SCHEME FOR A QUANTUM PHASE GATE BASED ON ELECTROMAGNETICALLY INDUCED TRANSPARENCY

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We present a scheme for the experimental realization of a quantum phase gate acting on the polarization degree of freedom of traveling single photon wave-packets. The scheme exploits the giant Kerr nonlinearities that can be achieved in dense atomic media showing electromagnetically induced transparency, and it may be useful in a variety of quantum communication schemes.

Photons are ideal carriers of quantum information since they travel at the speed of light and are negligibly affected by decoherence. In fact, quantum key distribution [1] and quantum teleportation [2,3] have been demonstrated using either single photon pulses, in which quantum information is encoded in polarization [1,2], or squeezed light, in which information is encoded in the field quadrature [3]. Photons have also been proposed for quantum computation schemes, even though the absence of significant photon-photon interactions is an obstacle for the realization of efficient quantum gates. Two different ways have been proposed to circumvent this problem: i) linear optics quantum computation [4], which is a probabilistic scheme implicitly based on the nonlinearity hidden in single-photon detectors; ii) enhancement of photon-photon interaction using either cavity QED configurations [5,6,7], or dense atomic media showing electromagnetically-induced transparency (EIT) [8]. The linear optics conditional scheme of [4] is scalable in principle, but it is limited by the requirement of very efficient single-photon sources and single-photon detectors. Here we shall follow the second approach, which is instead hindered by the difficulty in getting the desired strong optical nonlinearity simultaneously with negligible losses.

Single qubit gates and one universal two-qubit gate are needed for implementing universal quantum computation. The most common two-qubit gate in optical implementations is the quantum phase gate (QPG), in which one qubit gets a phase conditioned to the other qubit state, i.e., $|i\rangle_1|j\rangle_2 \rightarrow \exp\{i\phi_{ij}\}|i\rangle_1|j\rangle_2$, with $i,j=0,1$ denoting the logical qubit bases. This gate is just the manifestation of cross-phase modulation in optical Kerr media, at the single photon level, and it is universal when the conditional phase shift $\phi = \phi_{11}+\phi_{00}-\phi_{10}-\phi_{01} \neq 0$ [5,9,10]. Partial demonstrations of an optical QPG have been already performed. A conditional phase shift $\phi \simeq 16^\circ$ between two frequency-distinct high-Q cavity modes, due to the effective cross modulation mediated by a beam of Cs atoms, has been measured [5]. However, the complete truth table of the gate has not been determined in this experiment. A conditional phase shift $\phi \simeq 8°$ has been instead obtained between weak coherent pulses, using a second-order nonlinear crystal [11]. However, this experiment did not demonstrate a *bona fide* QPG because ϕ depends on the input states, and the gate works only for a restricted class of inputs (weak coherent states). Finally, a phase-tunable *mixed* QPG between a two-level Rydberg atom and the two lowest Fock states of a high-Q microwave cavity has been demonstrated [6].

Therefore a complete demonstration of a fully optical QPG is still lacking. Here we shall provide a new scheme for the realization of a QPG between the polarization degree of freedom of two traveling single-photon wave-packets [12]. Such a gate for traveling photonic qubits would be extremely useful for quantum communication schemes: for example it has been showed that perfect Bell-state discrimination and complete quantum teleportation become possible if a QPG with a conditional phase shift $\phi = \pi$ is used [13]. Our proposal is based on the giant Kerr nonlinearities that can be achieved in dense atomic media showing EIT [14], and that have been already demonstrated in "slow-light" experiments [15].

