Polarization Qubit Phase Gate in Driven Atomic Media

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We present here an all-optical scheme for the experimental realization of a quantum phase gate. It is based on the polarization degree of freedom of two traveling single-photon wave packets and exploits giant Kerr nonlinearities that can be attained in coherently driven ultracold atomic media.

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Photons are ideal carriers of quantum information as they travel at the speed of light and are negligibly affected by decoherence. In fact, quantum key distribution [1] and quantum teleportation [2,3] have been demonstrated using either single-photon pulses, which encode the quantum information in the photon polarization [1,2], or squeezed light encoding the information in the field quadrature [3]. The use of photons has also been suggested for quantum computation schemes even though the absence of significant photon-photon interactions becomes an obstacle toward the realization of efficient *quantum gates*. Two different ways have been proposed to circumvent this problem, namely, linear optics quantum computation [4] and nonlinear optical processes that involve few photons. While one is a probabilistic scheme implicitly based on the nonlinearity hidden in single-photon detectors, the other is based on the enhancement of photon-photon interaction achieved either in cavity QED configurations [5–7] or in dense atomic media exhibiting electromagnetically induced transparency (EIT) [8].

Single-qubit gates and one universal two-qubit gates are required for implementing universal quantum computation. The prototype optical implementation of a two-qubit gate is the quantum phase gate (QPG) in which one qubit gets a phase conditional to the other qubit state according to the transformation $|i\rangle_1|j\rangle_2 \rightarrow \exp\{i\phi_{ij}\}|i\rangle_1|j\rangle_2$, where $\{i, j\} = 0, 1$ denote the logical qubit bases. This gate becomes universal when $\phi = \phi_{11} + \phi_{00} - \phi_{10} - \phi_{01} \neq 0$ [5,9].

Partial demonstrations of an optical QPG have already been performed. A conditional phase shift $\phi \simeq 16°$ between two frequency-distinct cavity modes that experience an effective cross modulation mediated by a beam of Cs atoms was first measured nearly a decade ago [5]. The complete truth table of a QPG has not been determined as yet and an attempt in this direction has been made only very recently [10] , whereby a conditional phase shift $\phi \simeq 8°$ has been obtained between weak coherent pulses exploiting second-order nonlinearities in a crystal. This experiment, however, does not seem to demonstrate a bona fide QPG as ϕ depends on the input states and the gate works only for a restricted class of inputs. A phase-tunable *mixed* QPG between a two-level Rydberg atom and the two lowest Fock states of a high-Q microwave cavity has also been demonstrated [6].

A complete demonstration of a fully optical QPG is still lacking and we here envisage a new scheme for the realization of such a logic gate. Our proposal relies on the polarization degree of freedom of two traveling single-photon wave packets and exploits the giant Kerr nonlinearities that can be observed in dense atomic media under EIT [11]. A two-qubit gate for traveling photon qubits is useful not only for optical implementations of quantum computation, but also for quantum communication schemes. For example, perfect Bell-state discrimination for quantum dense coding and teleportation becomes possible if a QPG with a conditional phase shift $\phi = \pi$ could be used [12].

In our proposal the two qubits are a *probe* and a *trigger* polarized single-photon wave packet,

$$|\psi_i\rangle = \alpha_i^+|\sigma^+\rangle_i + \alpha_i^-|\sigma^-\rangle_i, \qquad i = \{P, T\}, \tag{1}$$

which can be written, in general, as a superposition of two circularly polarized states,

$$|\sigma^\pm\rangle_i = \int d\omega\, \xi_i(\omega)\hat{a}_\pm^\dagger(\omega)|0\rangle, \tag{2}$$

where $\xi_i(\omega) = (\tau_i^2/2\pi)^{1/4}\exp\{-\tau_i^2(\omega - \omega_i)^2/4\}$ is the frequency distribution of the incident wave packets centered on ω_i and with a time duration τ_i. In the interaction region of length l the electric field operator undergoes the following transformation:

$$\hat{a}_\pm(\omega) \rightarrow \hat{a}_\pm(\omega)\exp\left\{i\frac{\omega}{c}\int_0^l dz n_\pm(\omega, z)\right\}, \tag{3}$$

where $n_\pm$ is the real part of the refractive index which depends also on z when cross-phase modulation is present. Inserting Eq. (3) into Eq. (2) and assuming that the refractive index varies slowly over the bandwidth of the wave packets, one gets

$$|\sigma^{\pm}\rangle_i \rightarrow e^{-i(\omega_i/c)\int_0^l dz n_{\pm}(\omega_i, z)}|\sigma^{\pm}\rangle_i \equiv e^{-i\phi_{\pm}^{i}}|\sigma^{\pm}\rangle_i, \quad (4)$$

yielding a two-qubit gate in the form

$$|\sigma^{\pm}\rangle_P|\sigma^{\pm}\rangle_T \rightarrow e^{-i(\phi_{\pm}^{P}+\phi_{\pm}^{T})}|\sigma^{\pm}\rangle_P|\sigma^{\pm}\rangle_T. \quad (5)$$

