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Efficient modulation frequency doubling by induced modulation instability

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ABSTRACT

We show that inducing modulation instability with a weak modulation whose frequency is such that its second harmonic falls within the band of instability may lead to asynchronous Fermi–Pasta–Ulam recurrence and efficient transfer of power from the pump into the second harmonic of the modulation, resulting in a periodic modulation at the second harmonic with extinction ratios in excess of 30 dB.

Modulation instability (MI) [1,2] is a well-known physical process leading to the break-up of uniform waves in weakly nonlinear and dispersive transmission media, such as for example deep water [3] and silica optical fibers [4]. It is also well-known, since the pioneering work by Fermi, Pasta and Ulam (FPU) [5], that exciting a single mode in a system with many degrees of freedom does not lead to an irreversible energy transfer from the initial mode to other modes of the dynamical system. MI is one of the processes that explicitly belongs to the FPU-type [6]. Here, the continuous wave (CW) is not irreversibly transformed into a set of modulated waves. Quite to the opposite, FPU recurrence is observed, which means that upon propagation the input wave is once or periodically back-converted into the original CW. In other words, the process starts with all the energy in a single frequency mode and subsequently it is spread around many other modes with different frequencies. It was shown theoretically [7] and experimentally [8] that energy is returned back to the initial mode after a full cycle of nonlinear evolution.

When considering induced MI (IMI) with initial modulation frequencies Ω near the value, say, Ω_{max} that yields peak gain of the MI spectrum as it is obtained by the linear stability analysis of the CW, it is quite remarkable that the basic qualitative behaviours (e.g., its period) of the associated FPU recurrence may be well captured by means of a simple mode truncation involving just a pair of frequency modes, i.e. the input pump and the MI sideband mode [9,10]. In other words, the three-wave mixing (TWM) representation may provide a good description of the IMI process. This is a significant reduction in the number of degrees of freedom which allows us to write simple exact solutions for the TWM dynamics [11]. In this case, the FPU recurrence is a synchronous process where the periodic evolution of all higher order sidebands is locked to the same period that is obtained with the truncated two-mode description [8]. On the other hand, it is reasonable to expect that the simple TWM description is not adequate for relatively low initial modulation frequencies Ω. In that case, the second harmonic 2Ω may fall within the MI gain spectrum thus increasing the number of modes that effectively participate in the dynamics. Indeed, in this situation a relatively complex spectrum distribution is observed, and it is generally perceived that truncated descriptions in terms of a few spectral modes are of little use [12].

Quite to the contrary, in this work we show that for a significant range of IMI frequencies the FPU recurrence may be well described by simply adding a third spectral mode to the wave dynamics. Moreover, in the specified range of IMI frequencies, the observed FPU recurrence exhibits a qualitatively different behaviour from the case of Ref. [8], in that the evolution of different harmonics of the initial IMI frequency Ω becomes largely asynchronous. Indeed, we point out that a periodic energy conversion may occur from the pump into the second harmonic 2Ω, with almost no energy left in either the initial frequency mode at Ω nor in its higher harmonics. Potential applications of this effect to all-optical high-frequency light modulation will be discussed.

Let us consider wave propagation in weakly dispersive and nonlinear media as described in dimensionless units by the so-called focusing nonlinear Schrödinger equation (NLSE)

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\frac{\partial^2 u}{\partial t^2}+|u|^2u=0 \tag{1}$$

In the context of pulse propagation in optical fibers, z and t represent the distance along the fiber and retarded time, respectively. IMI corresponds to solving Eq. (1) with the initial condition

$$u(z=0)=1+\varepsilon\exp(i\phi)\cos(\Omega t), \tag{2}$$

where ε, ϕ and Ω are the initial modulation amplitude, phase and frequency, respectively. As it is well known, the linear stability analysis of Eq. (1) for perturbations of the type (2) with $\varepsilon << 1$ yields that instability occurs for $0 \leqslant \Omega \leqslant \Omega_c$, where $\Omega_c = 2$. Whereas peak IMI

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gain is obtained at $\Omega = \Omega_{\max} = \sqrt{2}$, and $\phi = \pm\pi/4$. Exact solutions of Eq. (1) with the initial conditions (2) that are periodic in t and localized (or periodic) in z have been found by Akhmediev et al. [7,13]. On the other hand, whenever $1 \leqslant \Omega \leqslant 2$, simple approximate solutions of the TWM equations can be found by representing the evolution of the field u in terms of just two Fourier modes, namely the pump and the sidebands at $\pm\Omega$ [9–11].

