Theory of lossless polarization attraction in telecommunication fibers: erratum

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An erroneous procedure of averaging the components of the Stokes vector of a polarization scrambled beam over the Poincaré sphere introduced in our earlier paper [J. Opt. Soc. Am. B **28**, 100–108 (2011)] has been corrected.

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The procedure of averaging the components of the Stokes vector of a scrambled beam over the Poincaré sphere, the one which we found to be erroneous, was explained in detail in Appendix A of [1]. Here we reproduce the relevant fragment of the text.

> As explained in the body of the text, $N = 110$ signal beams with different SOPs uniformly distributed over the Poincaré sphere are launched into the fiber, one at a time. For all of these $N = 110$ realizations, the pump beam with one and the same SOP was launched from the opposite end of the fiber. For each realization we measure $S_1^+(L)$, $S_2^+(L)$, and $S_3^+(L)$. At the end of the simulations we calculate the mean values
>
> $$\langle S_i^+(L)\rangle = \frac{1}{N}\sum_{j=1}^{N}[S_i^+(L)]_j, \tag{1}$$
>
> where $N = 110$ and $i = 1, 2, 3$ [1].

The formulas in Eq. (1) are inaccurate. Since in a lossless medium the Stokes vector evolves on the surface of the Poincaré sphere and the input state of polarization (SOP) distribution is provided on a rectangular grid in spherical coordinates (polar angle and azimuth), proper averaging should be carried out in terms of spherical coordinates. Let us refer to the angular representation of the Stokes vector components, which for the initial signal Stokes vector reads as

$$S_1^{(k,n)}(z = 0) = S_0^+ \sin\theta_k \cos\phi_n, \tag{2}$$

$$S_2^{(k,n)}(z = 0) = S_0^+ \sin\theta_k \sin\phi_n, \tag{3}$$

$$S_3^{(k,n)}(z = 0) = S_0^+ \cos\theta_k. \tag{4}$$

Here $\theta_k = (k - 1)\Delta\theta$ and $\phi_n = 2(n - 1)\Delta\phi$, where index k runs from 1 to N and index n independently runs from 1 to $N - 1$; $\Delta\theta = \Delta\phi = \pi/(N - 1)$ is the computational angular step.

The output Stokes components are subject to averaging; they are obtained as follows:

$$\langle S_j(z = L)\rangle = \frac{1}{I}\sum_{k=1}^{N}\sin\theta_k\Delta\theta\sum_{n=1}^{N-1}\Delta\phi S_j^{(k,n)}(z = L), \tag{5}$$

where $j = 1, 2, 3$, and the normalization factor $I = \sum_{k=1}^{N}\sin\theta_k\Delta\theta\sum_{n=1}^{N-1}\Delta\phi$. In the continuous limit, the previous discrete averaging reduces to the usual formula for the integration in spherical coordinates of the Stokes vector components on the Poincaré sphere:

$$\langle S_j(z = L)\rangle = \frac{1}{4\pi}\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi}d\phi S_j(z = L). \tag{6}$$

When applied to the calculation of the output degree of polarization (DOP), the incorrect averaging gives the results that are shown in Fig. 1 (this figure reproduces Fig. 2 in [1]). In its turn, the correct averaging procedure yields the results that are shown in Fig. 2.

The comparison between the two figures reveals a slight qualitative difference—the correct averaging procedure basically provides the same DOP, independently of the input pump SOP, a conclusion that is consistent with physical intuition, while the incorrect averaging yields different values of DOP for different input pump SOPs. However, the quantitative differences among the various DOP values remain relatively small (i.e., less than 10% for backward beam powers less than 4).

In order to have a more precise feeling about the actual numbers, we present Tables 1–3. In these tables the correct averaging 1 means that 200 numerical integration steps across the entire length of the fiber were used, while correct averaging 2 means that the number of steps is increased up to 300. In both cases the averaging was performed according

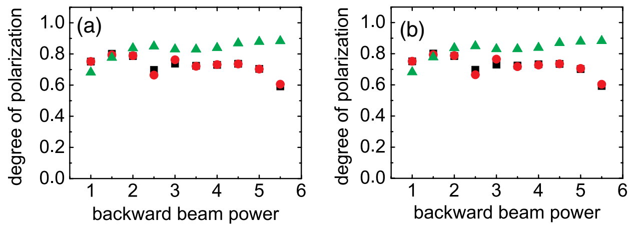

Fig. 1. (Color online) Figure reproduced from [1]. DOP of the output signal beam as a function of the relative pump beam power for six input SOPs of the pump beam: (a) $(-0.99, 0.01, 0.14)$ (black squares), $(0.01, -0.99, 0.14)$ (red circles), $(0.01, 0.01, -0.9999)$ (green triangles); (b) $(0.99, 0.01, 0.14)$ (black squares), $(0.01, 0.99, 0.14)$ (red circles), $(0.01, 0.01, 0.9999)$ (green triangles).

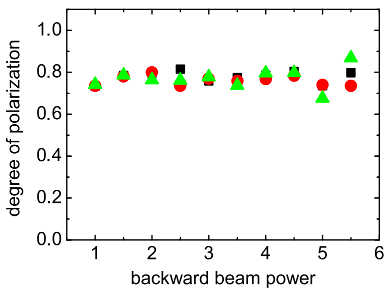

Fig. 2. (Color online) Correct averaging. DOP of the output signal beam as a function of the relative pump beam power for three input SOPs of the pump beam: $(-0.99, 0.01, 0.14)$ (black squares); $(0.01, -0.99, 0.14)$ (red circles); $(0.01, 0.01, -0.9999)$ (green triangles). The difference between black, red, and green points corresponding to the same power is due to numerical error.

to the formulas given in Eq. (5). The incorrect averaging means that we used the formulas of Eq. (1).

The values in all three tables exhibit some unessential statistical error. This error originates from three sources: (1) the discretization on both angles, (2) degradation of accuracy, as the pump power grows larger, and (3) the emergence of periodic solutions instead of steady-state solutions as the pump power grows larger; see [2] for details.

Table 1. Pump SOP $(-1, 0, 0)$

Pump Power	Incorrect DOP	Correct DOP 1	Correct DOP 2
1	0.751179397	0.734076679	0.734739184
1.5	0.801635683	0.78055 6083	0.782927394
2	0.785418212	0.798823893	0.800423384
2.5	0.697320759	0.784509778	0.78916496
3	0.730205655	0.741433024	0.777112782

Table 2. Pump SOP $(0, -1, 0)$

Pump Power	Incorrect DOP	Correct DOP 1	Correct DOP 2
1	0.752019525	0.735271871	0.735929012
1.5	0.793726444	0.787611425	0.784359455
2	0.788560688	0.785862684	0.777725637
2.5	0.664938748	0.744350493	0.777450621
3	0.765496135	0.764971554	0.782144129

Table 3. Pump SOP $(0, 0, -1)$

Pump Power	Incorrect DOP	Correct DOP 1	Correct DOP 2
1	0.682013988	0.741455972	0.74209404
1.5	0.777529538	0.786928654	0.777216017
2	0.837094963	0.763373315	0.782297432
2.5	0.848855495	0.761343956	0.844034374
3	0.830317974	0.778020144	0.785348535

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