Exact solution for the gigantic amplification of ultrashort pulses in counterpumped Raman amplifiers

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We exactly solve the initial-boundary value problem of interaction of three waves in the limit when one of these waves is strongly damped. The solution is applied to the characterization of transient effects in Raman amplifiers, with a special emphasis on the possibility of generating Stokes pulses with peak powers that are orders of magnitude higher than the input power of the pump beam. © 2011 Optical Society of America

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Fiber-optic Raman amplifiers are widely used in many contemporary high-speed optical networks. Because of their broad amplification bandwidth, Raman amplifiers successfully compete with erbium-doped-fiber amplifiers [1]. The counterpumped geometry is particularly advantageous in smoothing the fluctuations of the signal beam resulting from the transfer of the relative-intensity noise from the pump beam. When a Raman amplifier is integrated into an optical network, the power of the input signal beam coupled to the Raman amplifier changes due to channel add/drop modules or cable cuts. As a result of these changes transient effects at the output may occur, see [2–4]. The analytical study of these transient effects is the main objective of this Letter. Moreover, we suggest exploiting transient effects in Raman amplifiers for the generation of short optical pulses with high peak powers.

As a most representative example, let us consider the situation when initially (at $t = 0$) only a CW pump beam of peak power P, which enters the fiber at $z = 0$, is present across the entire fiber span of total length L. The Stokes signal beam is absent in the fiber medium for $t < 0$, and it enters the fiber from the opposite end at $z = L$ at time $t = 0$. In the case when the input Stokes signal has a step function time profile, the output signal acquires the characteristic shape of a giant spike followed by a low-intensity tail; see Fig. 1.

In optical communication networks, such spikes may appear when for some reason the Stokes signal into the amplifier is not present for a relatively long time. These sudden bursts of radiation can be detrimental for subsequent optical components. However, the disadvantage can be turned into an advantage, whenever the transient amplification regime is used for the generation of ultrashort pulses with peak powers at much higher levels than would be possible in the steady-state amplification regime.

For instance, consider a 30 dB linear gain Raman amplifier counterpumped by a 500 mW pump beam. In the steady-state regime, the power of the output signal cannot exceed 500 mW. However, it turns out that in the transient regime, the same Raman amplifier may deliver up to 1 kW peak power in a short Stokes pulse, assuming that the input power of the signal pulse is on the order of 1 W.

The physical mechanism leading to such gigantic amplification lies in the observation that the front edge of the signal pulse always propagates through an undepleted amplifier: thus it continuously experiences exponential linear gain. Potentially, the signal pulse can devour all of the energy that is supplied by the pump beam. In a loose sense, the energy of a long pump pulse (of duration estimated as the time which is needed for the pump beam to traverse the fiber of length L) is compressed into a signal pulse of much shorter temporal duration. A similar method was used for the generation of multi-MJ multi-exawatt-laser pulses in plasmas, reported in [5].

As is well-known for transient effects in fiber-optic Raman amplifiers, the material wave may be considered as strongly damped. This leads to two simple rate equations describing the unidirectional transfer of photons from the pump beam into the Stokes beam [6]. To the best of our knowledge, we solve the initial-boundary value associated with these rate equations exactly for the first time. Previously, only perturbative approaches were reported: first in a classical paper on backward Raman amplification [7], and later in [4]. In our minimal model, we only consider the first-order Raman scattering effect and disregard the presence of group-velocity dispersion, which can be of some relevance whenever the Stokes signal consists of picosecond or shorter

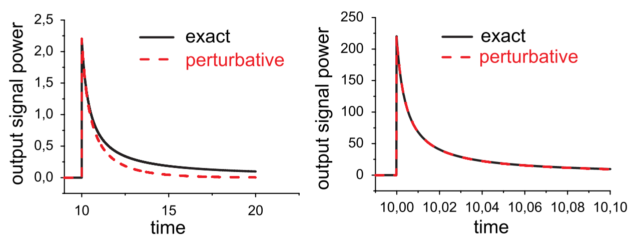

Fig. 1. (Color online) Step-like excitation. Output signal calculated with the exact formula (21) (black solid curve) and approximate formula (4.6) from [7] (red dashed curve). (left) $A_{\text{in}} = 10^{-4}$, (right) $A_{\text{in}} = 10^{-2}$, where A_{in} is the size of the initial step. For both cases, $P = 0.5$ and $L = 10$. When expressed in dimensional units, one dimensionless unit of power corresponds to 1 W, unit of time to 0.12 ms, and unit of length to 0.4 km. All are related to the example of a dispersion-compensating fiber considered in the body of the text.

pulses. We also do not account for the linear losses of the transmission fiber.

