POLE PLACEMENT TECHNIQUES FOR A CLASS OF IIR BLIND EQUALIZERS

M. C. Campi, R. Leonardi and L. A. Rossi
University of Brescia, DEA via Branze 38, I-25123, Brescia, Italy
FAX: +(39)-030-380014, Email: {campi, leon, lrossi}@ing.unibs.it

ABSTRACT

This work proposes a method for blind equalization of possibly non-minimum phase channels using particular infinite impulse response (IIR) filters. In this context, the transfer function of the equalizer is represented by a linear combination of specific rational basis functions. This approach estimates separately the coefficients of the linear expansion and the poles of the rational basis functions by alternating iteratively between an adaptive (fixed pole) estimation of the coefficients and a pole placement method. The focus of the work is mainly on the issue of good pole placement (initialization and updating).

1 INTRODUCTION

Blind equalization deals with the problem of estimating the unknown input to an unknown possibly non-minimum phase channel by the sole knowledge of the noisy channel output. There are many blind equalization approaches available for finite length tapped delay lines (TDL), e.g. [6]. However it should be noted that the impulse response of an ideal equalizer can in general be IIR. On the other hand, it has been shown that increasing the length of a TDL equalizer does not necessarily lead to better equalization performance, due to problems of estimation accuracy [3]. Therefore, blind equalization methods for IIR filters have been recently suggested [2, 8].

Our approach is based on a class of IIR filters whose transfer function is a linear combination of rational basis functions and as such is an IIR generalization of the TDL equalizers. The aim is to adopt lower order filters with good equalization performance. Thus there are two classes of parameters to be estimated: the weights of the linear expansion and the poles of the rational basis functions. The presented approach iterates between an adaptive (fixed pole) estimation of the weights and a set of pole placement methods. For the estimation of the equalizer weights (with fixed pole values), the Super-Exponential (SE) algorithm of Shalvi and Weinstein [6] was reformulated [1]. Pole locations are initially estimated by identifying the optimal moving average (MA) channel model thanks to an exploitations of properties of the SE method. A procedure for a progressive tuning of the pole values is then formulated.

The paper is organized as follows. In Section 2, the equalization problem is formulated. Section 3 presents the generalization of the SE algorithm. Section 4 describes the pole estimation methods and the complete equalization procedure. Finally, simulation results are presented in Section 5.

2 PROBLEM FORMULATION

Consider the discrete time model represented in Figure 1. The input sequence $a(k)$ is a zero mean and independent identically distributed (i.i.d.) sequence of non-Gaussian random variables. The channel $\{h(k)\}$ is stable and does not present zeros on the unit circle. The output $x(k)$ of the linear time invariant channel $\{h(k)\}$ is observed. We want to adjust the linear equalizer $\{e(k)\}$ so that its output $y(k)$ be equal to a delayed version (up to a constant phase shift) of the random signal $a(k)$. Note that the ideal equalizer is often an IIR system, as it has to invert the zeros of the channel.

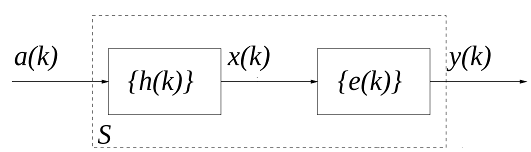

Figure 1: Channel-equalizer cascade

In our approach, the equalizer $\{e(k)\}$ is a causal stable possibly IIR filter (all poles lay within the unit circle) having the following transfer function:

$$E(z) = \sum_{k=0}^{L} e_B(k) B_k(z) = \sum_{k=0}^{\infty} e(k) z^{-k}, \qquad (1)$$

where the terms $B_k(z)$, $k = 0, 1, ..., L$, are rational stable linearly indipendent functions which can have poles in different locations of the unit circle. A particular well known case is the TDL equalizer, whose poles are all equal to zero and therefore $B_k(z) = z^{-k}$. Notice that the equalizer coefficients are represented here by $\{e_B(k)\}$, while the coefficients of its impulse response are $\{e(k)\}$.