The two qubits are represented by two polarized single photon wave-packets (probe and trigger), whose generic state can be written as $|\psi_i\rangle = \alpha_i^+|\sigma^+\rangle_i + \alpha_i^-|\sigma^-\rangle_i$, $i = P, T$, where we have chosen the two circularly polarized states

$$|\sigma^{\pm}\rangle_i = \int d\omega \xi_i(\omega) \hat{a}_{\pm}^{\dagger}(\omega)|0\rangle \tag{1}$$

as logical basis, with

$$\xi_i(\omega) = \left(\frac{L_i^2}{2\pi c^2}\right)^{1/4} e^{-\frac{L_i^2}{4c^2}(\omega-\omega_i)^2} \tag{2}$$

giving the spectral shape of the wave-packets, with spatial length (in vacuum) L_i, and carrier frequency ω_i. The desired gate transformation is realized when the two wave-packets simultaneously cross the dense atomic medium prepared in an appropriate configuration. In the interaction region of length l the optical field annihilation operators undergo the following transformation

$$\hat{a}_{\pm}(\omega) \to \hat{a}_{\pm}(\omega) \exp\left\{ i \int_0^l dz n_{\pm}(\omega, z) \frac{\omega}{c} \right\}, \tag{3}$$

where $n_{\pm}(\omega, z)$ is the refractive index, which depends upon z when cross phase modulation is present. Inserting this expression into Eq. (1) and considering pulses with a sufficiently narrow bandwidth so that the refractive indices vary in a negligible way within it, one gets

$$|\sigma^{\pm}\rangle_i \rightarrow \exp\left\{-i\int_0^l dz n_{\pm}(\omega_i, z)\frac{\omega_i}{c}\right\}|\sigma^{\pm}\rangle_i$$
$$\equiv \exp\left\{-i\phi^i_{\pm}\right\}|\sigma^{\pm}\rangle_i, \qquad (4)$$

yielding the following two-qubit gate

$$|\sigma^{\pm}\rangle_P|\sigma^{\pm}\rangle_T \rightarrow e^{-i(\phi^P_{\pm}+\phi^T_{\pm})}|\sigma^{\pm}\rangle_P|\sigma^{\pm}\rangle_T, \qquad (5)$$

which is a universal QPG when the conditional phase shift $\phi = (\phi^P_+ + \phi^T_-) + (\phi^P_- + \phi^T_+) - (\phi^P_- + \phi^T_-) - (\phi^P_+ + \phi^T_+) \neq 0$.

These conditions can be realized if a magnetically confined ultracold Rb gas (not necessarily in the condensed phase) is used as Kerr medium. This cold atomic gas has to be driven in such a way that a nontrivial cross-phase modulation between probe and trigger arise for only one of the four possible configurations of their polarization. A schematic description of such configuration is given in Fig. 1. All the initial atomic population is in the trapped state $|1\rangle = |5S_{1/2}, F = 1, m = -1\rangle$. A σ^+ polarized probe field at λ_P = 795 nm couples it to the excited state $|2\rangle = |5P_{1/2}, F = 1, m = 0\rangle$, which is in turn coupled to another Zeeman-splitted ground-state sublevel $|3\rangle = |5S_{1/2}, F = 2, m = 1\rangle$, by an intense σ^- polarized pump beam with Rabi frequency Ω_2. Ground state $|3\rangle$ is coupled to level $|4\rangle = |5P_{3/2}, F = 1, m = 0\rangle$ when a trigger beam at λ_T = 780 nm is σ^- polarized. As first showed by Schmidt and Imamoğlu [14], such a four-level N scheme yields an appreciable cross-phase modulation between probe and trigger when the probe is subject to EIT, and the trigger is sufficiently detuned from the $3 \leftrightarrow 4$ transition. Here we consider a modified, five-level, M scheme in which we add another σ^- polarized beam (tuner), with Rabi frequency Ω_4, between level $|4\rangle$ and a third ground-state sublevel, $|5\rangle = |5S_{1/2}, F = 1, m = 1\rangle$. Thanks to the tuner, also the trigger pulse is subject to EIT, and its group velocity is slowed down. This is crucial because the trigger pulse is not slowed down in the N scheme, and the group velocity mismatch significantly limits the achievable nonlinear phase shifts [16,17]. We shall see that, within the M scheme of Fig. 1, the parameters can be tuned so to make the group velocity mismatch negligible. In correspondence, the nonlinear cross phase shift will assume values of the order of π.

