Symmetrical Linear Prediction

C.Saraceno & R. Leonardi
Signals & Communications Lab.
DEA, University of Brescia
Brescia, I-25123 Italy
Ph: +(39-30) 371-5434 Fax: +(39-30) 380-014
email: leonardi@icil64.cilea.it

Abstract

In multidimensional signal processing, there has been a growing interest in extracting local features that exhibit certain geometrical properties. In particular, efforts have been made to extract local symmetries in 2-D images for purposes of Pattern Recognition [1] and more recently for Image Coding [2]. The ability to study symmetry in images finds its usefulness as symmetry occurs frequently in nature and as most man-made objects are quasi-symmetric. Often however symmetry does not appear in an exact mathematical sense. Imposing symmetry to a near symmetric object may induce a slight error to the real original signal. However, this may still be useful to model non-stationary signals such as speech or images.

If one assumes that an s^{th} order symmetry has been found in an original M-dimensional signal, it is meaningful to study models that can predict the signal given that symmetry information. We suggest here to establish linear predictive model that makes use of such symmetry information. The idea is to establish a symmetric correspondence between the sample that one wants to predict and a certain neighborhood of this sample taken symmetrically with respect to the center/axis of symmetry. Therefore, in the most general case, the current sample estimate $x[\mathbf{n}]$ is obtained through a linear combination of 3 sets of samples caracterized by 3 sets of parameters: $a_{\mathbf{i}}$, $b_{\mathbf{j}}$, and $c_{\mathbf{k}}$ with p, $q+1$, and $r+1$ different values for $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ respectively. Each set will refer respectively to the *past* output information (some "previous" neighborhood of $x[\mathbf{n}]$), the *past* and *present* input information $u[\mathbf{n}-\mathbf{j}]$ representing the innovation term, and finally the already known symmetrical information $\bar{x}[\mathbf{n}-\mathbf{k}]$, where $\bar{x}[\mathbf{n}]$ is the symmetrical sample of $x[\mathbf{n}]$ (see Fig. 1). From this model, it is possible to establish classes of symmetrical linear predictors, based on the fact that the different sets of parameters are or are not set to 0 in the linear prediction similarly to the concepts of moving average (MA), autoregressive (AR) and autoregressive moving average (ARMA) models [3]. We introduce here the terminology of symmetric model whenever there is at least one $c_{\mathbf{k}}$ different from 0. More specifically, one can write

$$x[\mathbf{n}] = \sum_{\mathbf{i}=\mathbf{i}_1}^{\mathbf{i}_p} a_{\mathbf{i}} x[\mathbf{n}-\mathbf{i}] + \sum_{\mathbf{j}=\mathbf{j}_0}^{\mathbf{j}_q} b_{\mathbf{j}} u[\mathbf{n}-\mathbf{j}] + \sum_{\mathbf{k}=\mathbf{k}_0}^{\mathbf{k}_r} c_{\mathbf{k}} \bar{x}[\mathbf{n}-\mathbf{k}] \quad (1)$$

If the symmetrical information is unknown, the linear prediction problem can be stated by a set of M equations (one for each dimension), the prediction starting close to the symmetry location, assuming that all states in the symmetry neighborhood are known. The prediction is then applied artenatively to each set of equation from one side of the symmetry to the opposite so as to construct the missing point information. For example, in the 1-D case with a very simple model, assuming that the symmetry occurs at the origin, we can write:

$$x[\mathbf{n}] = \quad a_1 x[n-1] + b_0 u[n] + c_1 x[1-n] \quad n > 0 \quad (2)$$

$$x[\mathbf{n}] = \quad a_1' x[n-1] + b_0' u[n] + c_0' x[-n] \quad n < 0 \quad (3)$$

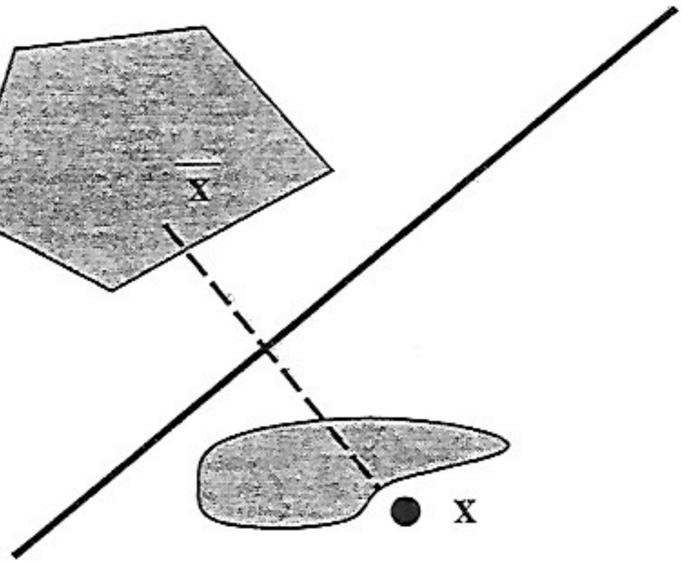

Figure 1: Prediction regions

where the optimal predictor could be estimated with the additional constraint that $a_1 = a'_1$ and $b_1 = b'_1$. The linear prediction is performed by alternating the estimation from one side of the origin to the other to get $x[1], x[-1], x[2], x[-2], \ldots$

In this paper we shall describe how to estimate the different parameters of the model, so as to minimize a quadratic error measure. Applications will be also shown for modeling symmetrical objects in images. Finally we shall show applications of this model to symmetry-based image compression.

References

[1] G. Marola, "On the detection of the axis of symmetry of symmetric and almost symmetric planar images", *IEEE Trans. on Pattern Anal. & Machine Intell.*, 11(1):104-108, Nov. 1989.
[2] P. Cicconi, R. Leonardi, & M. Kunt, "Symmetry-based image coding", *Proc. of the SPIE Conf. on Visual Comm. & Image Proc., Boston, 1992*, 1818(3):1312-1323, Nov. 1992.
[3] J. Makhoul, "Linear Prediction: A Tutorial", *Proc. of the IEEE, 63(4):561-580, Apr. 1975.*