Still Image Compression Using BSP Trees

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Abstract

Segmentation-based image coding methods divide the desired image into a finite number of regions, where the image signal within each region is smooth (or continuous). This approach provides very high compression ratios when compared with classical image coding methods such as transform and entropy-based techniques [Kunt]. Recently, we have introduced an efficient image representation method that is based on segmenting the image domain recursively using arbitrarily oriented lines [Radha 90] [Radha 93]. This recursive partitioning results in (i) a binary tree, known as the the *Binary Space Partitioning (BSP) tree* representation, and (ii) a set of convex (unpartitioned) polygons known as the BSP tree cells. The non-leaf nodes of the tree represent the partitioning lines, and the leaves represent the unpartitioned regions (cells).

The most challenging aspect of a segmentation-based coding approach is to balance between a small number of geometricly simple regions and the smoothness (or continuity) of the image signal within these regions. BSP tree-based image coding achieves this balance by using a simple, yet very flexible, description of the image regions. This represents an improvement over other segmentation-based methods (e.g., the contour-texture and quadtree approaches). Due to the flexible geometric description of the BSP tree approach, a small number of regions can be used to represent the image while maintaining simplicity for the regions' boundaries, and smoothness for the image signal within these regions.

A key requirement for achieving high compression ratios when using a BSP-tree based image coding system, is to encode the partitioning lines very efficiently. In this work:

- We describe a hierarchical method for coding the partitioning lines of the BSP tree representation of images. We focus on the normal (θ, ρ) parameterization of straight lines. We derive the number of bits required for encoding both parameters, where this number is proportional to the size of polygon under consideration. The idea, simply stated, is that lines which partition small polygons uses less bits than lines that partition larger polygons.

- We also describe how to efficiently encode the BSP tree cells using low order polynomials for approximating the image signal within these cells. Both zero and first-order polynomials are considered. The impact of encoding the polynomial coefficients in a differential manner is also discussed.

- In addition, we outline an optimum pruning algorithm used to reduce the bit rate of the encoded BSP tree while minimizing distortion. In this case, we develop a Generalized BFOS (G-BFOS) based pruning algorithm [Chou] for BSP tress. We show that, under certain conditions, the BSP tree representation of images meets the monotonicity constraint of the G-BFOS algorithm. This guarantees that the pruning algorithm, used in here, provides optimum sub-trees.

As will be shown, using this BSP tree-based image coding method provides high compression ratios in the range of 50-100 when applied on complex images.

References

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