When either the probe or the trigger polarizations (or both) are changed, the M scheme of Fig. 1 is not realized, and the phase shifts acquired by the two pulses are very different. In fact, if the probe is σ^- polarized, there is no sufficiently close level which the atoms in $|1\rangle$ can be driven to. Both probe and trigger do not interact with the medium and each wave-packet only acquires the trivial vacuum phase shift $\phi^i_0 = k_i l$ ($k_i = \omega_i/c$, $i = P, T$). When the probe is σ^+ polarized as in Fig. 1, and also the trigger is σ^+ polarized, the probe is subject to the EIT corresponding to the Λ scheme formed by the levels

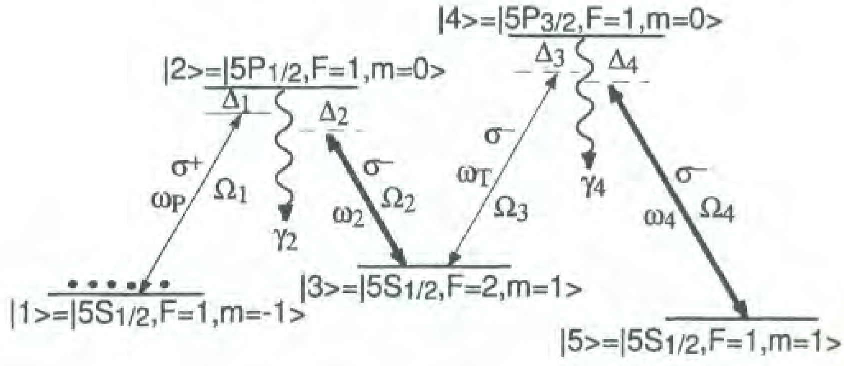

Figure 1. Energy level scheme for ^{87}Rb atoms. The probe field (with frequency ω_P and Rabi frequency Ω_1), and the trigger field (with frequency ω_T and Rabi frequency Ω_3) are weak coherent fields. The tuner (with frequency ω_4 and Rabi frequency Ω_4) and the σ^- polarized field at ω_2 is an intense pump beam. All the population is initially in level $|1\rangle$.

$|1\rangle - |2\rangle - |3\rangle$, while the trigger does not interact with the medium, because it is too far from resonance from any level. The trigger acquires again a vacuum phase shift ϕ_0^T, while the probe is slowed down by EIT and acquires a phase shift ϕ_Λ^P. We thus arrive at the following QPG

$$|\sigma^-\rangle_P|\sigma^-\rangle_T \to e^{-i\left(\phi_0^P+\phi_0^T\right)}|\sigma^-\rangle_P|\sigma^-\rangle_T \tag{6}$$

$$|\sigma^-\rangle_P|\sigma^+\rangle_T \to e^{-i\left(\phi_0^P+\phi_0^T\right)}|\sigma^-\rangle_P|\sigma^+\rangle_T \tag{7}$$

$$|\sigma^+\rangle_P|\sigma^+\rangle_T \to e^{-i\left(\phi_\Lambda^P+\phi_0^T\right)}|\sigma^+\rangle_P|\sigma^+\rangle_T \tag{8}$$

$$|\sigma^+\rangle_P|\sigma^-\rangle_T \to e^{-i\left(\phi_+^P+\phi_-^T\right)}|\sigma^+\rangle_P|\sigma^-\rangle_T \tag{9}$$

with conditional phase shift

$$\phi = \phi_+^P + \phi_-^T - \phi_\Lambda^P - \phi_0^T. \tag{10}$$

The nontrivial nonlinear phase shifts ϕ_+^P and ϕ_-^T have to be evaluated considering the full M scheme of Fig. 1, while ϕ_Λ^P can be evaluated neglecting the trigger and the tuner beams in Fig. 1.