This becomes a universal QPG [5,9] provided the conditional phase shift

$$\phi = (\phi_{+}^{P} + \phi_{-}^{T}) - (\phi_{-}^{P} + \phi_{-}^{T}) + \{+ \longleftrightarrow -\} \neq 0. \quad (6)$$

The two-qubit gate (5) could be implemented in a magnetically confined cold sample of ^{87}Rb atoms where two weak and well stabilized probe and trigger light beams exhibit a strong cross-Kerr effect in the five levels M configuration described in Fig. 1. A σ^+ polarized probe couples the excited state $|2\rangle$ to the ground $|1\rangle$ where all the atomic population is initially trapped. The other Zeeman-split ground state $|3\rangle$ is coupled to level $|4\rangle$ by a σ^- polarized *trigger* beam and to the excited state $|2\rangle$ by an intense σ^- polarized *pump*. A fourth σ^- polarized *tuner* beam couples level $|4\rangle$ and a third ground-state sublevel, $|5\rangle$. Owing to the tuner, the trigger group velocity can be significantly slowed down similarly to what happens to the probe. This represents an essential improvement over the four levels N scheme of Ref. [11] which does not involve the tuner and where the trigger pulse, which is not slowed down, leads to a group velocity mismatch that significantly limits the achievable nonlinear shifts [13,14]. We anticipate that in the present M scheme the group velocity mismatch can instead be reduced to zero and the cross-Kerr nonlinearity made large enough to yield cross-phase shift values of the order of π. Phase gating is realized when only one of the four possible probe and trigger polarization configurations in (5) exhibits a strong nonlinear cross-phase shift. For both σ^- polarized probe and trigger it can be seen, in fact, that for not too large detunings there is no sufficiently close excited state to which level $|1\rangle$couples and no population

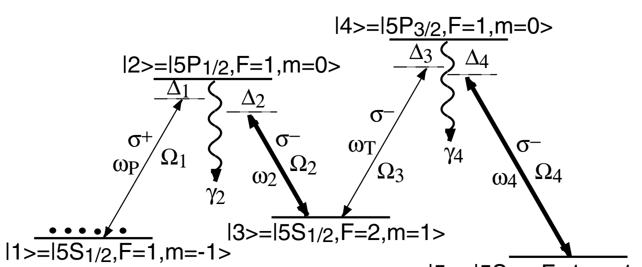

FIG. 1. Polarization phase gate in ultracold ^{87}Rb. The probe (ω_P, Ω_1) and trigger (ω_T, Ω_3) pulses impinging upon a Rb sample in the presence of a strong pump (ω_2, Ω_2) and a tuner (ω_4, Ω_4) realize the gating transformation (7)–(10). For a suitable choice of the four beam detunings ($\Delta_1, \Delta_2, \Delta_3, \Delta_4$) and intensities, the σ^+ and σ^- polarized probe and trigger can acquire a large cross-Kerr phase modulation. The two excited states decay with rates $\gamma_2 \simeq \gamma_4 = \gamma = 2\pi \times 6$ MHz.

in $|3\rangle$ to drive the relevant trigger transition. Likewise for a σ^- polarized probe and a σ^+ polarized trigger. In either case, probe and trigger only acquire the trivial vacuum phase shift $\phi_0^i = k_i l = \omega_i l/c$. When both probe and trigger are instead σ^+ polarized, the former, subject to the EIT produced by the $|1\rangle$–$|2\rangle$–$|3\rangle$ levels Λ configuration [15,16], acquires a nontrivial phase shift ϕ_{Λ}^{P} which can be evaluated by neglecting trigger and tuner altogether, while the latter, off any close resonant level, acquires again the vacuum shift ϕ_0^T. Finally, for a σ^+ and σ^- polarized probe and trigger, the two pulses will experience a substantial cross-Kerr effect acquiring nonlinear cross-phase shifts ϕ_{+}^{P} and ϕ_{-}^{T}. We arrive then at the following QPG table:

$$|\sigma^{-}\rangle_P|\sigma^{-}\rangle_T \rightarrow e^{-i(\phi_0^P+\phi_0^T)}|\sigma^{-}\rangle_P|\sigma^{-}\rangle_T, \quad (7)$$

$$|\sigma^{-}\rangle_P|\sigma^{+}\rangle_T \rightarrow e^{-i(\phi_0^P+\phi_0^T)}|\sigma^{-}\rangle_P|\sigma^{+}\rangle_T, \quad (8)$$

$$|\sigma^{+}\rangle_P|\sigma^{+}\rangle_T \rightarrow e^{-i(\phi_{\Lambda}^P+\phi_0^T)}|\sigma^{+}\rangle_P|\sigma^{+}\rangle_T, \quad (9)$$

$$|\sigma^{+}\rangle_P|\sigma^{-}\rangle_T \rightarrow e^{-i(\phi_{+}^P+\phi_{-}^T)}|\sigma^{+}\rangle_P|\sigma^{-}\rangle_T, \quad (10)$$

with a conditional phase- shift given by

$$\phi = \phi_{+}^{P} + \phi_{-}^{T} - \phi_{\Lambda}^{P} - \phi_{0}^{T}. \quad (11)$$

Let us now explicitly evaluate the phase shift appearing in the required gate transformation (7)–(10). We start by describing the system dynamics for the M configuration of Fig. 1 in terms of five coupled equations for the slowly varying atomic amplitudes c_i [11,17], i.e.,

$$i\dot{c}_1 = -\frac{\Omega_1^*}{2}c_2, \quad (12)$$

$$i\dot{c}_2 = \left(\Delta_1 - i\frac{\gamma_2}{2}\right)c_2 - \frac{\Omega_1}{2}c_1 - \frac{\Omega_2}{2}c_3, \quad (13)$$

$$i\dot{c}_3 = \Delta_{12}c_3 - \frac{\Omega_2^*}{2}c_2 - \frac{\Omega_3^*}{2}c_4, \quad (14)$$

$$i\dot{c}_4 = \left(\Delta_{13} - i\frac{\gamma_4}{2}\right)c_4 - \frac{\Omega_3}{2}c_3 - \frac{\Omega_4}{2}c_5, \quad (15)$$

$$i\dot{c}_5 = \Delta_{14}c_5 - \frac{\Omega_4^*}{2}c_4, \quad (16)$$

where the relative detunings $\Delta_{12} = \Delta_1 - \Delta_2$, $\Delta_{13} = \Delta_{12} + \Delta_3$, and $\Delta_{14} = \Delta_{13} - \Delta_4$ are defined in terms of the detunings $\Delta_1 = \omega_{21} - \omega_P$, $\Delta_2 = \omega_{23} - \omega_2$, $\Delta_3 = \omega_{43} - \omega_T$, $\Delta_4 = \omega_{45} - \omega_4$. We here examine ultracold atomic samples at temperatures $T < 1\ \mu$K so that Doppler broadenings and shifts can be neglected. We assume that decay only occurs from the two excited states $|2\rangle$ and $|4\rangle$ out of the system, with similar rates $\gamma_2 \simeq \gamma_4 = \gamma$. The pump and the tuner are taken as cw light beams with constant Rabi frequencies Ω_2 and Ω_4, while Ω_1 and Ω_3, referring to weak probe and trigger coherent pulses, are space and time dependent Rabi frequencies.

We determine the stationary state of Eqs. (12)–(16) by assuming that most of the population remains in the initially populated level $|1\rangle$; this occurs when the intensity of the pump is sufficiently larger than the probe intensity and the detunings as well, i.e., $|\Omega_2|^2 \gg |\Delta_{12}(\Delta_1 - i\gamma/2)|$. Under the further assumption that the pump be stronger than the trigger as well, the stationary probe and trigger susceptibilities can be rewritten as

$$\chi_P(z,t) \simeq \chi_{12}^{(1)} + \chi_{12}^{(3)}|E_T(z,t)|^2, \tag{17}$$

$$\chi_T(z,t) \simeq \chi_{34}^{(3)}|E_P(z,t)|^2. \tag{18}$$

Here E_P and E_T are the probe and trigger electric fields, while

$$\chi_{12}^{(1)} = -\frac{N}{V}\frac{|\mu_{12}|^2}{\hbar\epsilon_0}\frac{4\Delta_{12}}{|\Omega_2|^2}, \tag{19}$$

$$\chi_{12}^{(3)} = \chi_{34}^{(3)} = \frac{N}{V}\frac{4|\mu_{12}|^2|\mu_{34}|^2}{\hbar^3\epsilon_0|\Omega_2|^2}\left[\Delta_{13} - i\frac{\gamma}{2} - \frac{|\Omega_4|^2}{4\Delta_{14}}\right]^{-1} \tag{20}$$

are, respectively, the linear and nonlinear susceptibilities given in terms of the dipole matrix elements μ_{12} and μ_{34} and atomic density N/V. These expressions yield previous results as limiting cases. The third-order susceptibility for the N configuration assumed in [11] is obtained when $\Omega_4 = 0$, while the trigger susceptibility for the M configuration examined in [18] is obtained when $\Delta_{13} = 0$.

