on the prosumers' investment decisions, PV plant sizing as well as self-consumption and exchange P2P preferences.
The presented approaches, as well as the related findings, can be useful to quantify in detail the effects and the drivers of the prosumers' participation in REC under several perspectives.
With reference to energy savings, especially in the case of households, it is widely recognized that investments in PV technology are mainly driven by the opportunity that such agents have to save on energy costs through the achievement of the highest possible level of self-consumption.
However, if prosumers take into account the economic profitability concept as a whole, such self-consumption decision must be counterbalanced by the potential return that might arise from the sale of energy to the national grid.
Furthermore, the decision becomes more complex if the prosumers could have also the possibility to exchange energy one with each other.
As in the model of Chapter 2, those agents that are yet to invest in a PV plant and want to do that with the aim of self-consumption, selling to the national grid and exchange energy P2P, face a decision similar to those that are contemplating the idea to build a REC.
Cooperation in investment decision is required to allow exchange of energy peer-

[33] Caramizaru and Uihlein (2020) and CEER (2019)

[34] Also Frieden et al. (2020) remarks that "several case studies identified would not qualify as CEC or REC under the Clean Energy Package (CEP) framework because they involve an energy company or benefit from exemptions as pilot projects. This does not necessarily mean that such projects will be prohibited in a post-CEP framework but instead will simply not be able to explicitly claim the rights of energy communities."

to-peer and the use of a RO framework provides information about the value of flexibility, as well as the identification of the price threshold that triggers the investment decision.
To the best of current knowledge, there are no studies identifying at micro level the effect to exchange P2P introduction under uncertainty and in presence of cooperation in investment decision.
Indeed, this work tries to contribute to a relevant matter also raised by Caramizaru and Uihlein (2020). The authors underline that "more research is necessary to clarify and quantify the potential benefits that the energy communities could provide for supporting the EU's climate and energy goals" and that the energy communities' "long-term success will depend on their ability to operate energy networks in a cost efficient way, ensuring benefits for all customer and the whole energy system".
Indeed, the model that will be presented in Chapter 3 focuses more on the differences in prosumers' load profiles and investigates the self-consumption behaviors assuring the effective set up of the exchange P2P.
Under this framework, the outcomes of the model are able to provide information on the volume of exchanged energy among prosumers, as well as the level of self-consumption assuring the exchange P2P optimal set up.
These elements are among the key ones of the energy communities and still there are few studies able to provide insights on them. Those that focuses on the exchange dynamics within the energy community, exploit mainly algorithms and are not developed according to traditional economic modeling approach.

Chapter 2

Photovoltaic Smart Grids in the prosumers investment decisions: a real option model

Remarks The model presented in this Chapter is published by the *Journal of Economic Dynamics and Control* as Castellini et al. (2020)[*] coauthored with Francesco Menoncin[†], Michele Moretto[‡] and Sergio Vergalli[§].
A working paper version is available as FEEM Working Paper No. 28.2019, published by Fondazione Eni Enrico Mattei.[¶]

The model in brief This model provides a theoretical real option framework with the aim to model prosumers' decision to invest in photovoltaic power plants, assuming that they are integrated in a SG. Our main focus is to study the optimal plant size and the optimal investment threshold, in a context where exchange of energy among prosumers is possible. The model was calibrated and tested with data from the Northern Italy energy market. Our findings show that the possibility of selling energy between prosumers, via the SG, increases investment values. This opportunity encourages prosumers to invest in a larger plant compared with the case without exchange possibility and that there is a positive relation between optimal size and (optimal) investment timing. The effect of uncertainty is in line with the literature, showing increasing value to defer with volatility. Our comparative statics stress the need for policies to push the PV efficiency.

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[*] `https://doi.org/10.1016/j.jedc.2020.103988`

[†] Department of Economics and Management, Università degli Studi di Brescia, Italy.

[‡] Department of Economics and Management, Università degli Studi di Padova, Italy.

[§] Department of Economics and Management, Università degli Studi di Brescia, Italy. FEEM - Fondazione Eni Enrico Mattei, Milan, Italy.

[¶] `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3522990`

1 Introduction

In recent years climate change has become an important issue in the economic debate. The latest IPCC[1] report (IPCC, 2019) underlines how important is to control temperature levels by reducing or limiting CO2 emissions. This could avoid the occurrence of irreversible effects. Some mitigation paths are characterized by the reduction in energy demand, the decarbonization of electricity and other fuels, and the electrification of the final use of energy. In this line, the European Union 2030 climate and energy policy has set three macro targets: (i) the reduction of 40% in greenhouse gas emissions (with respect to 1990 levels), (ii) 32% of energy coming from renewable sources, and (iii) an improvement in energy efficiency of 32.5%. In addition to that the European Union long-term strategy aims to reach a climate neutral economy within 2050.[2]

Such policies require strong deployment of low carbon technologies as well as an adequate efficient environment.[3] A central role is played by the definition of new emerging power system, required to be decarbonized, decentralized, and digitized. Decarbonization is also related to the diffusion of renewable energy plants. Instead, decentralization refers to the growing role of new many electricity producers, with small-scale, decentralized, and intermittent periods of overproduction of electricity, mostly photovoltaic (PV, hereafter). Finally, digitization implies the innovation of the power system, a concept that has also been associated in the last years with the Smart Grids (SGs, hereafter) that are "robust, self-healing networks that allow bidirectional propagation of energy and information within the utility grid".[4] This last element plays an important role, since technological development enables also an affordable energy transition.

In this respect, the continuous integration of Distributed Energy Resources (DERs, hereafter), (Sousa et al. (2019); Bussar et al. (2016); Zhang et al. (2018)),[5] along with the advance in Information and Communication Technology (ICT) devices (Saad al sumaiti et al., 2014), are inducing a transformation of a share of electricity consumers who **pro**duce and con**sume** and share energy with other grid users. Such users are called "**prosumers**" (Luo et al. (2014); Sommerfeldt and Madani (2017); Espe et al. (2018); Zafar et al. (2018)).

Smart grids actually introduce the possibility of adopting new behaviors: while traditional consumers are characterized by a passive behavior in buying and receiving energy from the grid, prosumers are proactive in managing their consumption and production (Zafar et al., 2018). Indeed, they can reduce their energy consumption costs, by self consuming the energy produced by their PV plants (Luthander et al. (2015); Masson et al. (2016)). In addition to that, Espe et al. (2018) remark the importance of prosumers participation to the smart grid

[1]Intergovernmental Panel on Climate Change

[2]*Source*: `https://ec.europa.eu/clima/policies/strategies_en`

[3]Some laws of 2030 package refer in particular to the energy market, from the revision of the Renewables Directive to the update of the Energy Efficiency Directive(also called Energy Performance of buildings Directive).

[4]Smart Grid definition according EU. *Source* `https://ec.europa.eu/energy/en/topics/market-and-consumers/smart-grids-and-meters`

[5]e.g., from rooftop solar panels, storage and control devices

as critical for both the sustainability and the long term efficiency of the energy sharing process.

Furthermore, SGs allow instantaneous interactions between agents and the grid: depending on its needs, the grid can send signals (through prices) to the agents, and agents can respond to those signals and obtain monetary gains as a counterpart. These two characteristics (self-consumption and possible return energy exchange with national grid) can add flexibility that, in turn, increases the value of the investment (Bertolini et al., 2018). A third important characteristic, that depends on the development of new technologies and digitalization, is the possibility to exchange energy also between agents (InterregEU (2018); Luo et al. (2014); Alam et al. (2017); Zafar et al. (2018); Zhang et al. (2018)), in a Peer-to-Peer (P2P, hereafter) energy trading or in developing energy communities (Sousa et al., 2019).

P2P energy trading represents "direct energy trading between peers, where energy from small-scale DERs in dwellings, offices, factories, etc, is traded among local energy prosumers and consumers" (Alam et al. (2017); Zhang et al. (2018)). Energy communities can involve groups of citizens, social entrepreneurs, public authorities and community organizations participating directly in the energy transition by jointly investing in, producing, selling and distributing renewable energy. This can introduce further flexibility to the investment that could add value, depending on the adoption costs of the new technology and the shape of the load (demand) electricity curve of agents. Therefore, it is interesting to study whether this additional flexibility may have value, how it could affect the investment decisions, and whether it may be supported by data.

In this paper, we examine how the connection to the SG and the possibility to exchange energy among agents, may increase the investment value in a PV plant (i.e., investment profitability) and influence decisions regarding the optimal size of the plant. We model the investment decision of two small (price-taker) end-user households. Each agent is a prosumer that have the non-excluding possibility to: (i) self-consume its energy production, (ii) exchange energy with national grid, and (iii) exchange energy with the other agent.

Due to the irreversibility and high uncertainty over the demand evolution, the technological advances, and the ever changing regulatory environment (Schachter and Mancarella (2015); Schachter and Mancarella (2016); Cambini et al. (2016)), we implement a real option model to determine the optimal size and the overall investment value of a PV system characterized by the features previously described.

Because of the many opportunities, SGs may generate managerial flexibility which prosumers can exercise optimally when deciding to invest. This flexibility gives them the option to decide strategically the optimal production/consumption energy pattern and can significantly contribute to energy saving and hedging the investment risk. To capture the value of managerial flexibility, we calibrate and test our model using data from the Italian electricity market.

In our work we combine decisions on irreversible investments under uncertainty with connections to an SG and with possibility of exchange between prosumers.

This paper contributes to both the SG and real option literature. The first strand studies technologies (Kriett and Salani, 2012), prosumers' behavior in energy markets (Ottesen et al. (2016), Bayod-Rújula et al. (2017)), demand-side management (Oren (2001), Salpakari and Lund (2016)), demand-response (Schachter and Mancarella (2016), Sezgen et al. (2007)), P2P, and energy community.[6]

On the side of the real option literature, we complement the studies about the energy sector (Kozlova (2017), Ceseña et al. (2013)) and in PV plants (Martinez-Cesena et al. (2013),Tian et al. (2017)) with a novel application in which we introduce prosumers sharing an initial investment and exchanging energy in domestic PV systems. Among these contributions, the closest to ours are: Bertolini et al. (2018) where the size of the optimal plant is identified through a real options analysis; Luo et al. (2014), in which exchange P2P is deepen in a Microgrid context under the assumption of storage possibility and its dynamics is simulated to understand the impact of cooperative energy trading on renewable energy utilization; Zhang et al. (2018) who investigates the feasibility of P2P energy trading with flexible demand and focusing on the energy exchange between the Microgrid and the utility grid; Gonzalez-Romera et al. (2019) where the case of two households prosumers is investigated, even though the focus is on energy exchange minimization instead of energy cost. In this context the novelties of our paper are: (i) the study of the the value of flexibility introduced by P2P energy community, and (ii) the use of a real option approach.

Our findings show that at current prices, the introduction of the possibility of selling energy between agents encourages investment in larger plants. Moreover, both the exchange option and the investment deferral option have always a positive value. In addition to that, our results show a positive relation between plant optimal size and optimal investment timing (i.e., the greater the plant optimal size, the greater the investment deferral). About uncertainty, increasing volatility rises the option value to defer and, in turn, increases the investment value. At the same time, with high volatility, the PV plant is built for selling and not for exchanging energy. Thus, the energy community diffusion can be effectively pushed by stabilizing the energy prices', and reducing their volatility.

[6] A wide review of current literature in these topics is provided by Espe et al. (2018), focusing on prosumers community group and prosumers relationship, and Sousa et al. (2019) which deepen the aspects of the P2P energy market as *consumer-centric electricity market.* In both works relevant attention is drawn on the key role played by information and communication technology with two different perspectives: on the economics side, related to the definition of market structure and on the technological one, with reference to the concepts of the SG and Microgrid. Prosumers' behaviors in self consumption, exchange and investment choices are investigated through several optimization techniques (Zafar et al. (2018), Angelidakis and Chalkiadakis (2015), Razzaq et al. (2016)) and most of them focus on cost minimization (Liu et al., 2018). A different approach is provided instead by Gonzalez-Romera et al. (2019), in which the prosumers' benefit is determined by the minimizing of the exchange of energy instead of the energy cost and by Ghosh et al. (2018), where the price of exchanged P2P energy is defined with the aim to minimize the consumption of conventional energy, even though prosumers' aim is to minimize their own payoffs.

The rest of the paper is organized as follows. Section 2 describes the model set-up. Section 3 introduces the calibration of the parameters, and Section 4 provides our main results and comparative statics. Section 5 concludes. Some technicalities are left to the appendices.

2 The model

In this model we investigate the case of two prosumers (i, j)[7], currently connected to a national grid under a flat contract. Each agent has to decide whether and when to invest in a PV plant to cover part of its energy demand. Thus, the prosumer i has to decide the size of its plant α_i. Each prosumer may also decide to build a SG to connect its plant to the second prosumer and to the energy market, with the possibility of selling the energy produced to the other prosumer at price z_t and to the national provider at price v_t, where the latter is assumed to be stochastic. In addition to that, prosumers can also decide to buy energy directly from the national grid at a constant price c.[8] Before analyzing the investment decision, we introduce some assumptions.

2.1 Main assumptions

Assumption 1 (prosumer's energy demand). *The per period energy demand of prosumer i is constant over time, normalized to 1, and it can be covered as follows:*

$$\begin{aligned} 1 = & \textit{Energy produced and self-consumed} \\ & + \textit{Energy purchased from the other prosumer} \\ & + \textit{Energy purchased from the national grid.} \end{aligned}$$

Now, we use the following notation:

- α_i and α_j are the energy produced per unit of time t (henceforth, the size of the PV system) by prosumer i and j respectively,
- $\xi_i \in [0, \bar{\xi}_i]$ is the proportion of α_i destined to self-consumption, and $\bar{\xi}_i$ is the maximum self consumption and exchange levels that are reasonably achieved with the photovoltaic system,[9]

[7] In this framework prosumers are meant to behave as households.

[8] The price c represents the cost each prosumer pays to buy energy from an energy provider operating in the national grid. With this price we refer to the one set by a long term agreement between the two. For the sake of brevity, we will refer to the purchase from such energy provider as purchase from the national grid.

[9] Since the prosumer's self consumption depends on the load profile, the location and the renewable energy technology applied, in general it can be represented as a weakly concave function of α_i, i.e. $\xi_i(0)$, $\xi_i'(\alpha_i) > 0$ and $\xi_i''(\alpha_i) \leq 0$. However, as many technical reports show that this quota does not exceed 30% - 50% of production, the assumption of a linear function for $\xi_i(\alpha_i)$, with an upper bound $\bar{\xi}_i$, is non-restrictive and reasonably acceptable in real world situations.

- $\gamma_i \in [0, \bar{\gamma_i}]$ is the quantity supplied by agent j that prosumer i wants to buy; for what concerns $\bar{\gamma_i}$ we can apply the same argument used for $\bar{\xi_i}$; the energy shared between the two prosumers is accordingly $\gamma_i (1 - \xi_j) \alpha_j$,
- $b_i \in [\underline{b}_i, 1]$ is the amount of energy that prosumer i purchases from the national provider, and $\underline{b}_i$ is the night-time energy demand that necessarily needs to be purchased from the national grid.[10]

Given this notation, we can specify the equation in Assumption 1 as:

$$1 = \xi_i \alpha_i + \gamma_i (1 - \xi_j) \alpha_j + b_i, \qquad i \neq j, \tag{1}$$

Assumption 2 (prosumers' behavior in exchange of energy choices). *The two prosumers are assumed to be asymmetric in load curves, meaning that they behave complementarily in demand and supply of exchanged energy. Moreover, the demand of energy of prosumer i in exchange process is rationed by the supply of prosumer j.*

To better describe this assumption, we show in Figure 2.1 an example of daily load and production curves for two perfect asymmetric prosumers that share the same total energy demand and production (i.e. they are perfectly symmetric in the exchange). In the lower part of Figure 2.1, we show, for each agent, how the load curve is satisfied and how the exchange works between prosumers.

In detail, for prosumer i, we show the PV production represented by α_i, the self consumption quota, $\xi_i \alpha_i$, the energy shared between the two prosumers, $\gamma_i (1 - \xi_j) \alpha_j$, the quota bought from the national provider, b_i, and finally the excess of production that can be sold to the national grid. Since the two agents are perfectly asymmetric in load curves, they are able to exchange energy and, moreover, self consumption and exchange counterbalance sales to national grid.

[10] $\underline{b}_i$ corresponds to the interval in which the PV plant is not producing. In general, the value for $\underline{b}_i$ depends on the prosumer's daily load patterns, and can be positively affected (decrease) by PV installation Luthander et al. (2015).

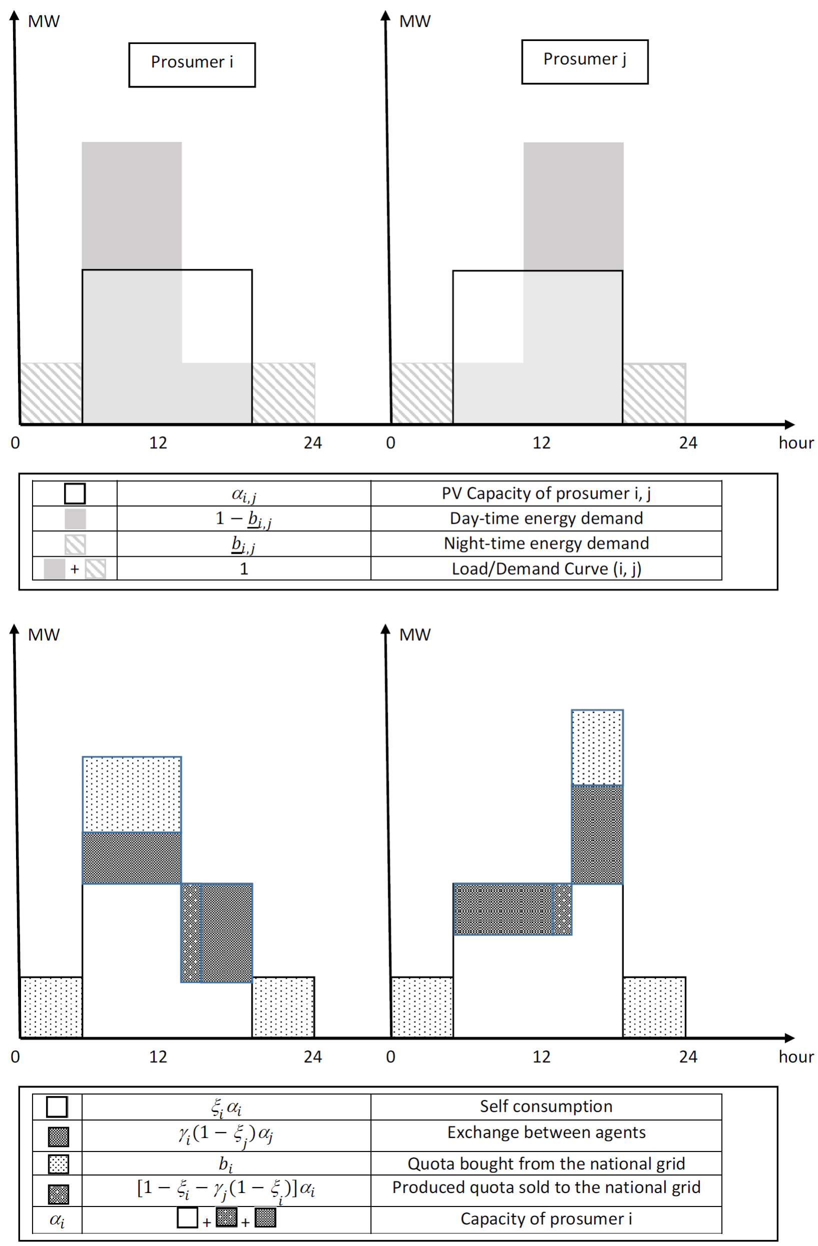

Figure 2.1: Daily load and production curves

Assumption 3 **(storage is not possible).** *According to De Sisternes et al. (2016), ESG (2016), and ESG (2018), storage technologies are still far from being cost effective, thus we assume that no battery is included in the investment.*

This choice allows to include a decrease in prosumers' managerial flexibility, since energy must be used as long as it is produced. From this follows $b_i \in [\underline{b}_i; 1]$ *.*

Assumption 4 (**investment cost function**). *Prosumers cooperate in investment decision*[11]*, meaning that at time* $t = \tau$*, where* $\tau \in [0, +\infty)$*, the investment cost function of the prosumers is*

$$I(\alpha_i, \alpha_j) := P + \frac{K}{2}\left(\alpha_i^2 + \alpha_j^2\right) + H(\alpha_i + \alpha_j), \tag{2}$$

in which P *is a fixed cost,* $\frac{K}{2}\left(\alpha_i^2 + \alpha_j^2\right)$ *is the sum of the plant costs, and* $H(\alpha_i + \alpha_j) < 0$ *is the saving gained thanks to the cooperation in investment decision.*[12]

Current literature on exchange of energy between prosumers is cast in a framework where exchange occurs only virtually (P2P Cloud), therefore we assume that P represents the sunk cost the prosumers have to pay to access the P2P energy community through the SG. The investment cost function $I(\alpha_i, \alpha_j)$ is assumed to be increasing and convex.[13]

Prosumers receive information on selling prices at the beginning of each time interval dt and make decisions on how much of the produced energy to self consume and how much to sell. There is only one hourly local spot market in which prosumers observe selling prices and instantaneously decide either to sell the production or not. Each prosumer's aim to minimize energy costs, thus investment decision depends on their energy demands and the ratio between the buying and selling prices of energy.

Assumption 5 (**energy selling price**). *We define with* dv_t *the price increment overtime of the stochastic energy selling price* (v_t) *which follows an Arithmetic Brownian Motion (ABM)*[14]

$$dv_t = \theta dt + \sigma dW_t, \tag{3}$$

[11] Apart from the one of building a PV plant, the prosumers' joint-investment decision implies several different simultaneous choices, that are: i) prosumers commit to each other to exchange energy in a way such that perfect complementary in exchange P2P is always assured, and ii) they agree on a specific price for the exchanged energy.

[12] MIT (2015) analyses the decline of PV system prices in US from 2004 to 2014 at residential (Systems up to 10 kilowatts –kW) and commercial (Systems ranging between 10 kW and 1 megawatt (MW) level. A 50% decline in the residential prices and 70% in utility prices was assessed. "Prices for commercial systems showed a similar decline, with the absolute price per watt tending to lie 10%–15% below the residential average during this period". $H(\alpha_i + \alpha_j)$ could take into account the economies of scale that may arise due to prosumers' cooperation in investment decision. If no cooperation occurs, the prosumer pays a "residential" price to invest in a PV plant. If instead cooperation occurs, prosumers may have access to "commericial" prices, since they will act as a single buyer and demand an overall plant size bigger than the one when cooperation not occurs. $H(\alpha_i + \alpha_j)$ takes into account this dynamic.

[13] Sunk costs are assumed to be quadratic, for the sake of simplification. None of the results are altered if investment costs are represented by a more general formulation: $I(\alpha_i, \alpha_j) = K\left(\alpha_i^\delta + \alpha_j^\delta\right)$ where $\delta > 1$.

[14] There is a wide literature on electricity prices. The most relevant for our work are Gianfreda and Grossi (2012), and Fanone et al. (2013), whereas Alexander et al. (2012) refer to the use of ABM stochastic process in real options theory.

where dW_t is the increment of Wiener's process (normally distributed with zero mean and variance dt), θ is the (constant) increment in the energy price over time (measured in monetary units), and σ is the instantaneous standard deviation of dv_t.

The price v_t and its expected value , given the initial price v_0, are

$$v_t = v_0 + \theta t + \sigma W_t, \tag{4}$$

$$\mathbb{E}_0 \left[v_t \right] = v_0 + \theta t. \tag{5}$$

Under the assumption of exchange possibility and cooperative investment, prosumers require to agree on the price of the energy exchanged (that we call z_t). It is more than reasonable to assume that such agreement is reached at the same moment in which the investment decision is jointly undertaken ($t = \tau$) and that prosumers decide to set this price equal to the one paid by the National Energy Market Operator (NEMO) for the energy the prosumers sell to the national grid (v_t).

Assumption 6 (**price of the exchanged energy**). *The prosumers exchange energy at the market price, i.e. $z_t = v_t$.*[15]

Assumption 7 (**plant maintenance cost**). *The plant maintenance cost is proportional to its capacity: $a\alpha_i$.*[16]

2.2 Prosumers net operative cost function under exchange scenario

Given the assumptions stated in the previous section, we can decompose the cost of prosumer i in the following components:

- the maintenance cost: $a\alpha_i$,

[15] Zafar et al. (2018) underline the importance of a negotiation process to determine the price of the exchanged energy, whereas Ilic et al. (2012) mention the example of EU project NOBEL where the price is determined in a stock exchange market structure. Alam et al. (2013) set the Microgrid energy price in range from 0 to the grid energy price level, whereas Mengelkamp et al. (2017) state that local prices should converge towards the grid prices under perfect information. Given that the two prosumers behave complementarily in demand and supply it is plausible to set $v_t = z_t$ because it maximizes the difference between c and v_t and therefore the cooperative gain of the exchange. If we remove the assumption the prosumers to be symmetric in exchange, thus to be characterized by different demands, each prosumer could start a negotiation on z probably obtaining different prices with respect to v_t. It could be a good extension for further research.

[16] Here, a represents the maintenance cost per unit of installed capacity and can be considered as the marginal cost of internal production. Since solar radiations represents the production input and are for free, the marginal production costs for the PV power plants may considered negligible, thus a will be set at nil (Bertolini et al., 2018, Tveten et al., 2013, Mercure and Salas, 2012).

- the cost of the energy bought from the grid: cb_i,
- the cost of the energy bought from the other prosumer: $v_t\gamma_i(1-\xi_j)\alpha_j$,

while it also has two positive cash flows:

- the return on the energy sold to the other prosumer: $v_t\gamma_j(1-\xi_i)\alpha_i$,
- the return on the energy sold to the grid: $v_t(1-\gamma_j)(1-\xi_i)\alpha_i$.

The overall cost function of prosumer i is

$$C_i(\xi_i,\gamma_i,\alpha_i) = \underset{\text{maintenance cost}}{a\alpha_i} + \underset{\text{energy bought from the grid}}{cb_i} + \underset{\text{energy bought from the other prosumer}}{v_t\gamma_i(1-\xi_j)\alpha_j}$$
$$\underset{\text{energy sold to the other prosumer}}{-v_t\gamma_j(1-\xi_i)\alpha_i} \quad \underset{\text{energy sold to the grid}}{-v_t(1-\gamma_j)(1-\xi_i)\alpha_i}.$$

Substituting $b_i = 1-\xi_i\alpha_i-\gamma_i(1-\xi_j)\alpha_j$ from (1) we obtain

$$\begin{aligned} C_i(\xi_i,\gamma_i,\alpha_i) =& a\alpha_i + c[1-\xi_i\alpha_i-\gamma_i(1-\xi_j)\alpha_j] + v_t\gamma_i(1-\xi_j)\alpha_j \\ & - v_t\gamma_j(1-\xi_i)\alpha_i - v_t(1-\xi_j)(1-\xi_i)\alpha_i. \end{aligned} \tag{6}$$

The net operative cost function $C_i(\xi_i,\gamma_i,\alpha_i)$ is decreasing in ξ_i and γ_i only if $v_t < c$,[17] i.e. when the price paid by the NEMO is lower than the one each prosumer pays to buy energy from it. This implies that the opportunities to self consume and exchange minimize energy costs only for $v_t < c$, leading to the following optimal self consumption and exchange behavior choices:

$$\begin{cases} v_t < c & \rightarrow \xi_i \in \left(0,\bar{\xi}_i\right], \gamma_i \in (0,\bar{\gamma}_i], \\ v_t \geq c & \rightarrow \xi_i,\gamma_i = 0, \end{cases} \tag{7}$$

and equation (6) becomes

$$C_i(\xi_i,\gamma_i,\alpha_i) = a\alpha_i + c - v_t\alpha_i - [\xi_i\alpha_i + (1-\xi_j)\alpha_j\gamma_i](c-v_t)\mathbb{I}_{v<c}, \tag{8}$$

in which $\mathbb{I}_\varepsilon$ is the indicator function of the event ε, whose value is 1 if the event occurs, and 0 otherwise.

To assure that the investment always minimizes net operative cost once the optimal timing $t = \tau$ is reached, the following conditions must hold simultaneously[18]

$$\begin{cases} C_i(\xi_i,\gamma_i,\alpha_i) < c, \\ C_i(\xi_i,\gamma_i,\alpha_i) < C_i(\xi_i,0,\alpha_i), & \text{iff} \quad v_t < c \\ C_i(\xi_i,\gamma_i,\alpha_i) \geq C_i(0,0,\alpha_i), & \text{iff} \quad v_t \geq c \end{cases} \tag{9}$$

The first inequality is always satisfied if and only if $v_t > 0$ and $\xi_i \neq \gamma_i(1-\xi_j)\frac{\alpha_j}{\alpha_i}$, whereas the second, which is always verified, assures that the possibility to exchange energy minimizes costs when self consumption occurs, thus when $v_t < c$. If instead $v_t \geq c$ net cost minimization is assured by the absence of self consumption and exchange $(\xi_i,\gamma_i = 0)$.

[17] The corresponding derivatives are $\frac{\partial C_i(\xi_i,\gamma_i,\alpha_i)}{\partial \xi_i} = (v_t-c)\alpha_i$ and $\frac{\partial C_i(\xi_i,\gamma_i,\alpha_i)}{\partial \gamma_i} = (v_t-c)(1-\xi_j)\alpha_j$.

[18] See Appendix A

2.3 Optimization

The agent minimizes its total net operative cost under the assumption of a cooperative investment decision between prosumers. The control variables are the optimal size of the PV plant of each prosumer $\left(\alpha_i^*, \alpha_j^*\right)$ and the time when to invest τ. This last control variable will be substituted by the price threshold that triggers the investment decision (v_τ).

Before the investment (which occurs at time τ), each prosumer pays a constant cost c for buying energy. When the investment is undertaken and, at the same moment in time, the optimal size is chosen, the prosumer pays the initial cost $I\left(\alpha_i, \alpha_j\right)$, and after that moment, it pays the cost C_i as defined in (6). Since the agents are assumed to solve the optimization problem together, we define the joint discounted cost function as follows:

$$\begin{aligned}\mathcal{C}\left(\alpha_i, \alpha_j, \tau\right) := & \int_0^\tau c e^{-rt} dt + \int_\tau^\infty C_i\left(\xi_i, \gamma_i, \alpha_i\right) e^{-rt} dt \\ & + \int_0^\tau c e^{-rt} dt + \int_\tau^\infty C_j\left(\xi_j, \gamma_j, \alpha_j\right) e^{-rt} dt \\ & + I\left(\alpha_i, \alpha_j\right) e^{-r\tau}.\end{aligned}$$

Thus, the cost minimizing problem for both prosumers together is

$$\min_{\alpha_i, \alpha_j, \tau} \mathbb{E}_0\left[\mathcal{C}\left(\alpha_i, \alpha_j, \tau\right)\right]. \tag{10}$$

After plugging all the equations into the joint cost function $\mathcal{C}\left(\alpha_i, \alpha_j, \tau\right)$, the problem becomes

$$\begin{aligned}\min_{\alpha_i, \alpha_j, \tau} & \left(H\left(\alpha_i + \alpha_j\right) + \frac{K}{2}\left(\alpha_i^2 + \alpha_j^2\right) + P\right) \mathbb{E}_0\left[\ e^{-r\tau}\ \right] \\ & \frac{2c}{r} + a\left(\alpha_i + \alpha_j\right) \mathbb{E}_0\left[\int_\tau^\infty e^{-rt} dt\right] - \left(\alpha_i + \alpha_j\right) \mathbb{E}_0\left[\int_\tau^\infty v_t e^{-rt} dt\right] \\ & - \left(\left(\xi_j + \left(1 - \xi_j\right)\gamma_i\right)\alpha_j + \left(\xi_i + \left(1 - \xi_i\right)\gamma_j\right)\alpha_i\right) \mathbb{E}_0\left[\ \int_\tau^\infty \left(c - v_t\right) \mathbb{I}_{v_t < c} e^{-rt} dt\ \right],\end{aligned} \tag{11}$$

where there are some expected value that we are able to compute. In particular, we know that

$$\mathbb{E}_0\left[e^{-r\tau}\right] = e^{-\beta_1\left(v^* - v_0\right)}, \tag{12}$$

in which v^* is the price threshold that triggers the investment, and $\beta_1 = -\frac{\theta}{\sigma^2} + \sqrt{\left(\frac{\theta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}$ is obtained through the martingale approach (Appendix B).