In our experiments we adopted the set of discrete-time functions $\{b_n(k)\}$ $k, n \in \mathrm{N}_0$ having a $\mathcal{Z}$ transform

$$B_n(z) = \frac{\sqrt{1-|p_n|^2}}{1-p_n z^{-1}} \prod_{j=0}^{n-1} \frac{z^{-1}-p_j^*}{1-p_j z^{-1}} \tag{2}$$

where $(\cdot)^*$ denotes conjugation, and $|p_n| < 1$ $,\forall n,$. Those are orthonormal on the unit circle and are called Kautz functions [5].

3 GENERALIZED SUPER-EXPONENTIAL METHOD

The solution the aforementioned equalization problem implies the optimal estimation of the poles and the coefficients of the basis expansion. As such, the problem clearly presents multimodal error surfaces and it is difficult to find the global optimum. As we said, the proposed procedure estimates separately the equalizer coefficients (i.e. the weights of the basis functions) and the poles (i.e. the parameters characterizing the shape of the basis functions) by alternating iteratively between an adaptive (fixed pole) estimation of the coefficients and a pole placement method. Therefore, once the pole values have been fixed, the equalization problem becomes linear in the parameters, as it is in the case of classical TDL equalizers, and therefore TDL algorithms can be adopted with some modifications. We will show that this approach prevents the risk of instability and presents a good convergence behaviour (Section 5). To the end of coefficient adjustment, we derived a generalized version of the SE method [1], that we shortly describe in the following paragraphes.

We recall that, the Shalvi-Weinstein algorithm [6], expressed in the domain of $\{s(k)\}$ coefficients, consists in the following two-step iterative procedure:

$$\begin{aligned} g(k) &= s(k)^p (s^*(k))^q, \\ s(k) &= \frac{1}{\| \mathbf{g} \|} g(k), \end{aligned} \tag{3}$$

where $\| \cdot \|$ denotes the euclidean norm, $\mathbf{g}$ is the vector of the $g(k)$ and p, q are positive integers such that $p+q > 2$.

Let us introduce some notation adopted for our formulation. The coefficients of the overall channel-equalizer cascade system impulse response $\{s(k)\}$ are given, in matrix notation, by

$$\mathbf{s} = \mathbf{H}\mathbf{e}, \tag{4}$$

where $\mathbf{H}$ is the channel convolutional matrix with $H_{(ij)} = h(i-j)$ and $\mathbf{e}$ is the vector of the equalizer impulse response coefficients. Then, we have to express the relation between equalizer coefficients $\{e_B(k)\}$ and equalizer impulse response coefficients $\{e(k)\}$. Given a set of poles $\{p_i\}$, let us indicate the infinite dimensional vector of the impulse response for the generalized basis of order j:

$$\mathbf{b}_j := \left[\begin{array}{ccc} b_j(0) & b_j(1) & \cdots \end{array}\right]^T, \tag{5}$$

with $(\cdot)^T$ transpose operator. Hence the equalizer impulse response may be expressed as

$$\mathbf{e} = \mathbf{B}\mathbf{e}_B, \tag{6}$$

where $\mathbf{B}$ is the $\infty \times (L+1)$ matrix

$$\mathbf{B} := \left[\begin{array}{cccc} \mathbf{b}_0 & \mathbf{b}_1 & \cdots & \mathbf{b}_L \end{array}\right]. \tag{7}$$

In [1], we showed that the algorithm in (3)) can be approximated in the domain of the coefficients $\{e_B(k)\}$ as follows:

$$\begin{aligned} \mathbf{e}'_B &= \left((\mathbf{HB})^+ \mathbf{HB}\right)^{-1} (\mathbf{HB})^+ \mathbf{g}, \\ \mathbf{e}_B &= \frac{\mathbf{e}'_B}{\sqrt{\mathbf{e}'^+_B (\mathbf{HB})^+ \mathbf{HB} \mathbf{e}'_B}}, \end{aligned} \tag{8}$$

where $(\cdot)^+$ is the transpose conjugate operator.