The above results (17)–(20) enable one to asses the group velocity mismatch between probe and trigger. As pointed out in [14], the two group velocities have to be comparable and small in order to achieve large cross-phase modulations. Unlike the six-level scheme studied in [19], in which cross-phase modulation takes place in a symmetric fashion so that the two group velocities are equal by construction, our present scheme is not symmetrical and hence probe and trigger group velocities are not, in general, equal. The group velocities follow from (19) and (20),

$$v_g^P \simeq \frac{\hbar c\epsilon_0}{8\pi|\mu_{12}|^2\omega_P(N/V)}\frac{|\Omega_2|^2}{1+\beta|\Omega_3|^2}, \tag{21}$$

$$v_g^T \simeq \frac{\hbar c\epsilon_0}{8\pi|\mu_{34}|^2\omega_T(N/V)}\frac{|\Omega_2|^2}{\beta|\Omega_1|^2}, \tag{22}$$

where

$$\beta = \frac{(1+\frac{|\Omega_4|^2}{4\Delta_{14}^2})[(\Delta_{13} - \frac{|\Omega_4|^2}{4\Delta_{14}})^2 - \frac{\gamma^2}{4}]}{[(\Delta_{13} - \frac{|\Omega_4|^2}{4\Delta_{14}})^2 + \frac{\gamma^2}{4}]^2}. \tag{23}$$

It follows that the two velocities can be made both small and equal by varying the probe and trigger relative intensities and the parameter β. Because of the tuner, our present configuration enables one to further control the group mismatch through β, which can be varied independently by adjusting the tuner intensity and its relative detuning Δ_{14}.

By comparing the qubit shifts in (4) with the solution

$$\varepsilon_i(z,t) = \varepsilon_i\left(0, t - \frac{z}{v_g^i}\right)\exp\left\{2\pi i k_i \int_0^z dz'\chi_i(z',t)\right\} \tag{24}$$

of the propagation equation [15] for the slowly varying electric field amplitudes $\varepsilon_i(z,t)$, where $\chi_i \simeq (n_i - 1)/2\pi$ are given in Eqs. (19) and (20) and v_g^i in Eqs. (21) and (22), the phase in Eq. (24) yields directly the required shifts for the phase-gating transformation (7)–(10). The *linear* phase shift ϕ_Λ^P acquired by a σ^+-polarized probe pulse moving in the z direction across a sample of optical thickness l then becomes

$$\phi_\Lambda^P = k_P l\{1 + 2\pi\chi_{12}^{(1)}\}, \tag{25}$$

while the *nonlinear* shift is obtained when the last contribution on the right-hand side of Eq. (17) is included. For a trigger Gaussian pulse [20] of peak Rabi frequency Ω_3^{pk} and moving within the sample with group velocity v_g^T, we arrive at an overall probe shift in the form

$$\begin{aligned}\phi_+^P &= \phi_\Lambda^P + 2\pi k_P\chi_{12}^{(3)}\int_0^l dz'|E_T(z',t)|^2 \\ &= \phi_\Lambda^P + k_P l\frac{\pi^{3/2}\hbar^2|\Omega_3^{pk}|^2}{4|\mu_{34}|^2}\frac{\mathrm{erf}[\zeta_P]}{\zeta_P}\mathrm{Re}\chi_{12}^{(3)},\end{aligned} \tag{26}$$

with $\zeta_P = (1 - v_g^P/v_g^T)\sqrt{2}l/v_g^P\tau_T$ and where τ_T is the trigger pulse time duration. By following the same procedure one has for the trigger phase shift

$$\phi_-^T = \phi_0^T + k_T l\frac{\pi^{3/2}\hbar^2|\Omega_1^{pk}|^2}{4|\mu_{12}|^2}\frac{\mathrm{erf}[\zeta_T]}{\zeta_T}\mathrm{Re}\chi_{34}^{(3)}, \tag{27}$$

where ζ_T is obtained from ζ_P upon interchanging $P \leftrightarrow T$.