The other expected values are[19]

$$\mathbb{E}_0\left[e^{-r\tau} v_\tau\right] = v^* \mathbb{E}_0\left[e^{-r\tau}\right], \tag{13}$$

$$\mathbb{E}_0\left[\int_\tau^\infty e^{-rt} dt\right] = \frac{1}{r} \mathbb{E}_0\left[e^{-r\tau}\right], \tag{14}$$

[19]Detailed computations available in Appendix B

$$\mathbb{E}_0\left[\int_\tau^\infty v_t e^{-rt}dt\right] = \frac{1}{r}\mathbb{E}_0\left[e^{-r\tau}v_\tau\right] + \frac{\theta}{r^2}\mathbb{E}_0\left[e^{-r\tau}\right], \tag{15}$$

and the expected value with the option is obtained according to Dixit and Pindyck (1994)

$$\begin{aligned}&\mathbb{E}_0\left[\ \int_\tau^\infty (c-v_t)\,\mathbb{I}_{v_t<c}e^{-rt}dt\ \right]\\ &=\left(\left(Ae^{\beta_1 v^*} - \frac{v^*}{r} + \frac{c}{r} - \frac{\theta}{r^2}\right)\mathbb{I}_{v^*<c} + Be^{\beta_2 v^*}\mathbb{I}_{v^*\geq c}\right)\mathbb{E}_0\left[e^{-r\tau}\right],\end{aligned} \tag{16}$$

where $\beta_2 = -\frac{\theta}{\sigma^2} - \sqrt{\left(\frac{\theta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}$, and (16) measures the value of the possibility to exchange energy P2P. The optimal capacity and price threshold for each prosumer are obtained by solving numerically the following system of first order conditions in the two different cases, for $v^* < c$ and $v^* \geq c$, respectively:

$$\alpha_i^*, \alpha_j^*, v_\tau^* : \begin{cases} \frac{\partial \mathbb{E}_0[\mathcal{C}(\alpha_i,\alpha_j,v_\tau)]}{\partial \alpha_i^*} = 0, \\ \frac{\partial \mathbb{E}_0[\mathcal{C}(\alpha_i,\alpha_j,v_\tau)]}{\partial \alpha_j^*} = 0, \\ \frac{\partial \mathbb{E}_0[\mathcal{C}(\alpha_i,\alpha_j,v_\tau)]}{\partial v^*} = 0. \end{cases} \tag{17}$$

If $v_\tau < c$, self consumption minimizes the prosumer net operative cost. Thus, the previous system of FOCs (17) can be written as

$$\left(\alpha_i^*, \alpha_j^*, v^*\right)_{v_\tau<c} : \begin{cases} 0 = & H + K\alpha_i^* + \frac{a}{r} - \frac{v^*}{r} - \frac{\theta}{r^2} - (\xi_i + (1-\xi_i)\gamma_j)\left(Ae^{\beta_1 v^*} - \frac{v^*}{r} + \frac{c}{r} - \frac{\theta}{r^2}\right) \\ 0 = & H + K\alpha_j^* + \frac{a}{r} - \frac{v^*}{r} - \frac{\theta}{r^2} - (\xi_j + (1-\xi_j)\gamma_i)\left(Ae^{\beta_1 v^*} - \frac{v^*}{r} + \frac{c}{r} - \frac{\theta}{r^2}\right) \\ 0 = & -\beta_1\left(P + H(\alpha_i+\alpha_j) + \frac{K}{2}\left(\alpha_i^2+\alpha_j^2\right)\right) - \frac{a}{r}(\alpha_i+\alpha_j)\beta_1 \\ & -\frac{1}{r}(\alpha_i+\alpha_j)(1-v^*\beta_1) + \frac{\theta}{r^2}(\alpha_i+\alpha_j)\beta_1 \\ & +((\xi_j+(1-\xi_j)\gamma_i)\alpha_j + (\xi_i+(1-\xi_i)\gamma_j)\alpha_i)\left(Ae^{\beta_1 v^*} - \frac{v^*}{r} + \frac{c}{r} - \frac{\theta}{r^2}\right)\beta_1 \\ & -((\xi_j+(1-\xi_j)\gamma_i)\alpha_j + (\xi_i+(1-\xi_i)\gamma_j)\alpha_i)\left(\beta_1 Ae^{\beta_1 v^*} - \frac{1}{r}\right). \end{cases} \tag{18}$$

Instead, when $v_\tau \geq c$, the prosumers minimize net operative costs by selling and buying energy to and from the national grid, and the system of first order conditions becomes:

$$\left(\alpha_i^*, \alpha_j^*, v^*\right)_{v_\tau\geq c} : \begin{cases} 0 = & H + K\alpha_i^* + \frac{a}{r} - \frac{v^*}{r} - \frac{\theta}{r^2} - (\xi_i + (1-\xi_i)\gamma_j)Be^{\beta_2 v^*} \\ 0 = & H + K\alpha_j^* + \frac{a}{r} - \frac{v^*}{r} - \frac{\theta}{r^2} - (\xi_j + (1-\xi_j)\gamma_i)Be^{\beta_2 v^*} \\ 0 = & -\beta_1\left(P + H(\alpha_i+\alpha_j) + \frac{K}{2}\left(\alpha_i^2+\alpha_j^2\right)\right) + \frac{a}{r}(\alpha_i+\alpha_j)\beta_1 \\ & -\frac{1}{r}(\alpha_i+\alpha_j)(1-v^*\beta_1) + \frac{\theta}{r^2}(\alpha_i+\alpha_j)\beta_1 \\ & +((\xi_j+(1-\xi_j)\gamma_i)\alpha_j + (\xi_i+(1-\xi_i)\gamma_j)\alpha_i)\beta_1 Be^{\beta_2 v^*} \\ & -((\xi_j+(1-\xi_j)\gamma_i)\alpha_j + (\xi_i+(1-\xi_i)\gamma_j)\alpha_i)\beta_2 Be^{\beta_2 v^*}. \end{cases} \tag{19}$$

In both systems (18) and (19), the intuition behind the two first lines is that we must equate the marginal cost of the investment (given by K weighted by α_i or α_j and H) with the marginal return of the investment. In fact, all the elements with the negative signs in the two first rows measure the marginal value of the option to invest and exchange.

The last row, instead, measures both the marginal cost and the marginal return to postpone the investment. We see that most of the terms are multiplied by either β_1 or β_2 since they measure the reaction of the expected value $\mathbb{E}_0\left[e^{-r\tau}\right]$ to a change in the threshold v^* (see Eq. (12)). This last row already takes into account both the value-matching and the smooth-pasting conditions respect to the option value to invest as expressed in B. The corresponding conditions can be found in equations (B.22) and (B.24), respectively.

3 Calibration of the model

For calibration we normalize demand of energy to 1 MWh/y and we assume that the two prosumers are perfectly asymmetric in load curves, (i.e. they are symmetric in the exchange). This implies that $\xi_i = \xi_j$ and $\gamma_i = \gamma_j$. Price calibration focuses on the Northern Italy electricity market over the time interval from 2012 to 2018. Parameters θ and σ of the price (v_t) paid to the prosumers by the NEMO for the energy sold to the national grid are obtained with the method of moments using Italian Zonal prices (geographical prices). The dataset is built starting from hourly prices of the physical national zone of Northern Italy available on the website of the Italian NEMO, GME (Gestore Mercati Energetici),[20] and taking into account the daily time interval from 8 a.m to 7 p.m as reference of the PV plant operating time. Average monthly prices are computed, seasonally adjusted and non-stationarity assumption is verified with Dickey Fuller test.[21] The initial price v_0 is 87.13 euro/MWh, the minimum $\left(v_t^{\min}\right)$ is 32.26 euro/MWh and the maximum $\left(v_t^{\max}\right)$ is 103.63 euro/MWh. The annual drift and standard deviation of the price v_t are $\theta = -3.19$ and $\sigma = 34.30$, respectively.

The price paid by the prosumers to buy energy from the national grid (c) is set equal to 154.00 euro/MWh, that is the average value of the electricity price paid by household consumers in the European Market.[22] As per assumption 6, the price agreed between prosumers for the exchanged energy z_t is set equal to v_t.

With reference to the PV plant investment cost $I\left(\alpha_i, \alpha_j\right)$, the parameter K is computed using the same approach described by Bertolini et al. (2018).[23]

[20] `https://www.mercatoelettrico.org/en/download/DatiStorici.aspx`

[21] Augmented Dickey-Fuller Test is performed in R with adf.test command, where the alternative hypothesis is stationarity. Test result is -2.0623 and p-value is equal to 0.5503. Thus we fail to reject the null hypothesis.

[22] Eurostat - Energy Statistics, Electricity prices for household consumers - bi-annual data (from 2007 onwards) [nrg_pc_204]. The data are in in Euro currency, refer to an annual consumption between 2 500 and 5 000 kWh (Band-DC, Medium), excluding taxes and levies.

[23] The unit of measure of the plant's size α_i is kWh/year. Indeed, it is always possible to obtain the average amount of energy produced by the PV plant over a certain time interval in kWh, i.e., in a year. Following Bertolini et al. (2018) (Appendix B), the plant energy output

The average plant life time interval is 25 years, thus T is set equal to 25,[24] whereas the levelized cost of energy ($LCOE$) for PV technology is set equal to 100 euro/MWh.[25] The discount rate r is defined as an average of the values used in Bertolini et al. (2018) and set equal to 0.05. The parameter H of the investment cost function represents the cost saved by the prosumers because of their decision to undertake the investment cooperatively. On the basis of MIT (2015), H is set equal to $-0.15K$,[26] whereas the sunk cost to access the Smart Grid P is set equal to $0.1K$,[27] and the PV plant maintenance cost a is set equal to 0.

Prosumers' self consumption behavior is described by parameter $\xi_i \in \left[0, \bar{\xi_i}\right]$, where ξ_i is set equal to 0.30.[28] Finally, for what concerns $\gamma_i \in [0, \bar{\gamma_i}]$, which measures the energy exchange P2P attitude, we set it equal to 0.10, because prosumers are assumed to be asymmetric in load curves.[29]

Table 3.1 gathers all the parameters used for model calibration.

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is "determined by multiplying the size in kWp by the local solar insolation that takes capacity factor into account in the units: kWh/kWp/year". Then, if the cost of the plant per kWp is known, it is also possible to trace, using LCOE, the cost of the plant as a function of the energy produced in a year, as in the following equation:

$$K = 2\frac{LCOE}{r}\left(1 - e^{-rT}\right)$$

This allows to construct a cost function in terms of kWh /year instead of kWp.

[24]Branker et al. (2011), Kästel and Gilroy-Scott (2015).

[25]IEA (2018) identifies an average value of the solar PV levelized cost of electricity in 2017 equal to 100 euro/MWh

[26]MIT (2015) analyses the decline of PV system prices in US from 2004 to 2014 at residential and commercial level. A 50% decline in the residential prices and 70% in utility prices was assessed. "Prices for commercial systems showed a similar decline, with the absolute price per watt tending to lie 10%–15% below the residential average during this period". We use this variation as a proxy of the cost saving prosumers can gain from cooperation. Thus we set $H = -0.15K$.

[27]With reference to Italy, we set parameter P as a share (0.1) of the capital cost K, as an average of two possible fees coming from two projects: "REGALGRID" (`https://www.regalgrid.com/`), where the average fee is 400 euro/year (Peloso, 2018) and "sonnenCommunity" (`https://sonnengroup.com/sonnencommunity/`), where the monthly fee is 20 euro/month.

[28]Kästel and Gilroy-Scott (2015), Ciabattoni et al. (2014), Cucchiella et al. (2017)

[29]Sousa et al. (2019), Zhang et al. (2018).

Parameter	*Description*	*Value*	*Source/Reference*
θ	drift	-3.19	Calibrated on Northern Italy zonal prices, NEMO GME
σ	volatility	34.30	Calibrated on Northern Italy zonal prices, NEMO GME
v_0	price v_t at the beginning of the time period	87.13	Northern Italy zonal prices, NEMO GME
c	cost to buy energy from the national grid	154.00	Eurostat
T	PV plant lifetime (years)	25	Branker et al. (2011), Kästel and Gilroy-Scott (2015)
r	discount rate	0.05	Bertolini et al. (2018)
$LCOE$	levelized cost of electricity for PV plants euro	100.00	IEA (2018)
K	PV plant cost of capital	2853.98	Computed, Bertolini et al. (2018)
a	PV plant maintenance cost	0	Bertolini et al. (2018),Mercure and Salas (2012), Tveten et al. (2013)
H	prosumers gain from cooperation	$-0.15K$	Computed, MIT (2015)
P	cost to access to virtual exchange platform	$0.10K$	Computed, Peloso (2018)
ξ_i	prosumers' self consumption parameter	0.30	Kästel and Gilroy-Scott (2015) Ciabattoni et al. (2014)
γ_i	prosumers' exchange parameter	0.10	Sousa et al. (2019), Zhang et al. (2018)

Table 3.1: Parameters

4 Main results and comparative statics

This section is devoted to the main results and comparative statics. We define the following four scenarios: $E(v_\tau < c)$ and $E(v_\tau > c)$ refer to the cases with exchange possibility (E, Exchange) and where v_τ is lower and higher than c respectively, whereas $NE(v_\tau < c)$ and $NE(v_\tau > c)$ refer to the cases in which there is no exchange possibility (NE, No Exchange). The NE cases are obtained by setting $\gamma_i = 0$.

Numerical solutions for $E(v_\tau < c)$ and $E(v_\tau > c)$ are obtained from equations (18) and (19), whereas $NE(v_\tau < c)$ and $NE(v_\tau > c)$ from equations (C.6) and (C.7) in Appendix C. In the following tables and figures, we show and comment the four scenarios. We also present the optimal size α_i^* and the selling price v^* which triggers investments.[30] Furthermore, for each case we show the optimal investment cost I_i^* for each prosumer and the overall net operative cost $\mathbb{E}_0[OC_i^*]$.[31] In case of multiple viable thresholds we will choose the scenario with the lowest $\mathbb{E}_0[OC_i^*]$.

In Table 4.1 we present the benchmark case, calculated by using the parameters of Table 3.1, where in bold we mark the optimal cases in terms of net operative cost minimization and the symbol "–" represents the unfeasible cases.[32]

Scenario	α_i^*	v^*	I_i^*	$\mathbb{E}_0[OC_i^*]$
$E(v_\tau > c)$	1.635163	259.119	3258.118	2247.551
$E(v_\tau < c)$	**0.948976**	**139.987**	1021.530	**1951.837**
$NE(v_\tau > c)$	–	–	–	–
$NE(v_\tau < c)$	0.699665	131.071	698.557	2267.01

Table 4.1: Optimal capacities, price thresholds, investment costs, and net operative costs, with $\xi_i = 0.30$, $\gamma_i = 0.10$, $c = 154$, $v_0 = 87.13$, $\theta = -3.19$, $\sigma = 34.30$, $r = 0.05$, $T = 25$, $LCOE = 100$, $P = 0.10K$, $H = -0.15K$

Table 4.1 shows three viable solutions of v^*. Two of them are for the scenarios E and one for scenarios NE. See also Figure 4.1 showing both c and the optimal triggers v^*.

[30] Optimal capacity is expressed in MWh/y, whereas price threshold, optimal investment and overall net operative cost in euro/MWh.

[31] where $I_i^* := \frac{1}{2} I\left(\alpha_i^*; \alpha_j^*\right)$ and $\mathbb{E}_0[OC_i^*] := \mathbb{E}_0\left[\int_0^\tau ce^{-rt}dt + \int_\tau^\infty C_i^*\left(\alpha_i^*, \alpha_j^*, v_\tau^*\right)e^{-rt}dt\right]$

[32] We define as unfeasible those cases where the value obtained for v^* does not lie in the price interval of the related scenario. For instance, we reject the solution of $v^* > c$ if we are in the scenario where $v_\tau < c$.

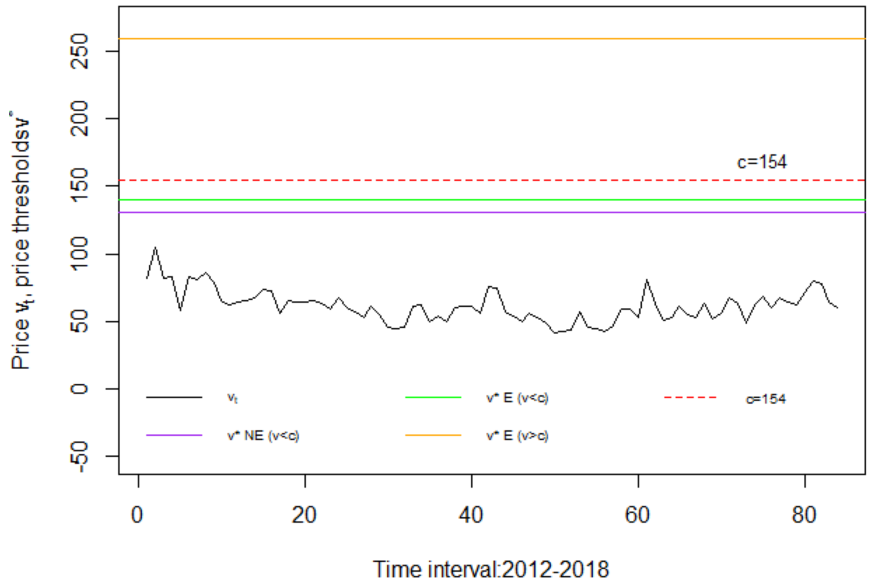

Figure 4.1: Northern Italy price and price thresholds comparison, with $\xi_i = 0.30$, $\gamma_i = 0.10$, $c = 154$, $v_0 = 87.13$, $\theta = -3.19$, $\sigma = 34.30$, $r = 0.05$, $T = 25$, $LCOE = 100$, $P = 0.10K$, $H = -0.15K$

In the benchmark, the lowest net operative cost $\mathbb{E}_0 [OC_i^*]$ is achieved in scenario $E(v_\tau < c)$, where the value of energy exchanged is always positive and it makes the agents better off. Furthermore, the possibility of selling energy between agents (i.e., the option to switch) encourages prosumers to invest in larger plants when compared with plants sized according to scenario NE.

Table 4.2 shows, for scenarios E, the comparative statics of a change in ξ_i and γ_i parameters. We move from "sales-oriented profile" agents characterized by low values of both ξ_i and γ_i to "exchange-oriented profile" agents with higher values of ξ_i and γ_i. Such higher values represent the case in which the load/demand curves of the agents allow them to exchange and self consume a bigger share of their production. A "sales-oriented profile" agent would like to invest for selling energy to the national grid, gaining from the difference between v_t and c in sales. This is coherent with the result in Table 4.2 where the viable scenario is $E(v_\tau > c)$. On the contrary, an "exchange-oriented profile" agent invests for reducing the cost of energy by increasing self-consumption and exchange. This is coherent with the result in Table 4.2 where the viable scenario is $E(v_\tau < c)$.

Moreover, comparing the net operative cost $\mathbb{E}_0 [OC_i^*]$ between scenarios

$E(v_\tau < c)$ and $E(v_\tau > c)$, we observe that the cost related to $E(v_\tau < c)$ is always smaller regardless of the shape of the load/demand curve. Furthermore, this is true although the optimal size of the plant is a negative function of both ξ_i and γ_i. That is to say that "exchange-oriented profile" agents are able to use more efficiently their PV plants, i.e. to invest earlier and with a lower optimal size of the plant. We can interpret the difference between the net operative cost $\mathbb{E}_0[OC_i^*]$ in the case where $\xi_i = 0.10$ and $\gamma_i = 0.05$, and the net operative cost in the case $\xi_i = 0.35$ and $\gamma_i = 0.15$, as the maximum amount that each agent would be willing to pay for a technology able to increase self-consumption and exchange (i.e. home automation).

Parameters	Scenario	α_i^*	v^*	I_i^*	$\mathbb{E}_0[OC_i^*]$
$\xi_i = 0.10$; $\gamma_i = 0.05$	$E(v_\tau > c)$	1.408054	235.698	2369.090	2269.625
	$E(v_\tau < c)$	–	–	–	–
$\xi_i = 0.30$; $\gamma_i = 0.10$	$E(v_\tau > c)$	1.635163	259.119	3258.118	2247.551
	$E(v_\tau < c)$	0.948976	139.987	1021.530	1951.837
$\xi_i = 0.35$; $\gamma_i = 0.15$	$E(v_\tau > c)$	1.701074	265.964	3543.692	2248.722
	$E(v_\tau < c)$	**0.847737**	**106.233**	805.303	**1745.941**

Table 4.2: Optimal capacities and price thresholds as a function of ξ_i and γ_i with $\boldsymbol{\xi_i = 0.30}$, $\boldsymbol{\gamma_i = 0.10}$, $c = 154$, $v_0 = 87.13$, $\theta = -3.19$, $\sigma = 34.30$, $r = 0.05$, $T = 25$, $LCOE = 100$, $P = 0.10K$, $H = -0.15K$

Table 4.3 shows the comparative statics with respect to σ. Three comments are in order for this table: (i) in line with standard results in the Real Option literature on investment timing flexibility, the greater the volatility of prices, the greater the option value to defer the investment and, in turn, the greater the investment value (see Bar-Ilan and Strange (1999); Dangl (1999); Hagspiel et al. (2016)), (ii) with high volatility the PV plant is built for selling; indeed, the viable scenario is $E(v_\tau > c)$ for $\sigma = 40$, whereas it is $E(v_\tau < c)$ for $\sigma = 30$ and $\sigma = 20$ (see Figure 4.2), and (iii) there is a positive relation between α_i^* and v^*. In order to invest in a larger plant, prosumers wait longer to be profitable. When σ is high, the option to delay prevails over the option to exchange and each agent delays to make the sale convenient. In other words, if a policymaker would like to push towards energy community, it should try to stabilize the energy prices, thus reducing σ.

Parameters	Scenario	α_i^*	v^*	I_i^*	$\mathbb{E}_0\left[OC_i^*\right]$
$\sigma = 40$	$E\left(v_\tau > c\right)$	**1.863268**	**292.383**	4299.221	**1945.291**
	$E\left(v_\tau < c\right)$	–	–	–	–
	$NE\left(v_\tau > c\right)$	0.890449	162.066	1131.461	2022.237
	$NE\left(v_\tau < c\right)$	–	–	–	–
$\sigma = 30$	$E\left(v_\tau > c\right)$	1.471143	234.923	2601.290	2445.350
	$E\left(v_\tau < c\right)$	**0.792776**	**111.669**	700.1702	**2065.294**
	$NE\left(v_\tau > c\right)$	–	–	–	–
	$NE\left(v_\tau < c\right)$	0.569402	108.798	462.657	2406.617
$\sigma = 20$	$E\left(v_\tau > c\right)$	1.1337197	183.533	1491.496	2812.596
	$E\left(v_\tau < c\right)$	**0.535413**	**60.139**	322.562	**1901.717**
	$NE\left(v_\tau > c\right)$	–	–	–	–
	$NE\left(v_\tau < c\right)$	0.313163	61.265	139.9473	2535.985

Table 4.3: Optimal capacities and price thresholds as a function of σ with $\xi_i = 0.30$, $\gamma_i = 0.10$, $c = 154$, $v_0 = 87.13$, $\theta = -3.19$, $\sigma = \mathbf{34.30}$, $r = 0.05$, $T = 25$, $LCOE = 100$, $P = 0.10K$, $H = -0.15K$

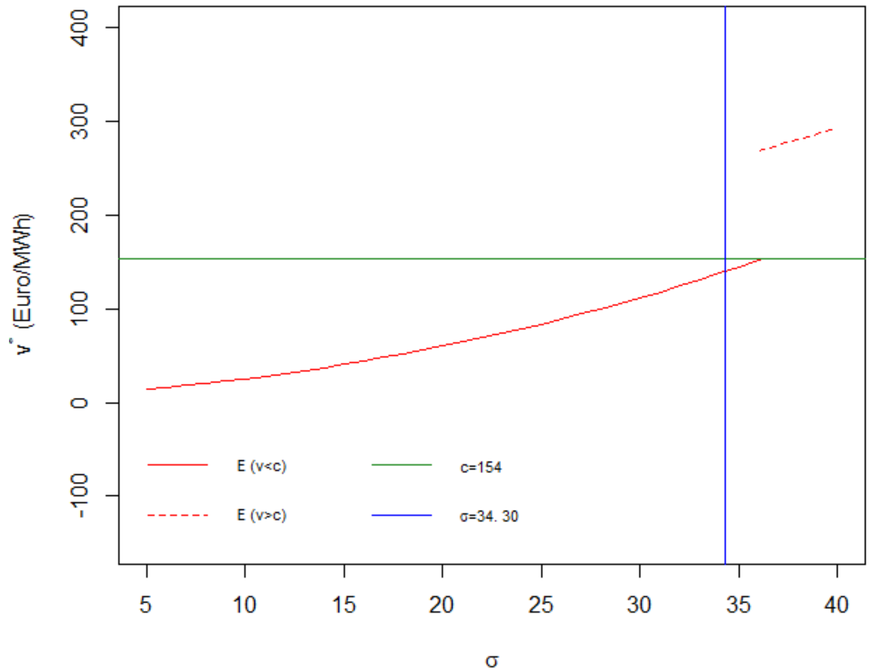

Figure 4.2: Price thresholds as a function of σ, with $\xi_i = 0.30$, $\gamma_i = 0.10$, $c = 154$, $v_0 = 87.13$, $\theta = -3.19$, $\sigma = \mathbf{34.30}$, $r = 0.05$, $T = 25$, $LCOE = 100$, $P = 0.10K$, $H = -0.15K$

Table 4.4 and Table 4.5 show comparative statics with respect different values for $LCOE$ (110 and 80) and lifetime plant T (20 and 30). An increase in $LCOE$ implies an increase in investment timing and a reduction in plant size. Intuitively, higher $LCOE$ implies higher investment costs which, in turn, cause a generalized investment delay. This delay can be reduced by reducing the plant size. A change of plant lifetime T generates a similar effects: when T increases, ceteris paribus, plant size decreases and the selling price triggering the investment increases (i.e., the agent invests later).

Parameters	Scenario	α_i^*	v^*	I_i^*	$\mathbb{E}_0 [OC_i^*]$	K
$LCOE =$ 110	$E(v_\tau > c)$	1.498441	258.816	2975.807	2315.272	3139.38
	$E(v_\tau < c)$	**0.882003**	**141.184**	962.736	2038.753	3139.38
	$NE(v_\tau > c)$	–	–	–	–	–
	$NE(v_\tau < c)$	0.636059	131.071	635.052	2340.926	3139.38
$LCOE =$ 80	$E(v_\tau > c)$	2.013789	260.065	4054.034	2062.670	2283.18
	$E(v_\tau < c)$	**1.139501**	**138.566**	1206.220	1714.136	2283.18
	$NE(v_\tau > c)$	–	–	–	–	–
	$NE(v_\tau < c)$	0.874582	131.071	873.197	2063.774	2283.18

Table 4.4: Optimal capacities and price thresholds as a function of $LCOE$ with $\xi_i = 0.30$, $\gamma_i = 0.10$, $c = 154$, $v_0 = 87.13$, $\theta = -3.19$, $\sigma = 34.30$, $r = 0.05$, $T = 25$, $\mathbf{LCOE = 100}$, $P = 0.10K$, $H = -0.15K$

Parameters	Scenario	α_i^*	v^*	I_i^*	$\mathbb{E}_0 [OC_i^*]$
$T = 30$	$E(v_\tau > c)$	1.512445	258.845	3004.556	2308.303
	$E(v_\tau < c)$	**0.888809**	**141.036**	968.507	**2029.809**
	$NE(v_\tau > c)$	–	–	–	–
	$NE(v_\tau < c)$	0.642589	131.071	641.571	2333.340
$T = 20$	$E(v_\tau > c)$	1.829729	259.601	3665.021	2152.153
	$E(v_\tau < c)$	**1.046068**	**139.004**	1113.085	**1829.298**
	$NE(v_\tau > c)$	–	–	–	–
	$NE(v_\tau < c)$	0.789735	131.071	788.484	2162.362

Table 4.5: Optimal capacities and price thresholds as a function of T with $\xi_i = 0.30$, $\gamma_i = 0.10$, $c = 154$, $v_0 = 87.13$, $\theta = -3.19$, $\sigma = 34.30$, $r = 0.05$, $\mathbf{T = 25}$, $LCOE = 100$, $P = 0.10K$, $H = -0.15K$

Table 4.6 shows comparative statics with respect to different values of P (0, $0.10K$ and $0.15K$), the cost the prosumers pay to access to the virtual exchange platform. The case with $P = 0.10K$ is our benchmark (Figure 4.2).

In Table 4.6 we add the value of b_i corresponding to our optimal solutions. P is a sunk cost and its reduction leads prosumers to invest earlier and in a smaller plant size. Conversely, an increase in P leads prosumers to postpone investment decision, at a higher threshold and in a bigger plant size.

With respect to the investment timing, a significant difference arises from the comparison between the level of v^* obtained with $P = 0$ or $P = 0.15K$ and the level of v_0 (set to 87.13). The optimal delay could be very long depending on the parameters of the process. Nevertheless, the optimal plant size increases as well as the self consumption level of the two agents.

The last column of Table 4.6, shows the corresponding values of b_i, which represent the energy the two agents respectively must buy from the national grid (assumption 1). The higher is b_i, compared to the optimal size α_i^*, the lower are the self consumption and the levels of energy exchanged P2P. Furthermore, if we measure efficiency by the ratio b_i/α_i^*, the higher is b_i, the lower is the efficiency of the PV system. In addition to that, we observe that high levels of connection costs reduce b_i. This effect is due to the fact that an increase in P postpones the investment, increases the size α_i^*, which, in turn, leads to an increase in self-consumption and in energy exchanged P2P.

A policy maker whose aim is to increase both PV investments and related efficiency (in terms of lower b_i with respect to α_i^*), needs to identify the appropriate instruments: a subsidy on the investment sunk cost is not sufficient to increase the efficiency of the PV plant in terms of the b_i/α_i^* ratio. The subsidy should be also accompanied by policies aimed at increasing the use of the energy produced by the PV plant.

Parameters	Scenario	α_i^*	v^*	I_i^*	$\mathbb{E}_0\left[OC_i^*\right]$	b_i
$P=0$	$E\left(v_\tau > c\right)$	1.593685	252.412	2942.063	2223.366	0.410
	$E\left(v_\tau < c\right)$	**0.806438**	**111.910**	582.798	**1966.711**	0.702
$P=0.10K$	$E\left(v_\tau > c\right)$	1.635163	259.119	3258.118	2247.551	0.395
	$E\left(v_\tau < c\right)$	**0.948976**	**139.987**	1021.530	**1951.837**	0.649
$P=0.15K$	$E\left(v_\tau > c\right)$	1.655391	262.376	3415.788	2259.376	0.387
	$E\left(v_\tau < c\right)$	**1.012054**	**151.9826**	1242.390	**1960.238**	0.625

Table 4.6: Optimal capacities and price thresholds as a function of P with $\xi_i = 0.30$, $\gamma_i = 0.10$, $c = 154$, $v_0 = 87.13$, $\theta = -3.19$, $\sigma = 34.30$, $r = 0.05$, $T = 25$, $LCOE = 100$, $\mathbf{P = 0.10K}$, $H = -0.15K$

Parameters	α_i^*	b_i	$b_i - \underline{b}_i$	$1 - b_i$	$\alpha_i^* - (1 - b_i)$
$P=0$	0.806	0.702	$\{0.202, 0.302\}$	0.298	0.508
$P=0.10K$	0.949	0.649	$\{0.149, 0.249\}$	0.351	0.598
$P=0.15K$	1.012	0.625	$\{0.125, 0.225\}$	0.375	0.637

Table 4.7: Optimal capacities and price thresholds as a function of P with $\xi_i = 0.30$, $\gamma_i = 0.10$, $c = 154$, $v_0 = 87.13$, $\theta = -3.19$, $\sigma = 34.30$, $r = 0.05$, $T = 25$, $LCOE = 100$, $\mathbf{P = 0.10K}$, $H = -0.15K$, $\underline{b}_i \in \{0.40, 0.50\}$

Finally, in Table 4.7 taking as reference $\underline{b}_i \in \{0.40, 0.50\}$, from Table 4.6 we are able to calculate: the quantity of energy bought during the day $(b_i - \underline{b}_i)$, the energy self-consumed plus the energy bought from the other prosumer $(1 - b_i)$ and the energy produced and sold to the national grid $(\alpha_i^* - (1 - b_i))$ for different plant sizes.

In all cases, the energy produced by the PV plant and sold to the national grid remains very high whereas the self-consumption plus the exchanged energy remains lower than 40% of the prosumers' energy demand. This result, again, stresses the need for policies to push the PV efficiency. To support the PV market, a policy maker should foster storage adoption in addition to exchange of energy P2P.

5 Conclusions

In this work, we model two prosumers' investment decisions in a photovoltaic (PV) plant connected to the Smart Grid (SG). Each prosumer can: (i) self-consume its energy production, (ii) exchange energy with national grid, and/or (iii) exchange energy with the other agent. According to the characteristics of each load/demand factor, we distinguish between "sales-oriented profiles" that would like to invest for selling energy to the national grid and "exchange-oriented profiles" that invest with the aim to reduce the cost of energy by increasing self-consumption and exchange P2P.

Our findings show that: (i) in the benchmark case, the value of the exchange is positive, (ii) the option value to defer investment is positive, (iii) the possibility of selling energy between agents encourages investment in larger plants, compared with the cases with self consumption and no exchange, (iv) the "exchange-oriented profile" agents invest earlier and with a lower optimal size of the plant, and (v) there is a positive relation between plant optimal size and optimal investment timing (i.e. the greater the plant optimal size, the greater the investment deferral). About the volatility effect, on the one hand it is perfectly in line with current literature, and on the other hand it shows interesting results. The greater the volatility, the higher the option value to defer and, in turn, the greater the investment value. At the same time, with high volatility, the PV plant is built for selling and not for exchange purpose. Thus, an interesting policy implication for a policymaker that would like to push energy community diffusion is the stabilization of the energy prices' volatility.

The comparative statics performed on the SG connection costs, show that higher costs postpone the investment decision, increase the optimal size of the PV plant, and lead to an increase in self-consumption and energy exchanged levels. This, in turn, reduces the amount of energy that each prosumer purchases from the national provider. Nevertheless, in all the analyzed cases, the self-consumption plus the exchanged energy remains lower than 40% of the demand. Moreover, the energy produced by the PV plant and sold to the national grid remains very high. This stresses the need for policies to push the PV efficiency. To support the PV market, a policy maker could foster storage adoption in addition to exchange P2P.

Lastly, two possible extensions of our research could be: (i) to relax the assumption on the load factors studying different possibilities with totally asymmetric prosumers and calculating which profile is more viable, and (ii) to apply our approach to the PV plant disposal problem in order to understand policy implications related to this topic.