We adopt as an underlying model an ensemble of non-interacting two-level molecules, where each molecule with susceptibility α is described by the Hamiltonian $H = H_0 - \frac{1}{2}(\partial\alpha/\partial q)qE^2$ [7]. Here q is the oscillator displacement, $E = E_s\cos(\omega_s t + k_s z) + E_p\cos(\omega_p t - k_p z)$ is the applied electric field with real slowly varying Stokes (signal) and pump amplitudes E_s and E_p centered at corresponding frequencies $\omega_{s,p}$ and wave vectors $k_{s,p}$, H_0 is the total molecular Hamiltonian in the absence of the fields, and the two levels are eigenstates of H_0. In the rate equation approximation and under the assumption that the molecular vibration ground state population is not significantly changed, while anti-Stokes and higher-order Stokes generation are ignored, one obtains the system of rate equations for the signal and pump powers P_s and P_p

$$-\partial_X P_s + (n/c)\partial_T P_s = g_R P_s P_p, \tag{1}$$

$$\partial_X P_p + (n/c)\partial_T P_p = -g_R P_s P_p, \tag{2}$$

where n is the refractive index of the Raman-active medium, c is the speed of light, Z is the propagation distance, T is time, and g_R is the Raman gain coefficient. After introducing dimensionless variables $x = X/L_R$, $t = cT/nL_R$, $A = P_s/P_0$, and $B = P_p/P_0$ with $L_R^{-1} = g_R P_0$ and $P_0 = 1\,\mathrm{W}$, we arrive at the equations

$$\partial_t A - \partial_x A = 2AB, \tag{3}$$

$$\partial_t B + \partial_x B = -2AB, \tag{4}$$

which are the basis of our analysis. Initial and boundary conditions for the pump beam read as

$$B(0,t) = P, \qquad 0 \le t < \infty, \tag{5}$$

$$B(x,0) = P, \qquad 0 \le x \le L, \tag{6}$$

and the initial and boundary conditions for the signal beam are

$$A(L,t) = A_L(t), \qquad 0 < t < \infty, \tag{7}$$

$$A(x,0) = 0, \qquad 0 \le x \le L. \tag{8}$$

The general solution to Eqs. (3) and (4) can be written in terms of two unknown functions $a(\xi)$ and $b(\eta)$, each depending on a single variable:

$$A(x,t) = \frac{\partial_\xi a(\xi)}{a(\xi) + b(\eta)}, \tag{9}$$

$$B(x,t) = -\frac{\partial_\eta b(\eta)}{a(\xi) + b(\eta)}. \tag{10}$$

Here $\eta = t - x$ and $\xi = t + x$. The exact form of these functions depends on the initial and boundary conditions, and our goal is to find out how.

Let us use the general solution for writing down the temporal variations of the signal and pump beams at $x = 0$:

$$A_0(t) = \frac{\partial_t a(t)}{a(t) + b(t)}, \tag{11}$$

$$P = -\frac{\partial_t b(t)}{a(t) + b(t)}. \tag{12}$$

The problem is solved when $A_0(t) \equiv A(t,0)$ is found as function of $A_L(t) \equiv A(t,L)$. By combining the previous two expressions we can formulate the equation

$$\frac{d}{dt}[a(t) + b(t)] = [A_0(t) - P][a(t) + b(t)]. \tag{13}$$

In the interval $0 \le t \le L$, $A_0(t)$ is zero, since the signal pulse did not have enough time to propagate from $x = L$ to $x = 0$. Thus we can write the solution of Eq. (13) as