We demonstrated in [1], that the algorithm, reformulated in terms of the equalizer input and output cumulants and for $p = 2$ and $q = 1$, turns out to be:

$$\mathbf{e}'_B = \left(\mathbf{R}^{Bx}\right)^{-1} \mathbf{c}^{yBx}_{3,1}, \quad \mathbf{e}_B = \frac{\sigma_a}{\sqrt{\mathbf{e}'^+_B \mathbf{R}^{Bx} \mathbf{e}'_B}} \mathbf{e}'_B, \tag{9}$$

where σ_a is the standard deviation of the input signal $a(k)$, $\mathbf{R}^{Bx}_{(nm)}$ is the $(L+1)\times(L+1)$ matrix whose elements are

$$C_2^{Bx}(m,n) := R^{Bx}_{(nm)} = \mathrm{cum}\left(x_n(k); x^*_m(k)\right) \tag{10}$$

and $\mathbf{c}^{yBx}_{3,1}$ is the $L+1$ vector whose elements are

$$C_3^{yBx}(n) := c^{yBx}_{3,1(n)} = \mathrm{cum}\left(y(k):2; y^*(k); x^*_n(k)\right). \tag{11}$$

We haveindicated the intermediate basis outputs as $x_n(k)$, i.e.,

$$x_n(k) := x(k) * b_n(k) = \sum_l b_n(l) x(k-l) \tag{12}$$

with $b_n(k)$ generic basis sequence of order n. For notational convenience,

$$\mathrm{cum}\left(x(k):p; \cdots\right) := \mathrm{cum}(\underbrace{x(k); \cdots; x(k);}_{p \text{ times}} \cdots).$$

Note that if the terms $x(k-n)$ substitute the terms $x_n(k)$, the GSE algorithm turns out to be the SE algorithm.

As in the SE case, this fact implies the convergence close to the optimal minimum mean square error (MMSE) solution for the Generalized Super-Exponential (GSE) method, once the poles have been fixed.

4 POLE SELECTION PROCEDURES

The GSE algorithm must be adopted jointly with a suitable pole estimation method. Since an equalizer transfer function is an approximation in the MMSE sense and with a certain delay of the channel inverse system, the poles could be initially placed in correspondence of the biggest channel zeros within the unit circle. (the proposed equalizer is a causal and stable filter). The pole values may be then initialized, with an identification of the MA channel model. Such a MA identification can be achieved at low additional computational cost by exploiting some SE method properties, once the convergence has been reached [1]. Thus our equalization procedure consists in preliminary SE iterations for initializing pole values (i.e. GSE iterations with null poles), then followed by GSE iterations.

However from experimental evidences, it has been noted that the poles should be progressively updated in order to improve equalization performance. For such purpose, we propose a method based on a MA to AR conversion of the equalizer truncated impulse response.

4.1 Pole initialization

At convergence of the algorithm formulated in (3), $\mathbf{g} \approx \boldsymbol{\delta}^{(k_d)}$, the SE solution approximates the optimal MMSE solution, that is

$$\mathbf{e}_{\mathrm{SE}} \approx \mathbf{e}_{\mathrm{MMSE}} = \left(\mathbf{R}^x\right)^{-1} \mathbf{c}_2^{ax}, \tag{13}$$

where $\mathbf{R}^x$ is the covariance matrix of the received sequence $x(k)$ and $\mathbf{c}_2^{ax}$ is the $(L+1)$ dimensional crosscorrelation vector between channel input and output. It is easy to show that

$$\mathbf{c}_2^{ax} = \sigma_a^2 \left[\; h^*(k_d) \quad h^*(k_d-1) \quad \cdots \quad h^*(k_d-L), \;\right]^T \tag{14}$$

where $h(k)$ are the channel impulse response coefficients. Therefore the channel and thus its zeros can be estimated from the cross-cumulant vector $\mathbf{c}_{3,1}^{yBx}$ at convergence of the GSE method with null poles (i.e. the SE algorithm), because (13) implies that

$$\mathbf{c}_{3,1}^{yBx} \approx K \cdot \mathbf{c}_2^{ax}, \tag{15}$$

where K is a constant depending on channel input cumulants. After some SE iterations (i.e. GSE with zero value poles), the pole values will be initialized as the zeros of the polynomial associated with the elements of $\mathbf{c}_{3,1}^{yx}$ placed in reverse order. Note that (15) is no more valid for the GSE algorithm with non zero poles. Hence this procedure cannot be adopted for pole updating.