A Appendix: cost minimization conditions

In order to assure that once the optimal timing $t = \tau$ is reached the investment always minimizes net operative cost, the following conditions must hold simultaneously

$$\begin{cases} C_i(\xi_i, \gamma_i, \alpha_i) < c, \\ C_i(\xi_i, \gamma_i, \alpha_i) < C_i(\xi_i, 0, \alpha_i) & \text{iff} \quad v_t < c, \\ C_i(\xi_i, \gamma_i, \alpha_i) \geq C_i(0, 0, \alpha_i) & \text{iff} \quad v_t \geq c. \end{cases} \tag{A.1}$$

First condition assures that once the threshold is reached the investment always minimizes prosumers' energy costs

$$\begin{aligned} C_i(\xi_i, \gamma_i, \alpha_i) <& c \\ a\alpha_i + c - v_t\alpha_i - [\xi_i\alpha_i + (1-\xi_j)\alpha_j\gamma_i](c - v_t)\mathbb{I}_{v<c} <& c \\ a <& v_t + \left[\xi_i + (1-\xi_j)\frac{\alpha_j}{\alpha_i}\gamma_i\right](c - v_t)\mathbb{I}_{v<c}, \end{aligned} \tag{A.2}$$

which can be rewritten as follows

$$\begin{cases} a < v_t + \left[\xi_i + \gamma_i(1-\xi_j)\frac{\alpha_j}{\alpha_i}\right](c - v_t) & v_t < c, \\ a < v_t & v_t \geq c. \end{cases} \tag{A.3}$$

Since a represents the PV plant maintenance cost and we assume it to be nil ($a = 0$), the previous system can be rewritten as follows

$$\begin{cases} v_t > -\left[\xi_i + \gamma_i(1-\xi_j)\frac{\alpha_j}{\alpha_i}\right](c - v_t), & v_t < c \\ v_t > 0, & v_t \geq c \end{cases} \tag{A.4}$$

and if $v_t < c$, the RHS is always negative, first inequality is always satisfied iff $v_t > 0$ and $\xi_i \neq \gamma_i(1-\xi_j)\frac{\alpha_j}{\alpha_i}$[33]. *Second condition* assures that exchange possibility introduction minimizes prosumers' energy costs and it is satisfied only if self consumption occurs, thus when $v_t < c$

$$\begin{aligned} C_i(\xi_i, \gamma_i, \alpha_i) < C_i(\xi_i, 0, \alpha_i) \\ -(1-\xi_j)\alpha_j\gamma_i < 0, \end{aligned} \tag{A.5}$$

if instead $v_t > c$,follows (*third condition*)

$$\begin{aligned} C_i(\xi_i, \gamma_i, \alpha_i) \geq C_i(0, 0, \alpha_i) \\ a\alpha_i + c - v_t\alpha_i \geq a\alpha_i + c - v_t\alpha_i, \end{aligned} \tag{A.6}$$

which is always true.

[33] if $\xi_i = \gamma_i(1-\xi_j)\frac{\alpha_j}{\alpha_i}$, $v_t = 0$ and this solution is not admissible if $v_t > c$

B Appendix: expected values computation

The following expected value

$$\mathbb{E}_0\left[e^{-r\tau}\int_{\tau}^{\infty} e^{-r(t-\tau)}dt\right], \tag{B.1}$$

can be simplified by using the so-called tower property of (iterated) expected values. Thus, we write a new expected value inside the initial one, by using a larger filtration:

$$\mathbb{E}_0\left[e^{-r\tau}\int_{\tau}^{\infty} e^{-r(t-\tau)}dt\right] = \mathbb{E}_0\left[\mathbb{E}_\tau\left[e^{-r\tau}\int_{\tau}^{\infty} e^{-r(t-\tau)}dt\right]\right], \tag{B.2}$$

and since $e^{-r\tau}$ is known at time τ, this term can be collected outside the inner expected value:

$$\begin{aligned}\mathbb{E}_0\left[e^{-r\tau}\int_{\tau}^{\infty} e^{-r(t-\tau)}dt\right] &= \mathbb{E}_0\left[e^{-r\tau}\mathbb{E}_\tau\left[\int_{\tau}^{\infty} e^{-r(t-\tau)}dt\right]\right] \\ &= \mathbb{E}_0\left[e^{-r\tau}\frac{1}{r}\right]. \end{aligned}\tag{B.3}$$

In the other expected value

$$\mathbb{E}_0\left[e^{-r\tau}\int_{\tau}^{\infty} v_t e^{-r(t-\tau)}dt\right], \tag{B.4}$$

we initially use the same approach:

$$\begin{aligned}\mathbb{E}_0\left[e^{-r\tau}\int_{\tau}^{\infty} v_t e^{-r(t-\tau)}dt\right] &= \mathbb{E}_0\left[\mathbb{E}_\tau\left[e^{-r\tau}\int_{\tau}^{\infty} v_t e^{-r(t-\tau)}dt\right]\right] \\ &= \mathbb{E}_0\left[e^{-r\tau}\mathbb{E}_\tau\left[\int_{\tau}^{\infty} v_t e^{-r(t-\tau)}dt\right]\right] \\ &= \mathbb{E}_0\left[e^{-r\tau}\int_{\tau}^{\infty} \mathbb{E}_\tau\left[v_t\right] e^{-r(t-\tau)}dt\right]. \end{aligned}\tag{B.5}$$

Now we recall that, for any $t > \tau$

$$\mathbb{E}_\tau\left[v_t\right] = v_\tau + \theta\left(t-\tau\right), \tag{B.6}$$

and so

$$\begin{aligned}\mathbb{E}_0\left[e^{-r\tau}\int_{\tau}^{\infty} v_t e^{-r(t-\tau)}dt\right] &= \mathbb{E}_0\left[e^{-r\tau}\int_{\tau}^{\infty} \left(v_\tau + \theta\left(t-\tau\right)\right)e^{-r(t-\tau)}dt\right] \\ &= \mathbb{E}_0\left[e^{-r\tau}v_\tau\int_{\tau}^{\infty} e^{-r(t-\tau)}dt\right] + \mathbb{E}_0\left[\theta e^{-r\tau}\int_{\tau}^{\infty} \left(t-\tau\right)e^{-r(t-\tau)}dt\right] \\ &= \mathbb{E}_0\left[e^{-r\tau}\frac{v_\tau}{r}\right] + \mathbb{E}_0\left[\frac{\theta}{r^2}e^{-r\tau}\right] \\ &= \frac{1}{r}\mathbb{E}_0\left[e^{-r\tau}v_\tau\right] + \frac{\theta}{r^2}\mathbb{E}_0\left[e^{-r\tau}\right]. \end{aligned}\tag{B.7}$$

B.1 Expected value with the option

The expected value with the option $\mathbb{E}_0\left[\ \int_\tau^\infty (c - v_t)\,\mathbb{I}_{v_t<c} e^{-rt} dt\ \right]$ is obtained according to Dixit and Pindyck (1994)

$$\begin{aligned}
\mathbb{E}_0\left[\ \int_\tau^\infty (c - v_t)\,\mathbb{I}_{v_t<c} e^{-rt} dt\ \right] =& \mathbb{E}_0\left[e^{-r\tau}\int_\tau^\infty (c - v_t)\,\mathbb{I}_{v_t<c} e^{-r(t-\tau)} dt\right] \\
=& \mathbb{E}_0\left[\mathbb{E}_\tau\left[e^{-r\tau}\int_\tau^\infty (c - v_t)\,\mathbb{I}_{v_t<c} e^{-r(t-\tau)} dt\right]\right] \\
=& \mathbb{E}_0\left[e^{-r\tau}\mathbb{E}_\tau\left[\int_\tau^\infty (c - v_t)\,\mathbb{I}_{v_t<c} e^{-r(t-\tau)} dt\right]\right].
\end{aligned} \tag{B.8}$$

Now, we set

$$V_t = \mathbb{E}_t\left[\int_t^\infty (c - v_s)\,\mathbb{I}_{v_s<c} e^{-r(s-t)} ds\right], \tag{B.9}$$

whose value V_t must solve the following PDE

$$\frac{\partial V_t}{\partial v_t}\theta + \frac{1}{2}\frac{\partial^2 V_t}{\partial v_t^2}\sigma^2 - rV_t + (c - v_t)\,\mathbb{I}_{v_t<c} = 0. \tag{B.10}$$

which can be split into two PDEs

$$\begin{cases} \frac{\partial V_t}{\partial v_t}\theta + \frac{1}{2}\frac{\partial^2 V_t}{\partial v_t^2}\sigma^2 - rV_t + c - v_t = 0 & v_t < c, \\ \frac{\partial V_t}{\partial v_t}\theta + \frac{1}{2}\frac{\partial^2 V_t}{\partial v_t^2}\sigma^2 - rV_t = 0 & v_t \geq c. \end{cases} \tag{B.11}$$

If $v_t \geq c$ the guess function is $V_t = Be^{\beta v_t}$ and the corresponding PDE can be written as

$$\beta Be^{\beta v_t}\theta + \frac{1}{2}\beta^2 Be^{\beta v_t}\sigma^2 - rBe^{\beta v_t} = 0, \tag{B.12}$$

from which

$$\beta_{1,2} = -\frac{\theta}{\sigma^2} \pm \sqrt{\left(\frac{\theta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}. \tag{B.13}$$

This equation has two solutions but we take only the negative one.

Thus, we set

$$V_{1,t} = Be^{\beta_2 v_t}. \tag{B.14}$$

If $v_t < c$ the guess function is

$$V_t = Ae^{\beta v_t} + Dv_t + E, \tag{B.15}$$

and when this function is plugged into the PDE we get

$$\theta\beta Ae^{\beta v_t} + \theta D + \frac{\sigma^2}{2}\beta^2 Ae^{\beta v_t} - r\left(Ae^{\beta v_t} + Dv_t + E\right) + c - v_t = 0, \tag{B.16}$$

which can be split into three equations

$$Ae^{\beta v_t}\left(\frac{1}{2}\beta^2\sigma^2 + \theta\beta - r\right) = 0, \tag{B.17}$$

$$-v_t(1 + rD) = 0, \tag{B.18}$$

$$\theta D - rE + c = 0, \tag{B.19}$$

where the first equation is satisfied for the same value of β already presented above. In this case, instead, we take the positive value β_1.

The solution to the second equation is $D = -\frac{1}{r}$, and the solution to the last equation is $E = \frac{c}{r} - \frac{\theta}{r^2}$. Finally, the solution to the second PDE is

$$V_{2t} = Ae^{\beta_1 v_t} - \frac{1}{r}v_t + \frac{c}{r} - \frac{\theta}{r^2}. \tag{B.20}$$

Taking into account both price scenarios, the equation of V_t can be rewritten as follows

$$V_t = \begin{cases} V_{2,t} = Ae^{\beta_1 v_t} - \frac{v_t}{r} + \frac{c}{r} - \frac{\theta}{r^2}, & v_t < c, \\ V_{1,t} = Be^{\beta_2 v_t}, & v_t \geq c. \end{cases} \tag{B.21}$$

Constants A and B are obtained combining the value matching and the smooth pasting conditions. The first condition asks for $V_{1,t}$ to be the same as $V_{2,t}$ when $v_t = c$:

$$Ae^{\beta_1 c} - \frac{c}{r} + \frac{c}{r} - \frac{\theta}{r^2} = Be^{\beta_2 c}, \tag{B.22}$$

$$Ae^{\beta_1 c} - Be^{\beta_2 c} = \frac{\theta}{r^2}. \tag{B.23}$$

The second condition asks for the derivatives of V_t w.r.t. v_t are the same when $v_t = c$, i.e.

$$A\beta_1 e^{\beta_1 c} - \frac{1}{r} = B\beta_2 e^{\beta_2 c}, \tag{B.24}$$

$$A\beta_1 e^{\beta_1 c} - B\beta_2 e^{\beta_2 c} = \frac{1}{r}. \tag{B.25}$$

Combing the two conditions gives:

$$\begin{cases} Ae^{\beta_1 c} - Be^{\beta_2 c} = \frac{\theta}{r^2} \\ A\beta_1 e^{\beta_1 c} - B\beta_2 e^{\beta_2 c} = \frac{1}{r}, \end{cases} \tag{B.26}$$

we find that the constants are

$$A = e^{-\beta_1 c}\frac{1}{r}\frac{1 - \beta_2\frac{\theta}{r}}{\beta_1 - \beta_2}, \tag{B.27}$$

$$B = e^{-\beta_2 c}\frac{1}{r}\frac{1 - \beta_1\frac{\theta}{r}}{\beta_1 - \beta_2}. \tag{B.28}$$

B.2 Real Option through martingale approach

Starting from $\mathbb{E}_0\left[e^{-r\tau}\right]$, given that $v_\tau - v_0 = \theta\tau + \sigma W_\tau$ and under the assumption that $W_0 = 0$, the Martingale approach exploits the property for which a process without drift is a martingale. Given a process x_t such that $dx_t = \beta x_t dW_t$

$$d\ln x_t = \left(0 + \frac{1}{x_t}0 + \frac{1}{2}\left(-\frac{1}{x_t^2}\right)\beta^2 x_t^2\right)dt + \frac{1}{x_t}\beta x_t dW_t \tag{B.29}$$

$$= -\frac{1}{2}\beta^2 dt + \beta dW_t, \tag{B.30}$$

and

$$\int_0^t d\ln x_s = -\int_0^t \frac{1}{2}\beta^2 ds + \int_0^t \beta dW_s, \tag{B.31}$$

$$\ln x_t - \ln x_0 = -\frac{1}{2}\beta^2 t + \beta\left(W_t - W_0\right), \tag{B.32}$$

$$\frac{x_t}{x_0} = e^{-\frac{1}{2}\beta^2 t + \beta(W_t - W_0)}, \tag{B.33}$$

$$x_t = x_0 e^{-\frac{1}{2}\beta^2 t + \beta W_t}, \tag{B.34}$$

and its expected value is

$$\mathbb{E}_0\left[x_t\right] = x_0, \tag{B.35}$$

$$\mathbb{E}_0\left[x_0 e^{-\frac{1}{2}\beta^2 t + \beta W_t}\right] = x_0, \tag{B.36}$$

$$\mathbb{E}_0\left[e^{-\frac{1}{2}\beta^2 t + \beta W_t}\right] = 1. \tag{B.37}$$

Considering now $W_\tau = \frac{v_\tau - v_0 - \theta\tau}{\sigma}$[34], where v_τ represents the price threshold

$$\mathbb{E}_0\left[e^{-\frac{1}{2}\beta^2\tau + \beta\left(\frac{v_\tau - v_0 - \theta\tau}{\sigma}\right)}\right] = 1, \tag{B.38}$$

$$\mathbb{E}_0\left[e^{-\left(\frac{1}{2}\beta^2 + \beta\frac{\theta}{\sigma}\right)\tau + \beta\frac{v_\tau - v_0}{\sigma}}\right] = 1, \tag{B.39}$$

$$\mathbb{E}_0\left[e^{-\left(\frac{1}{2}\beta^2 + \beta\frac{\theta}{\sigma}\right)\tau}\right]e^{\beta\frac{v_\tau - v_0}{\sigma}} = 1, \tag{B.40}$$

$$\mathbb{E}_0\left[e^{-\left(\frac{1}{2}\beta^2 + \beta\frac{\theta}{\sigma}\right)\tau}\right] = e^{-\beta\frac{v_\tau - v_0}{\sigma}}, \tag{B.41}$$

where $\beta = -\frac{\theta}{\sigma} \pm \sqrt{\left(\frac{\theta}{\sigma}\right)^2 + 2r}$ is the solution of the equation $\frac{1}{2}\beta^2 + \beta\frac{\theta}{\sigma} = r$. From this follows

$$\mathbb{E}_0\left[e^{-r\tau}\right] = e^{-\beta\frac{v_\tau - v_0}{\sigma}}. \tag{B.42}$$

[34] obtained from $v_\tau - v_0 = \theta\tau + \sigma W_\tau$

C Appendix: model with self consumption and no exchange

This scenario is investigated in order to identify the value of flexibility provided by prosumers' cooperative investment and exchange possibility. Under this context, two additional assumptions are introduced:

- the absence of exchange with $\gamma_i = 0$
- prosumers' investment decision is no longer undertaken cooperatively.

In the latter case the investment cost function becomes

$$I(\alpha_i) = \frac{K}{2}\alpha_i^2, \tag{C.1}$$

whereas the new prosumer demand function is

$$\int_0^{24} l(s)\, ds = 1MWh = \xi_i \alpha_i + b_i, \tag{C.2}$$
$$\text{where} \quad \xi_i \in [0,1].$$

The net operative cost function of prosumer i in absence of exchange becomes

$$C_i(\xi_i, \alpha_i) = a\alpha_i + c - v_t\alpha_i - \xi_i\alpha_i(c - v_t)\mathbb{I}_{v_t<c}, \tag{C.3}$$

and each prosumer i solves the following minimization problem

$$\min_{\alpha_i,\tau} \mathbb{E}_0\left[\int_0^{\tau} ce^{-rt}dt + \int_{\tau}^{\infty} C_i(\xi_i,\alpha_i)\, e^{-rt}dt + I(\alpha_i)\, e^{-r\tau}\right]. \tag{C.4}$$

Introducing the extended form of $C_i(\xi_i, \alpha_i)$ and $I(\alpha_i)$, the minimization problem can be rewritten as follows

$$\begin{aligned}\min_{\tau\in[0,\infty],\alpha\geq 0} & \frac{c}{r} - \frac{\alpha_i}{r} v_\tau \mathbb{E}_0\left[e^{-r\tau}\right] + \mathbb{E}_0\left[e^{-r\tau}\right]\frac{K}{2}\alpha^2 \\ & + \alpha_i \mathbb{E}_0\left[e^{-r\tau}\right]\left[\frac{a}{r} - \frac{\theta}{r^2} - \xi_i\left(Ae^{\beta_1 v_\tau} + \frac{c}{r} - \frac{\theta}{r^2} - \frac{v_\tau}{r}\right)\mathbb{I}_{v_\tau<c} - \xi_i B e^{\beta_2 v_\tau}\mathbb{I}_{v_\tau\geq c}\right].\end{aligned} \tag{C.5}$$

The optimal capacity (α_i^*) and the price threshold (v_τ) that triggers the investment for the prosumer i in absence of exchange are defined in two different cases. If $v_\tau < c$, self consumption minimizes the prosumer net operative cost and the optimal capacity and price threshold that triggers the investment are obtained solving numerically the following system:

$$(\alpha_i^*, v^*)_{v_\tau<c} : \begin{cases} \alpha_i^* - \frac{1}{K}\left[\frac{v^*}{r} - \frac{a}{r} + \frac{\theta}{r^2} + \xi_i\left(Ae^{\beta_1 v^*} + \frac{c}{r} - \frac{\theta}{r^2} - \frac{v^*}{r}\right)\right] = 0 \\ -\frac{1}{r} + \frac{v^*}{r}\beta_1 - \frac{K}{2}\alpha_i^*\beta_1 \\ \qquad - \left[\frac{a}{r} - \frac{\theta}{r^2} - \xi_i\left(Ae^{\beta_1 v^*} + \frac{c}{r} - \frac{\theta}{r^2} - \frac{v^*}{r}\right)\right]\beta_1 - \xi_i\left(\beta_1 A e^{\beta_1 v^*} - \frac{1}{r}\right) = 0. \end{cases} \tag{C.6}$$

If $v_\tau \geq c$, the prosumer i minimizes its net operative cost by selling and buying energy to and from the national grid. Also in this case, optimal capacity and price thresholds are obtained solving numerically the following system

$$(\alpha_i^*, v^*)_{v_\tau \geq c} : \begin{cases} \alpha_i^* - \frac{1}{K}\left[\frac{v^*}{r} - \frac{a}{r} + \frac{\theta}{r^2} + \xi_i B e^{\beta_2 v^*}\right] = 0 \\ -\frac{1}{r} + \frac{v^*}{r}\beta_1 - \frac{K}{2}\alpha_i^* \beta_1 - \left[\frac{a}{r} - \frac{\theta}{r^2} - \xi_i B e^{\beta_2 v^*}\right]\beta_1 - \xi_i \beta_2 B e^{\beta_2 v^*} = 0. \end{cases} \tag{C.7}$$

Chapter 3

Exchange of energy among prosumers under prices uncertainty

Remarks The model presented in this Chapter is coauthored with Luca Di Corato[‖], Michele Moretto[**] and Sergio Vergalli[††].
It is submitted to *Energy Economics*, under review.

The model in brief This model provides a theoretical RO framework for modeling prosumers' investment decisions in PV plants in a SG context, when exchange P2P is possible. We focus on the optimal size of their PV plants and on the self-consumption profiles the prosumers must comply with to assure the demand and supply matching in P2P exchange. The model was calibrated to the Northern Italy energy market. We investigate the investment decision under different prosumers' behaviors, taking into account all the possible combinations of their energy demand and supply. Our findings show that the existence of the exchange P2P is not assured in all the cases we have focused on, but depends on the shape and relationship between the supply and demand curves of the two prosumers. The best situation is when the two prosumers have an excess of supply and asymmetric and perfectly complementary demand curves. Sub-optimal cases occur when the exchange P2P and the sell to the national grid are exploited advantageously. This scenario is profitable if there is efficient co-operation between the two agents in exchange choices. Furthermore, prosumers invest in the highest capacity when they are characterized by different exchange P2P and self-consumption profiles, and they reach the maximum gain from the investment in a context characterized by excess supply in exchange P2P.

[‖]Department of Economics, Ca' Foscari University of Venice, Italy
[**]Department of Economics and Management, Università degli Studi di Padova, Italy.
[††]Department of Economics and Management, Università degli Studi di Brescia, Italy. FEEM - Fondazione Eni Enrico Mattei, Milan, Italy.

1 Introduction

The last decade has been characterized by the increasing use of renewable energy sources as alternative to fossil fuels. Such a process has been widely encouraged by policymakers to achieve decarbonization targets. In this context, both in Italy and in other EU countries, a number of distributed power plants have been installed, even though much effort is still required to achieve a sustainable energy future. [1]

Compared to fossil fuels, renewable energy sources are known to be beneficial in terms of environmental impact, but are often characterized by inflexible production compared to load curves. In particular, photovoltaic (PV, hereafter) production shows a certain variability depending on daily and seasonal solar irradiation, but, above all, its production is concentrated in certain daily time slots, leaving night-time demand uncovered and showing problems in managing peak demand.

This makes challenging the management of the electricity grid (for instance in terms of inefficiency, congestion rents, power outages, etc.) which may benefit from the introduction of digitalization for becoming a *smarter* electricity grid[2]. This implies the innovation of the power system, a concept that has also been associated in the last years with the Smart Grids (SG, hereafter) which can be defined as "robust, self-healing networks that allow bidirectional propagation of energy and information within the utility grid".[3]

Such a technological transformation is characterized by three fundamental elements: i) the continuous integration of Distributed Energy Resources (DERs, hereafter), (Sousa et al. (2019); Bussar et al. (2016); Zhang et al. (2018)),[4]; ii) the massive introduction of Information and Communication Technology (ICT) devices (Saad al sumaiti et al., 2014); iii) the central role of the prosumers'[5] production and consumption choices (Luo et al. (2014); Sommerfeldt and Madani (2017); Espe et al. (2018); Zafar et al. (2018)).

The SG context allows and leads the players of the energy markets to adopt new behaviors. With reference in particular to traditional consumers, charac-

[1] International Renewable Energy Agency (IRENA) remarks, in its *Roadmap to 2050*, the importance to boost investments in clean energy technologies since still two-thirds of global greenhouse gas emissions stem from energy production and use.

[2] Campagna et al. (2020) describe the idea of the smart grids as "the merge of digital technology, DES and ICT for energy consumption optimization, which provides and enhances the traditional power grid in terms of flexibility, reliability and safety". Feng et al. (2016) remark the contribution of smart grids in "reducing power outage, lowering delivery costs, encouraging more energy conscious behaviors from consumers" as well as in the transition towards low-carbon economic growth. Moreno et al. (2017) describe in details the evolving landscape from conventional electricity systems to low-carbon smart grids, underlining the transition of distribution networks from passive structures to active systems and the evolution of end-users, which "will become active participants in system and market operation" as well as remarking the "opening up opportunities for aggregating and coordinating consumers and system needs".

[3] Smart Grid definition according EU. *Source* `https://ec.europa.eu/energy/en/topics/market-and-consumers/smart-grids-and-meters`

[4] e.g., from rooftop solar panels, storage and control devices

[5] Consumers who **pro**duce, con**sume** and share energy with other grid users.

terized by a passive behavior in buying and receiving energy from the grid, they gain the opportunity to become proactive in managing their consumption and production (Zafar et al., 2018), reducing their energy consumption costs, by self consuming the energy produced by their PV plants (Luthander et al. (2015); Masson et al. (2016)) as well as integrating effectively and efficiently into the electricity markets (Parag and Sovacool (2016)).[6]

Indeed, the EU's *Clean energy for all Europeans package* [7] has set a new legal framework for the internal energy market and particular attention has been devoted to the benefits of consumers, from both environmental and economic perspectives. The EU Directive 2018/2001[8] formally introduces the *renewables self-consumers* and sets the elements needed to ensure the spread of this status as much as possible.

As widely acknowledged by researchers in this field, the SG deployment, as well as its evolution, is also strictly related to the Peer-to-Peer (P2P, hereafter) energy trading concept .[9]

P2P represents "direct energy trading between peers, where energy from small-scale DERs in dwellings, offices, factories, etc, is traded among local energy prosumers and consumers" (Alam et al. (2017); Zhang et al. (2018))[10].

Exchange P2P can involve households, firms as well as public authorities , participating directly in the energy transition by jointly investing in, producing, selling and distributing renewable energy. The benefits in the energy markets arising for these new players range from their positive contribution in helping utilities to solve the energy management issues (Zafar et al. (2018)) as well as boosting investments in renewables' energy plants, thanks to the potential savings gained from cooperation in investment decisions and from the new flexibility in energy sourcing options. However, it is important to remark that such positive impact strictly depends on the adoption costs of the technology and

[6]SG allow instantaneous interactions between agents and the grid: depending on its needs, the grid can send signals (through prices) to the agents, and agents can respond to those signals and obtain monetary gains as a counterpart. These two characteristics (self-consumption and possible return energy exchange with national grid) can add flexibility that, in turn, increases the value of the investment ((Bertolini et al., 2018), Castellini et al. (2020)).

[7]The EU's *Clean energy for all Europeans package* sets the new energy union strategy with eight legislative acts, where the main pillars are: energy performance in buildings, renewable energy, energy efficiency, governance regulation, electricity market design. The recast of EU Directive 2018/2001 aims "at keeping the EU a global leader in renewable" and sets new binding targets on renewable energy. Directive 2019/944 focuses on the new common rules for the internal market for electricity, where the "consumer is put at the center of the clean energy transition" and new rules are defined with the aim to enable their active participation in this process

[8]`https://eur-lex.europa.eu/legal-content/en/TXT/?uri=CELEX%3A32018L2001`

[9](InterregEU (2018); Luo et al. (2014); Alam et al. (2017); Zafar et al. (2018); Zhang et al. (2018), (Sousa et al., 2019)).

[10]In detail: "peer-to-peer trading of renewable energy means the sale of renewable energy between market participants by means of a contract with pre-determined conditions governing the automated execution and settlement of the transaction, either directly between market participants or indirectly through a certified third-party market participant, such as an aggregator. The right to conduct peer-to-peer trading shall be without prejudice to the rights and obligations of the parties involved as final customers, producers, suppliers or aggregators" (EU (2018)).

the shape of the load (demand) electricity curve of the agents.
With reference to the effects of the direct exchange of energy among prosumers on SG deployment, researchers have analyzed and developed this topic with different perspectives and exploiting various approaches.[11] A wide strand of this literature focuses on the study of the *Microgrids,* as small communities of prosumers, with particular attention to their relationship with the electricity network, also deepening the prosumers' behavioral aspects. Significant importance has been also recognized by researchers to the need of a proper market design for the *prosumer era* (Parag and Sovacool (2016), Morstyn et al. (2018)). Several optimization techniques have been used to investigate prosumers' behaviors in self-consumption, exchange and investment choices (Zafar et al. (2018), Angelidakis and Chalkiadakis (2015), Razzaq et al. (2016)) and most of them focus on cost minimization (Liu et al., 2018). A different approach is provided instead by Gonzalez-Romera et al. (2019), in which the prosumers' benefit is determined minimizing the exchange of energy, instead of the energy cost and by Ghosh et al. (2018) where the price of P2P exchanged energy is defined with the aim of minimizing the consumption of conventional energy, even though prosumers' aim is to minimize their own payoffs.
Yet, there are still several interesting themes related to this topic that requires further development, such as: whether the additional flexibility provided by the exchange P2P may have value, how it could affect the investment decisions, and whether it may be supported by data. Some of the literature has tried to answer these questions: studying the possible combinations of agents in a micro-grid context (Mishra et al. (2019)), or focusing on decentralized energy systems under different supply scenarios (Ecker et al. (2017)); Talavera et al. (2019), investigate the PV plant sizing problem under cost competitiveness and self-consumption maximization perspective whereas Jiménez-Castillo et al. (2019) exploit the net present value (NPV) technique with a similar purpose but focusing also on economic profitability. To the best of our knowledge, problems concerning the possibility of matching load and supply curves in an uncertain environment, as well as in a exchange P2P framework, are yet to be investigated under this perspective.

[11] Among other, a comprehensive review is provided by Hernández-Callejo (2019).

This paper contributes to SG and exchange P2P research as well as to the real option literature in the energy field. [12]
Among these contributions, the closest to ours are: Bertolini et al. (2018) and Castellini et al. (2020) in the field of the optimal plant sizing and investment decisions under uncertainty; Luo et al. (2014) which focuses on the impact of co-operative energy trading on renewable energy utilization in a Microgrid context; Zhang et al. (2018) who investigate the feasibility of P2P energy trading with flexible demand; Gonzalez-Romera et al. (2019) where a minimization problem is developed with the aim to minimize the energy exchange in a framework of two prosumers households; Bellekom et al. (2016) who developed an agent-based model in a residential community context under different prosumption scenario. Our paper provides a theoretical framework for modeling the decision of two agents[13] to invest in a PV plant, assuming they are integrated into an intelligent network (i.e. in a SG context), where exchange P2P is possible. Each agent can produce and consume the energy produced by the PV plant and clear any gap between its production and consumption by trading with both the national grid (N, hereafter) and the other agent. Uncertainty is taken into account by the dynamics of the price paid by the National Energy Market Operator (NEMO, hereafter), managing the Day ahead energy Market[14],to the prosumers for the energy sold to N, which is assumed to be stochastic. Each agent can buy energy from an energy provider that operates on N, paying a different stable price[15], while the price for the exchange of energy P2P (between the two prosumers) is modeled as a weighted average of the two prices for buying and selling energy from and to N. The investment decision is taken cooperatively to allow prosumers to exchange energy P2P. Due to the uncertainty over the demand evolution and market prices, the technological advances, and the ever changing

[12] Mondol et al. (2009), Paetz et al. (2011), Kriett and Salani (2012), Pillai et al. (2014), Moreno et al. (2017), Farmanbar et al. (2019) and Campagna et al. (2020), among others, focuses on technological aspects of SG. Sun et al. (2013), Ciabattoni et al. (2014), Kästel and Gilroy-Scott (2015), Luthander et al. (2015), Ottesen et al. (2016), Bayod-Rújula et al. (2017) investigate the role of prosumers' behaviors, whereas Oren (2001), Salpakari and Lund (2016), Sezgen et al. (2007) study demand-side management and demand-response. With reference to exchange P2P, we recall, among others Angelidakis and Chalkiadakis (2015), Zafar et al. (2018), Ghosh et al. (2018), Liu et al. (2018), Gonzalez-Romera et al. (2019) and Hahnel et al. (2020); whereas in the EC field Mengelkamp et al. (2017), Razzaq et al. (2016), Moret and Pinson (2018),Gui and MacGill (2018), Espe et al. (2018), Morstyn et al. (2018), Sousa et al. (2019) and van Summeren et al. (2020). On the side of the real option literature, we complement the studies about the energy sector, among which Boomsma et al. (2012), Ceseña et al. (2013), Martinez-Cesena et al. (2013), Feng et al. (2016), Kozlova (2017), Tian et al. (2017),Schachter et al. (2016), Schachter and Mancarella (2016), Ioannou et al. (2017).

[13] Such agents are intended as two small households that are willing to become prosumers.

[14] In detail, we use data of the Italian market and for the sake of simplicity we consider the day-ahead prices for a specific zone of Italy. See among others, detailed discussion provided by Gianfreda and Grossi (2012), Andreis et al. (2020). The Italian National Energy Market Operator (NEMO) is Gestore Mercati Elettrici (GME).

[15] We assume that each prosumer enters into a long term contract with a generic energy provider and pays a constant price to buy energy from it to satisfy the energy demand that is not covered by the renewable energy produced by its own PV plant and with the exchange P2P. For the sake of brevity, we will refer to the purchase of energy from the energy provider as purchase of energy from N.

regulatory environment (Schachter and Mancarella (2015); Schachter and Mancarella (2016); Cambini et al. (2016)), we implement a real option (RO) model to capture the value of managerial flexibility associated with the operation of the plant. In a two agents context, our purpose is to understand which characteristics of their supply-demand profiles favor the exchange of energy and if they are compatible with the real existence of an exchange P2P framework. Secondly, we identify the size of the PV plant which maximizes the joint benefit of the two agents and finally focus on the amount of energy exchanged P2P and the self-consumption shares which allow prosumers to reach the highest economic saving.

While the value of self-consumption and exchange (Bertolini et al. (2018), Castellini et al. (2020)) are two topics already studied in the literature, to the best of our knowledge, the conditions for the existence of an exchange P2P structure in a two-agent RO framework and the calculation of exchange energy rates are a novelty.