Let us now explicitly evaluate the nonlinear phase shift of both probe and trigger for the M scheme of Fig. 1. This scheme has been only recently studied (see [18,19]) for its capability to give high-order nonlinearity via multi-photon coherence. We describe the dynamics of the M system in terms of five coupled equations for the slowly varying atomic amplitudes c_i, in which relaxation processes are introduced phenomenologically [14,20], i.e.,

$$i\dot{c}_1 = -\frac{\Omega_1^*}{2}c_2 \tag{11}$$

$$i\dot{c}_2 = \left(\Delta_1 - i\frac{\gamma_2}{2}\right)c_2 - \frac{\Omega_1}{2}c_1 - \frac{\Omega_2}{2}c_3 \tag{12}$$

$$i\dot{c}_3 = \Delta_{12}c_3 - \frac{\Omega_2^*}{2}c_2 - \frac{\Omega_3^*}{2}c_4 \quad (13)$$

$$i\dot{c}_4 = \left(\Delta_{13} - i\frac{\gamma_4}{2}\right)c_4 - \frac{\Omega_3}{2}c_3 - \frac{\Omega_4}{2}c_5 \quad (14)$$

$$i\dot{c}_5 = \Delta_{14}c_5 - \frac{\Omega_4^*}{2}c_4, \quad (15)$$

where $\gamma_2 \simeq \gamma_4 = \gamma = 2\pi \times 6$ MHz are the decay rates of the two excited states $|2\rangle$ and $|4\rangle$, and the relative detunings $\Delta_{12} = \Delta_1 - \Delta_2$, $\Delta_{13} = \Delta_{12} + \Delta_3$, $\Delta_{14} = \Delta_{13} - \Delta_4$, are defined in terms of the detunings $\Delta_1 = \omega_{21} - \omega_P$, $\Delta_2 = \omega_{23} - \omega_2$, $\Delta_3 = \omega_{43} - \omega_T$, $\Delta_4 = \omega_{45} - \omega_4$. We assume an ultracold Rb gas ($T < 1$ μK) and consequently neglect Doppler broadenings and shifts. As we have seen above (see Eq. (4)), the nonlinear phase shifts are determined by the classical nonlinear refractive index, and therefore we describe the four fields in terms of the four Rabi frequencies Ω_i, $i = 1, \ldots, 4$. The two pump fields can be taken as cw fields, so that Ω_2 and Ω_4 are constants, while Ω_1 and Ω_3 describe the probe and trigger weak coherent pulses, and therefore are space and time dependent functions. The stationary state of Eqs. (11)-(15) can be easily determined assuming that the pump is much stronger than the probe ($\Omega_2 \gg \Omega_1$), so that most of the population remains in level $|1\rangle$. From the stationary polarization, one gets the probe and trigger susceptibilities, which we rewrite as

$$\chi_P(z,t) = \chi_{12}^{(1)} + \chi_{12}^{(3)}|E_T(z,t)|^2 \quad (16)$$

$$\chi_T(z,t) = \chi_{34}^{(3)}|E_P(z,t)|^2, \quad (17)$$

where $E_i(z,t)$ $(i = P, T)$ are the probe and trigger electric fields,

$$\chi_{12}^{(1)} = \frac{N}{V}\frac{|\mu_{12}|^2}{\hbar\epsilon_0 D}\Delta_{12}\left[\Delta_{14}\left(\Delta_{13} - i\frac{\gamma_4}{2}\right) - \frac{|\Omega_4|^2}{4}\right] \quad (18)$$

$$\chi_{12}^{(3)} = -\frac{N}{V}\frac{|\mu_{12}|^2|\mu_{34}|^2}{\hbar^3\epsilon_0}\frac{\Delta_{14}}{D} \quad (19)$$

$$\chi_{34}^{(3)} = -\chi_{12}^{(3)}\frac{|\Omega_2|^2}{4D^*}\left[\Delta_{14}\left(\Delta_{13} + i\frac{\gamma_4}{2}\right) - \frac{|\Omega_4|^2}{4}\right]. \quad (20)$$