4.2 Pole tuning

The basic idea behind this approach is that poles represent the "time constants" of the equalizer impulse response. Hence more appropriate pole values may be

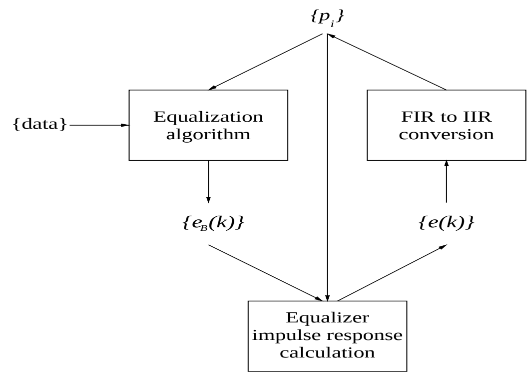

Figure 2: Method for pole tuning

estimated by studying the truncated equalizer impulse response and identifying its "time constants". This process can be seen like a MA to auto-regressive (AR) model conversion (Figure 2).

In [9], a MA to AR conversion algorithm, suitable for the aforementioned purposes, is described. It is derived from the Steiglitz-McBride algorithm for system identification [7]. Given a MA system $\boldsymbol{f}$, this procedure minimizes the cost function

$$J = \| \boldsymbol{f} - \hat{\boldsymbol{f}}^{p_1,p_2,\cdots,p_n} \|^2$$

where $\hat{\boldsymbol{f}}^{p_1,p_2,\cdots,p_n}$ is an AR approximating system.

Complete equalization procedure

The proposed algorithms can be adopted as follows. First, some iterations of the SE method are performed in order to estimate the initial values for the equalizer poles. Then, having fixed a set of poles, channel equalization is performed by means of the GSE method.

For improving the equalization performance by adjusting pole values, the procedure described in Section 4.2 can be adopted. This new pole assignment must be alternated with the blind GSE coefficient estimation approach till a convergence criterion is reached.

5 SIMULATIONS

In the first experiment, it is shown how the pole updating algorithm (Figure 3) can improve the performance after the pole initialization (Figure 4). 100 sequences of 600 samples of a 2PAM signal and a 13 coefficient equalizer with 3 poles have been simulated. The 'o's represent the true channel zeros. The channel output was corrupted by additive white Gaussian noise (SNR=15dB).

In the second experiment, a sequence of 1200 samples of a 16-QAM random sequence was transmitted through a radio channel whose magnitude and phase are represented Figure 5. The equalizer has 15 coefficients and 4 poles different from zero. In Figure 6, the resulting MSE in terms of SNR (continuous line), averaged over 100 Monte Carlo trials, is compared with the one of a 15 tap TDL equalizer (dotted line).

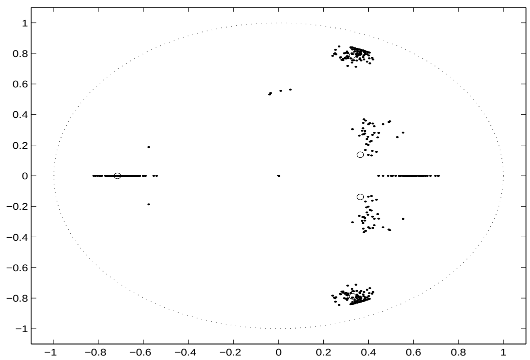

Figure 3: Results of the pole initialization procedure

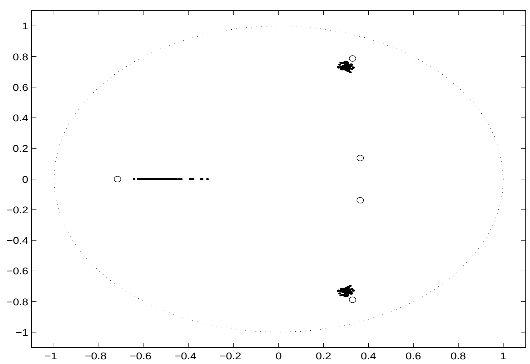

Figure 4: Results of the pole updating procedure

From Figure 6, it can be noted that our method allows an improvement of equalization performance without increasing the number of equalizer taps. This fact is important since it has been shown that a longer TDL equalizer cannot necessarily outperform a shorter one [3, 6].