In order to do this, we study the investment decision under different prosumers' behaviors, taking into account all the possible combinations of energy demand and supply for the two agents in exchange P2P. These are summarized in four scenarios we focus on. Scenario 1 refers to the case of excess of supply from both prosumers. Scenario 2 instead focuses on excess of demand. Scenario 3 shows the case where prosumer 1 needs not more than what the other prosumer could provide, while prosumer 2 needs more than the what prosumer 1 could provide. Scenario 4 instead analyzes the case in which prosumer 2 needs not more than what prosumer 1 could provide, while prosumer 1 needs more than what prosumer 2 could provide. Each scenario is therefore characterized by constraints in terms of energy exchange among the prosumers, leading to specific conditions under which the prosumers' self-consumption behaviors must comply to assure the feasibility of the scenario. In order to calculate the feasibility of our scenarios, we calibrate our model by using Italian energy market data. Model calibration is performed on a dataset built using Italian Zonal Electricity Prices to obtain the parameters of the stochastic price paid by the NEMO to the prosumers for the energy sold to N. The cost of the investment is determined using the methodology of Bertolini et al. (2018) and other parameters refer to data provided by EUROSTAT, IRENA and International Energy Agency (IEA). The main findings of our work are briefly listed here below.

- In all four scenarios there are mathematically feasible conditions for having convenient energy exchange among agents and thus it is optimal to have an exchange P2P structure;

- Among these mathematical conditions, only some are feasible in reality, as only in specific cases the solutions have economic significance and correspond to load and supply curves that can occur in the profile of an agent exchanging energy over the 24 hours;

- The situation which guarantees the existence of an exchange P2P framework and, at the same time, generate the maximum saving is one per scenario;

- Among these, the profiles assuring the maximum benefit (NPV of the generated savings), are characterized by perfectly asymmetric and mutually complementary demand functions: agents produce, consume and exchange energy in such a way as to cover each other's opposite daytime demand functions. If they have an oversupply (as in the case of scenario 1) they also sell some of their production to N in order to maximize the benefit. If they have excess demand (as in the case of scenario 2), they sell nothing to N but cover all their daytime demand with their own energy production.

- The scenarios showing the lowest savings are the two asymmetric scenarios (3 and 4), characterized by excess demand for one agent and excess supply for the other one and viceversa. The combination which guarantees the existence of the exchange P2P framework is the one where an agent produces to self-consume and sell, and the other agent buys the surplus of the other agent and sells all of his production to the grid. The maximum savings are guaranteed by the cooperation of the two agents in such a way that one of them allows the other to maximize its own earnings. Under a cooperative perspective, the gain is shared between the agents. In this context, it is observed that one agent invests in an over-sized PV plant, while the other one chooses a size similar to the ones identified in scenarios 1 and 2.

- In all scenarios, although prosumers are characterized by different supply-demand profiles, very similar total savings are achieved. This depends on the possible combinations that the agents manage to create. In some cases, making the most of mutual exchange, in other cases, producing and exchanging with N , so as to reduce energy costs. The best case, however, is the one where prosumers are characterized by excess supply and asymmetric and complementary load curves.

The novelty of our work can be summarized in two main points: (i) RO methodology is used to identify the optimal size of the PV plant, the quantity of P2P-traded and self-consumed energy; (ii) by studying the different characteristics of supply and demand, four scenarios can be identified. Comparison of the feasible mathematical solutions and the daily 24-hour load curves allow, for each scenario, to identify the optimal combinations to maximize prosumers' savings. The reminder of the paper is the following: in Section 2, we present the basic setup of our model. In Section 3, we identify the expected net energy cost to be borne by each prosumer once the PV project has been activated. In Section 4, we set the optimization problem with the aim to identify the prosumers' optimal capacities of the PV system and describe our four exchange P2P scenarios. For each of the latter, we find analytically the respective prosumers' optimal capacities (detailed in Appendix A.4). In Section 5, we present the model calibration. Section 6 shows our main results and discussion. Section 8 concludes.

2 The basic setup

Consider two households ($i = 1, 2$) who currently purchase energy from a national provider (N, hereafter) at a constant unit energy price $p > 0$[16].
The two agents contemplate the opportunity of setting up an exchange P2P framework, where they would act as *prosumers*. In order to do so, they must cooperatively invest in a project[17] for the installation of i) two individual PV systems and ii) a SG, allowing them to exchange energy with each other, i.e. energy exchange P2P, and with N.[18]
To set up our model, we introduce the following assumptions: [19]

Assumption 1 ***(project time horizon)****. The investment project, once undertaken, lasts forever.*

Assumption 2 ***(individual energy demand).*** *The energy demand of each prosumer i is constant overtime, normalized to 1 and it is covered as follows:* [20]*:*

$$1 = \xi_i \cdot \alpha_i + \gamma_i + b_i \qquad \textit{with } i = 1, 2, \tag{1}$$

where

- α_i represents the capacity power[21] of the PV system installed by each prosumer i per unit of time (henceforth, the PV plant size).Note that, at no loss for what may concern our results, we assume that the PV system, once installed, delivers at each generic time period t an amount of energy equal to the power capacity.

- $\xi_i \in [0, 1]$ is the proportion of α_i used to self-consumption.[22]

[16]This price represents the one agreed between each prosumer and his/her energy provider under a long-term contract.

[17]Cooperation in investment decision affects not only the physical investment, but implies also that once the prosumers decide to build their PV systems, they have to reach an agreement on the price of the energy exchanged P2P. Further discussion will be provided in following Assumption 3.

[18]For the sake of brevity, we will define the purchase of energy from the energy provider, as "purchase of energy from N".

[19]Note that, in terms of model set-up, we share some of our assumptions with Castellini et al. (2020), such as our assumptions 7, 8, 9.

[20]Considering the day (i.e., 24 h) as time reference, equation 1 may be rewritten as follows:

$$\xi_i \cdot \alpha_i + \gamma_i + b_i \ = 1 = \int_0^{24} l(s)\, ds \tag{1.1}$$

where $l(s)$ denotes the instantaneous consumption of energy at each time $s \in [0, 24]$.

[21]α_i is the average production per unit of time and accounts for potential production losses due to variation in temperature, low irradiance, shading and albedo (Bertolini et al., 2018).

[22]The prosumer's instantaneous self-consumption depends on i) the load profile, ii) the location and iii) the renewable energy technology applied and it is, in general, represented as a weakly concave function of the power capacity α_i, i.e. $\xi_i(0)$, $\xi_i'(\alpha_i) > 0$ and $\xi_i''(\alpha_i) \leq 0$. However, based on scientific evidence by, among others, Bellekom et al. (2016), Velik and Nicolay (2016),Pillai et al. (2014) and Mondol et al. (2009), the assumption of a linear function is not too restrictive and provides a reasonable representation of the reality.

- γ_i is the amount of energy that each prosumer i purchases from the other prosumer j, with $i, j = \{1, 2\}$ and $i \neq j$.
- $b_i \geq \bar{b} > 0$ is the amount of energy that prosumer i purchases from N, where $\bar{b} > 0$ is the night-time individual energy demand that must necessarily be covered by purchasing energy from N.[23]

Hence, summing up, the individual energy demand at each time period t can be covered as follows:

$$\begin{aligned} 1 = & \text{ Energy produced and self-consumed, i.e. } \xi_i \cdot \alpha_i \\ & + \text{ Energy purchased from the other prosumer, i.e. } \gamma_i \\ & + \text{ Energy purchased fromt the national grid, i.e. } b_i, \quad \text{with } i = 1, 2. \end{aligned}$$

Assumption 3 ***(energy prices).*** *On the energy market, the prosumers can: i) purchase energy only from N at a constant price $p > 0$[24] and ii) sell the energy produced by their own PV systems only to N at price q_t.[25] We assume that the selling price q_t is stochastic and evolves overtime according to the following Geometric Brownian Motion (GBM):[26]*

$$dq_t/q_t = \theta dt + \sigma d\omega_t, \quad \textit{with } q_0 = q. \tag{2}$$

[23]The amount of energy $\bar{b}$ corresponds to the time interval in which the PV plant is not operating. Note that, in general, its magnitude may depend on the prosumer's daily load patterns, and may be lowered by installing a PV systemLuthander et al. (2015).

[24]We assume each prosumer i pays a constant price p to buy energy from a generic energy provider to satisfy the energy demand that is not covered by PV plant production and with the exchange P2P . The price p is assumed to be constant (see also Bertolini et al. (2018)) because our framework is conceived in a context where the prosumers are households, which sign a long term contract with the generic energy provider. In addition to that, the price p is set differently with respect to the price the prosumers receive from the sell to N, as "the price that results from daily exchanges on the electricity markets is only a fraction of the whole electricity price paid by end-users, both industrial users and domestic ones. The price paid by consumers includes other variables that should be considered"(Biondi and Moretto, 2015).

[25]Note that we are implicitly assuming that the prosumers are price-taker. This is justified by the focus set on investment decisions taken by agents who, as households and due to the small size of their PV plants, are not able to influence the market's price. The price q_t is the one the prosumers receive from the NEMO (National Energy Market Operator), which manages the national Day ahead energy Market. With reference to the Italian market, the NEMO is Gestore Mercati Elettrici (GME). Further details can be found the works of Gianfreda and Grossi (2012), Andreis et al. (2020) and will be provided in the Section devoted to calibration (Section 5).

[26]The GBM is largely used in the field of Real Options and renewable energy (see review of the literature provided by Kozlova (2017)). However, it is important to underline the discussion provided by Borovkova and Schmeck (2017). In their work, they state that a Brownian motion alone, neither in an arithmetic nor geometric form, would be appropriate as the basis model, since electricity prices exhibit more complex features than stock prices. On the other hand, Andreis et al. (2020) provide a complete discussion on the approximation of electricity spot prices with a GBM, clarifying that, even though this process does not provide a realistic representation of the electricity price dynamics, it represents one of the best solution to derive explicit pricing formulae for call options, allowing to present in the most clear way the main features of the model. Since the aim of our work requires closed form solutions to investigate in depth the research question, we acknowledge the discussion on the GBM process and stick to the perspective provided by Andreis et al. (2020).

where θ is the drift rate, σ is the volatility rate, and $d\omega_t$ is the increment of the standard Wiener's process satisfying $\mathbb{E}[d\omega_t] = 0$ and $\mathbb{E}[d\omega_t^2] = dt$.
Process (2) implies that at a generic $t \geq 0$, the price level q_t is log-normally distributed with mean equal to $\ln q + (\theta - \frac{\sigma^2}{2})t$ and variance equal to $\sigma^2 t$. Furthermore, note that as process (2) is memoryless (i.e. Markovian), the observed q_t is the best predictor of future prices available at time t.

Assumption 4 ***(information on prices).*** *The prosumers receive information about selling market price at the beginning of each time period t. For the sake of simplicity, we assume that they can only trade energy on the energy market at this specific time point.*

By Assumption 4, once informed about the selling price, the prosumers decide whether they should sell i) the entire amount of energy produced by their own PV system to N or ii) only part of it, keeping the residual for self-consumption or for the exchange P2P.

Assumption 5 ***(exchange P2P price).*** *The prosumers agree to exchange energy at the price v_t, which is defined as follows:*

$$v_t = mp + (1-m)q_t \qquad \text{with} \quad 0 < m < 1, \tag{3}$$

where, as showed in Appendix A.1, by m and $1-m$, with $m \in (0,1)$, we denote the seller's and buyer's strength exerted in the price bargaining.[27] Note that, when the buying price,p, is higher than the selling price q_t the exchange P2P is always more convenient than purchasing from/selling energy to N since $v_t < p$ and $q_t < v_t$, respectively.

Assumption 6 ***(the investment cost function).*** *Prosumers take the investment decision cooperatively, meaning that at a certain point in time they decide jointly to undertake the investment, paying a sunk cost $I(\alpha_1, \alpha_2)$ for the PV plant set up and securing a total expected production equal to $\alpha_1 + \alpha_2$.*[28]

[27]Zafar et al. (2018) state that the energy price's negotiation is a challenging part of the SG set-up. The model presented by Alam et al. (2013) sets the energy price of the micro-grid in a specific time slot to vary from 0 to the grid energy price. Mengelkamp et al. (2017) design the P2P market such that prosumers and consumers trade with each other individually and in a randomized order on a pay-as bid basis and local prices (thus prices within the micro-grid) are expected to converge to grid prices under perfect information.

It is important to remark that, in our framework, cooperation in investment decision implies also the need of prosumers to reach an agreement on the price of the energy exchanged P2P. From our perspective, it is more than reasonable such event to occur at the same moment in time in which investment decision occurs and that cooperation, and not competition, is the driver of the negotiation on v.

[28]Since we are in context with two prosumers, cooperation in investment decision assures the set up of the exchange P2P framework.

The investment cost function is: [29]

$$I(\alpha_1, \alpha_2) = K_A + K_B \cdot \sum_{i=1}^{2} \frac{\alpha_i^2}{2} \tag{4}$$

where $K_A > 0$ represents the cost to be undertaken in order to install the SG and $K_B > 0$ is a dimensional cost parameter associated with the installation of each individual PV system.
Note that, as for the set-up of the PV system, the investment cost is increasing and convex in the amount of energy produced by each prosumer, i.e. α_i. Differently, the cost associated with the installation of the SG is not affected by the amounts of energy produced by the two prosumers.[30]

Assumption 7 ***(the cost of solar energy).*** *The unit cost of producing solar energy is nil.*[31]

Assumption 8 ***(the discount rate).*** *Prosumers discount future payoffs using risk adjusted interest rate r, where $r > \theta$.* [32]

Assumption 9 ***(no storability)****. The energy produced by the PV plant at each time period t cannot be stored.*

Storability would be highly beneficial for the two prosumers as it would provide additional flexibility in the destination of the energy produced. By Assumption (9), we exclude the possibility of storing energy since, in spite of some promising progresses, storage technologies are still far from being cost effective.[33]

3 The expected energy cost after the activation of the PV project

In this Section, we determine the expected energy cost to be borne by each prosumer once the PV project has been activated. Before proceeding, the following set of feasibility constraints is needed in order to fully characterize the exchange P2P:

[29] We consider a quadratic function for the sake of simplicity. None of our results would be affected if a more general formulation, such as $I(\alpha_1, \alpha_2) = K_A + K_B \cdot \sum_{i=1}^{2} \frac{\alpha_i^\delta}{\delta}$ with $\delta > 1$ is assumed.

[30] As the number of EC members increase, each individual member may benefit from economies of scale for what concerns the fixed cost component K_A.

[31] Since solar radiations represent the production input and are for free, the marginal production costs for the PV power plants may considered negligible (Bertolini et al., 2018, Tveten et al., 2013, Mercure and Salas, 2012).

[32] The discount rate refers the case of a household, who partially covers investment costs by debt (i.e bank loan). Convergence of the model requires the trend in the price evolution not to exceed the discount rate.

[33] See De Sisternes et al. (2016), ESG (2016) and ESG (2016)

i) No prosumer can purchase from the other prosumer more than the amount that the other prosumer does not self-consume, that is:

$$\gamma_i \leq (1-\xi_j) \cdot \alpha_j, \quad \text{with } i, j = \{1,2\} \text{ and } i \neq j. \tag{5}$$

ii) Each prosumer does not purchase from the other prosumer more that s/he actually needs, that is:[34]

$$0 < \gamma_i \leq (1-\bar{b}) - \xi_i \cdot \alpha_i, \quad \text{with } i, j = \{1,2\} \text{ and } i \neq j. \tag{6}$$

Let's denote by c_i the net energy cost of prosumer i at the generic time period t. The following two scenarios must be considered:

1. *No self-consumption and mutual exchange (NSCE):*

$$c_i^{NSCE}(q_t; \alpha_i) = p - \alpha_i q_t, \quad \text{for } i = \{1,2\}; \tag{7}$$

2. *Self-consumption and mutual exchange (SCE):*

$$\begin{aligned} c_i^{SCE}(q_t; \alpha_i, \gamma_i, \gamma_j) &= (1 - \xi_i\alpha_i - \gamma_i)\, p + (\gamma_i - \gamma_j)[mp + (1-m)q_t] + \\ &\quad - (\alpha_i - \xi_i\alpha_i - \gamma_j)\, q_t \\ &= p - \alpha_i q_t + S_i(q_t; \alpha_i, \gamma_i, \gamma_j)(q_t - p), \qquad (8) \\ \text{for} &\quad i, j = \{1,2\} \text{ with } i \neq j. \\ \text{where} &\quad S_i(q_t; \alpha_i, \gamma_i, \gamma_j) = \xi_i\alpha_i + (1-m)\gamma_i + m\gamma_j. \quad (9) \end{aligned}$$

Note that, as for the amount of energy produced by her/his own PV system, each prosumer chooses how much energy should be sold to N rather than be self-consumed or sold to the other prosumer. Hence, at any instant, the prosumer energy cost, c_i, can be minimized by solving the following problem:[35]

$$\begin{aligned} c_i(q_t; \alpha_i, \alpha_j, \gamma_i, \gamma_j) &= \min[c_i^{NSCE}(q_t; \alpha_i), c_i^{SCE}(q_t; \alpha_i, \gamma_i, \gamma_j)] \\ &= p - \alpha_i q_t + \min\{0, S_i(q_t; \alpha_i, \gamma_i, \gamma_j)(q_t - p)\}. \quad (10) \end{aligned}$$

The solution of Problem (10) is:

$$c_i(q_t; \alpha_i, \alpha_j, \gamma_i, \gamma_j) = \begin{cases} c_i^{NSCE}(q_t; \alpha_i), & \text{for } q_t > p, \\ c_i^{SCE}(q_t; \alpha_i, \gamma_i, \gamma_j), & \text{for } q_t \leq p, \end{cases} \tag{11}$$

since:

$$\begin{aligned} c_i^{NSCE}(q_t; \alpha_i) &< c_i^{SCE}(q_t; \alpha_i, \gamma_i, \gamma_j) \quad \text{for} \quad q_t > p \\ c_i^{NSCE}(q_t; \alpha_i) &\geq c_i^{SCE}(q_t; \alpha_i, \gamma_i, \gamma_j) \quad \text{for} \quad q_t \leq p \end{aligned}$$

[34] When $q_t < p$, $b_i = \bar{b}$ since purchasing energy from the other prosumer at price v_t is cheaper than purchasing it from N at price p.

[35] Note that in the following we omit for notational convenience that all the equations holds for $i, j = \{1,2\}$ with $i \neq j$.

Let's now firstly consider the range of values $q_t > p$ and denote by $C_i^{NSCE}(q_t;\alpha_i)$ the expected present value taken at the generic time period $t \geq 0$ of the flow of periodic net energy costs to be paid over the assumed time horizon. Using standard arguments, $C_i^{NSCE}(q;\alpha_i)$ solves the following Bellman equation:

$$C_i^{NSCE}(q_t;\alpha_i) = c_i^{NSCE}(q_t;\alpha_i)\,dt + \mathbb{E}_t\left[e^{-rdt}C_i^{NSCE}(q_{t+dt};\alpha_i)\right], \qquad (12)$$

where the first term is the net energy cost borne over the generic time interval $(t, t+dt)$ and the second term is the continuation value.
By a straightforward application of the Ito's Lemma to Eq. (12), $C_i^{NSCE}(q;\alpha_i)$ can be determined by solving the following differential equation:

$$\Gamma C_i^{NSCE}(q_t;\alpha_i) = -c_i^{NSCE}(q_t;\alpha_i), \quad \text{for } q_t > p, \qquad (11.1)$$

where $\Gamma = -r + \theta q \frac{\partial}{\partial q_t} + \frac{1}{2}\sigma^2 q_t^2 \frac{\partial^2}{\partial q_t^2}$ is a the differential operator.
Let's now turn to the range of values $q_t < p$ and denote by $C_i^{SCE}(q_t;\alpha_i,\gamma_i,\gamma_j)$, the expected present value taken at the generic time period $t \geq 0$ of the flow of periodic net energy costs to be paid over the assumed time horizon. As above, $C_i^{NSCE}(q;\alpha_i)$ is the solution of the following Bellman equation:

$$\begin{aligned} C_i^{SCE}(q;\alpha_i,\alpha_j,\xi_i,\gamma_i,\gamma_j) &= c_i^{SCE}(q_t;\alpha_i,\gamma_i,\gamma_j)\,dt \\ &\quad + \mathbb{E}_t\left[e^{-rdt}C_i^{SCE}(q_{t+dt};\alpha_i,\gamma_i,\gamma_j)\right] \end{aligned} \qquad (13)$$

where the first term is the net energy cost borne over the generic time interval $(t, t+dt)$ and the second term is the continuation value.
By applying the Ito's Lemma to Eq. (12), $C_i^{NSCE}(q;\alpha_i)$ can be determined by solving the following differential equation:

$$\Gamma C_i^{SCE}(q_;\alpha_i,\gamma_i,\gamma_j) = -c_i^{SCE}(q_t;\alpha_i,\gamma_i,\gamma_j), \text{for } q_t < p \qquad (12.1)$$

The solutions of Eqs. (11.1) and (12.1) are subject to the following boundary Conditions:

$$\lim_{q_t \to \infty} C_i^{NSCE}(q_t;\alpha_i) = \frac{p}{r} - \alpha_i \frac{q_t}{r-\theta}, \qquad (11.2)$$

and

$$\lim_{q_t \to 0} C_i^{SCE}(q_t;\alpha_i,\gamma_i,\gamma_j) = \frac{p}{r} - \alpha_i \frac{q_t}{r-\theta} - S_i(q_t;\alpha_i,\gamma_i,\gamma_j)\left(\frac{p}{r} - \frac{q_t}{r-\theta}\right) \qquad (12.2)$$

respectively. The term $\frac{p}{r} - \alpha_i \frac{q_t}{r-\theta}$ represents the expected present value of the flow of the net energy costs conditional on i) purchasing all the energy needed by prosumer i from N and ii) selling all the energy produced by his/her the PV system to N. This is, of course, the case when $q_t > p$. Further, note that, if the capacity installed is sufficiently high, i.e. $\alpha_i > \frac{p}{r} / \frac{q_t}{r-\theta}$, the prosumer earns a profit. In contrast, when $q_t < p$, self-consumption and mutual exchange of energy are more convenient than trading energy (selling to and buying from) with N. The expected present value of the flow of periodic gains associated with

self-consumption and mutual exchange of energy is equal to $S_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)(\frac{p}{r}-\frac{q_t}{r-\theta})$ which is, consistently, decreasing in q_t.
As shown in Appendix A.2, by the linearity of Eq. (11.1)and (12.1) and taking into account Condition (11.2) and (12.2), the solution of the prosumer's cost minimization problem, i.e.

$$\begin{array}{ll}\Gamma C_i^{NSCE}\left(q_t;\alpha_i\right)=-c_i^{NSCE}\left(q_t;\alpha_i\right), & \text{for } q_t>p,\\ \Gamma C_i^{SCE}\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)=-c_i^{SCE}\left(q_t;\alpha_i,\gamma_i,\gamma_j\right), & \text{for } q_t<p,\end{array} \tag{14}$$

is:

$$C_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)=\begin{cases} C_i^{NSCE}\left(q_t;\alpha_i\right)=\frac{p}{r}-\alpha_i\frac{q_t}{r-\theta} \\ \qquad +S_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)X^{NSCE}\left(\frac{q_t}{p}\right)^{\beta_2} \\ \qquad\qquad \text{for} \quad q_t>p, \\ C_i^{SCE}\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)=\frac{p}{r}-\alpha_i\frac{q_t}{r-\theta} \\ \qquad -S_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)\left[\left(\frac{p}{r}-\frac{q_t}{r-\theta}\right)-Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right] \\ \qquad\qquad \text{for} \quad q_t<p, \end{cases} \tag{15}$$

where $\beta_2<0$ and $\beta_1>1$ are the roots of the characteristic equation $\Phi\left(x\right)\equiv\frac{1}{2}\sigma^2x\left(x-1\right)+\theta x-r$ and

$$X^{NSCE} = \frac{p}{r-\theta}\frac{r-\theta\beta_1}{r\left(\beta_2-\beta_1\right)}\leq 0, \tag{16}$$

$$Y^{SCE} = \frac{p}{r-\theta}\frac{r-\theta\beta_2}{r\left(\beta_2-\beta_1\right)}\leq 0. \tag{17}$$

In the first branch of $C_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)$, the term $S_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)X^{NSCE}\left(\frac{q_t}{p}\right)^{\beta_2}$ represents the expected present value of the option to switch from the $NSCE$ to the SCE scenario as soon as $q_t<p$. Note that the closer q_t to p, the lower the stochastic discount factor $\left(\frac{q_t}{p}\right)^{\beta_2}$ and, consequently, the higher the value of the option to switch. This is because the expected amount of time that the prosumer must wait before switching is lower.

Turning to the second branch of $C_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)$, the term $S_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}$ represents the value associated with the flexibility to switch from the SCE to the $NSCE$ scenario as soon as $q_t>p$. As above but moving from below this time, the closer $q_t$to p, the lower the stochastic discount factor $\left(\frac{q_t}{p}\right)^{\beta_1}$ and the higher the value of the flexibility to switch. This is because the switch will occur earlier in expected terms.

4 The optimal PV system's capacities

In this Section, we determine the optimal PV system's capacities that each prosumer should install in order to maximize the value of the joint investment project. Let's start by identifying the project's value considering, for the sake of simplicity, a scenario where self-consumption and exchange P2P would be, once the investment is activated, immediately convenient, i.e. when $q_t < p$.
A necessary condition for investing in the project is the arising of a benefit from it, with respect the status quo scenario, that is, not producing her/his own energy and covering her/his own needs by purchasing energy from N at price p. In Appendix A.3, we show that this condition is met since:

$$\Delta C_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)=\frac{p}{r}-C_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)>0, \tag{18}$$

that is, the energy cost associated with the status quo scenario, i.e. $\frac{p}{r}$, which, once invested, is implicitly saved, and it is higher than the expected energy cost associated with the PV project, i.e. $C_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)$.
By Assumption (6), the two prosumers take the investment decision cooperatively, which implies that they determine jointly the optimal capacities of their PV systems. The optimal pair, (α_1^*,α_2^*) must be such that the expected net present value of the PV project is maximized. Formally:

$$\begin{aligned}(\alpha_1^*,\alpha_2^*) &= \arg\max \mathcal{O}\left(\alpha_1,\alpha_2\right),\\ &\quad \text{s.t.}\quad (5)\quad \text{and}\quad (6)\quad \text{hold}\end{aligned} \tag{19}$$

and where

$$\begin{aligned}O\left(\alpha_1,\alpha_2\right) &= \Delta C_1\left(q_t;\alpha_1,\gamma_1,\gamma_2\right)+\Delta C_2\left(q_t;\alpha_2,\gamma_2,\gamma_1\right)-I(\alpha_1,\alpha_2)\\ &= \left(\xi_1\alpha_1+\gamma_1+\xi_2\alpha_2+\gamma_2\right)\left[\frac{p}{r}-\frac{q_t}{r-\theta}-Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right]+\\ &\quad +\left(\alpha_1+\alpha_2\right)\frac{q_t}{r-\theta}-I\left(\alpha_1,\alpha_2\right)\end{aligned} \tag{20}$$

is the expected net present value of the PV project.
We now investigate the investment decision under four different *P2P exchange scenarios*. Each of them is characterized by different constraints in terms of energy exchanged P2P, leading to specific feasibility conditions. Next, we present the overall framework for each scenario, while in Appendix A.4 we show the respective feasible mathematical solutions of Problem (19), distinguishing the internal solutions and the corner solutions. However, it must be stressed that the mathematical solutions are not always feasible in a real context, as they may identify daily supply and demand pairings that cannot be realized over a 24-hour period for two representative agents. In Section 6 we provide discussion on the real feasibility of the scenarios according to the outcomes obtained from the calibration of the model and in line with the mathematical results found in Appendix A.4.

Scenario 1: excess supply in the energy exchange P2P. In Scenario 1 we focus on the case of excess supply from both prosumers in exchange P2P and the constraint presented in Eq. (6) is detailed as follows[36]:

$$0 < (1-\bar{b}) - \xi_1\alpha_1 < (1-\xi_2)\alpha_2, \tag{21}$$

$$0 < (1-\bar{b}) - \xi_2\alpha_2 < (1-\xi_1)\alpha_1 \tag{22}$$

In the mid of both Inequalities (21) and (22), we find the quantity of energy that each prosumer demand from the other prosumer, i.e. $(1-\bar{b}) - \xi_1\alpha_1$ and $(1-\bar{b}) - \xi_2\alpha_2$, that is, the residual quantity of energy needed once i) purchased the amount $\bar{b}$ from N [37] and ii) consumed his/her own produced energy, i.e. $\xi_1\alpha_1$ and $\xi_2\alpha_2$. Both amounts must, of course, be positive. On the RHS we find instead the quantity of energy that the other prosumer could actually supply, that is, the residual quantity of energy produced not self-consumed, i.e. $(1-\xi_2)\alpha_2$ and $(1-\xi_1)\alpha_1$. As it can be immediately seen, under this scenario, the exchange P2P is characterized by an excess supply since $(1-\bar{b}) - \xi_i\alpha_i] < (1-\xi_j)\alpha_j$ for $i, j = 1, 2$ with $i \neq j$. In other words, the quantity of energy demanded by each prosumer is lower than the quantity that the other prosumer could actually provide.

Scenario 2: excess demand in the energy exchange P2P. In Scenario 2 there is excess of demand from both prosumers and Eq. (6) becomes:

$$(1-\bar{b}) - \xi_1\alpha_1 \geq (1-\xi_2)\alpha_2 > 0, \tag{23}$$

$$(1-\bar{b}) - \xi_2\alpha_2 \geq (1-\xi_1)\alpha_1 > 0. \tag{24}$$

If Inequalities (23 and/or (24) hold strictly, the quantity of energy that each prosumer demand to the other prosumer, i.e. $(1-\bar{b}) - \xi_i\alpha_i$, is higher than the quantity of energy that each prosumer may actually supply, i.e. $(1-\xi_j)\alpha_j$. This implies that the exchange P2P is characterized by an excess demand since $(1-\bar{b}) - \xi_i\alpha_i] > (1-\xi_j)\alpha_j$ for $i, j = 1, 2$ and $i \neq j$. Otherwise, if (23 and/or (24) hold with the equality, the quantity of energy demanded equals the quantity of energy supplied.

Scenario 3: non complementarity in the energy exchange P2P. Under Scenario 3, prosumer 1 demand less energy than the quantity that prosumer 2 could provide while prosumer 2 may need i) more energy than the quantity that prosumer 1 could provide or ii) exactly the quantity that prosumer 1 could provide. The constraint characterizing this scenario are the following:

[36]Eq. (21) refers to prosumer 1 and (22) to prosumer 2. The same occurs in the following scenarios.

[37]We remind that when $q_t < p$, $b_i = \bar{b}$ since purchasing energy from the other prosumer at price v_t is cheaper than purchasing it from N at price p.

$$0 < (1-\bar{b}) - \xi_1\alpha_1 < (1-\xi_2)\alpha_2, \tag{25}$$

$$(1-\bar{b}) - \xi_2\alpha_2 \geq (1-\xi_1)\alpha_1 > 0. \tag{26}$$

Scenario 4: non complementarity in the energy exchange P2P. Scenario 4 is symmetric to scenario 3. In fact, in this case, prosumer 2 demand less energy than the amount that prosumer 1 could provide while prosumer 1 may need i) more energy than the quantity that prosumer 2 could provide or ii) exactly the quantity that prosumer 2 could provide.

$$(1-\bar{b}) - \xi_1\alpha_1 \geq (1-\xi_2)\alpha_2 > 0, \tag{27}$$

$$0 < (1-\bar{b}) - \xi_2\alpha_2 < (1-\xi_1)\alpha_1. \tag{28}$$

5 Calibration of the model

Concerning the unit price q_t paid to the prosumers selling energy to N, the dataset is built using hourly Italian Zonal Prices for Northern Italy from 2012 to 2018. The dataset is built using Italian Zonal Prices[38], where q_t refers to Northern Italy region and time interval is set from 2012 to 2018. We take into account only the prices relative to the hours where the PV plant is operating, that is, from 8 a.m. to 7 p.m.. Average quarterly prices are then computed and seasonally adjusted.
To test whether the price q_t follows a GBM with drift, non stationarity is checked using the *Shapiro Test* [39] and the *Augmented Dickey-Fuller Test* (ADF) [40].
The drift rate, θ, and the volatility rate, σ, of the process for the price q_t are computed using the method of moments. Their estimates (θ, σ) are obtained by plugging the sample mean $(\hat{\theta})$ and variance $(\hat{\sigma})$ into $\theta = \left(\hat{\theta} + \frac{1}{2}\hat{\sigma}^2\right) dt$ and $\sigma = \frac{\hat{\sigma}}{\sqrt{dt}}$. The annual drift θ and the volatility σ are equal to 0.01 and 0.32, respectively.[41]
The value of the price q_t for both prosumers in assumed to be the average value over the reference time interval and it is set equal to 58.86 euro/Mwh.
The price paid by the prosumers to buy energy from N (p) is set equal to 154.00

[38] The Spot Electricity Market (MPE) is part of the Italian wholesale electricity market, or IPEX (Italian Power Exchange). It consists of the Day ahead Market (MGP), the Intra-day Market, (MA o MI) and the Ancillary Services Market (MSD). The MGP is a single implicit auction market where market zonal market clearing prices are determined and it is managed by the National Energy Market Operator (NEMO) *Gestore Mercati Energetici* (GME). A detailed discussion about the Italian zonal market framework is provided by Gianfreda and Grossi (2012).

[39] Shapiro-Wilk normality test: W = 0.94926, p-value = 0.2057

[40] Dickey-Fuller =-1.8958, Lag order = 3, p-value = 0.6124, alternative hypothesis: stationary. ADF test null hypothesis is failed be to rejected, thus non stationarity assumption is confirmed.