μ_{12} and μ_{34} are the dipole matrix elements, N/V is the atomic number density, and

$$D = -\Delta_{14}\left(\Delta_1 - i\frac{\gamma_2}{2}\right)\frac{|\Omega_3|^2}{4} + \left[\Delta_{12}\left(\Delta_1 - i\frac{\gamma_2}{2}\right) - \frac{|\Omega_2|^2}{4}\right]\left[\Delta_{14}\left(\Delta_{13} - i\frac{\gamma_4}{2}\right) - \frac{|\Omega_4|^2}{4}\right], \quad (21)$$

which is time-independent (as well as $\chi_{12}^{(1)}$, $\chi_{12}^{(3)}$ and $\chi_{34}^{(4)}$) in the case of a weak trigger pulse, $\Omega_3 \ll \Omega_2$. These general expressions reproduce previous results as limiting cases. The N scheme third-order susceptibility of [14] is obtained when $\Delta_{12} = 0$ and $\Omega_3, \Omega_4 \ll \Omega_2$, while the M scheme trigger susceptibility of [18] is obtained when $\Delta_{12} = \Delta_{13} = 0$ and $\Omega_3 \ll \Omega_2$.

It is straightforward to obtain from Eqs. (16)-(21) the probe and trigger group velocities, which are important for the determination of the nonlinear phase shifts. In fact, as shown in [16], cross phase modulation becomes relevant just when the probe is in the EIT condition, and its group velocity becomes very slow. If however the trigger is not slowed down, the nonlinear phase shift is limited by the time duration of the trigger pulse [16]. A much larger phase shift can be obtained when the two group velocities are slow and equal, so that the phase shift becomes proportional to the time spent within the medium. Two schemes have been proposed to achieve this goal: i) using a mixture of two atomic species [17] so that one kind of atoms realizes a N scheme, and the other one realizes a Λ scheme able to slow down the trigger pulse; ii) considering a six level scheme in which probe and trigger are affected by EIT and cross-phase modulation in a symmetric fashion, so that the two group velocities are equal by construction [21]. The present M scheme is not symmetrical, and therefore it does not give in general equal probe and trigger group velocities. However the two group velocities can be tuned and made equal simply by tuning the frequencies and intensities of the four input light beams. This way of achieving a zero group velocity mismatch has clear advantages over the above schemes [17,21]. Our scheme could be directly applied to a magnetically confined cold sample of ^{87}Rb atoms while the scheme in [17] requires an accurate (and difficult) control of the atomic densities in order to get equal group velocities. The symmetric scheme studied in [21] yields equal group velocities automatically. Yet, the initial atomic population is here to be put in a Zeeman-split $m = 0$ ground state sublevel which cannot be easily done in a magnetically confined atomic sample requiring more sophisticated optical trapping techniques.

Using $v_g(\omega) = c/(1 + n_g(\omega))$, with $n_g(\omega) = 2\pi\omega\partial \mathrm{Re}\chi(\omega)/\partial\omega$, and Eqs. (16)-(21), one has, when $\Delta_{12} = 0$ and $\Omega_3 \ll \Omega_2$

$$n_g(\omega_P) = n_g^P \simeq \frac{N}{V}\frac{8\pi|\mu_{12}|^2\omega_P}{\hbar\epsilon_0|\Omega_2|^2}\left(1 + |\Omega_3|^2\beta\right) \tag{22}$$

$$n_g(\omega_T) = n_g^T \simeq \frac{N}{V}\frac{8\pi|\mu_{34}|^2\omega_T}{\hbar\epsilon_0|\Omega_2|^2}|\Omega_1|^2\beta, \tag{23}$$

where

$$\beta = \frac{\left(1 + \frac{|\Omega_4|^2}{4\Delta_{14}^2}\right)\left[\left(\Delta_{13} - \frac{|\Omega_4|^2}{4\Delta_{14}}\right)^2 - \frac{\gamma_4^2}{4}\right]}{\left[\left(\Delta_{13} - \frac{|\Omega_4|^2}{4\Delta_{14}}\right)^2 + \frac{\gamma_4^2}{4}\right]^2}. \tag{24}$$

The two group velocities can be both made small and essentially equal by adjusting the tuner Rabi frequency Ω_4 and the parameter Δ_{14}.