$$a(t) + b(t) = \mathrm{e}^{-Pt}, \qquad 0 \le t \le L, \tag{14}$$

and for $t \ge L$, we get

$$a(t) + b(t) = \mathrm{e}^{\alpha_0(t) - Pt}, \tag{15}$$

where $\alpha_0(t) \equiv \int_0^t \mathrm{d}t' A_0(t')$. We are now ready to find $a(t)$ and $b(t)$ separately, by using Eqs. (14) and (15). For $0 \le t \le L$, we get $a(t) = 0$ and $b(t) = \exp(-Pt)$, and

$$a(t) = \mathrm{e}^{\alpha_0(t) - Pt} - \mathrm{e}^{-Pt} \tag{16}$$

for $L \le t \le 2L$. Thus, we have found $a(t)$ for $t \le 2L$ and $b(t)$ for $t \le L$ as a function of $\alpha_0(t)$, which is the quantity to be determined. In order to link this quantity to the known function $A_L(t)$, by using Eqs. (11) and (15) we can write

$$A_0(t+L) = \frac{\partial_t a(t+L)}{\exp[\alpha_0(t+L) - P(t+L)]}, \tag{17}$$

and in parallel, by using Eq. (9), we get

$$A_L(t) = \frac{\partial_t a(t+L)}{a(t+L) + \exp[-P(t-L)]}. \tag{18}$$

By eliminating $\partial_t a(t+L)$ from these two equations, we obtain

$$A_0(t+L)\mathrm{e}^{\alpha_0(t+L)} = A_L(t)[\mathrm{e}^{P(t+L)} a(t+L) + \mathrm{e}^{2PL}]. \tag{19}$$

The expression for $a(t+L)$ can be found by solving Eq. (18)

$$a(t+L) = \mathrm{e}^{\alpha_L(t)} \int_0^t \mathrm{d}t' A_L(t') \mathrm{e}^{-\alpha_L(t') - P(t'-L)}, \tag{20}$$

with $\alpha_L(t) = \int_0^t \mathrm{d}t' A_L(t')$. With Eq. (20) in Eq. (19), and observing that $A_0\exp(\alpha_0) = \partial_t \exp(\alpha_0)$, upon integration

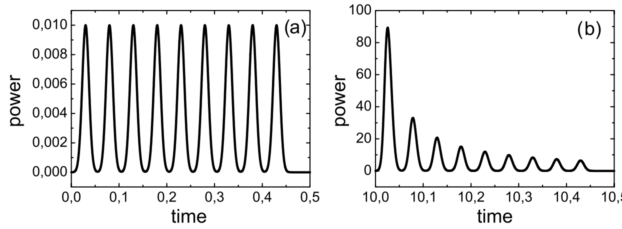

Fig. 2. Illustration of the Raman amplification of a sequence of short pulses. Parameters are $P = 0.5$, $L = 10$, the peak intensity of each input pulse 0.01: (a) input, (b) output.

of both sides of Eq. (19), we finally obtain the shape of the output signal for $L \leq t \leq 2L$ as

$$A_0(t) = \frac{A_L(t-L)\exp(2PL)f(t-L)}{1+\exp(2PL)\int_0^{t-L} \mathrm{d}t' A_L(t'-L)f(t'-L)}, \quad (21)$$

where $f(t) \equiv 1 + \exp[g(t)] \int_0^t \mathrm{d}t' \exp[-g(t')]A_L(t')$ with $g(t) \equiv \alpha_L(t) + Pt$. This expression, which links the output Stokes temporal shape to its input shape, completes the analytical part of our study.

Figure 1 illustrates the application of formula (21) to the case of a steplike excitation. Also shown is the comparison with the perturbative formula derived in [7]. Since we are only interested in the characterization of the spike, the behavior of the signal beam for $t > 2L$ is unimportant. The most impressive feature of the output signal shown on the right panel of Fig. 1 is that the peak power is 44 times larger than the pump power. Pulses are amplified in a similar fashion; see Fig. 2. As in the example of the steplike excitation, the signal is absent for $t < 0$. Then a pulse sequence consisting of nine identical pulses, each described by $A_L(t) = \exp[-(t-t_0)^2/T_p^2]$, where t_0 is initial time shift and T_p the pulse duration, is applied at $t = 0$ until $t = 0.5$. The longer the pulse in the sequence, the lower its output peak power, as illustrated in Fig. 3(a).