6 CONCLUSION

This work proposes a blind equalization method based on IIR generalization of the TDL equalizers. The proposed approach adopts a HOS based coefficients adjustment procedure, derived from the Shalvi-Weinstein method. The above method is combined with a recursive two step algorithm for pole estimation. Simulation results have shown the performance of the whole equalization approach, evidentiating improvement with respect to FIR blind equalizers.

References

[1] M. C. Campi, R. Leonardi, and L. A. Rossi. Generalized super-exponential method for blind equalization using Kautz filters. In *Proc. IEEE SP Workshop on Higher Order Statistics*, Caesarea, Israel, June 1999.

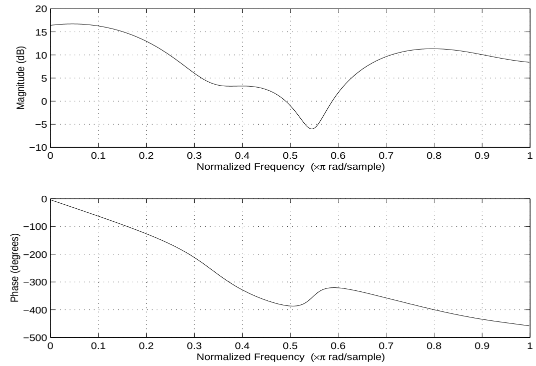

Figure 5: Channel zeros

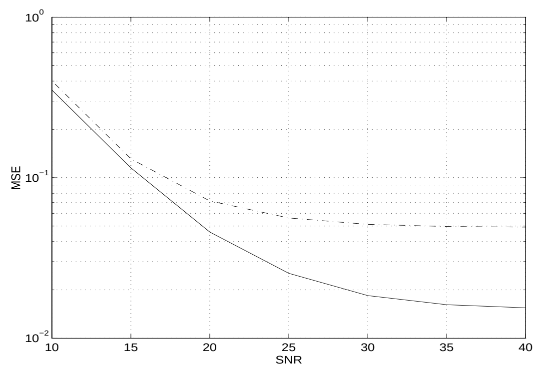

Figure 6: MSE in terms of SNR

[2] S. Lambotharan and J. A. Chambers. A new blind equalization structure for deep-null communication channels. *IEEE Trans. Circuits and Systems-II: Analog and Digital Signal Processing*, 45(1):108–114, January 1998.

[3] Y. Li and K. J. R. Liu. Static and dynamic convergence behavior of adaptive blind equalizers. *IEEE Trans. Signal Processing*, 44(11):2736–2745, November 1996.

[4] J. Mendel. Tutorial on higher-order-statistics (spectra) in signal processing and system theory: theoretical results and some applications. *Proc. of the IEEE*, 79(3):278–305, Mar. 1991.

[5] B. Ninness and F. Gustafsson. A unifying construction of orthonormal bases for system identification. *IEEE Trans. Automat. Contr.*, 42(4):515–521, April 1997.

[6] O. Shalvi and E. Weinstein. Super-exponential methods for blind deconvolution. *IEEE Trans. Inform. Theory*, 39(2):504–519, March 1993.

[7] K. Steiglitz and L. McBride. A technique for the identification of linear systems. *IEEE Trans. Automat. Contr.*, AC-10:461–464, October 1997.

[8] F.-B. Ueng. Adaptive lattice-form IIR blind equaliser. *IEEE Electronics Letters*, 32(20):1869–1870, September 1996.

[9] G. A. Williamson and S. Zimmerman. Globally convergent adaptive IIR filters based on fixed pole locations. *IEEE Trans. Signal Processing*, 44(6):1418–1427, June 1996.