We have therefore all the ingredients for the evaluation of the phase shifts ϕ_+^P and ϕ_-^T of Eq. (9), and ϕ_Λ^P of Eq. (8) for the proposed QPG. In fact, the propagation equation for the slowly varying electric field amplitudes $\varepsilon_i(z,t)$ (such that $E_i(z,t) = \varepsilon_i(z,t)\exp\{ik_iz - i\omega_it\} + c.c$) is given by $(i = P, T)$

$$\left(\frac{\partial}{\partial z} + \frac{1}{v_g^i}\frac{\partial}{\partial t}\right)\varepsilon_i(z,t) = 2\pi i k_i \chi_i(z,t)\varepsilon_i(z,t), \tag{25}$$

whose solution is

$$\varepsilon_i(z,t) = \varepsilon_i(0, t - \frac{z}{v_g^i})\exp\left\{2\pi i k_i \int_0^z dz'\chi_i(z',t)\right\}. \tag{26}$$

The phase shifts at the outer boundary $z = l$ of the gas sample are therefore given by $\phi_+^P = \phi_0^P + 2\pi k_P \int_0^z dz'\chi_P(z',t)$ and $\phi_-^T = \phi_0^T + 2\pi k_T \int_0^z dz'\chi_T(z',t)$. As discussed above, in the case when both probe and trigger are σ^+-polarized, one has only a Λ-scheme for the probe, and therefore one has the linear phase shift $\phi_\Lambda^P = \phi_0^P + 2\pi k_P l \chi_{12}^{(1)}(\Omega_3 = \Omega_4 = 0)$. Using Eqs. (16) and (17), and the fact that for the single photon Gaussian wavepacket of Eq. (2), moving with group velocity v_g^i and with transverse area S_i, one has

$$|E_i(z,t)|^2 = (\hbar\omega_i/\sqrt{2\pi}S_iL_i\epsilon_0)\exp\left\{-2c^2\left(t - z/v_g^i\right)^2/L_i^2\right\}, \tag{27}$$

one finally gets the probe and trigger phase shifts,

$$\phi_+^P = \phi_0^P + 2\pi k_P l \chi_{12}^{(1)} \tag{28}$$
$$+k_P\chi_{12}^{(3)}\frac{\hbar\omega_T\pi}{2S_Tc\epsilon_0}\left|\frac{1}{v_g^P} - \frac{1}{v_g^T}\right|^{-1}\mathrm{erf}\left[\frac{\sqrt{2}lc}{L_T}\left|\frac{1}{v_g^P} - \frac{1}{v_g^T}\right|\right]$$

$$\phi_-^T = \phi_0^T + k_T\chi_{34}^{(3)}\frac{\hbar\omega_P\pi}{2S_Pc\epsilon_0}\left|\frac{1}{v_g^P} - \frac{1}{v_g^T}\right|^{-1} \tag{29}$$
$$\times\mathrm{erf}\left[\frac{\sqrt{2}lc}{L_P}\left|\frac{1}{v_g^P} - \frac{1}{v_g^T}\right|\right].$$

These expressions are the central result of the paper and can be used for an accurate estimation of the conditional phase shift of the proposed QPG. Large nonlinear shifts take place when probe and trigger velocities are very much alike and for appreciably large values of the two nonlinear susceptibilities real parts. At the same time, their imaginary parts have to be kept small so as to avoid absorption, which may hamper the efficiency of the gating mechanism. Assuming a perfect EIT regime for the probe, *i.e.* $\Delta_1 = \Delta_2 = 0$, it is easily seen from Eqs. (16)-(21) that one can attain imaginary parts that are two orders of magnitude smaller than their real parts for suitable values of the tuner intensity and provided that trigger and tuner are both strongly detuned and by nearly equal amounts, i.e. $\Delta_3 \simeq \Delta_4$. Such a choice further leads to values of β that yield equal group velocities. By taking, e.g., $\Delta_3 \simeq \Delta_4 = 20\gamma$