Large nonlinear shifts take place when probe and trigger velocities are very much alike, i.e., when $\zeta \to 0$, in which case the $\mathrm{erf}[\zeta]/\zeta$ reaches the maximum value $2/\sqrt{\pi}$, and for appreciably large values of the two nonlinear susceptibilities' real parts. At the same time, their imaginary parts have to be kept small so as to avoid absorption, which may hamper the efficiency of the gating mechanism. Assuming a perfect EIT regime for the probe, i.e., $\Delta_1 = \Delta_2 = 0$, it is easily seen from Eq. (20) that one can attain imaginary parts that are 2 orders of magnitude smaller than their real parts for suitable values of the tuner intensity and provided that trigger and tuner are both strongly detuned and by nearly equal amounts, i.e., $\Delta_3 \simeq \Delta_4$. Such a choice further leads to values of β that yield equal group velocities. By taking, e.g., $\Delta_3 \simeq \Delta_4 = 20\gamma$ with $\Delta_{14} = 10^{-2}\gamma$, and $\Omega_4 \simeq \gamma$, $\Omega_1 \simeq 0.08\gamma$, $\Omega_3 \simeq 0.04\gamma$, $\Omega_2 \simeq 2\gamma$, one has at typical densities of $N/\mathrm{V} = 3\times10^{13}\ \mathrm{cm}^{-3}$ group velocities

$v_g^P \simeq v_g^T \simeq 10$ m/s along with over 65% average transmission [21] and a conditional phase shift $\phi \simeq \pi$ over an interaction length $l \simeq 1.8$ mm. This set of Rabi frequencies corresponds to single-photon probe and trigger pulses for tightly focused beams (several microns) with time duration ~ 1 μs. The non-negligible absorption accompanying the nonlinear phase shift does not hinder the proposed QPG mechanism. A demonstration of the proposed QPG may be done by using post selection of single-photon coherent pulses instead of single-photon wave packets. In this case, the phase-gating mechanism described by Eqs. (7)–(10) is carried out by considering the four possible configurations for the input polarizations, measuring the phase shifts with a Mach-Zender interferometer setup [10], and post selecting only the events with a coincident detection of one photon out of each probe and trigger pulse. Non-negligible absorption implies then only a smaller value of probe and trigger transmitted amplitudes with a concomitant lower probability (by 40%) to detect a two-photon coincidence between probe and trigger.

Laser pump intensity and frequency fluctuations may increase absorption and phase-shift fluctuations. The gate fidelity may then be hampered though in the proposed post-selection scheme; the fidelity is essentially affected only by the fluctuations of the shifts ϕ_Λ^P, ϕ_-^T, and ϕ_+^P. On general ground one estimates that a 1% intensity fluctuation yields an error probability of about 3% though relative detuning fluctuations of the order of $10^{-5}\gamma$ can make the error probability become as large as 10% [22].

It is worthwhile to note that a classical phase gate could be implemented by using more intense probe and trigger pulses. In fact, a conditional phase shift $\phi \simeq \pi$ could be achieved with the same atomic density but over a shorter interaction length, $l \simeq 10$ μm, along with 80% average transmission, by choosing $\Omega_1 \simeq 1.4\gamma$, $\Omega_3 \simeq 0.16\gamma$, $\Omega_4 \simeq \gamma$, $\Omega_2 \simeq 7\gamma$ and by slightly decreasing the detunings Δ_3 and Δ_4.

We here propose in conclusion a feasible scheme for an all-optical quantum phase gate that uses traveling single-photon pulses in which quantum information is encoded in the polarization degree of freedom. Unlike a similar scheme already investigated in [18,23] and where the issue of the two probe and trigger pulses group velocities mismatch was not addressed, we here observe that a π phase shift is obtained only when the probe and trigger group velocities are both small and almost equal. We show, within the framework of the present model, that this can be realized simply by tuning the frequencies and intensities of the four input light beams. This way of achieving a zero group velocity mismatch has clear advantages over other schemes that have been recently discussed [14,19]. The proposed scheme could be directly applied, in fact, to a magnetically confined cold sample of ^{87}Rb atoms and does not require a cold trapped mixture of two atomic species as in [14], where the two species realizing an N and a Λ scheme, respectively, require an accurate control of the atomic densities in order to get equal group velocities. The scheme studied instead in [19] is symmetric for probe and trigger and therefore yields equal group velocities automatically. Yet, the initial atomic population is here to be put in a Zeeman split $m = 0$ ground-state sublevel which cannot be easily done in a magnetically confined atomic sample requiring more sophisticated optical trapping techniques.

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