[41] The estimates were computed on the basis of quarterly average prices and then put in annual terms.

euro/Mwh, that is the average value of the electricity prices payed by Italian households consumers over the reference time interval according Eurostat.[42]
The discount rate r results from the average of the values used in Bertolini et al. (2018) and it is set equal to 0.05.
The model calibration is performed normalizing the demand of energy to 1Mwh/y. The dimensional investment cost parameter K_B of the investment cost function $I(\alpha_1, \alpha_2)$ is computed following Bertolini et al. (2018). The unit of measure of the PV plant's size α_i is kWh/year. It is always possible to obtain the average amount of energy produced by the PV plant over a certain time interval in kWh, i.e., in a year. Following Bertolini et al. (2018) (Appendix B), the plant energy output is the product of the size (kWp) and the local solar insolation that takes capacity factor into account (kWh/kWp/year). If the cost of the plant per kWp is known, it is also possible to trace, using LCOE, the cost of the plant as a function of the energy produced in a year, as in the following equation: $K_B = 2\frac{LCOE}{r}\left(1 - e^{-rT}\right)$. This allows to construct a cost function in terms of kWh/year instead of kWp.
The assumed average plant life time, T, is set equal to 25 years.[43] The levelized cost of energy (LCOE) for the PV technology is set equal to 80 euro/MWh.[44]
The parameter K_A represents the cost the prosumers pay to be connected to the SG and we set it equal to $0.15K_B$.[45]
Table 1 summarizes all the parameters used for the model calibration

[42]Eurostat - Energy Statistics, Electricity prices for household consumers - bi-annual data (from 2007 onwards) [nrg_pc_204]. The data are in Euro currency, refer to an annual consumption betwween 2 500 and 5000 kWh (Band-DC, Medium), excluding taxes and levies.

[43]See Branker et al. (2011),Kästel and Gilroy-Scott (2015).

[44]Lazard (2020) ranges the LCOE (unsubsidized) values for Solar PV Rooftop Residential from 154 to 227 USD/MWh, for Solar PV Rooftop CI from 74 to 179 USD/MWh, and for Solar PV Community from 63 to 94 USD/MWh.

[45]With reference to Italy, we set parameter K_A on the basis of the fees of these two projects: "REGALGRID"(`https://www.regalgrid.com/`), where the average fee is 400 euro/year (Peloso, 2018) and "sonnenCommunity" (`https://sonnengroup.com/sonnencommunity/`), where the monthly fee is 20 euro/month.

Parameter	Description	Value	Source/Reference
θ	drift	0.01	Calibrated on Northern Italy zonal prices, NEMO GME.
σ	volatility	0.32	Calibrated on Northern Italy zonal prices, NEMO GME.
q	average level of the price q_t over time period	58.85	Northern Italy zonal prices, NEMO GME.
p	cost to buy energy from the national grid	154.00	Eurostat, Energy Statistics, Electricity prices for household consumers.
$\bar{b}$	minimum amount of energy prosumers buy from the national grid	0.40	Luthander et al. (2015), Weniger et al. (2014).
T	PV plant lifetime (years)	25	Branker et al. (2011), Kästel and Gilroy-Scott (2015).
r	discount rate	0.05	Bertolini et al. (2018).
$LCOE$	levelized cost of electricity for PV plants euro	80.00	Lazard (2020).
K_A	cost to set up the SG	342.48	Own computation, Peloso (2018).
K_B	PV dimensional investment cost parameter	2283.18	Own computation, Bertolini et al. (2018).
β_1	Root	1.41	Own computation.
β_2	Root	-0.67	Own computation.

Table 1: Parameters

6 Results

In this Section, we present the main findings obtained running our model according to the calibration presented in Section 5. For each scenario[46], our aims are as follows: i) to investigate the role of self-consumption as a driver for setting up the exchange P2P (ξ_1, ξ_2), ii) to determine the optimal capacity of the individual PV system, i.e. (α_1^*, α_2^*), and iii) to determine the expected NPV of the PV project, i.e. $(O(\alpha_1^*, \alpha_2^*))$.
The solutions of Problem (19) lead to several feasible outcomes. However, some of them, even if mathematically sound, are not realistic. This is, for instance, the case for outcomes where both prosumers exchange all the energy individually produced, i.e. no self-consumption, or they self-consume all the energy individually produced, i.e. no energy exchange (see appendix A.5).
Another case is the situation in which the division of the day into production

[46]Note that we provide only the findings relative to Scenarios 1, 2 and 3. Scenario 4 is excluded since findings would be symmetric with respect to those obtained in Scenario 3.

and hourly consumption does not allow supply and demand to meet in the same time slot, even if this equilibrium is mathematically contemplated in model like ours, where an entire day is compressed in an unique time point. Once again, this implies that, although some solutions are mathematically feasible, they identify supply and demand pairs that cannot occur over the course of a day, since they would ideally imply an instantaneous exchange of all quantities consumed during the day.
In the light of these remarks, in Table 2, we show the outcomes that, in our view, are the most representative of our four scenarios. Our selection takes into account the following requirements: 1. the outcomes are all mathematically feasible, as we show in the Appendix A.4; 2. we identify those outcomes that are consistent with realistic daily supply and demand curves; 3. we focus on those characterized by the highest NPV. The first block of Figure 1 shows the different self-consumption sets, while the second how the prosumers' demands are covered under the different exchange scenarios. In the latter, the optimal capacity levels are included as well.
Interestingly, the outcomes we show have similar NPVs despite presenting very different supply and demand functions. Furthermore, we discuss the circumstances under which an exchange P2P framework may be set up and the roles played in this process by both prosumers and the NEMO.
Computational details concerning each considered scenario are presented in Appendix A.5.

Parameters	*Scenario 1*	*Scenario 2*	*Scenario 3*
$\xi_1 \in$	$[0.43; 0.58]$	$(0.50; 1]$	$[0.51; 0.52]$
$\xi_2 \in$	$[0.43; 0.58]$	$(0.50; 1]$	$[0; 0.02]$
α_1^*	0.710	0.600	1.152
α_2^*	0.710	0.600	0.720
$\xi_1\alpha_1^*$	0.360	0.426	0.593
$\xi_2\alpha_2^*$	0.360	0.426	0.007
γ_1^*	0.240	0.173	0.007
γ_2^*	0.240	0.173	0.559
$\mathcal{O}\left(\alpha_1^*, \alpha_2^*\right)$	3301	3098	3012

Table 2: *Results*

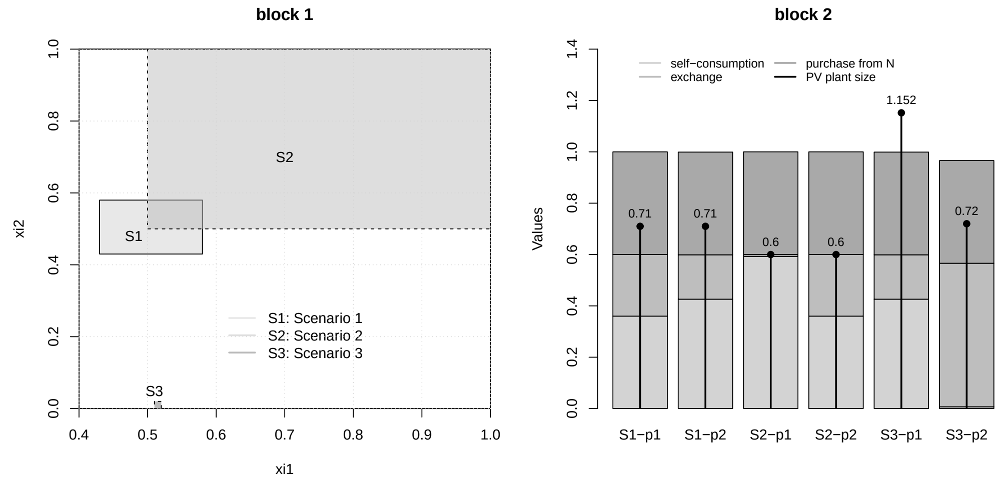

Figure 1: Block 1 - Self-consumption sets under different scenarios, where with S1 we refer to scenario 1, S2 to scenario 2 and with S3 to scenario 3.
Block 2 - Prosumers' demand coverage and optimal capacity levels, where with P1 we refer to prosumer 1 and with P2 to prosumer 2.

In the following, we show the representative outcomes of the different scenarios, studying their characteristics in order to understand which is the best one and the key elements that makes it better than the other cases.
According to the requirements we have listed above, in the second column of Table 2, we present the outcome from scenario 1 characterized by the highest NPV and, Figure 2, we show a realistic combination of supply and demand that can support it.

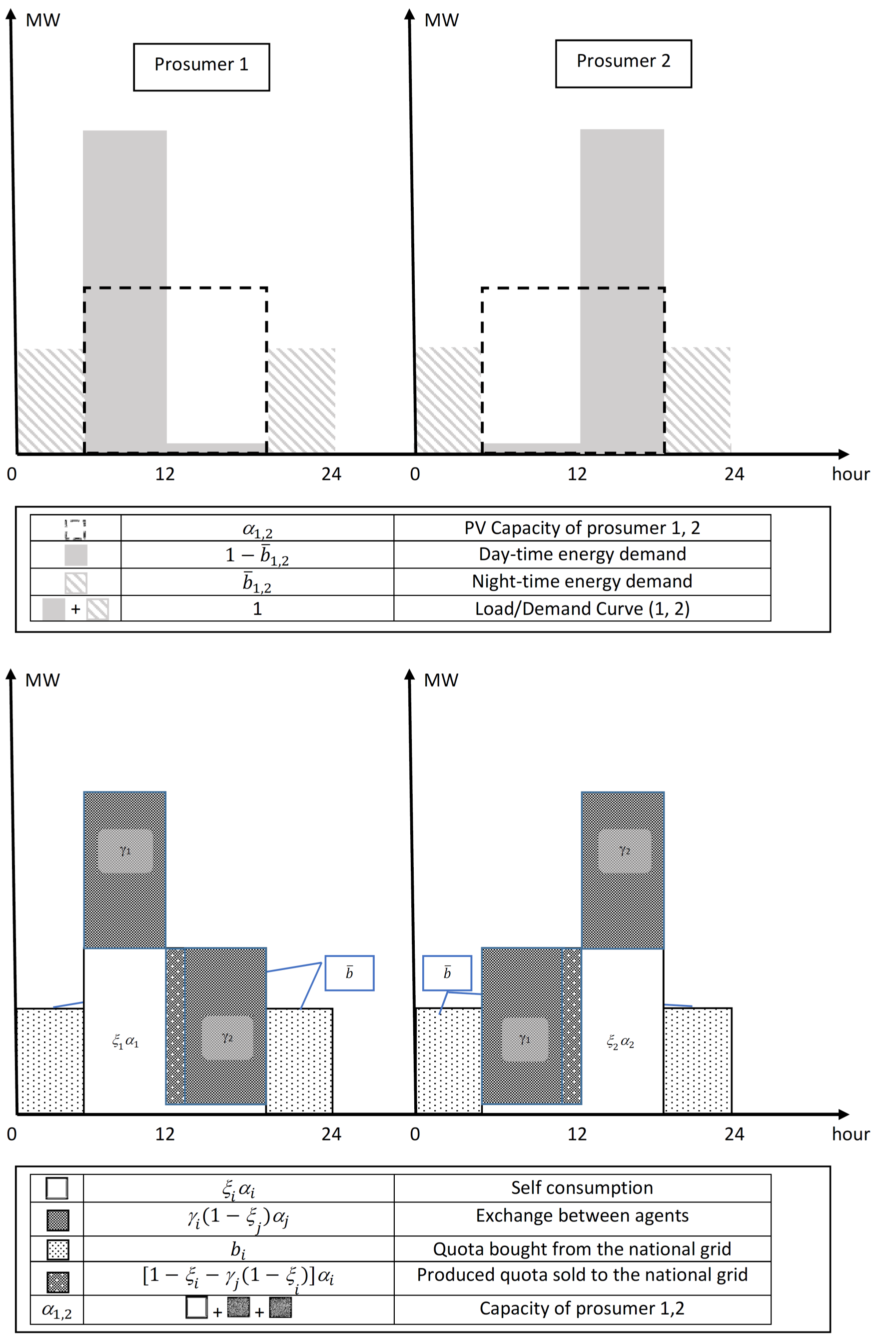

Figure 2: *Scenario 1* - Load and supply curves and distribution of energy trade and consumption.

In greater detail, at the top of Figure 2 we find, for each agent, the daily load curves over 24 hours and the amount of energy produced by the PV system of each prosumer. The dashed areas correspond to the night demand, while the gray ones correspond to the day demand. The dashed frame represents the capacity of the PV system. In the lower part of the diagram we show how the PV system's production is split between self-consumption, energy exchange with the other prosumer and sold to N. In the following, we show how the individual demand is covered through self-consumption, energy exchange with the other prosumer and purchases from N. The dark gray areas represent the energy exchange between the two agents, i.e. γ_1 and γ_2.
From Table 2, we can observe that the two prosumers have an energy production of the same size (0.710) and asymmetric-complementary demand functions. In this way, one prosumer manages to sell its excess production to the other, exactly when the other agent needs it. The two prosumers, by acting cooperatively, manage to have an optimal symmetrical plant size (0.710) that allows avoiding the purchase of daytime energy from N. We remind that this scenario is characterized by excess supply. Therefore, the two prosumers are able to fully meet their own energy needs, without buying daytime energy from N and, at the same time, each being able to sell 0.110 to it. Self-consumption is about 50% of PV production, which corresponds to 36% of the total demand. On the other hand, the two self-consumption profiles must be quite similar and closer as the optimal capacity increases.[47]. As Table 4 (Appendix A.4.1) shows, a negative relation occurs between the self-consumption and exchange. A decrease in the price q_t lowers the prosumers' optimal capacities, as well as the NPV, while the self-consumption maximum level increases, although the prosumers must be characterized by much similar profiles.[48] The effect of a lowering of the $LCOE$ (i.e a decrease in the cost of the PV project) increases the optimal capacity, allows prosumers' self consumption profile to diverge more but reduces its maximum achievable levels. [49]
In the third column, we find the outcome from scenario 2. The NPV is very close to the one in scenario 3. We notice also that these two scenarios are very similar in terms of demand and supply composition. As can be seen, self-consumption is about 42.6% of the total demand, or about 71% of PV production. This scenario is characterized by excess demand from both prosumers. Among the feasible outcomes, the one having the highest NPV, is actually a corner solution in which the two agents manage to fully cover their demand with a mutual energy exchange[50]. Again, the two demand functions are asymmetrical, as shown by Figure 3.
Compared to scenario 1, the two prosumers do not sell any energy to N and we

[47]In Figure 6, presented Appendix A.4.1, the distance between the self-consumption parameters, thus $\xi_1 - \xi_2$, represents the heterogeniety in the prosumers' self consumption profile. The higher is such difference, more the prosumers' self- consumption choices are allowed to diverge.

[48]See Table 3 and Figure 8 in Appendix A.4.1.

[49]See Table 3 and Figure 9 in Appendix A.4.1.

[50]We refer to corner solution 3. See Figure 3 and Table 6 in Appendix A.4.2.

are in a situation in which the PV plant size is set at the maximum daytime consumption. In this way the two agents can minimize their costs, but they do not get an extra profit by selling excess of energy production to N, as is the case in scenario 1. See for example, in the following figure 4, the results of the corner solution 1 of the scenario 2 .[51] The two PV plants' sizes are equal to 0.535 and 0.488, while self-consumptions are 0.228 and 0.294, respectively. This case shows that although there is also an exchange of the produced energy of 0.194 and 0.306, the two prosumers cannot satisfy all their demand. They have to buy from N an amount of 0.177, which is 18% of the total demand of one agent. This combination leads to a lower $\mathcal{O}\left(\alpha_1^*, \alpha_2^*\right)$ which is equal to 2823.

[51] See the second column of the table 6 in the Appendix A.5.

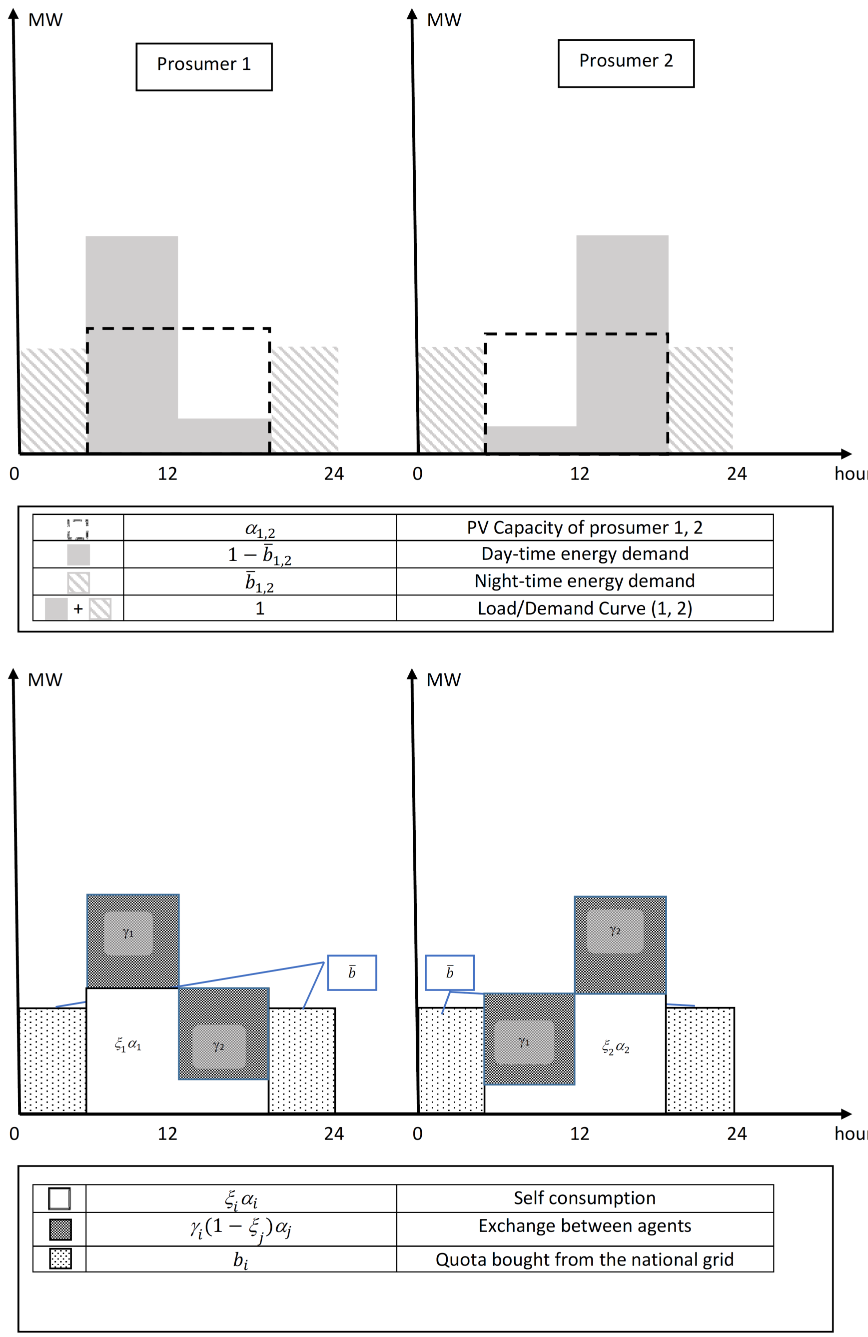

Figure 3: *Scenario 2, Corner Solution 3* - Load and supply curves and distribution of energy trade and consumption.

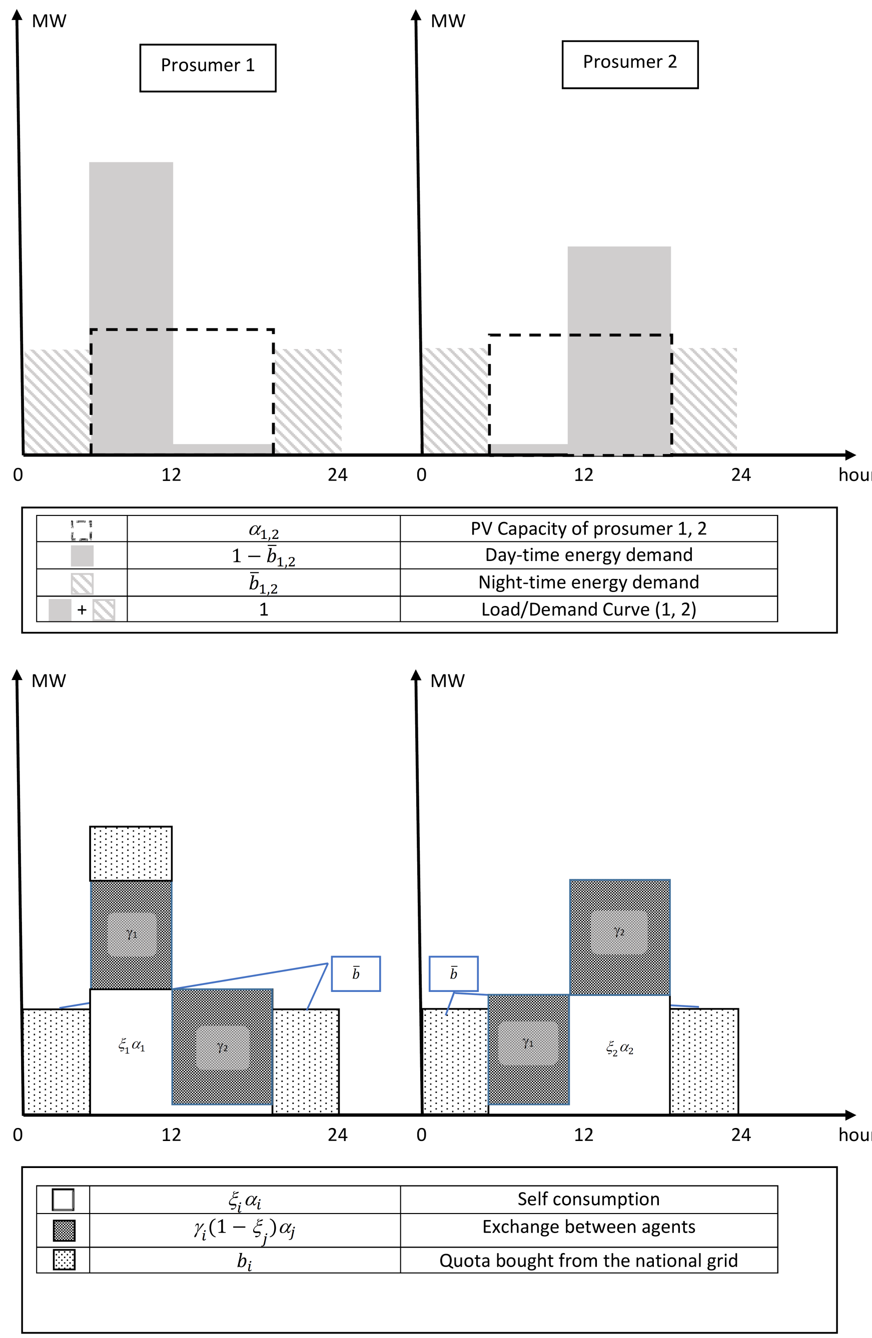

Figure 4: *Scenario 2* - Load and supply curves and distribution of energy trade and consumption.

About the scenario 2, we can conclude that the best outcomes are only feasible if the self-consumption levels (ξ_1 and ξ_2) are quite high, in particular higher than 0.50. This requires a relatively small PV systems' capacity. Otherwise, the agents would have too much energy to sell to N and this would be sub-optimal. Indeed the PV plants' sizes are equal to 0.60. This allows, in proportion a high level of self-consumption, as the results show. The consideration of all these features justify a lower NPV. With reference finally to the comparative statics (see Appendix A.4.2), performed changes in price q_t, volatility σ and $LCOE$,do not affect the feasibility of the optimal solution.[52] and corner solution 3 still represents the best solution.[53] Scenario 3 shows the non complementarity case. Our expectation is an asymmetric solution, because we are in a context where prosumer 1 needs in exchange not more than what the other prosumer could provide, while the prosumer 2 needs more than what the prosumer 1 could provide. The results in the last column of Table 2, show that agent 1 installs a PV capacity larger than its demand, i.e. 1.152, while prosumer 2, installs a PV plant of a size very similar to that obtained in scenario 1, i.e. 0.720. The interesting result is that, despite having a very different supply-demand structure, compared to the previous cases, the value levelsgenerated are not very far from those obtained in scenario 2. In fact, we get an $\mathcal{O}\left(\alpha_1^*, \alpha_2^*\right)$ equal to 3012. Let's present the main insight behind this outcome.

The interesting result is that prosumer 1 self-consumes a little more than half of its production (0.593), while all the rest of the production is sold to the second agent (0.559). Prosumer 2, on the other hand, buys all the energy sold by agent 1 and sells almost all of its production to N, thus its self-consumption is almost nil.

By doing so, the two prosumers manage to maximize their joint pay-offs, even though they are in a situation characterized by supply-demand asymmetry. Prosumer 2 sells to N and purchases from prosumer 1 almost the same quantity of energy. For prosumer 2, this exchange is unprofitable compared to the self-consumption hypothesis, because his savings are lower. However, thanks to cooperation, the exchange is profitable in terms of agents' overall total value . Prosumer 1 earns more than the lower savings of agent 2. The net effect is a NPV equal to 3012, very close to the ones of the other scenarios. Despite this, it is still lower as the case where the two agents work in complement and is still more advantageous. In this scenario, they exploit their own asymmetry by transforming an agent into a pure link between production and sale and playing on the difference in prices. All of this can only work with perfect coordination and cooperation between prosumers.

It is interesting at this point to consider whether it is worth the set up of the exchange P2P with these characteristics.

[52] All values of the optimal capacities presented in Table 5 of Appendix A.4.2, are still not feasible respect to the constraints described in Figure 10.

[53] Figures 15, 16 and 17 in Appendix A.4.2 show comparative statics performed for corner solution 1. The figures one related to corner solution 2 are omitted because they are symmetric to the once provided for corner solution 2 while, with reference to corner solution 3, none of the parameters q_t, σ and $LCOE$ affect the related optimal capacities.

In addition to that, a decrease of the price the prosumers receive for the energy sell to N, yields into two different effect respect to their self consumption behavior: it first increases the self-consumption ranges for both prosumers, even though optimal capacities decrease, leading to a potential average lowering of the self-consumption level. The effect on the energy exchange P2P is the opposite. [54] The lowering of the volatility of the price q_t (σ) leads to a widening of the prosumers' self-consumption possibilities. The optimal capacity of the prosumer 1[55] decreases while the one of prosumer 2 increases.[56] The same effect occurs for the prosumers' self-consumption level, while the opposite for the respective exchange P2P volumes. [57] The effect of a reduction in the $LCOE$ makes this scenario unfeasible.[58]
Let us now summarize our results by trying to reflect on the conditions that make convenient the set up of the exchange P2P. First of all, we have verified that not all the results that are mathematically feasible, and which we report in the related appendixes, make sense in reality. In fact, it is not always possible to find load curves satisfying the symmetry of the results with the asymmetry of the prosumers. We have shown that in scenario 1 (in scenario 2) exchange P2P can exist only if the prosumers are almost perfectly asymmetrical and with self-consumption levels of about 40-50% of the PV production (70% for scenario 2) and the same day/night distribution. Therefore the exchange P2P only makes sense under certain conditions and with particular combinations of supply and demand. It could also be relevant in a context similar to scenario 3, where there is an asymmetrical structural situation and the two agents try to maximize the joint value of the PV project. Perfect cooperation between prosumers is crucial in this context.

[54] See Figure 19 and Table 7 in Appendix A.4.3.

[55] We recall that under this scenario prosumer 1 demand less energy in exchange P2P respect to the one prosumer 2 is willing to supply.

[56] We recall that prosumer 2 needs more energy in exchange P2P respect to the amount the other prosumer can supply

[57] See Figure 20 and Table 7 in Appendix A.4.3.

[58] See Figure 21 in Appendix A.4.3.

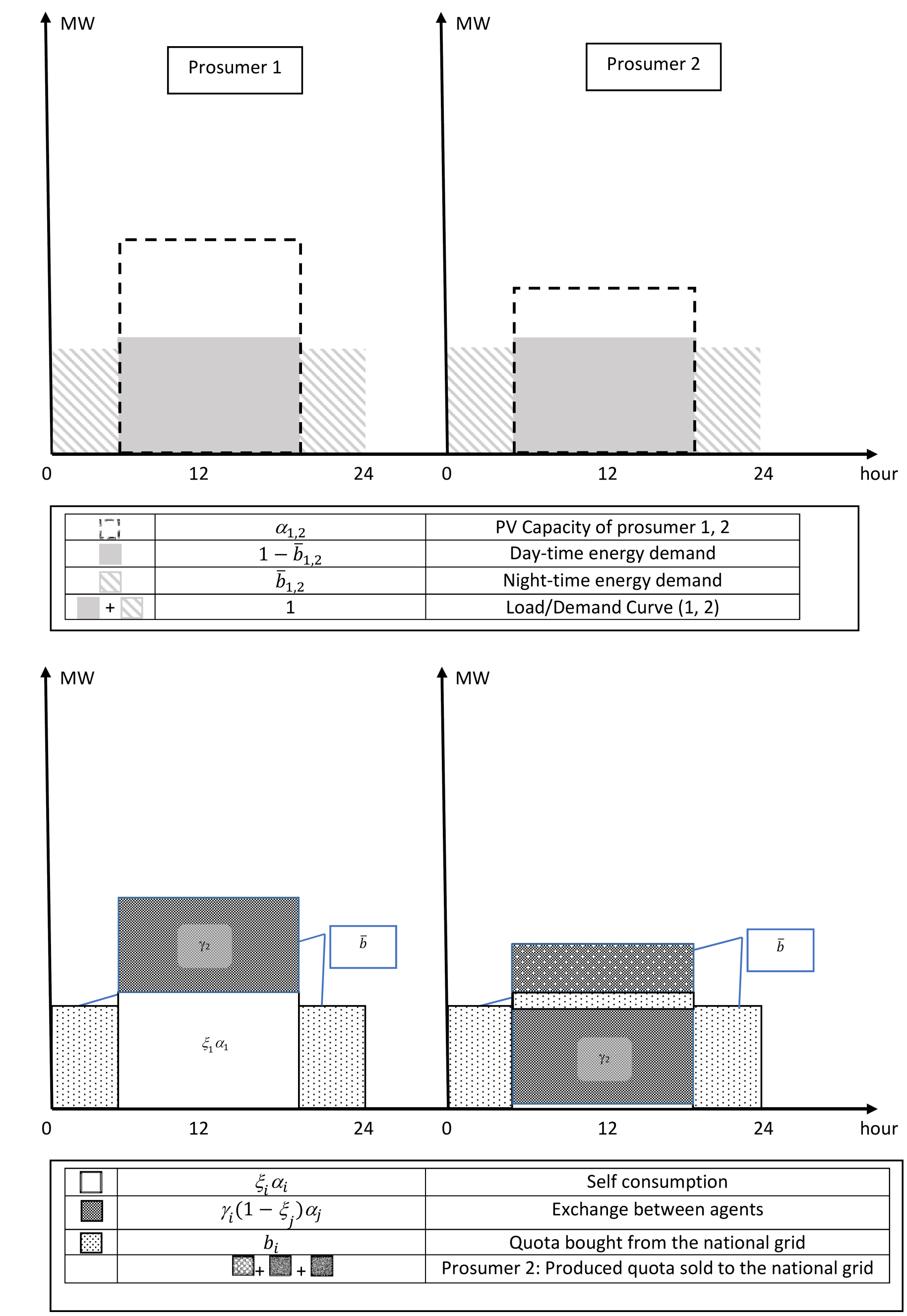

Figure 5: *Scenario 3* - Load and supply curves and distribution of energy trade and consumption.

7 Conclusions

In this work, we have modeled the investment decision of two prosumers in a PV system in a SG framework. Each prosumer can: (i) self-consume its energy production, (ii) exchange energy with the national grid, and/or (iii) exchange energy with the other agent. Uncertainty is taken into account by the dynamics of the price the prosumers receive for the energy sold to the NEMO, which is assumed to be stochastic. We investigate the cooperative investment decision under different prosumers' behaviors in exchange P2P, taking into account all the possible combinations of energy demand and supply for the two prosumers. These are summarized in four different exchange scenarios.

Our findings show that not in all the cases it is convenient to develop an exchange P2P framework. Indeed, after having calibrated our model on the Northern Italy energy market, we have calculated the mathematical feasibility of our investment decisions model under the four different scenarios. Among all these outcomes only some are also realistic, because not always it is possible to find load curves satisfying the symmetry of the results with the asymmetry of the prosumers.

We have found the prosumers' supply and demand profiles for which it makes sense to build an exchange P2P structure. The best case is when the two prosumers have excess demand in P2P exchange, and characterized by perfectly asymmetric and complementary supply and load curves. In this case, the two prosumers build two symmetrical PV plants of a size smaller compared to their demand, where: a share of the energy production is self-consumed, a share is exchanged P2P with the aim to match the hourly consumption demand reciprocally and a share is sold to N. Nothing is bought in daytime consumption form N.

A second feasible scenario refers to the case where the two prosumers are characterized by excess demand. Both produce and consume with a smaller plant respect to the previous one and set at the daytime demand level. Nothing is sold and purchased to and from the national grid in the daytime. The exchange P2P is also convenient with asymmetry between the two agents. Indeed, if one prosumer has excess demand and the other has excess supply, our model find a positive NPV, when an agent produces to self-consume and sell, and a second one buys the surplus of the other and sells all of his production to N. The maximum savings are guaranteed by the cooperation in investment decisions of the two agents in such a way that one allows the other to maximize its own earnings. In a cooperative view, the gain is shared between the agents. In this context, one prosumer over-sizes his PV plant, while the second one builds it with a capacity lower than his demand. Therefore the exchange P2P framework only makes sense under certain conditions and with particular combinations of supply and demand, although we found that the overall exchange structure could have a closer NPV while showing different and opposite supply and demand profiles. Much depends on the degree of self-consumption, the size of the PV system and the level of cooperation between agents.