According to our normalization, the values $P = 0.5$ and $L = 10$ correspond to a pump power of 500 mW and a 4 km length of dispersion compensating fiber with $g_R = 2.5\,(\mathrm{kmW})^{-1}$. The linear gain of this amplifier is $\exp(2PL) = 43\,\mathrm{dB}$. The green dotted curve in Fig. 3(b), plotted for $T_p = 0.0005$ (corresponding to 6 ns duration), suggests that for pulses shorter than 1 ns, the output power reaches the 10 kW level for an input signal power of 10 W. In this case, the striking feature is the gigantic amplification of 10 W signal pulses by a pump beam with a power as low as 500 mW. In order to get such a high level of amplification for each pulse in the sequence, the pulses should be separated in distance at least by L, which corresponds to the repetition rate of 800 kHz in our example. Note that Maier *et al.* [7] reported signal power that was 20 times the peak pump power after the signal was amplified in CS_2.

For high-power Stokes pulses, cascade second- and higher-order Raman scattering may become important. Interesting physics is expected for sub-100 fs pulses, for which the intrapulse Raman conversion of

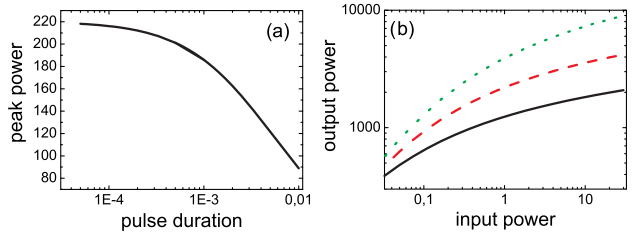

Fig. 3. (Color online) (a) Output intensity of the leading pulse in the sequence as function of initial pulse duration T_p, (b) output intensity versus input intensity for three values of the pulse duration T_p: 0.002 (black solid curve), 0.001 (red dashed curve), 0.0005 (green dotted curve). Parameters are $P = 0.5$, $L = 10$.

energy may lead to the effective generation of a supercontinuum. Note also that similar transient effects are expected in copumped Raman amplifiers, given a sizable mismatch of group velocities of the pump and signal beams. However, in fibers, the mismatch cannot be very large, and therefore transients are not as pronounced as in the counterpumped case (see [8]).

In conclusion, we exactly solved the initial-boundary value problem of three-wave interaction, with one wave strongly damped. We applied the solution of this problem to the characterization of transient effects in counterpumped Raman amplifiers. This solution can be useful in the analysis of telecom networks. We emphasized the fact that the peak power of the output Stokes pulse may significantly exceed the peak power of the CW pump beam.

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References

1. C. Headley and G. P. Agrawal, *Raman Amplification* (Elsevier, 2005).
2. P. M. Krummrich, E. Gottwald, A. Mayer, R. E. Neuhauser, and G. Fischer, in *Optical Amplifiers and Their Applications*, Vol. 44 of OSA Trends in Optics and Photonics Series (Optical Society of America, 2000), paper OTuC6.
3. C.-J. Chen and W. S. Wong, Electron. Lett. **37**, 371 (2001).
4. A. Bononi and A. Papararo, in *Optical Fiber Communications Conference*, Vol. 70 of OSA Trends in Optics and Photonics Series (Optical Society of America, 2002), paper ThR1.
5. V. M. Malkin, G. Shvets, and N. J. Fisch, Phys. Rev. Lett. **82**, 4448 (1999).
6. Y. R. Shen, *The Principles of Nonlinear Optics* (Wiley, 2002).
7. M. Maier, W. Kaiser, and J. A. Giordmaine, Phys. Rev. **177**, 580 (1969).
8. A. Hook, D. Anderson, and M. Lisak, J. Opt. Soc. Am. B **6**, 1851 (1989).