Rogue Gap Soliton

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Extreme waves occur in many scientific and social contexts, from hydrodynamics and oceanography to geophysics, plasma physics, Bose-Einstein condensates, financial markets and nonlinear optics. A typical example of rogue wave is the sudden appearance in the open sea of an isolated giant wave, whose height and steepness are much larger than the average sea values. A universal model for the dynamics of rogue waves is the one-dimensional nonlinear Schrödinger (NLS) equation in the self-focusing regime [1]. The mechanism leading to the generation of NLS rogue waves requires nonlinear interaction and modulation instability (MI) of the continuous wave (CW) background. Indeed, the nonlinear development of MI may be described by families of exact solutions such as the Akhmediev breathers. A special member of this solution family is the Peregrine soliton [2], which describes a wave that appears from nowhere and disappears without a trace. The Peregrine soliton was only recently experimentally observed in optical fibers [3].

Here we present the rogue wave solution of the classical massive Thirring model (MTM), a two-component nonlinear wave evolution equation that is completely integrable by the inverse scattering transform technique [4]. The MTM is a particular case of the coupled mode equations (CMEs) that describe pulse propagation in periodic or Bragg nonlinear optical media [5]. Soliton solutions of the MTM can be mapped into Bragg or gap solitons, that lead to pulse reshaping and dispersionless slow light generation in fiber Bragg gratings [6].

Let us express the MTM equations for the forward and backward waves with envelopes U and V as: $U_\xi = -i\nu V - \frac{i}{\nu}|V|^2 U\,; V_\eta = -i\nu U - \frac{i}{\nu}|U|^2 V$. Here the light-cone coordinates ξ , η are related to the space coordinate z and time variable t by the relations $\partial_\xi = \partial_t + c\partial_z$ and $\partial_\eta = \partial_t - c\partial_z$, where $c > 0$ is the linear group velocity. By developing a novel form of the Darboux transform method [7], we obtained the MTM rogue soliton solution

$$U = ae^{-i\omega t}\frac{\mu^*}{\mu}[1-\frac{4}{\mu^*}q^*(q+i)] \ , \ V = -ae^{-i\omega t}\frac{\mu}{\mu^*}[1-\frac{4}{\mu}q^*(q+i)]$$

where: $\omega = -\nu(1-\frac{a^2}{\nu^2})$, $q = -\frac{a^2}{\nu c}(ip(z-z_0)-c(t-t_0))$, $p = \sqrt{\frac{\nu^2}{a^2}-1}$, $\mu = 2|q|^2 - iq + iq^* + 1 - \frac{1}{p}(q-q^*+i)$, and z_0 and t_0 are arbitrary space and time shifts.

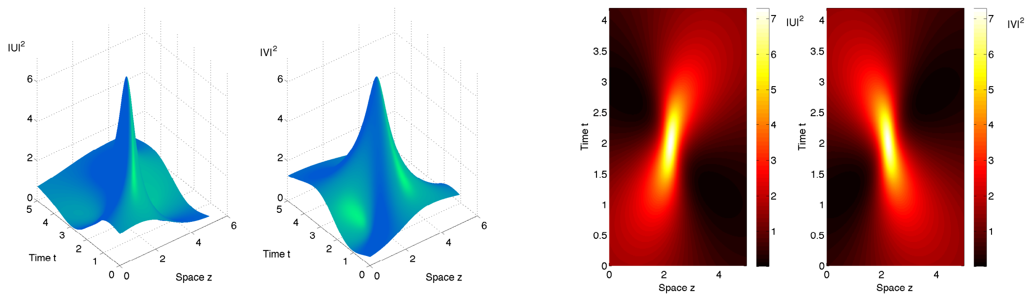

Fig. 1 Space-time evolution of intensities in forward and backward rogue components: surface (left) or contour (right) plots.

After discussing the analytical rogue wave solution (see an example in Fig.1), we numerically confirm its stability, and show that it may also be applied to describe the generation of extreme waves in the more general context of nonlinear grating propagation as described by the CMEs. Finally, we discuss the physical implementation of MTM rogue waves by using electromagnetically induced transparency, which leads to giant enhancement of cross-phase modulation and suppression of self-phase modulation [8].

References

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