

# Bridging $k$ -sum and CVaR optimization in MILP

## Supplementary Material

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In this supplementary material, we show that the solutions returned by model MKP(0.30) depend of the set of scenarios used.

In Table A, we integrate the data reported in Table 2 of the paper with information about the expect profit of the different solutions. We take the case of  $S = 100$  scenarios, always solved to optimality by model MKP(0.30), as the base case. Column “EP”, referred to the base case, contains the expected total profit of the optimal solutions. Here, the expected profits are computed according to the theoretical distribution that generates scenarios, namely, the expected profit of object  $j$  is  $0.125 \cdot c_j$  for all  $j = 1, \dots, 100$ . For  $S = 200, \dots, 500$ , we report, in column “% diff.” the percentage differences of the expected profit of the obtained solution with respect to the expected profit of the solution in the base case. All figures written in *italic* refers to instances that are not solved to optimality within the time limit (cf. Table 2 in the paper).

The main insight given by Table A is that, by increasing the number of scenarios, model MKP(0.30) return different solutions (the values of expected profit are always different with respect to the base case).

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Table A: Multidimensional knapsack instances with uncertain profits and increasing number of scenarios: expected values of the solutions returned by model MKP(0.30).

No. of scenarios $S =$	100	200	300	400	500	
$\alpha$	#Inst	EP	% diff.	% diff.	% diff.	% diff.
0.25	1	<b>2608</b>	-1.76	-2.81	-3.05	-0.99
	2	<b>2460</b>	2.52	3.73	0.99	6.91
	3	<b>2535</b>	-4.98	-0.51	-2.36	-7.78
	4	<b>2612</b>	-7.96	-5.24	1.27	-2.80
	5	<b>2389</b>	3.52	6.24	2.28	9.58
	6	<b>2548</b>	0.65	1.84	3.85	3.22
	7	<b>2622</b>	7.84	4.77	8.17	8.67
	8	<b>2269</b>	-3.30	2.29	0.54	6.71
	9	<b>2380</b>	1.11	5.02	6.85	9.55
	10	<b>2680</b>	-2.82	2.28	4.51	4.80
0.5	11	<b>4703</b>	1.27	2.92	5.13	4.51
	12	<b>4486</b>	0.35	6.28	6.14	7.33
	13	<b>4564</b>	1.67	0.96	3.66	1.46
	14	<b>4913</b>	0.84	3.82	2.42	6.35
	15	<b>4555</b>	-0.54	2.48	3.84	1.23
	16	<b>4605</b>	0.67	0.74	1.97	3.18
	17	<b>4524</b>	-0.78	-1.57	0.41	2.19
	18	<b>4905</b>	-2.52	4.39	4.25	6.65
	19	<b>4708</b>	4.17	4.65	8.04	7.40
	20	<b>4839</b>	-4.17	-2.30	-0.50	-1.14
0.75	21	<b>7064</b>	-3.20	-1.21	-0.63	0.19
	22	<b>7121</b>	-0.07	1.67	2.12	1.29
	23	<b>6847</b>	-0.48	1.19	2.00	2.64
	24	<b>7101</b>	0.57	0.43	0.04	-0.42
	25	<b>6941</b>	3.91	2.62	2.39	2.03
	26	<b>6844</b>	1.04	1.79	3.30	2.07
	27	<b>7059</b>	0.02	1.46	1.96	2.55
	28	<b>6883</b>	2.48	3.82	5.11	4.92
	29	<b>6545</b>	2.68	3.09	5.83	5.03
	30	<b>6894</b>	-0.06	2.40	3.33	4.00