with $\Delta_{14} = 10^{-2}\gamma$, and $\Omega_4 \simeq \gamma$, $\Omega_1 \simeq 0.08\ \gamma$, $\Omega_3 \simeq 0.04\ \gamma$, $\Omega_2 \simeq 2\gamma$, one has at typical densities of $N/V = 3 \times 10^{13}$ cm^{-3} group velocities $v_g^P \simeq v_g^T \simeq 10$ m/s along with over 65 % average transmission and a conditional phase shift $\phi \simeq \pi$ over an interaction length $l \simeq 1.8$ mm. This set of Rabi frequencies corresponds to single photon probe and trigger pulses for tightly focused beams (several microns) with time duration $\sim 1\ \mu$s.

The 65 % average transmission means that the desired nonlinear phase shift can be obtained only in conjunction with a non-negligible absorption. However, this does not mean that the proposed QPG does not work, but simply that one has a *probabilistic* QPG, giving single-photon probe and trigger pulses with the correct phase shifts at the output, in only 42% of the cases. Moreover, the easiest way to demonstrate experimentally the present QPG is to use post-selection of single–photon coherent pulses rather than single photon wave-packets, which are difficult to generate. In this case, the phase gating mechanism described by Eqs. (6)-(9) is carried out by considering the four possible configurations for the input polarizations, measuring the phase shifts with a Mach-Zender interferometer set-up [11], and post-selecting only the events with a coincident detection of one photon out of each probe and trigger pulse. Non-negligible absorption implies then only a smaller value of probe and trigger transmitted amplitudes with a concomitant lower probability (by 42%) to detect a two-photon coincidence between probe and trigger.

Laser pump intensity and frequency fluctuations may increase absorption and phase-shift fluctuations. The gate fidelity may then be hampered though in the proposed post-selection scheme, the fidelity is essentially affected only by the fluctuations of the shifts ϕ_Λ^P, ϕ_-^T and ϕ_+^P. On general ground one estimates that a 1% intensity fluctuation yields an error probability of about 3% though relative detuning fluctuations of the order of $10^{-5}\gamma$ can make the error probability to become as large as 10%. Yet, error probability as small as 1% or less are achieved when, e.g., Δ_{12} is stabilized at $10^{-6}\gamma$ or more. Such a requirement, though stringent, is within experimental reach provided all lasers are tightly phase-locked to each other [12].

Another important source of absorption is given by the decay of coherence between the ground state levels $|1\rangle$, $|3\rangle$, and $|5\rangle$, yielding a finite lifetime of the dark state at the basis of EIT [8]. However, these ground state dephasing processes are not included in the amplitude equation description of Eqs. (11)-(15), and their effect can be exactly described only within a Bloch equation treatment. We have therefore compared the predictions of the amplitude equation treatment of Eqs. (16)-(21) with the susceptibilities obtained from the numerical solution of the complete Bloch equations of the M scheme of Fig. 1. We have checked that the two predictions essentially coincide in a wide range of parameters (provided that the fundamental condition $\Omega_1, \Omega_3 \ll \Omega_2$ is satisfied) if the dephasing rates are zero. When the ground state dephasing rates are nonzero, probe and trigger absorption increase, even though the effect is not significant if these rates are not too large, i.e., they are kept smaller than 100 Hz. These moderate dephasing rates could be achieved using not too dense media, as for example bosonic cold gases above T_c.

It is worthwhile to note that a classical phase gate could be implemented by using more intense probe and trigger pulses. In fact, a conditional phase shift $\phi \simeq \pi$ could be achieved with the same atomic density but over a shorter interaction length, $l \simeq 10\mu$m, along with 80 % average transmission, by choosing $\Omega_1 \simeq 1.4\,\gamma$, $\Omega_3 \simeq 0.16\,\gamma$, $\Omega_4 \simeq \gamma$, $\Omega_2 \simeq 7\,\gamma$ and by slightly decreasing the detunings Δ_3 and Δ_4.

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