To conclude, since it is widely recognized that policymakers support the deploy-

ment of the exchange P2P due to their promising positive impact in terms of i) achievement of the decarbonization goals, ii) potential in the improvement in the electricity network's management, and iii) active involvement of the prosumers in the energy market, on the basis of our findings, it is important to remark that further research must be developed on the conditions assuring the optimal set up of the exchange P2P. Aspects like uncertainty, demand and supply matching in exchange and PV plant optimal sizing must be deepened with the aim to support policymakers in their future task to provide an enabling regulatory framework for the energy transition.
Lastly, possible extensions of our research could be focused on deepening: i) the main drivers of uncertainty in a exchange P2P framework, ii) the topic of different exchange P2Pnetwork's structures, in terms of existence conditions as well as of optimization in an uncertain framework and iii) study in greater detail the effect of a possible stochastic exchange price.

A Appendix

A.1 Nash price bargaining

Let's consider the bargaining process leading to the definition of the energy price v_t on the basis of a mutually convenient agreement between seller and buyer when $p > q_t$. If, at the generic time period $t > 0$, the seller, S, and the buyer, B, agree on a certain energy price v_t, they will obtain the following payoffs, respectively:

$$W^S(v_t; q_t, p) = v_t, \quad \text{and} \quad W^B(v_t; q_t, p) = -v_t$$

If either party decides to quit the negotiation, the buyer's and the seller's outside payoffs would be:

$$\underline{W}^S(v_t; q_t, p) = q_t \quad \text{and} \quad \underline{W}^B(v_t; q_t, p) = -p$$

Assume now that S and B engage in a Nash Bargaining game with outside options. As standard, this game can be solved using the Nash Bargaining solution concept (Nash (1950), Nash (1953), Harsanyi (1977)).
A feasible Nash Bargaining solution, v_t^* solves the following maximization problem:

$$\begin{aligned} \max_{v_t \geq 0} \Omega \quad &= \quad \left(W^S(v_t; q_t, p) - \underline{W}^S(v_t; q_t, p)\right)^m \cdot \left(W^B(v_t; q_t, p) - \underline{W}^B(v_t; q_t, p)\right)^{1-m} \\ \text{s.t.} \quad & \quad W^S(v_t; q_t, p) \geq \underline{W}^S(v_t; q_t, p) \qquad \text{and} \\ & \quad W^B(v_t; q_t, p) \leq \underline{W}^B(v_t; q_t, p) \end{aligned} \tag{A.1.1}$$

where by m and $1 - m$ with $m \in (0, 1)$ we denote the seller's and buyer's strength exerted in the bargaining.
The first-order Condition for the maximization problem (A.1.1) is: [59]

$$\left.\frac{d\Omega}{dv_t}\right|_{v_t = v_t^*} = (v_t^* - q_t)^{m-1}(p - v_t^*)^{-m}[v_t - mp - (1 - m)q_t] = 0 \tag{A.1.2}$$

Solving Eq. (A.1.2) we obtain

$$v_t^* = m \cdot p + (1 - m) \cdot q_t \tag{A.1.3}$$

[59] where the second-order Condition holds always.

A.2 Expected energy cost under the PV project

The general solutions to the differential equations (11.1) and (12.1) are (see Dixit (1989) pp. 624-628):[60]

$$C_i^{NSCE}(q_t;\alpha_i) = \frac{p}{r} - \alpha_i \frac{q_t}{r-\theta} + \widehat{X}_i^{NSCE} q_t^{\beta_2}, \quad \text{for} \quad q_t > p, \qquad \text{(A.2.1)}$$

$$C_i^{SCE}(q_t;\alpha_i,\gamma_i,\gamma_j) = \frac{p}{r} - \alpha_i \frac{q_t}{r-\theta} - S(q_t;\alpha_i,\gamma_i,\gamma_j)\left(\frac{p}{r} - \frac{q_t}{r-\theta}\right) + \widehat{Y}_i^{SCE} q_t^{\beta_1}, \quad \text{for} \quad q_t < p, \qquad \text{(A.2.2)}$$

where $\beta_2 < 0$ and $\beta_1 > 1$ are the roots of the characteristic equation $\Phi(x) \equiv \frac{1}{2}\sigma^2 x(x-1) + \theta x - r$. The terms $\widehat{X}_i^{NSCE} q_t^{\beta_2}$ and $\widehat{Y}_i^{SCE} q_t^{\beta_1}$ represents the value associated with the flexibility to switch to a regime reducing the total energy cost. Hence, to be consistent, the constants $\widehat{X}_i^{NSCE}$ and $\widehat{Y}_i^{SCE}$ must be non-positive. At $q_t = p$, the standard pair of Conditions for an optimal switching policy must hold, that is, the following:

value-matching Condition

$$C_i^{NSCE}(p;\alpha_i) = C_i^{SCE}(p;\alpha_i,\gamma_i,\gamma_j), \qquad \text{(A.2.3)}$$

smooth-pasting Condition

$$\left.\frac{dC_i^{NSCE}(q_t;\alpha_i)}{dq_t}\right|_{q_t=p} = \left.\frac{dC_i^{SCE}(q_t;\alpha_i,\gamma_i,\gamma_j)}{dq_t}\right|_{q_t=p}. \qquad \text{(A.2.4)}$$

Solving the program [A.2.3 - A.2.4] yields

$$\widehat{X}_i^{NSCE} = S(q_t;\alpha_i,\gamma_i,\gamma_j)\frac{p}{r-\theta}\frac{r-\theta\beta_1}{r(\beta_2-\beta_1)}p^{-\beta_2} = S(q_t;\alpha_i,\gamma_i,\gamma_j)X^{NSCE}p^{-\beta_2}$$

$$\widehat{Y}_i^{SCE} = S(q_t;\alpha_i,\gamma_i,\gamma_j)\frac{p}{r-\theta}\frac{r-\theta\beta_2}{r(\beta_2-\beta_1)}p^{-\beta_1} = S(q_t;\alpha_i,\gamma_i,\gamma_j)Y^{SCE}p^{-\beta_1}$$

which are linear in α_i and α_j and non-positive.

[60] Note that the general solution to Eq. (11.1) should take the form

$$C_i^{NSCE}(q_t;\alpha_i) = \frac{c}{r} - \frac{\alpha_i q_t}{r-\theta} + \widehat{X}_i^{NSCE} q_t^{\beta_2} + \widehat{Y}_i^{NSCE} q_t^{\beta_1}.$$

However, since the value of the flexibility to switch to the regime contemplating self-consumption vanishes as $q_t \to \infty$, we then set $\widehat{Y}_i^{NSCE} = 0$. Similarly, the general solution to Eq. (12.1) should be

$$C_i^{SCE}(q_t;\xi_i,\alpha_i) = \frac{(1-\xi_i\alpha_i)p}{r} - \frac{(1-\xi_i)\alpha_i q_t}{r-\theta} + \widehat{X}_i^{SCE} q_t^{\beta_2} + \widehat{Y}_i^{SCE} q_t^{\beta_1}.$$

However, the flexibility to switch to the regime where all the energy produced is sold becomes valueless as $q_t \to 0$ and then we set $\widehat{X}_i^{SCE} = 0$.

A.3 The value of the PV investment project

Let's prove that

$$\Delta C_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)=\frac{p}{r}-C_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)>0,\quad \text{for any } q_t<p \tag{A.3.1}$$

Substituting Eq.(15) into the inequality (A.3.1) yields:

$$\alpha_i\frac{q_t}{r-\theta}+S\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)H\left(q_t\right)>0 \tag{A.3.2}$$

where

$$H\left(q_t\right)=\left(\frac{p}{r}-\frac{q_t}{r-\theta}\right)-Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}. \tag{A.3.3}$$

Note that

i) $H\left(0\right)=\frac{p}{r}>0$,

ii) $H\left(p\right)=\frac{p}{r}\frac{r-\beta_1\theta}{(r-\theta)(\beta_1-\beta_2)}>0$,

iii) $H\left(0\right)>H\left(p\right)$, and

iv) $\frac{d^2H(q_t)}{dq_t^2}=\frac{\beta_1(\beta_1-1)}{r-\theta}\frac{r-\theta\beta_2}{r(\beta_1-\beta_2)}\left(\frac{q_t}{p}\right)^{\beta_1-2}\frac{1}{p}>0$.

Hence, in order to prove that $H\left(q_t\right)>0$ and, consequently, $\Delta C_i\left(q_t;\alpha_i,\gamma_i,\gamma_j\right)>0$ it suffices showing that the first derivative of $H\left(q_t\right)$, i.e.,

$$\frac{dH\left(q_t\right)}{dq_t}=-\frac{1}{r-\theta}-\frac{\beta}{r-\theta}\frac{r-\theta\beta_2}{r\left(\beta_2-\beta_1\right)}\left(\frac{q_t}{p}\right)^{\beta_1-1}$$

takes a negative sign at both $q_t=0$ and $q_t=p$, which, as shown in the following, is always the case:

$$\left.\frac{dH\left(q_t\right)}{dq_t}\right|_{q_t=0}=-\frac{1}{r-\theta}<0$$

$$\left.\frac{dH\left(q_t\right)}{dq_t}\right|_{q_t=p}=\frac{\beta_2}{r-\theta}\frac{r-\beta_1\theta}{r(\beta_1-\beta_2)}<0$$

A.4 The energy exchange P2P scenarios

A.4.1 Scenario 1: excess supply in the energy exchange P2P.

Suppose that:

$$0 < (1-\bar{b}) - \xi_1\alpha_1 < (1-\xi_2)\alpha_2, \quad \text{(A.4.1)}$$

$$0 < (1-\bar{b}) - \xi_2\alpha_2 < (1-\xi_1)\alpha_1. \quad \text{(A.4.2)}$$

When $q_t < p$, as the exchange P2P is more convenient than trading energy with N, the two prosumers exchange the following quantities of energy:

$$\gamma_1 = (1-\bar{b}) - \xi_1\alpha_1, \quad \text{(A.4.3)}$$

$$\gamma_2 = (1-\bar{b}) - \xi_2\alpha_2. \quad \text{(A.4.4)}$$

As for the individual excess supply, each prosumer has no other alternative than selling this energy to N at price q_t.
Substituting Eqs. (A.4.3) and (A.4.4) into Eq. (20) and solving Problem (19) yields:

$$\alpha_1^* = \alpha_2^* = \alpha^* = \frac{1}{K_B}\frac{q_t}{r-\theta} > 0. \quad \text{(A.4.5)}$$

The optimal pair (α_1^*, α_2^*) must be consistent with the feasibility constraints (A.4.3) and (A.4.4). As it can be easily shown, this requires that the following restrictions:

$$-(1-\frac{1-\bar{b}}{\alpha^*}) < (\xi_1-\xi_2) < 1-\frac{1-\bar{b}}{\alpha^*}, \quad \text{(A.4.6.1)}$$

$$\xi_1\alpha^* + \bar{b} < 1, \quad \text{(A.4.6.2)}$$

$$\xi_2\alpha^* + \bar{b} < 1, \quad \text{(A.4.6.3)}$$

$$\alpha^* + \bar{b} > 1, \quad \text{(A.4.6.4)}$$

hold together, otherwise, the pair (α_1^*, α_2^*) is not feasible. Last, substituting Eq. (A.4.5) into (20) yields the expected net present value of the PV project, that is:

$$O(\alpha_1^*, \alpha_2^*) = \alpha^{*2}K_B + 2(1-\bar{b})\left[\frac{p}{r} - \frac{q_t}{r-\theta} - Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right] - K_A. \quad \text{(A.4.7)}$$

A.4.2 Scenario 2: excess demand in the energy exchange P2P.

Suppose that:

$$
\begin{aligned}
(1-\bar{b})-\xi_1\alpha_1 &\geq (1-\xi_2)\alpha_2 > 0, &\text{(A.4.8)}\\
(1-\bar{b})-\xi_2\alpha_2 &\geq (1-\xi_1)\alpha_1 > 0. &\text{(A.4.9)}
\end{aligned}
$$

Internal solution. Let's start by considering the case where

$$
\begin{aligned}
(1-\bar{b})-\xi_1\alpha_1 &> (1-\xi_2)\alpha_2 > 0, &\text{(A.4.10)}\\
(1-\bar{b})-\xi_2\alpha_2 &> (1-\xi_1)\alpha_1 > 0. &\text{(A.4.11)}
\end{aligned}
$$

When $q_t < p$, as the exchange P2P is more convenient than trading energy with N, the two prosumers exchange the following quantities of energy:

$$
\begin{aligned}
\gamma_1 &= (1-\xi_2)\alpha_2, &\text{(A.4.12)}\\
\gamma_2 &= (1-\xi_1)\alpha_1. &\text{(A.4.13)}
\end{aligned}
$$

As for the excess demand, each prosumer has no other alternative than purchasing energy from N at price p.
Substituting Eqs. (A.4.12) and (A.4.13) into (20) and solving Problem (19) yields:[61]

$$
\alpha_1^* = \alpha_2^* = \alpha^* = \frac{1}{K_B}\left[\frac{p}{r} - Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right] > 0. \quad \text{(A.4.14)}
$$

At (α_1^*, α_2^*), to be consistent with the feasibility constraints (A.4.8) and (A.4.9), the following restrictions:

$$
\begin{aligned}
-\left(\frac{1-\bar{b}}{\alpha^*}-1\right) &< (\xi_1-\xi_2) < \frac{1-\bar{b}}{\alpha^*}-1 &\text{(A.4.15.1)}\\
\alpha^* + \bar{b} &< 1, &\text{(A.4.15.2)}
\end{aligned}
$$

must hold together, otherwise, the solution is not feasible.
Last, under this scenario, the expected net present value of the PV project is equal to:

$$
O\left(\alpha_1^*, \alpha_2^*\right) = \alpha^{*2}K_B - K_A. \quad \text{(A.4.16)}
$$

Corner solution 1. Consider the case where

$$
\begin{aligned}
(1-\bar{b})-\xi_1\alpha_1 &> (1-\xi_2)\alpha_2 > 0, &\text{(A.4.17)}\\
(1-\bar{b})-\xi_2\alpha_2 &= (1-\xi_1)\alpha_1 > 0. &\text{(A.4.18)}
\end{aligned}
$$

Combining Inequality (A.4.17) and Eq. (A.4.18) yields

$$
\begin{aligned}
\alpha_1 &= \frac{(1-\bar{b})-\xi_2\alpha_2}{1-\xi_1}, &\text{(A.4.19)}\\
\alpha_1+\alpha_2 &< 2(1-\bar{b}). &\text{(A.4.20)}
\end{aligned}
$$

[61] We show in Appendix A.3 that $\frac{p}{r} - Y^{SCE}(\frac{q_t}{p})^{\beta} > \frac{q_t}{r-\theta} \geq 0$ when $q_t < p$.

Prosumer 1 and prosumer 2 find convenient exchanging the following amounts of energy:

$$\begin{aligned} \gamma_1 &= (1-\xi_2)\alpha_2, && \text{(A.4.21)} \\ \gamma_2 &= (1-\xi_1)\alpha_1, && \text{(A.4.22)} \end{aligned}$$

respectively. Substituting Eqs. (A.4.21) and (A.4.22) into $O(\alpha_1, \alpha_2)$ and solving Problem (19) yields:

$$\alpha_1^* = \frac{(1-\bar{b})(1-\xi_1)}{(1-\xi_1)^2+\xi_2^2} - \frac{\xi_2(1-\xi_1-\xi_2)}{(1-\xi_1)^2+\xi_2^2}\frac{\frac{p}{r} - Y^{SCE}(\frac{q_t}{p})^{\beta_1}}{K_B}, \quad \text{(A.4.23)}$$

$$\alpha_2^* = \frac{(1-\bar{b})\xi_2}{(1-\xi_1)^2+\xi_2^2} + \frac{(1-\xi_1)(1-\xi_1-\xi_2)}{(1-\xi_1)^2+\xi_2^2}\frac{\frac{p}{r} - Y^{SCE}(\frac{q_t}{p})^{\beta_1}}{K_B}. \quad \text{(A.4.24)}$$

The feasibility of the optimal pair (α_1^*, α_2^*) requires that the following restrictions:

$$\begin{aligned} \alpha_1^* &> 0, && \text{(A.4.25.1)} \\ \alpha_2^* &> 0, && \text{(A.4.25.2)} \\ \alpha_1^* + \alpha_2^* &< 2(1-\bar{b}), && \text{(A.4.25.3)} \end{aligned}$$

hold together, otherwise, the pair (α_1^*, α_2^*) is not feasible. Under this scenario, the expected net present value of the PV project is equal to:

$$O(\alpha_1^*, \alpha_2^*) = (\alpha_1^* + \alpha_2^*)\left[\frac{p}{r} - Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right] - I(\alpha_1^*, \alpha_2^*). \quad \text{(A.4.26)}$$

Corner solution 2. Suppose that

$$\begin{aligned} (1-\bar{b}) - \xi_1\alpha_1 &= (1-\xi_2)\alpha_2 > 0, && \text{(A.4.27)} \\ (1-\bar{b}) - \xi_2\alpha_2 &> (1-\xi_1)\alpha_1 > 0. && \text{(A.4.28)} \end{aligned}$$

Combining Eq. (A.4.27) and Inequality (A.4.27) yields

$$\begin{aligned} \alpha_2 &= \frac{(1-\bar{b}) - \xi_1\alpha_1}{(1-\xi_2)}, && \text{(A.4.29)} \\ \alpha_1 + \alpha_2 &< 2(1-\bar{b}). && \text{(A.4.30)} \end{aligned}$$

Prosumer 1 and prosumer 2 find convenient exchanging the following amounts of energy:

$$\begin{aligned} \gamma_1 &= (1-\xi_2)\alpha_2, && \text{(A.4.31)} \\ \gamma_2 &= (1-\xi_1)\alpha_1, && \text{(A.4.32)} \end{aligned}$$

respectively. Substituting Eqs. (A.4.31) and (A.4.32) into $O(\alpha_1, \alpha_2)$ and solving Problem (19) yields:

$$\alpha_1^* = \frac{(1-\bar{b})\xi_1}{(1-\xi_2)^2+\xi_1^2} + \frac{(1-\xi_2)(1-\xi_1-\xi_2)}{(1-\xi_2)^2+\xi_1^2}\frac{\frac{p}{r} - Y^{SCE}(\frac{q_t}{p})^{\beta_1}}{K_B}, \quad \text{(A.4.33)}$$

$$\alpha_2^* = \frac{(1-\bar{b})(1-\xi_2)}{(1-\xi_2)^2+\xi_1^2} - \frac{\xi_1(1-\xi_1-\xi_2)}{(1-\xi_2)^2+\xi_1^2}\frac{\frac{p}{r} - Y^{SCE}(\frac{q_t}{p})^{\beta_1}}{K_B}. \quad \text{(A.4.34)}$$

The feasibility of the optimal pair (α_1^*, α_2^*) requires that the following restrictions:

$$\alpha_1^* > 0, \tag{A.4.35.1}$$
$$\alpha_2^* > 0, \tag{A.4.35.2}$$
$$\alpha_1^* + \alpha_2^* < 2(1-\bar{b}), \tag{A.4.35.3}$$

hold together, otherwise, the pair (α_1^*, α_2^*) is not feasible. Under this scenario, the expected net present value of the PV project is equal to:

$$O(\alpha_1^*, \alpha_2^*) = (\alpha_1^* + \alpha_2^*)\left[\frac{p}{r} - Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right] - I(\alpha_1^*, \alpha_2^*). \tag{A.4.36}$$

Corner solution 3. Suppose that

$$(1-\bar{b}) - \xi_1\alpha_1 = (1-\xi_2)\alpha_2, \tag{A.4.37}$$
$$(1-\bar{b}) - \xi_2\alpha_2 = (1-\xi_1)\alpha_1. \tag{A.4.38}$$

Solving the System [A.4.37-A.4.38] yields

$$\alpha_1^* = (1-\bar{b})\frac{1-2\xi_2}{1-\xi_2-\xi_1}, \tag{A.4.39}$$
$$\alpha_2^* = (1-\bar{b})\frac{1-2\xi_1}{1-\xi_2-\xi_1}. \tag{A.4.40}$$

The following restrictions are needed in order to secure that $\alpha_1^* > 0$ and $\alpha_2^* > 0$:

$$\xi_1 + \xi_2 < 1, \quad \xi_1 < 1/2, \quad \xi_2 < 1/2, \tag{A.4.41.1}$$
$$\xi_1 + \xi_2 > 1, \quad \xi_1 > 1/2, \quad \xi_2 > 1/2. \tag{A.4.41.2}$$

Last, under this scenario, the expected net present value of the PV project is equal to:

$$O(\alpha_1^*, \alpha_2^*) = 2(1-\bar{b})\left[\frac{p}{r} - Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right] - I(\alpha_1^*, \alpha_2^*). \tag{A.4.42}$$

A.4.3 Scenario 3: non complementarity in the energy exchange P2P.

Suppose that:

$$0 < (1-\bar{b}) - \xi_1\alpha_1 < (1-\xi_2)\alpha_2, \quad \text{(A.4.43)}$$
$$(1-\bar{b}) - \xi_2\alpha_2 \geq (1-\xi_1)\alpha_1 > 0. \quad \text{(A.4.44)}$$

Internal solution. Consider the case where:

$$0 < (1-\bar{b}) - \xi_1\alpha_1 < (1-\xi_2)\alpha_2, \quad \text{(A.4.45)}$$
$$(1-\bar{b}) - \xi_2\alpha_2 > (1-\xi_1)\alpha_1 > 0. \quad \text{(A.4.46)}$$

Prosumer 1 and prosumer 2 find convenient exchanging the following quantitites of energy:

$$\gamma_1 = (1-\bar{b}) - \xi_1\alpha_1, \quad \text{(A.4.47)}$$
$$\gamma_2 = (1-\xi_1)\alpha_1, \quad \text{(A.4.48)}$$

respectively. Prosumer 2 will then sell the residual quantity of energy, $(1-\xi_2)\alpha_2-(1-\bar{b})-\xi_1\alpha_1$, to N at price q_t and purchase the quantity of energy $(1-\bar{b})-\xi_2\alpha_2-\alpha_1(1-\xi_1)$ from N at price p.
Substituting Eqs. (A.4.47) and (A.4.48) into Eq. (20) and solving Problem (19) yields:

$$\alpha_1^* = \frac{1}{K_B}\left\{\xi_1\frac{q_t}{r-\theta} + (1-\xi_1)\left[\frac{p}{r} - Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right]\right\} > 0, \quad \text{(A.4.49)}$$
$$\alpha_2^* = \frac{1}{K_B}\left\{(1-\xi_2)\frac{q_t}{r-\theta} + \xi_2\left[\frac{p}{r} - Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right]\right\} > 0. \quad \text{(A.4.50)}$$

The feasibility of the optimal pair (α_1^*, α_2^*) requires that the following restrictions:

$$(1-\bar{b}) < \xi_1\alpha_1^* + (1-\xi_2)\alpha_2^*, \quad \text{(A.4.51.1)}$$
$$(1-\bar{b}) > (1-\xi_1)\alpha_1^* + \xi_2\alpha_2^*, \quad \text{(A.4.51.2)}$$
$$\xi_1\alpha_1^* + \bar{b} < 1, \quad \text{(A.4.51.3)}$$

hold together, otherwise, the pair (α_1^*, α_2^*) is not feasible.
Last, under this scenario, the expected net present value of the PV project is equal to:

$$O(\alpha_1^*, \alpha_2^*) = \frac{K_B}{2}(\alpha_1^{*2} + \alpha_2^{*2}) + (1-\bar{b})\left[\frac{p}{r} - \frac{q_t}{r-\theta} - Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right] - K_A. \quad \text{(A.4.52)}$$

Corner solution. Suppose that

$$0 < (1-\bar{b}) - \xi_1\alpha_1 < (1-\xi_2)\alpha_2, \quad \text{(A.4.53)}$$
$$(1-\bar{b}) - \xi_2\alpha_2 = (1-\xi_1)\alpha_1 > 0. \quad \text{(A.4.54)}$$

Combining Inequality (A.4.53) and Eq. (A.4.54) yields

$$\alpha_1 = \frac{(1-\bar{b}) - \xi_2\alpha_2}{1-\xi_1}, \quad \text{(A.4.55)}$$
$$\alpha_1 + \alpha_2 > 2(1-\bar{b}). \quad \text{(A.4.56)}$$

Prosumer 1 and prosumer 2 find convenient exchanging the following quantities of energy:

$$\gamma_1 = (1-\bar{b}) - \xi_1\alpha_1, \quad \text{(A.4.57)}$$

$$\gamma_2 = (1-\bar{b}) - \xi_2\alpha_2, \quad \text{(A.4.58)}$$

respectively. Substituting Eqs. (A.4.57) and (A.4.58) into $O(\alpha_1, \alpha_2)$ and solving Problem (19) yields:

$$\alpha_1^* = \frac{(1-\bar{b})(1-\xi_1)}{(1-\xi_1)^2+\xi_2^2} - \frac{\xi_2(1-\xi_1-\xi_2)}{(1-\xi_1)^2+\xi_2^2}\frac{\frac{q_t}{r-\theta}}{K_B}, \quad \text{(A.4.59)}$$

$$\alpha_2^* = \frac{(1-\bar{b})\xi_2}{(1-\xi_1)^2+\xi_2^2} + \frac{(1-\xi_1)(1-\xi_1-\xi_2)}{(1-\xi_1)^2+\xi_2^2}\frac{\frac{q_t}{r-\theta}}{K_B}. \quad \text{(A.4.60)}$$

The feasibility of the optimal pair (α_1^*, α_2^*) requires that the following restrictions:

$$\begin{aligned} \alpha_1^* &> 0, \\ \xi_1\alpha_1^* + \bar{b} &< 1, \\ \alpha_1^* + \alpha_2^* &> 2(1-\bar{b}), \end{aligned}$$

hold together, otherwise, the pair (α_1^*, α_2^*) is not feasible. Under this scenario, the expected net present value of the PV project is equal to:

$$\begin{aligned} O(\alpha_1^*, \alpha_2^*) &= (\alpha_1^* + \alpha_2^*)\frac{q_t}{r-\theta} - I(\alpha_1^*, \alpha_2^*) \\ &\quad +2(1-\bar{b})\left[\frac{p}{r} - \frac{q_t}{r-\theta} - Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right]. \end{aligned} \quad \text{(A.4.61)}$$

A.4.4 Scenario 4: non complementarity in the energy exchange P2P.

Suppose that:

$$
\begin{aligned}
(1-\bar{b})-\xi_1\alpha_1 &\geq (1-\xi_2)\alpha_2>0, && \text{(A.4.62)}\\
0 &< (1-\bar{b})-\xi_2\alpha_2<(1-\xi_1)\alpha_1. && \text{(A.4.63)}
\end{aligned}
$$

Internal solution. Consider the case where:

$$
\begin{aligned}
(1-\bar{b})-\xi_1\alpha_1 &> (1-\xi_2)\alpha_2>0, && \text{(A.4.64)}\\
0 &< (1-\bar{b})-\xi_2\alpha_2<(1-\xi_1)\alpha_1. && \text{(A.4.65)}
\end{aligned}
$$

Prosumer 1 and prosumer 2 find convenient exchanging the following quantities of energy:

$$
\begin{aligned}
\gamma_1 &= (1-\xi_2)\alpha_2 && \text{(A.4.66)}\\
\gamma_2 &= (1-\bar{b})-\xi_2\alpha_2 && \text{(A.4.67)}
\end{aligned}
$$

Substituting Eqs. (A.4.66) and (A.4.67) into (20) and solving Problem (19) yields:

$$
\begin{aligned}
\alpha_1^* &= \frac{1}{K_B}\left\{(1-\xi_1)\frac{q_t}{r-\theta}+\xi_1\left[\frac{p}{r}-Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right]\right\}>0 && \text{(A.4.68)}\\
\alpha_2^* &= \frac{1}{K_B}\left\{\xi_2\frac{q_t}{r-\theta}+(1-\xi_2)\left[\frac{p}{r}-Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right]\right\}>0 && \text{(A.4.69)}
\end{aligned}
$$

In order to have a feasible pair (α_1^*,α_2^*), the following restrictions

$$
\begin{aligned}
(1-\bar{b}) &> \xi_1\alpha_1^*+(1-\xi_2)\alpha_2^* && \text{(A.4.70.1)}\\
(1-\bar{b}) &< (1-\xi_1)\alpha_1^*+\xi_2\alpha_2^* && \text{(A.4.70.2)}\\
\xi_2\alpha_2^*+\bar{b} &< 1 && \text{(A.4.70.3)}
\end{aligned}
$$

must hold together, otherwise, the pair (α_1^*,α_2^*) is not feasible.
Last, substituting Eqs. (A.4.68) and (A.4.69) into (20) yields

$$
O\left(\alpha_1^*,\alpha_2^*\right)=\frac{K_B}{2}(\alpha_1^{*2}+\alpha_2^{*2})+(1-\bar{b})\left[\frac{p}{r}-\frac{q_t}{r-\theta}-Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right]-K_A. \quad \text{(A.4.71)}
$$

Corner solution. Suppose that

$$
\begin{aligned}
(1-\bar{b})-\xi_1\alpha_1 &= (1-\xi_2)\alpha_2>0, && \text{(A.4.72)}\\
0 &< (1-\bar{b})-\xi_2\alpha_2<(1-\xi_1)\alpha_1. && \text{(A.4.73)}
\end{aligned}
$$

Combining Eq. (A.4.72) and Inequality (A.4.72) yields

$$
\begin{aligned}
\alpha_2 &= \frac{(1-\bar{b})-\xi_1\alpha_1}{(1-\xi_2)}, && \text{(A.4.74)}\\
\alpha_1+\alpha_2 &> 2(1-\bar{b}). && \text{(A.4.75)}
\end{aligned}
$$

Prosumer 1 and prosumer 2 find convenient exchanging the following quantities of energy:

$$\gamma_1 = (1-\bar{b}) - \xi_1\alpha_1, \quad \text{(A.4.76)}$$

$$\gamma_2 = (1-\bar{b}) - \xi_2\alpha_2, \quad \text{(A.4.77)}$$

respectively. Substituting Eqs. (A.4.76) and (A.4.76) into $O(\alpha_1, \alpha_2)$ and solving Problem (19) yields:

$$\alpha_1^* = \frac{(1-\bar{b})\xi_1}{(1-\xi_2)^2+\xi_1^2} + \frac{(1-\xi_2)(1-\xi_1-\xi_2)}{(1-\xi_2)^2+\xi_1^2}\frac{\frac{q_t}{r-\theta}}{K_B}, \quad \text{(A.4.78)}$$

$$\alpha_2^* = \frac{(1-\bar{b})(1-\xi_2)}{(1-\xi_2)^2+\xi_1^2} - \frac{\xi_1(1-\xi_1-\xi_2)}{(1-\xi_2)^2+\xi_1^2}\frac{\frac{q_t}{r-\theta}}{K_B}. \quad \text{(A.4.79)}$$

The feasibility of the optimal pair (α_1^*, α_2^*) requires that the following restrictions:

$$\begin{aligned} \alpha_2^* &> 0, \\ \xi_2\alpha_2^* + \bar{b} &< 1, \\ \alpha_1^* + \alpha_2^* &> 2(1-\bar{b}), \end{aligned}$$

hold together, otherwise, the pair (α_1^*, α_2^*) is not feasible. Under this scenario, the expected net present value of the PV project is equal to:

$$\begin{aligned} O(\alpha_1^*, \alpha_2^*) &= (\alpha_1^* + \alpha_2^*)\frac{q_t}{r-\theta} - I(\alpha_1^*, \alpha_2^*) \\ &\quad + 2(1-\bar{b})\left[\frac{p}{r} - \frac{q_t}{r-\theta} - Y^{SCE}\left(\frac{q_t}{p}\right)^{\beta_1}\right]. \end{aligned} \quad \text{(A.4.80)}$$

A.5 Numerical results

A.5.1 Scenario 1: excess supply in the energy exchange P2P.

In Figure 6, we include the Constraints (A.4.6.1), (A.4.6.2), (A.4.6.3) and (A.4.6.4). Then, we isolate the feasible area (in gray) as resulting from the consideration of those constraints. This leads, on the Y-axis, to the indication of the gap between the two self-consumption parameters $(\xi_1 - \xi_2)$ that may secure the feasibility of the solution found.

Under this Scenario, both optimal capacities (α_1^*, α_2^*) (A.4.5) and the expected net present value of the PV project, $O(\alpha_1^*, \alpha_2^*)$, (A.4.7) do not depend on the prosumers' self-consumption levels (ξ_i). Based on the parameters chosen for our calibration, we find that $\alpha_1^* = \alpha_2^* = \alpha^* = 0.71$ MWh and $O(\alpha_1^*, \alpha_2^*) = 3301$ Euro, respectively (see Table 3).

The solution $\alpha^* = 0.71$ is feasible conditional on letting the gap between ξ_1 and ξ_2 range within ± 0.15. This implies that the prosumers' self-consumption profiles must not be too distant.

In general, the gap may be larger as it is, for instance, the case for $\alpha^* \in [0.60; 1.20]$, where it may range within $\pm$ 0.50. Further, we notice that when α^* is higher than 1.20, the allowed gap starts shrinking as the optimal capacity increases. Finally, Figure 7 shows the set of (ξ_1, ξ_2) satisfying the Constraints above when each prosumer install a capacity, α^*, equal to 0.71.[62]

The quantity of self-consumed energy $(\xi_i \alpha^*)$ and exchanged energy (γ_i) are determined over some feasible ranges of ξ_1 and ξ_2 (marked in dark gray in Figure 7). The corresponding figures are presented in Table 4. As it can be immediately seen, the quantity of self-consumed energy and exchanged energy are negatively related.

