Symmetry-Based BSP Tree Coding of Images

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Abstract

In image compression, there has been recently a great interest in segmentation-based representation of images, because of its capability to adapt to the physical nature of real scenes. On the other hand, efforts have been made to use local features that exhibit certain geometrical properties.

In particular the use of symmetry information in images has been suggested as a mean to reduce redundancy [2,4,7]. This seems quite adeguate as symmetric or quasi-symmetruc features are frequently present in nature or in most man-made objects, even if not necessarily in a stric mathematical sense.

With this idea in mind, it is suggested here how to compress visual information using symmetry information in conjunction with a segmentation process. The following approach has been used.

Considering a 2D image as an object, it can be demonstrated that an axis which divides the image into two mostly symmetric areas can be found using the Principle Axes of Inertia (PAI) of a rigid body. It is known that the PAI's of a rigid body are identified by the eigenvectors of its inertia matrix. For a 2D object with a mass distribution defined by the luminance function of the image, the inner matrix can be expressed by:

$$I = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{yx} & I_{yy} \end{bmatrix} \quad (1)$$

where $I_{ij} = \sum_{(x,y)\in D} i.j I(x,y)$ are the moments of inertia of the image with respect to the two reference axes, $i, j \in \{x, y\}$, and $I(x, y)$ us the luminance function of the image, and D is the support o the image.

In practice, PAI's are calculated on the basis of the edge information rather than the luminance funtion. This choice has been made to detect a contour symmetry of the object in the picture, without any disturbance introduced by some shadowing artefact. The PAI extraction procedure defines two orthogonal directions. The one corresponding to the smallest eigenvalue is then selected as it is the most likely axis of symmetry of the image passing through the center of mass of the 2D object.

To decide whether the two generated sub-regions could be considered symmetric or almost symmetric, a measure of symmetry must be used [1-5], using the image luminance function. If the measure of symmetry is high enough, one sub-region can be predicted from it symmetric counterpart. To improve the reconstruction, residue information of the prediction can be used [3,6,7].

As more than one symmetry could occur in the image, more than a single symmetry should be detected. A recursive procedure can be built for this purpose, combining the symmetry search with a segmentation of the image.

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A Binary Space Partitioning (BSP) tree has been chosen to represent the segmentation obtained with the recursive procedure. A BSP tree is an abstract data type that provides a representation of an n-dimensional space by iteratively dividing such space with arbitrarily oriented hyperplanes. This defines a recursive partitioning process which defines a binary tree.

In this context, the BSP tree is generated as follows:

- at the top of the tree, the root node is assigned the initial PAI that has partitioned the total image into two "mostly symmetric" sub-domains. If the symmetry measure exceeds a certain threshold, one of the two sub-domains refered by the right and left children respectively, is labelled as symmetric and becomes a node of the tree. The other is a general node of the tree which can be then processed as it were the root node. If there is not a sufficient symmetry, both children are treated as general nodes.
- the splitting process can either stop if a child of a node is considered symmetric with respect the other child of the same node, or when the region corresponding to it has uniform characteristics or its size has become too small.

This strategy was outlined in [7]. The key is to define a good way to encode the information obtained with the decomposition presented this far. For this sake, one needs to consider the 3 following stages in the coding process:

1. Coding the binary tree structure: This involves one bit per tree node unless the corresponding region is small enough so that the decoder knows a priori that it does not need any further splitting.
2. Coding the region boundary information: This involves defining the partitioning line of each non-leaf node. This information is obtained by quantizing the parametric representation of each line, and can be optimized by taking into account [8] that for nodes corresponding to small regions, fewer potential partitioning lines need to be considered.
3. Coding the leaf node information, i.e. the unpartitioned regions of the image, in the BSP tree representation. First, to each leaf node a label has to be assigned to identify whether it is a symmetric region or not. If this is not the case, the corresponding region is reconstructed through a region-based coding strategy (e.g., polynomial, DCT-basede representation). Otherwise, a symmetrical predictive coding strategy is developped, which combines quantization of the prediction coefficients and of the residue information [6].

Ways to find the optimal trade-off between this 3-step coding strategy will be discussed, using an optimum pruning of the tree-like data structures. Optimum pruning is an effective tool used to reduce the bit rate of a tree-based representation to meet a certain budget constraint while minimizing distortion. It has been used successfully in conjunction with tree-based signal coding applications such as Tree-Structured Vector Quantization (TSVQ), quadtree encoding, and wavelet packet-based compression. Here, we'll use the Generalized Breiman, Friedman, Olshen, and Stone (G-BFOS) optimum pruning algorithm [8] to show results for very low bit-rate applications.

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