Figures 8 and 9, show the effects of a reduction in q_t and $LCOE$ on the feasible pairs of the prosumers' self-consumption parameters, respectively. A decrease in the price paid for the energy sold to N lowers i) the optimal capacity, α^*, and ii) the expected net present value,[63] $O(\alpha_1^*, \alpha_2^*)$ (See Table 3). We notice also that, with respect to the benchmark case, the prosumers' self-consumption profiles must be closer[64]. However, the resulting set of (ξ_1, ξ_2) associated with a feasible solution allows for higher levels of self-consumption. A decrease in the $LCOE$, which implies, ceteris paribus, a lower cost of the PV project, makes convenient installing an higher capacity with respect to the benchmark and increases the expected net present value of the PV project. The feasible area widens in terms of allowed gap between ξ_1 and ξ_2 but their allowed maximum level decreases.

Finally, lowering the volatility level to $\sigma = 0.25$ affects only the expected net present value of the PV project which is lower than in the benchmark case.

[62] The set is obtained by letting each ξ_i $(i = 1, 2)$ vary between 0 to 1. In block 1, we have the ξ_1 and ξ_2 such that $\xi_1 - \xi_2 < \left(1 - \frac{1-\bar{b}}{\alpha^*}\right)$ and satisfying Eq. (A.4.6.2) and (Eq. A.4.6.3) whereas in block 2 those such that $\xi_1 - \xi_2 > -\left(1 - \frac{1-\bar{b}}{\alpha^*}\right)$ and satisfying Eq. (A.4.6.2) and (Eq. A.4.6.3). Finally, block 3, resulting from the combination of both the first and the second block, shows and show the set of all the feasible (ξ_1, ξ_2).

[63] This is because the gains from energy sold to N are lower.

[64] As it can be also immediately seen in Figure 6.

Parameters	*Benchmark case*	$q_t = 54$	$\sigma = 0.25$	$LCOE = 70$
α^*	0.710	0.650	0.710	0.810
$O(\alpha_1^*, \alpha_2^*)$	3301	3194	3194	3509

Table 3: *Scenario 1* - Benchmark results and comparative statics.

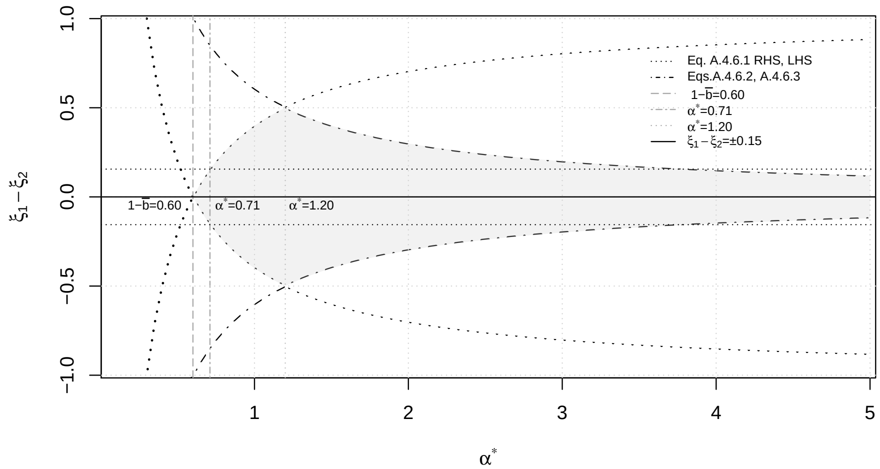

Figure 6: *Scenario 1* - The set of (ξ_1, ξ_2) associated with a feasible solution.

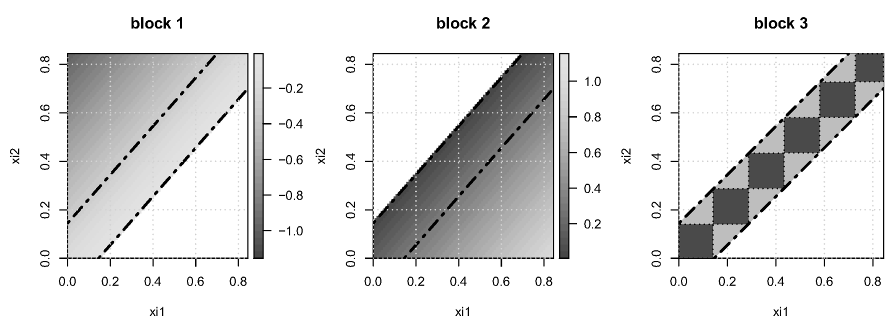

Figure 7: *Scenario 1* - The set of (ξ_1, ξ_2) associated with $\alpha^* = 0.71$.

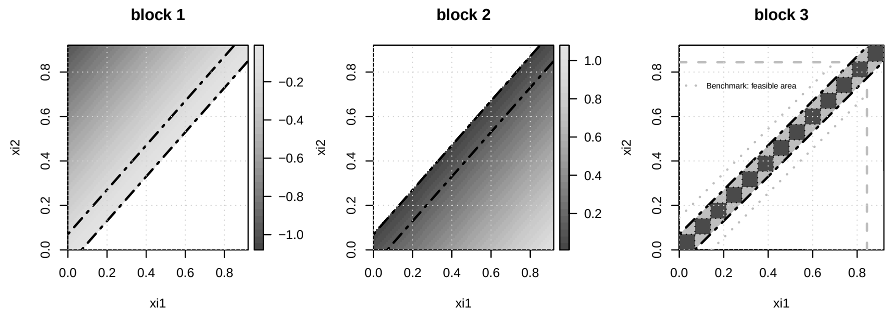

Figure 8: *Scenario 1* - The set of (ξ_1, ξ_2) associated with α^*: comparative statics on q.

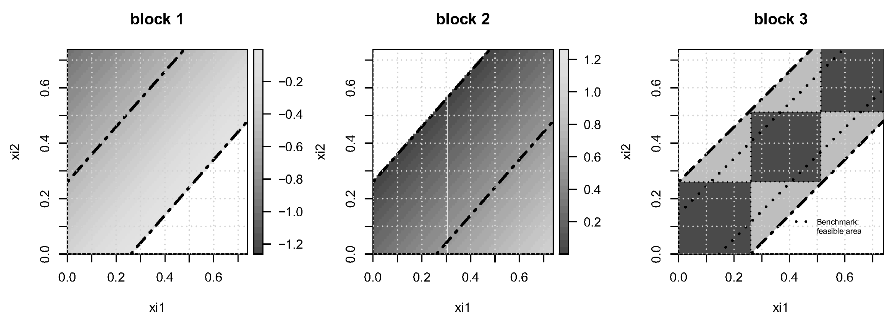

Figure 9: *Scenario 1* - The set of (ξ_1, ξ_2) associated with α^*: comparative statics on $LCOE$.

Parameters	FS1	FS2	FS3	FS4	FS5	FS6
$\xi_1, \xi_2 \in$	$[0; 0.14)$	$[0.14; 0.28)$	$[0.28; 0.43)$	$[0.43; 0.58)$	$[0.58; 0.72)$	$[0.72; 0.83]$
$\xi_1\alpha^*$	0.050	0.151	0.256	0.360	0.465	0.555
$\xi_2\alpha^*$	0.050	0.151	0.256	0.360	0.465	0.555
γ_1	0.550	0.448	0.344	0.240	0.136	0.045
γ_2	0.550	0.448	0.344	0.240	0.136	0.045

Table 4: *Scenario 1* - Self-consumed $(\xi_i\alpha^*)$ and exchanged (γ_i) quantities of energy in the benchmark case over several feasible sets (FS) (dark gray squares in Figure 7).

A.5.2 Scenario 2: excess demand in the energy exchange P2P

In Figure 10, we include the Constraints (A.4.15.1) and (A.4.15.2). Then, we isolate the feasible area (in gray) as resulting from the consideration of those constraints.

Under this Scenario, the optimal capacities, (α_1^*, α_2^*), (A.4.14) and the expected net present value of the project, $O(\alpha_1^*, \alpha_2^*)$, (A.4.16) do not depend on the prosumers' self-consumption levels (ξ_i). Based on the parameters chosen for our calibration, we find that $\alpha_1^* = \alpha_2^* = \alpha^* = 1.62$ MWh and $O(\alpha_1^*, \alpha_2^*) = 5647$ Euro, respectively (see Table 5).
As it can be immediately seen in Figure 10, the capacity level $\alpha^* = 1.62$ is not feasible. Thus, we move on considering the corner solutions (Appendix A.4).
Figures 11, 12 and 13 provide graphical representations of each set of scenario's constraints [65] and the resulting ranges of ξ_1 and ξ_2 associated with a feasible solution for each corner solution. The expected net present values of the PV project associated with each corner solution are presented in Figure 14.
Table 6 summarizes the findings associated with each corner solution.
In corner solution 1, the sets of ξ_1 and ξ_2 which allows reaching the highest level of expected net present value are $\xi_1 \in [0.30, 0.53]$ and $\xi_2 \in [0.52, 0.70]$. When considering instead corner solution 2, we have $\xi_1 \in [0.52, 0.70]$ and $\xi_2 \in [0.30, 0.53]$. In both cases, we notice that i) one prosumer must be more self-consumption oriented than the other, i) the average expected net present value is lower than under Scenario 1, iii) a lower q_t or a lower σ widens the feasible area, whereas a decrease in $LCOE$ shrinks it, but these changes do not affect the sets ξ_1 and ξ_2 which allows reaching the highest level of expected net present value.
The impact of changes in q_t, σ and $LCOE$ when considering the corner solution 1 are presented in Figures 15, 16, 17, respectively. [66]

In corner solution 3, the sets of ξ_1 and ξ_2 associated with a feasible solution are $\xi_1, \xi_2 \in [0; 0.50)$ (Eq. A.4.41.1) and $\xi_1, \xi_2 \in (0.50; 1]$ (Eq. A.4.41.2) (see Figure 13). This implies that, with respect to Scenario 1, the prosumers' self-consumption profile are allowed to be more than distant.

Parameters	*Benchmark case*	$q_t = 54$	$\sigma = 0.25$	$LCOE = 70$
α^*	1.620	1.590	1.550	1.850
$O(\alpha_1^*, \alpha_2^*)$	5647	5420	5146	6546

Table 5: *Scenario 2* - Benchmark results and comparative statics

[65] where the first and second blocks represent also the prosumers' optimal capacities
[66] For the sake of brevity, we do not present the comparative statics relative to corner solution 2 since they are specular to those relative to corner solution 1.

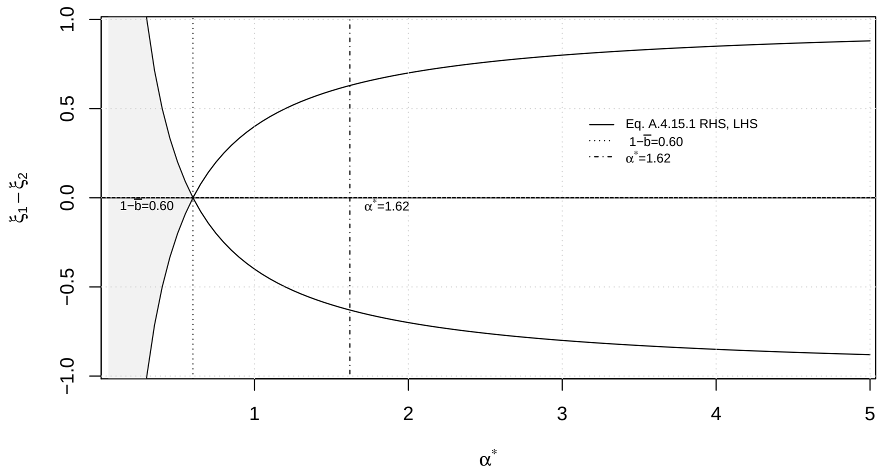

Figure 10: *Scenario 2* - The set of (ξ_1, ξ_2) associated with a feasible solution.

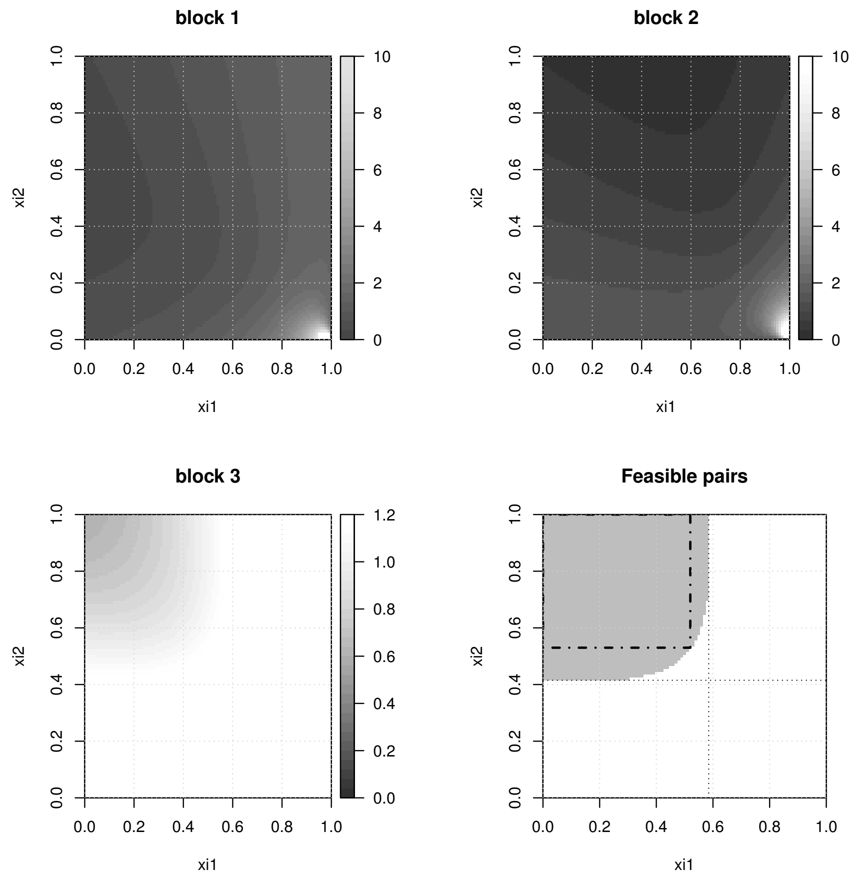

Figure 11: *Scenario 2* - Corner solution 1: constraints and pairs of ξ_1 and ξ_2 associated with the optimal solution. Blocks 1 and 2 results from considering Eqs. (A.4.25.1) and (A.4.25.2) respectively. Block 3 results from considering Eq. (A.4.25.3). The last block shows the pairs of ξ_1 and ξ_2 associated with the optimal solution.

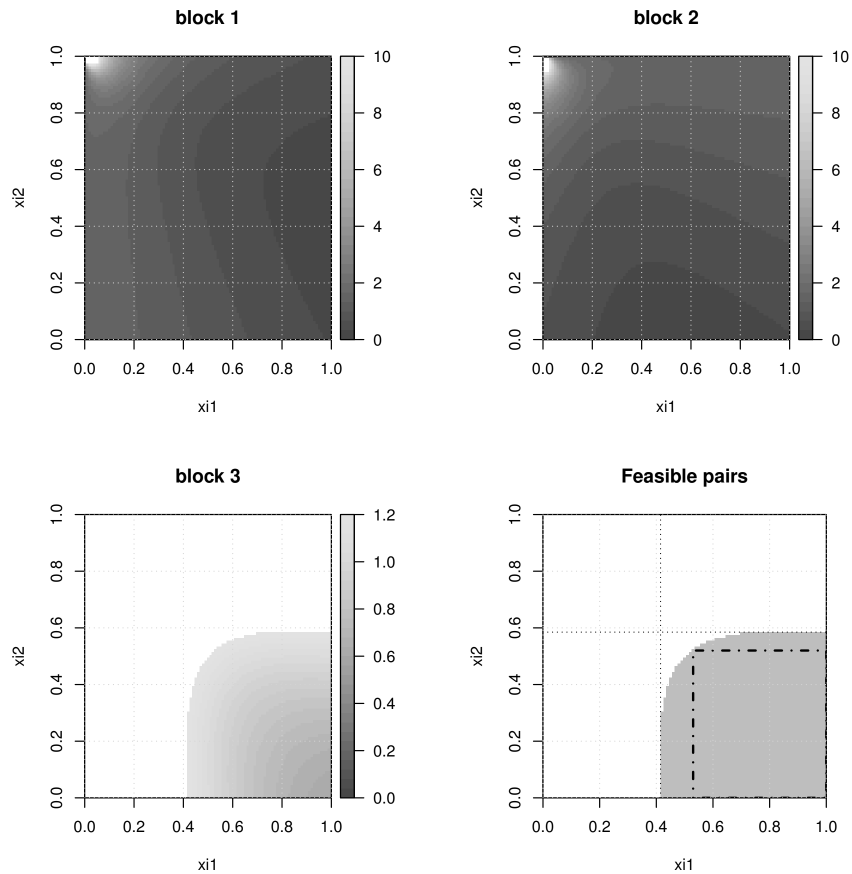

Figure 12: *Scenario 2* - Corner solution 2: constraints and pairs of ξ_1 and ξ_2 associated with the optimal solution. Blocks 1 and 2 results from considering Eqs. (A.4.35.1) and (A.4.35.2), respectively. Block 3 results from considering Eq. (A.4.35.3). The last block shows the pairs of ξ_1 and ξ_2 associated with the optimal solution.

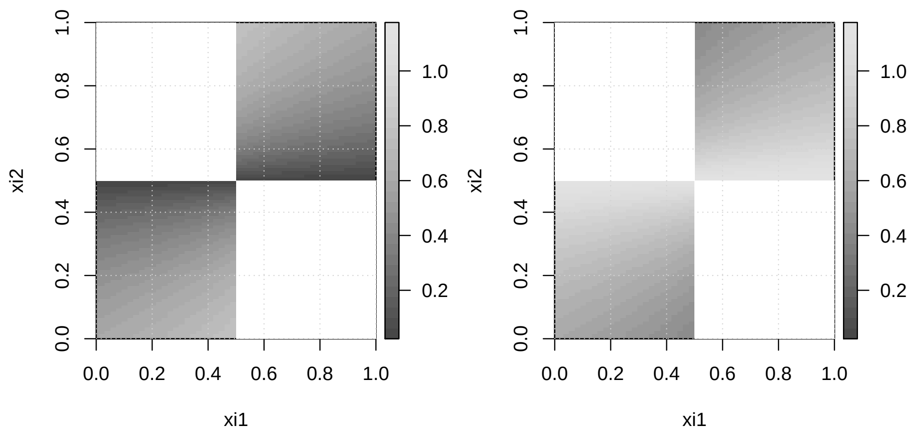

Figure 13: *Scenario 2* - Corner solution 3: constraints and pairs of ξ_1 and ξ_2 associated with the optimal solution. Blocks 1 and 2 results from considering Eqs. (A.4.37) and (A.4.38), respectively.

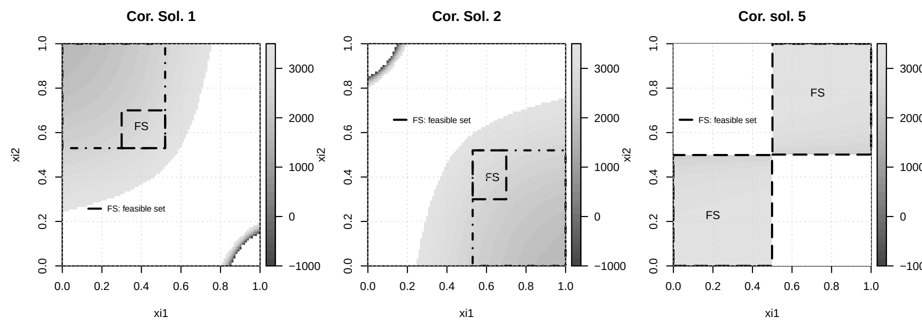

Figure 14: *Scenario 2* - Expected net present values. Blocks 1, 2 and 3 refer to corner solution 1,2 and 3 respectively. The feasible sets (FS) are identified considering only the pairs of the ξ_i associated with the highest level of expected net present value.

Parameters	*Cor. sol. 1*	*Cor. sol. 2*	*Cor. sol. 3*	*Cor. sol. 3*
$\xi_1 \in$	$[0.30; 0.53]$	$[0.52; 0.70]$	$[0; 0.50)$	$(0.50; 1]$
$\xi_2 \in$	$[0.52; 0.70]$	$[0.30; 0.53]$	$[0; 0.50)$	$(0.50; 1]$
α_1^*	0.535	0.488	0.600	0.600
α_2^*	0.488	0.535	0.600	0.600
$\xi_1\alpha_1^*$	0.228	0.295	0.174	0.426
$\xi_2\alpha_2^*$	0.294	0.229	0.174	0.426
γ_1^*	0.194	0.305	0.426	0.173
γ_2^*	0.306	0.194	0.426	0.173
$\mathcal{O}\left(\alpha_1^*, \alpha_2^*\right)$	2823	2823	3098	3098

Table 6: *Scenario 2* - Main findings by Corner solution

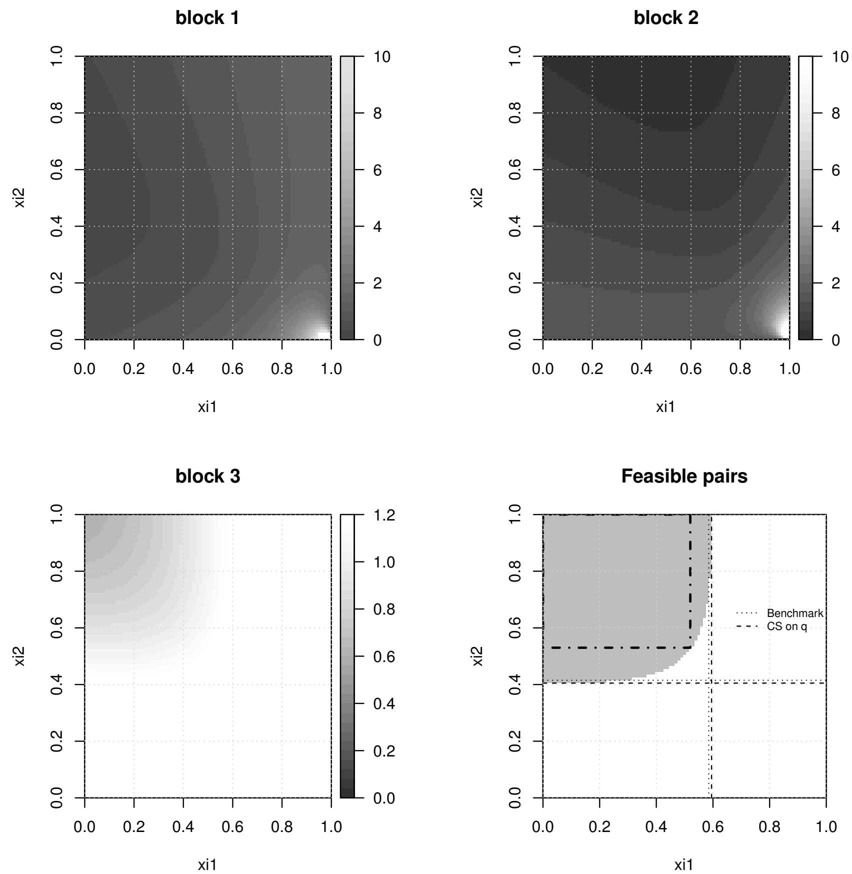

Figure 15: *Scenario 2* - Corner solution 1: constraints and pairs of ξ_1 and ξ_2 associated with the optimal solution when $q_t = 54$.

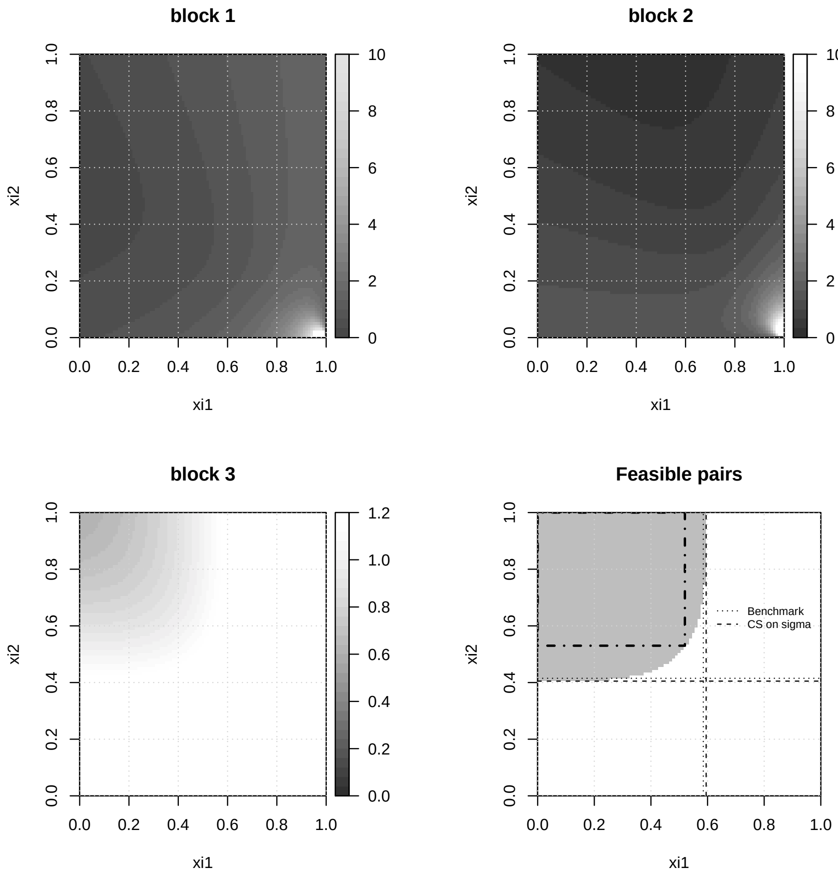

Figure 16: *Scenario 2* - Corner solution 1: constraints and pairs of ξ_1 and ξ_2 associated with the optimal solution when $\sigma = 0.25$.

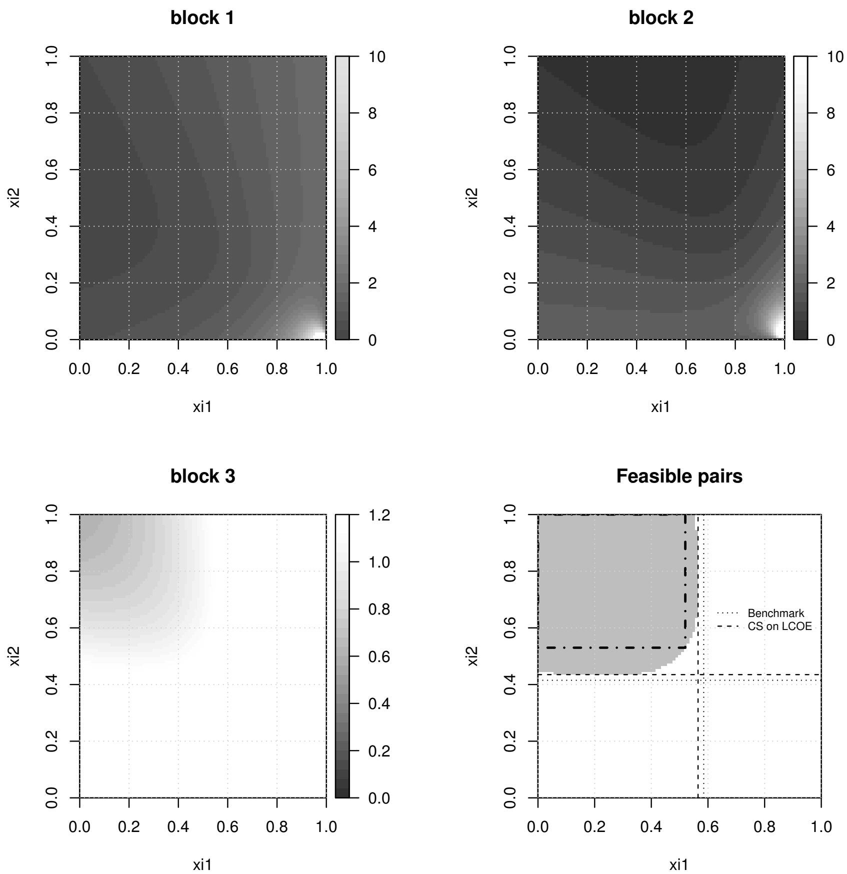

Figure 17: *Scenario 2* - Corner solution 1: constraints and pairs of ξ_1 and ξ_2 associated with the optimal solution when $LCOE = 70$.

A.5.3 Scenario 3: non complementarity in the energy exchange P2P.

Under this Scenario, the optimal capacities, (α_1^*, α_2^*) (A.4.49,A.4.50) and the expected net present value of the project, $O(\alpha_1^*, \alpha_2^*)$, (A.4.52) depend on the prosumers' self-consumption levels (ξ_i).
In Figure 18, we include the scenario's constraints as a function of ξ_1 and ξ_2, with the aim to identify the ranges over which they are all satisfied. [67] The area satisfying the constraint (A.4.51.2) satisfies also constraint (A.4.51.1). The Constraint (A.4.51.3) is satisfied if ξ_1 ranges from 0 to 0.53 (gray area). The fourth block of the Figure 18 shows the set of ξ_i associated with a feasible solution, that is $\xi_1 \in [0.51; 0.52]$ and $\xi_2 \in [0; 0.02]$. This means that the scenario's constraints are satisfied only when prosumer 1 has a relatively high level of *self-consumption* while prosumers 2 has an almost null level of *self-consumption*.

Figures 19,20 and 21 present how scenario's feasible ranges vary in response to a decrease in q_t, in σ and in $LCOE$, respectively.
Table 7 shows the optimal capacities, the quantity of self-consumed energy, the quantity of exchanged energy and the expected net present values in the benchmark case and when allowing for a change in q_t, in σ and in $LCOE$.
A reduction in q_t widens the set of the pairs of the ξ_i associated with an optimal solution, allowing prosumer 1 to reach higher levels of self-consumption. Further, the optimal capacities decrease, prosumer 1 self consumes less while prosumer 2 self consumes more. The effect on exchanged quantities is the opposite. Overall, prosumers gain less from investing in the PV project.
A decrease in σ widens the set of the pairs of the ξ_i associated with an optimal solution. The capacity installed by prosumer 2 increases, whereas the one installed by prosumer 1 decreases. The same occurs for self-consumption, while exchanged volume increases for prosumer 1 and decreases for prosumer 2. Also in this case, prosumers gain less from investing in the PV project.
Finally, any feasible solution may be found when lowering the $LCOE$ to 70.

[67] Eq. (A.4.51.1) in block 1, (A.4.51.2) in block 2 and (A.4.51.3) in block 3.
Constraints presented in Eq. (A.4.51.1) and (A.4.51.2) have been respectively rearranged as follow: $\xi_1\alpha_1^* + (1-\xi_2)\alpha_2^* - (1-\bar{b}) > 0$ and $(1-\xi_1)\alpha_1^* + \xi_2\alpha_2^* - (1-\bar{b}) < 0$. The constraints' graphical representation is obtained by letting ξ_1 and ξ_2 vary over the range from 0 to 1.

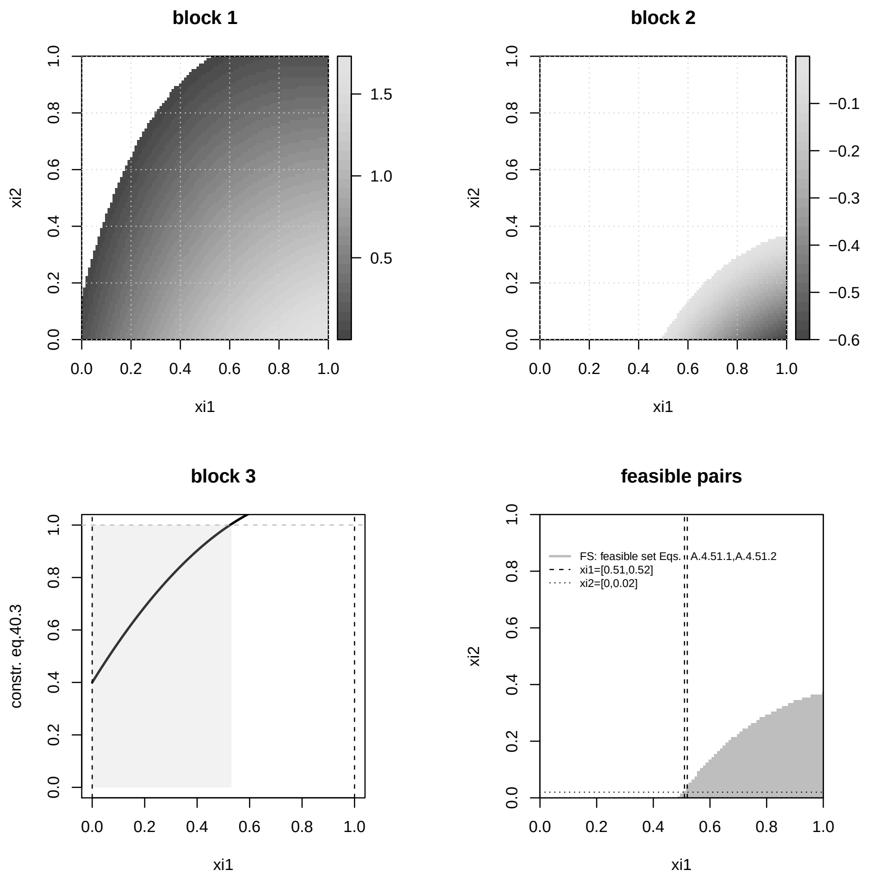

Figure 18: *Scenario 3* - Constraints and pairs of ξ_1 and ξ_2 associated with the optimal solution. Block 1 results from considering Eq. (A.4.51.1), block 2 results from considering Eq. (A.4.51.2) and block 3 results from considering Eq. (A.4.51.3). Block 4 shows the pairs of ξ_1 and ξ_2 associated with the optimal solution.

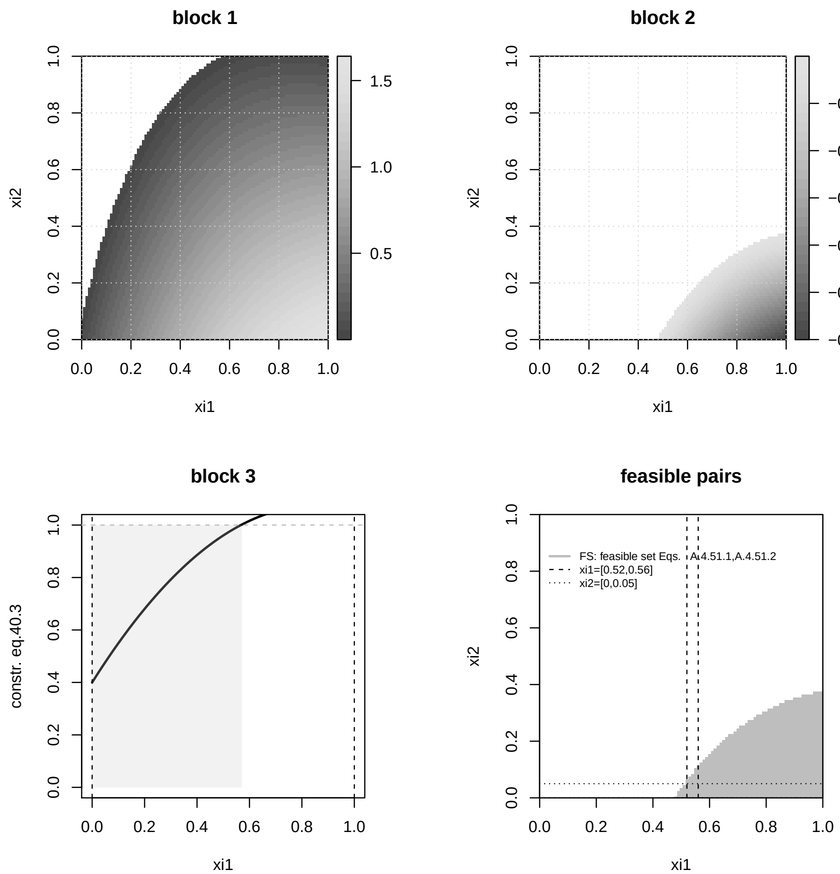

Figure 19: *Scenario 3* - Constraints and pairs of ξ_1 and ξ_2 associated with the optimal solution when $q_t = 54$. Block 1 results from considering Eq. (A.4.51.1), block 2 results from considering Eq. (A.4.51.2) and block 3 results from considering Eq. (A.4.51.3). Block 4 shows the pairs of ξ_1 and ξ_2 associated with the optimal solution.

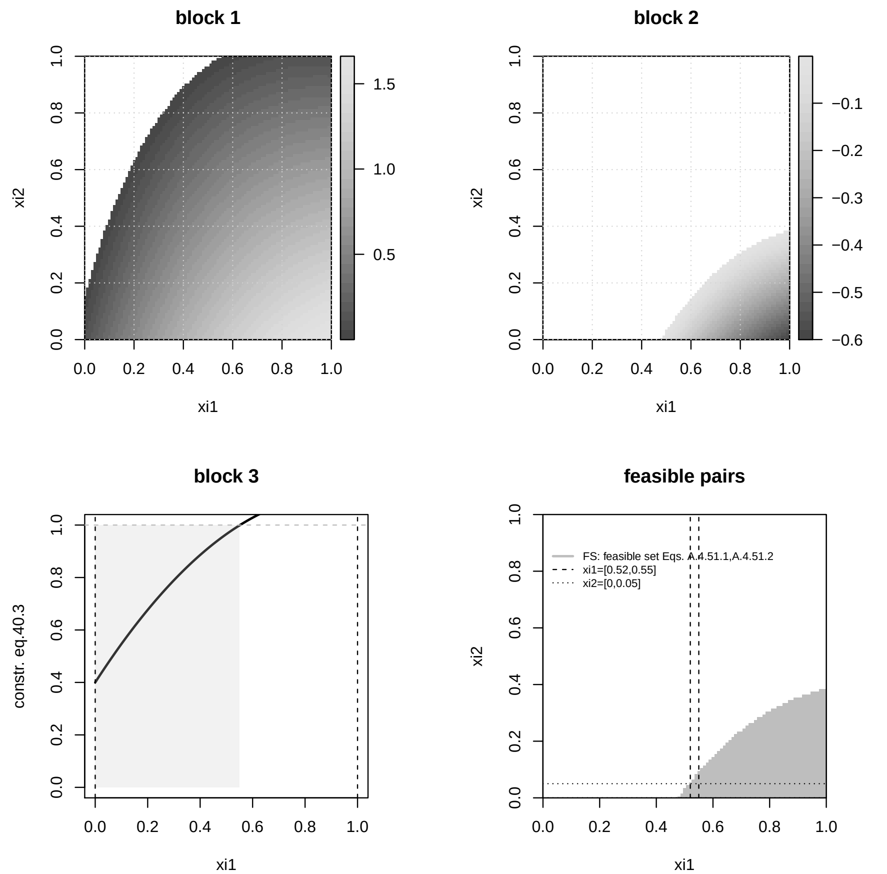

Figure 20: *Scenario 3* - Constraints and pairs of ξ_1 and ξ_2 associated with the optimal solution when $\sigma = 0.25$. Block 1 results from considering Eq. (A.4.51.1), block 2 results from considering Eq. (A.4.51.2) and block 3 results from considering Eq. (A.4.51.3). Block 4 shows the pairs of ξ_1 and ξ_2 associated with the optimal solution.

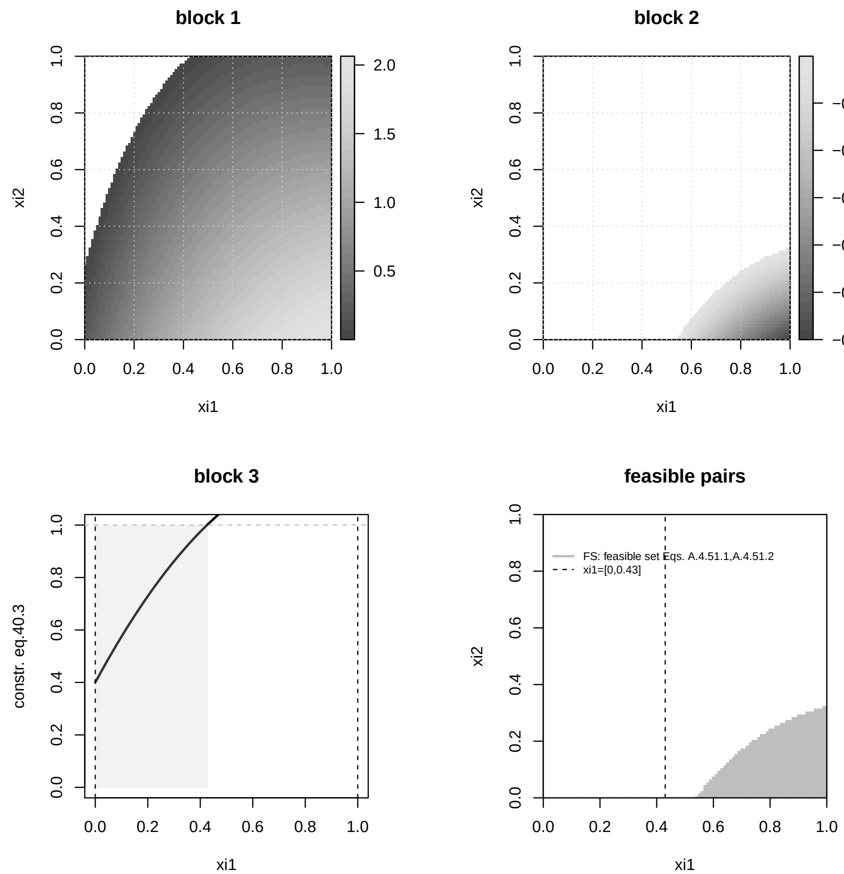

Figure 21: *Scenario 3* - Constraints and pairs of ξ_1 and ξ_2 associated with the optimal solution when $LCOE = 70$. Block 1 results from considering Eq. (A.4.51.1), block 2 results from considering Eq. (A.4.51.2) and block 3 results from considering Eq. (A.4.51.3). Block 4 shows the pairs of ξ_1 and ξ_2 associated with the optimal solution.

Parameters	*Benchmark*	$q_t = 54$	$\sigma = 0.25$	$LCOE = 70$
$\xi_1 \in$	$[0.51; 0.52]$	$[0.52; 0.56]$	$[0.52; 0.55]$	-
$\xi_2 \in$	$[0; 0.02]$	$[0; 0.05]$	$[0; 0.05]$	-
α_1^*	1.152	1.083	1.101	-
α_2^*	0.720	0.675	0.731	-
$\xi_1 \alpha_1^*$	0.5930	0.5845	0.589	-
$\xi_2 \alpha_2^*$	0.007	0.017	0.019	-
γ_1	0.007	0.016	0.011	-
γ_2	0.559	0.498	0.512	-
$\mathcal{O}\left(\alpha_1^*, \alpha_2^*\right)$	3012	2808	2811	-

Table 7: *Scenario 3* - Benchmark results and comparative statics

Chapter 4

Discussion

On the basis of the framework described in Chapter 1, the two models presented in Chapters 2 and 3 try to deepen some of the most relevant aspects of the exchange energy P2P in an SG context. In both of them, the analysis is undertaken in an uncertain framework where prosumers' investment decision occurs cooperatively and with the aim to optimize their economic joint payoffs.[1]

A brief discussion of the two models is presented in Section 1. Since the two works are linked together and refer to a common dataset, the discussion related to the calibration and numerical analysis is presented in Section 2. With reference to policy implications (Section 3) and conclusions (Section 4), a wider perspective is presented with the aim to understand the implications of the findings on the exchange P2P topic with reference to the energy communities' context.

1 The modeling framework

In the model presented in Chapter 2 specific attention has been devoted to a framework characterized by perfect complimentarity in prosumers' demand and supply in exchange P2P. The overall aim of the model is to analyze the impact of the new possibility prosumers have in terms of decision of the size of their PV plant as well as the investment timing.[2] To do that, the model was also solved under the assumption of no exchange possibility[3], thus in a context of no cooperation in the investment decision. Unfortunately, no analytical closed form solution was possible to be found and the discussion on the results was performed on the basis of the model's outcomes after the calibration.

[1]In the first model prosumers minimize their inter-temporal joint cost, whereas in the second they maximize their overall net cost saving.

[2]The investment timing has been observed through the analysis of the investment's threshold, i.e the optimal level of the price the prosumers receive from the energy sold to the national grid.

[3]see Appendix C of Chapter 2

The main features of the model presented in Chapter 2 are summarized here below:

- The prosumers' load curves are assumed to be asymmetric[4]. This feature allows to set another important assumption for the model, related to the exchange P2P, that is presented in the next bullet point.

- Perfect complementarity in demand and supply P2P. Such assumption assures the clearing of the exchange P2P market. It is important to acknowledge the stringency of such assertion, even though, since the SG framework relies on strong ICT diffusion in the energy network, as well as in the prosumers' PV system, the presence of smart devices may allow such prosumers' profile in exchange P2P.

- The cooperation in investment decision. In this framework, the prosumers' joint-investment decision implies several different simultaneous choices, apart from the one of building a PV plant, that are: i) prosumers commit to each other to exchange energy in a way such that perfect complementarity in exchange P2P is always assured, and ii) they agree on a specific price for the exchanged energy. As already mentioned above, the first action is strictly related to the prosumers' asymmetry in load curves. The second action, in turn, is a consequence of the prosumers' exchange commitment, which occurs only if the load curves are asymmetric.

On the other hand, the main elements of novelty with respect to the current state of art are:

- First application of the RO methodology in the exchange P2P topic.

- The use of the ABM to model the price increment overtime of the stochastic energy selling price to the national grid.[5]

- The price of the exchanged energy is set equal to the price the prosumers' receive for the energy sold to the national grid. As already discussed when presenting the model's feature of cooperation, there is a strict relation between the latter and the price of the energy exchanged P2P. Once the prosumers decide to invest in the PV plant, they also have to reach an agreement on the price of the exchanged energy. It is more than reasonable that this event occurs at the same timing of the investment decision and that cooperation, and not competition, is the driver of this negotiation. There is still no clear position in scientific research about the level of such price. With reference to the framework of the model, this price must be lower than the one the prosumers pay to purchase energy from the national grid, otherwise it becomes convenient to buy energy from the

[4]see Figure 1 in Chapter 2

[5]Negative spikes of the electricity prices are a recent phenomenon and are mainly caused by power generation's sources that cannot be switched off (Borovkova and Schmeck, 2017), such as wind and tidal among the renewables as well nuclear one, due to the high cost of turning the related plants off.

national grid. In addition to that, this price must be also higher than the price the prosumers receive for the energy sold to the national grid, to make exchange P2P more profitablethan the former. Since prosumers are assumed to be symmetric in energy exchange behavior (i.e. they are perfectly asymmetric in their load curves and hence this means that the energy sold and purchased between the two prosumers is of the same quantity), any level lower than the price paid for the energy bought from the national grid and higher than the one received for the energy sold to the national grid has no impact on the investment decision, because the exchange is a mere barter. Thus, in order to maximize the difference between the purchase and selling price from and to the national grid, the maximum gain is obtained if the price of exchange energy P2P is equal to the one of the energy sold to the national grid.[6] Furthermore, this choice allows to introduce stochasticity in the price of exchange energy, which in turn represents a novelty with respect to the current literature and allows the model to adress also this new perspective.[7]

The main outcomes of the model are:

- Exchange introduction, combined with self-consumption, assures the prosumers' net operative cost minimization[8], increases the size of the PV plant the prosumers choose to invest in[9], which is also closer to the energy demand of each agent.

- Prosumers characterized by *exchange-oriented profile*[10] are those gaining more from the investment, in terms of net operative costs.

- In line with RO literature, uncertainty has a positive relation with the investment timing. With high volatility levels, prosumers gain more from the investment if it is undertaken for selling purpose, meaning that prosumers characterized by *selling-oriented profile* gain more with respect to the *exchange-oriented profile* ones.

- PV technology's overall cost has been characterized by decreasing trend in the last years[11]. Such trend has a positive effect on both optimal size (increase) and investment threshold (decrease).

[6]this is also in line with the literature reviewed on the topic, and in particular following Alam et al. (2013) andMengelkamp et al. (2017).

[7]There are several scientific papers that investigate negotiations within the energy communities and many of them analysis bidding processes on several perspectives (energy exchange minimization, profit maximization...) and most part of them exploits numerical simulations of several scenarios of energy exchange P2P dynamic.

[8]For *net operative cost* definition and analytical form see Section 2.2 and footnote 31 of Chapter 2

[9]compared to the case where exchange P2P is not possible.

[10]The higher are the self consumption and exchange parameters, the lower are the prosumers' net operative costs

[11]In the model, such cost depends on the LCOE

- A subsidy on the sunk cost the prosumers pay to build the virtual infrastructure to exchange P2P, should be accompanied by instruments aimed at increasing the prosumers' use of the energy produced by the PV plant, to assure the increase in the size and in the efficiency of the system.

With reference to the model presented in Chapter 3, the problem of optimal PV plant sizing is solved again with the RO methodology, in a context characterized by uncertainty and exchange P2P. The attention now focuses on all possible prosumers' behaviors in exchange P2P. To do that, Assumption 2 of the model presented in Chapter 2 is removed with the aim to assess different exchange P2P scenarios [12].
Scenario 1 refers to the case of excess of supply from both prosumers. Scenario 2 instead focuses on excess of demand. Scenario 3 shows the case where prosumer 1 needs not more than what the other prosumer could provide, while prosumer 2 needs more than what prosumer 1 could provide. Scenario 4 instead analyzes the case in which prosumer 2 needs not more than what prosumer 1 could provide, while prosumer 1 needs more than what prosumer 2 could provide.
Each scenario is therefore characterized by constraints in terms of energy exchange between the prosumers, leading to specific conditions under which the prosumers' self-consumption behaviors must comply to assure the feasibility of the scenario. In other words, this work shows also how the prosumers' self-consumption behaviors affect the set up of the exchange P2P and the related existence conditions, taking into account the economic perspective, uncertainty and identifies the prosumers' different loads profile enabling it.
Optimal capacities under each scenario are identified in a closed form solution. Through numerical analysis, the levels of prosumers' self-consumption assuring the feasibility of each scenario, i.e. the existence of the exchange P2P, are identified (Appendix A.5 of Chapter 3). On the basis of such levels, results are analyzed focusing on those potentially in line with real prosumers' load curves.
The main features of this model are listed here below:

- Four different exchange P2P scenarios. With respect to the model presented in Chapter 2, this new framework allows to widen the perspective on the exchange P2P transaction, by studying the different characteristics of the supply and demand in exchange P2P.

- The prosumers' self-consumption profiles are the drivers of the exchange P2P existence's conditions. For each scenario, the prosumers undertake self-consumption choices such that related scenario's exchange features are satisfied. Such behavior is then compared with real load curves to verify the effective admissibility of the solutions.

- The increment overtime of the stochastic energy selling price to the national grid is assumed to follow a GBM. Such assumption allows to derive

[12] The framework of the model presented in Chapter 2 (complementarity in demand and supply of exchange P2P) is now included in one of the fourth scenarios.

closed form solutions[13], which are fundamental to identify the exchange P2P existence conditions.

The main elements of novelty of this new model are summarized here below:

- The RO methodology is applied to find the optimal size of the PV plants, the quantity of energy traded P2P and self-consumed by the prosumers.
- For each exchange P2P scenario, the optimal capacities of the prosumers' PV plants are found in a closed form solution.
- The price of the exchanged energy P2P is a weighted average of the two prices for buying and selling energy from and to the national grid. Such price is the result of a bargaining process between the two prosumers.[14] With this new price set up, the work provides an additional contribution to the discussion on the price of exchanged energy. To assure self-consumption and exchange P2P, the price of exchange must be set higher than the one the prosumers receive for the energy sold to the national grid and lower than the one they pay to buy from it.
- The existence conditions of the exchange P2P are identified in terms of prosumers' self-consumption behaviors. After having solved numerically all the feasible mathematical solutions, comparison with the daily 24-hour load curves was performed with the aim to identify the optimal combinations in terms of both saving maximization and real feasibility.

The main outcomes of the work are described here below:

- Mathematically feasible conditions for having convenient energy exchange between agents, and thus it isoptimal to set up and exchange P2P structure, are found for each scenario.
- Although each scenario is characterized by different supply-demand profiles, very similar total savings are achieved. The best solution is the one characterized by excess supply and asymmetric and complementary load curves.
- The prosumers' profiles which guarantee the maximum benefit (NPV of the generated savings), are characterized by perfectly asymmetric and mutually complementary demand functions: agents produce, consume and exchange energy in such a way as to cover each other's opposite daytime demand functions. If they have an oversupply (scenario 1) they also sell some of their production to the national grid in order to maximize the benefit. If they have excess demand (scenario 2), they sell nothing to the national grid but cover all their daytime demand with their own production.

[13]For discussion on this matter is presented in footnote 19 of Chapter 3
[14]see Appendix A.1 of Chapter 3

2 Numerical analysis

The empirical parts of the two models share some elements, both in terms of the parameters' calibration choices as well as the methodology used to estimate the parameters of the over time increment of the stochastic price the prosumers receive for the energy sold to the national grid.

With reference to the latter, the method of moments is applied to estimate the constant increment in the stochastic energy price over time and the instantaneous standard deviation of the price increment, while the dataset is built on the basis of the prices provided by the Italian energy provider [15]. It is also important to acknowledge that the same dataset is used in both works, even though the models are characterized by two different types of Brownian Motions (ABM in the first and GBM in the second). As already discussed in previous Section 1, the analytical tractability of the GBM is required in the model presented in Chapter 3 to obtain closed form solutions for the optimal capacities and identify exchange P2P optimal existence conditions. On the other hand, the ABM fits better to the data[16] and allows to take into account also the possibility of negative energy prices.[17]

Specific attention must also be drawn on the parameters describing the prosumers' self-consumption and exchange P2P behaviors.

In the model presented in Chapter 2, the parameter describing the prosumers' self-consumption behavior (ξ_i) is set making reference to the current literature. The same for the parameter related to prosumers' exchange P2P (γ_i), even though specific computations were required since there is still few empirical evidence about the quantity of energy the prosumers' may be willing to exchange P2P. To do that, one specific scientific publication was used as reference. In Zhang et al. (2018) a P2P energy trading platform was designed and a P2P energy trading was simulated using game theory. Table 3 compares the exchange of energy between a micro-grid and the utility grid over one day, under the assumption that the micro-grids' prosumers own PV plants. The introduction of the exchange P2P reduces the energy exchange with the utility grid of 9.19%. This data is used as a proxy of the share of energy that the prosumers decide to exchange instead of selling it to the utility grid. However, it is important to underline that, to the best of the current knowledge, there are no scientific publications providing specific insights on this matter. For sure, further research must be developed on this topic. Indeed, the model presented in Chapter 3

[15] Detailed description is provided in the two Calibration Sections of the two models.

[16] As also underlined by Borovkova and Schmeck (2017), "despite the voluminous literature on modeling electricity prices, there is no clear *winner* model". With reference to the price evolution overtime, the authors state that "mean reverting and jumps should be inherent features of such a model". Similar considerations were drawn by Gianfreda and Grossi (2012) and both Edoli et al. (2017) and Andreis et al. (2020) refer to such process in their discussions on modelling electricity prices. However, the mathematical tractability of the mean reverting process is complex and, under the framework of the first model, such process' features not allow any solution of the model.

[17] See also Fanone et al. (2013)

tries to provide some insights on this matter.[18] In its Section 6 and the related Appendix A.5, for each scenario the quantity of exchanged energy between prosumers is computed over the self-consumption set assuring the exchange P2P optimal existence.

With reference finally to the exchange P2P existence conditions, their identification relies on one of the main differences, in terms of numerical part, between the models. In the one presented in Chapter 3, self-consumption parameter (ξ_i) is set free to vary over the interval from 0 to 1. The prosumers' self-consumption behavior must be bounded to fit the constraints set by the features of each exchange P2P scenario. Further details on this overall framework and the numerical and graphical resolutions are available in Appendix A.5 of Chapter 3.

In the first model, the prosumers' self-consumption level is set equal to 0.30, and the main assumption, on the exchange P2P side, is the perfect complementarity in exchange P2P. A similar framework is the one presented in scenario 2 (excess of demand), corner solution 3[19]. From Table 6 (Appendix A.5), the ranges of the self-consumption parameter ξ_i satisfying the scenario's constraints are $\xi_i \in [0, 0.50)$ and $\xi_i \in (0, 0.50]$. The first interval includes the value associated to ξ_i in the first model (0.30). However, such level is not a feasible solution of the exchange P2P existence in scenario 3[20]. Indeed, Table 6 shows the outcomes that are the most representative[21] of all the four scenarios.[22] Unfortunately, there is no common value, or pair, for the parameter ξ_i across all the scenarios. From this perspective, one relevant element which deserves attention to better design policies related to the exchange P2P in an environment characterized by uncertainty, is the understanding of prosumers' exchange preferences, which in turn are affected by their self-consumption ones, as well as by the potential gain from trading with the grid. Since exchange P2P existence conditions rely on the self-consumption levels, policy makers should design instruments aimed at shaping the prosumers' behaviors towards its highest value, but also taking into account the exchange P2P existence conditions, so that environmental goals and economic optimality are both achieved.

This rationale must also be applied to the optimal PV plant sizing problem as well as investment timing. Prosumers' investment decisions are driven by many aspects, such as environmental concern, willingness of energy self-sufficiency, economic benefits, available wealth, among others.[23] On the other hand, policymakers need to achieve the decarbonization goal and a central role is acknowl-

[18] In the model presented in Chapter 2, the parameter γ_i determines the share of energy that each prosumer is willing to buy in exchange P2P on the basis of the energy produced by the PV plant of the other prosumers and that he/she do not self consume. In the one presented in Chapter 3, γ_i is the overall quantity bought in exchange by each prosumer.

[19] Each prosumer purchases in exchange exactly the quantity of energy that the other prosumer not self consumes.

[20] see Table 2 of the model presented in Chapter 3

[21] The selection of the outcomes for the results' discussion basis on the following elements: i) mathematical feasibility, ii) consistency with reality, and iii) economic profitability.

[22] Numerical solution of scenario 4 is not performed in the Appendix 5 and discussed in detail in the Results' section because it is symmetric to scenario 3.

[23] A detailed list of factors affecting the interest in small-scale generation is also provided by Mandelli et al. (2016), Table 2.

edged to the renewables' deployment.
With reference to the model presented in Chapter 2, the exchange P2P introduction always increases the size of the PV plant, but prosumers' investment threshold increases too, due to the higher sunk cost. This would imply that policy makers should support exchange P2P together with measures aimed at decreasing the investment thresholds, such as subsidies that lower the increase of the initial sunk cost, due to bigger PV plant sizes. Table 4.6 and 4.7 provide some insights about this matter. The parameter P represents the cost the prosumers pay to have access to the exchange P2P through the SG and the comparative statics summarized in the tables allow to better understand the effect of a possible measure in such direction. Indeed, a subsidy focused on the reduction of the sunk cost is not sufficient, since specific attention must be paid on the efficiency[24] of the PV system. The subsidy must be combined with policies aimed at increasing the use of energy produced by the PV plant, which could be the case of facilitating storage introduction.
This consideration becomes more complex under the framework described by the model presented in Chapter 3. Prosumers' invest in the highest capacity when they are characterized by non complementarity in exchange P2P (scenario 3). In addition to that, the self-consumption profile of the two is different: one prosumer builds the PV plant to self-consume, whereas the other to exchange. As already stated before, the exchange P2P existence conditions can set specific benchmarks for such policies, because if prosumers are subsidized to self-consume all the energy produced by their PV plant, nothing is left for the exchange P2P and so, what is the sense of supporting such new framework if a policy like this one is introduced? Of course this is a simplistic discussion of this topic and it must be acknowledged that removing no storage possibility changes deeply the overall framework and its perspective.

3 Policy implications and recommendations in the energy communities' field

On the basis of the main framework presented in Chapter 1, it is clear that a complex policy effort will be required by the EU itself, as well as by the Members States, to design proper measures aimed at supporting the deployment of the SG, exchange P2P and energy communities as well.
With reference to the first, cooperation between the national regulating authorities and the energy providers is required to boost the modernization, as well as digitization, of the whole electricity network. As also remarked by EU (2019), consumers should be able to adjust "their consumption according to market signals and, in return, benefitting from lower electricity prices or other incentive payments". To do that, specific solutions should be developed to overcome the

[24]in terms of lowering the energy purchased from the national grid respect to optimal capacity level. With reference to the notation of the model presented in Chapter 2, efficiency is assessed in terms of b_i/α_i^* ratio

lack of real-time or near real-time information about their energy consumption as well as on prices. Indeed, as also widely acknowledged by RO literature in energy field, and stressed in the first model, decreasing uncertainty on energy prices speeds investments timing. The achievement of such goal requires targeted interventions, which must deeply rely on the technical improvements and digitization of the overall energy network.[25]

With reference to exchange P2P and energy communities, policymakers are called to design the legal framework and related economic measures in a field which is still actually under-developed in reality. Some knowledge is provided by the study of pilot projects, even though many aspects are yet to be understood both on the technical as well as economic side.[26]

In the case of energy communities, Frieden et al. (2020) analyze several pilot projects and categorize them on the basis of three main elements: i) community owned generation assets[27], ii) virtual energy sharing[28] and iii) sharing of local production through community grids.[29]

The last two categories represent the most interesting for this work. Some of these "initiatives are driven by the local communities' wish to consume local energy", while others are promoted by "energy companies in a drive to innovate in the smart grid space and to create microgrids" (Frieden et al., 2020).

CEER (2019) underlines the importance of local matching within the energy communities. It is very likely that members try to match local generation with local demand[30] to increase the ability of consuming the energy generated within the energy community. This consideration shows the importance of the exchange P2P understanding. In addition to that, the positive technical impacts of the energy exchange, whether occurring virtually or physically, can be achieved only if combined by incentives aimed at changing the participants' consumption and production patterns in a way that is consistent with the needs of the whole energy system (CEER, 2019).

The arising of this complex framework has led to a growing interest by researchers and practitioners in the topic of the *energy sharing schemes*, focusing on the prosumers' behaviors in exchange P2P on one side, and the relation of the energy community with the national grid, on the other.

One of the main future challenges will be the development of new load manage-

[25]European Court of Auditors (2019) provides a detailed analysis of the current state of art of the wind and solar power generation system in the EU. Specific attention has also been devoted to the elements hindering growth of electricity from renewable sources. Among others, insufficient actions and practical barriers for the prosumers and the energy communities are described making reference to real cases.

[26]Among others, Roberts et al. (2014), CEER (2019) Caramizaru and Uihlein (2020), Frieden et al. (2020) and D'Alpaos and Andreolli (2020), which provide a wide overview these topics, advising also specific policy suggestions.

[27]The members do not self-consume the energy produced, but sell it to a supplier. The income is shared with members and/or re-invested in energy projects.

[28]This point refers to the case where the energy produced by the energy community is shared among its member through a common supplier, who takes care of the matching between production and consumption and supplies additional energy needed.

[29]Energy is physically shared through a community grid.

[30]where with "local" term CEER (2019) refers to the network within the energy community.

ment solutions for the local sharing. These new approaches are also expected to take into account the opportunities and constraints provided by the national grid. In the case of REC, the role of the latter will remain fundamental, as supplier of the prosumers' energy need that cannot be covered with the renewable energy local production.[31] On the other side, the expected changes in the energy markets of the future will imply new challenges for the traditional energy providers. Indeed, they will have to deal with a new structure of energy demand, mainly characterized by presumption and energy sharing P2P as well as new organizational frameworks like energy communities.[32]
The two models presented in this work can provide some insights on these new challenges. On the basis of the review of the literature related to the models and their outcomes, the main policy challenges that could arise in the future are listed here below:

- Policymakers should ground their decisions on the basis of the previous experience in renewable support schemes. Prosumers have already knowledge of these instruments and have adjusted their self-consumption and investment behaviors over time. The design of new policy instruments to support the exchange P2P and the diffusion of energy communities must take these adjustments in the prosumers' behaviors into account .

- With reference to the exchange P2P, the key element on which they should focus is the prosumers' cooperation dynamics, which involves the initial investment decision (i.e. the set up of the exchange P2P framework as well as the energy communities ones, in the type of REC), the supply and demand matching in exchange P2P and the creation of the exchange P2P market, which in turn allow to identify the price of the exchanged energy.

- Prosumers' self-consumption behaviors should be the core of the policy design. As shown in particular in the model presented in Chapter 3, the prosumers' self-consumption is one of the elements able to affect the effectiveness of the exchange P2P. In a certain manner, it is possible to say that the same rationale could be applied to the case of REC.

- Storage possibility may represent a turning point for the prosumers. However, the related high sunk cost and the absence of proper ICT infrastructure in their facilities makes this possible solution still far from being adopted and, if so, managed efficiently.

Thus, to sum up, policy makers must focus on a policy mix that should make reference to the following elements: i) energy prices, ii) prosumers self-consumption

[31]"The possibility of local exchange of energy, be it through collective elf-consumption, sharing the output of co-owned production asset, or peer-to-peer trading, rises the question of the relationship between the supplier and the local source of supply. Locally shared production may provide for part of the consumption, but in most cases, a back up supplier will still be needed to meet demand when the local production is not generating"(CEER, 2019).

[32]"The aggregation of demand and shifting of demand patterns can also allow energy communities to consume when spot markets offer lower prices, if they have access to market prices signals" (CEER, 2019)

behavior, iii) prosumers engagement activities towards the exchange P2P and the energy communities concept[33] and iv) storage adoption.

4 Conclusions

The main goal of the thesis, in which different perspectives on SG and exchange P2P were provided, is to complement a research strand which is yet to be developed in an uncertain framework. In addition to that, some insights provided by the models are analyzed with the aim to study the arising of energy communities. Indeed, prosumers' individual investment choices and willingness to cooperate in collective self-consumption [34], the engagement in energy sharing and the organization in energy communities, will be key elements of the future energy markets.

One of the main conclusions that can be drawn, on the basis also of the analysis of the existing research, is that there are still many aspects of these topics that need to be understood and further developed.

The economic analysis based on static models must be complemented with dynamic ones. Much modeling effort must be undertaken in this context, taking into account all possible aspects of uncertainties.

The design of this *new energy market* should ground on research findings developed taking into account also the traditional economic market theory, especially at micro level. This will allow to better understand the key elements and drivers of the exchange P2P as well as its implications in the arising of energy communities.

Further economic research must be also performed on the price the prosumers agree to exchange energy one with each other. On this side, a central role will be played by the organizational structure that will characterize the energy communities. The development of these new entities will deeply affect the context and the dynamics under which the price of the energy exchanged will be defined.

In addition to that, not so much is said about the environmental impact of the PV technology diffusion and REC as well. While usually assumed to be “clean”, the solar energy is not a zero environmental impact solution. In fact, both the production and the disposal of solar panels cause environmental damages. With reference to the optimal PV plant sizing problem, much attention should be devoted to the PV dismantling and recycling matters and on the impact that the energy communities diffusion may entail under this perspective.

Furthermore, the prosumers and social planner's perspectives must be taken into account in a simultaneous way to assure the positive and effective impact of the SG, exchange P2P and REC deployment.

[33] see also detailed discussion on empowerment and social innovation provided by Caramizaru and Uihlein (2020)

[34] Frieden et al. (2020) underline that such term is frequently associated to the "jointly acting renewables self-consumers" and that it could occur as "specific activity in the context of an energy community".

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