In this work, we restrict our attention to the IMI frequency range $0.5 < \Omega < 1$. To this end, we numerically solved Eqs. (1)-(2) with a standard Fourier split-step method and periodic boundary conditions. Fig. 1a illustrates the evolution with distance z of the field amplitude $|u|$ with $\varepsilon = 0.01$, $\Omega = 0.8718$ and an initial amplitude modulation (AM), that is $\phi = 0$: here four periods of the modulation are shown. As it can be seen, the initial weak modulation grows until at $z \cong 5$ the input CW pump is nearly fully depleted into the sideband at the input detuning Ω and its harmonics. Next, the energy is fully returned into the input CW, and conventional FPU recurrence of the modulation process is periodically observed along z (with the next occurrence of the full transfer from the pump into the modulation occurring at $z \cong 20$). However, quite surprisingly, Fig. 1a also shows that at approximately the mid-point $z \cong 13$ of the usual FPU recurrence, a modulation develops with a frequency that is twice the initial modulation, i.e. equal to 2Ω. By comparison with Fig. 1b, where a pure frequency modulation (FM) was injected at the input (i.e., $\phi = \pi/2$ in Eq. (2)), we may point out that the development of the frequency-doubled modulation is strongly sensitive to the relative phase between the pump and the IMI sidebands. In fact, Fig. 1b shows that no frequency-doubled modulation develops in the FM case.

A better insight into the spectral content of the development of the nonlinear modulations of Fig. 1 is provided by the plots of Fig. 2, where we display the evolution with distance z of the amplitude of the CW pump (solid curve), as well as of the initial modulation sideband at $+\Omega$ (dashed curve) and of its second harmonic at $+2\Omega$ (dot–dashed curve). Clearly the field spectrum of Eqs. (1)-(2) is frequency-symmetric, so that the amplitude of the sidebands with either positive or negative frequency shifts are equal to each other. In Fig. 2a we show the AM case as in Fig. 1a, whereas in Fig. 2b we consider the FM case as in Fig. 1b. Fig. 2 clarifies that the frequency-doubled modulation develops at $z \cong 13$ with almost no accompanying energy left in the original sidebands at $\pm\Omega$, nor in higher order sidebands. Indeed, Fig. 3 compares the field amplitude $|u|$ at $z = 13.1$ with either AM as in Fig. 1a (solid curve) or FM as in Fig. 1b (dashed curve) input perturbations. As it can be seen, in the FM case the field has returned back to its nearly original weakly modulated state, whereas in the AM case a fully sinusoidal modulation at frequency 2Ω has developed (with 40 dB power gain) from the initial weak modulation at frequency Ω. The extinction ratio between the maxima and the minima of the periodic signal shown by the solid curve of Fig. 3 is as high as 38 dB.

As we have seen, a dramatic difference exists in the nonlinear development of the IMI (2) depending upon relative phase ϕ of the modulation with respect to the CW pump. Therefore we have studied the full dependence upon ϕ of the solutions of Eqs.(1)-(2). By varying $0 \leqslant \phi \leqslant \pi$ in Eq. (2), we found that the sinusoidal modulation at frequency 2Ω always develops unless the input modulation is purely FM ($\phi = \pi/2$). However, the spatial period of both this, say, secondary modulation as well as of the, say, primary FPU recurrent modulation at frequency Ω may vary with ϕ. For example, Fig. 3a reports the case with $\phi = 0.64\pi$: in this case the period associated with the primary FPU has grown larger until its value almost coincides with the period of the secondary modulation, so that a merging of the two modulations is observed at $z \cong 13$. On the other hand, Fig. 3b shows that for $\phi = 0.8375\pi$ it is the spatial period of the secondary modulation that has grown

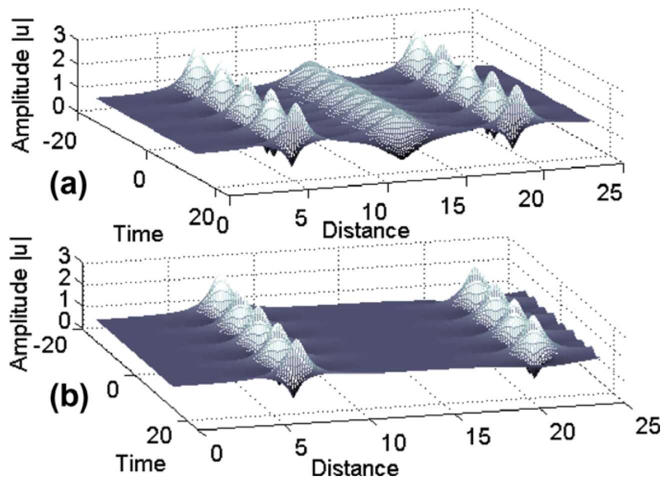

Fig. 1. Evolution with distance of the field amplitude $|u|$. Initial modulation with $\Omega = 0.8718$, $\varepsilon = 0.01$, of (a) AM ($\phi = 0$ or (b) FM ($\phi = \pi/2$) type.

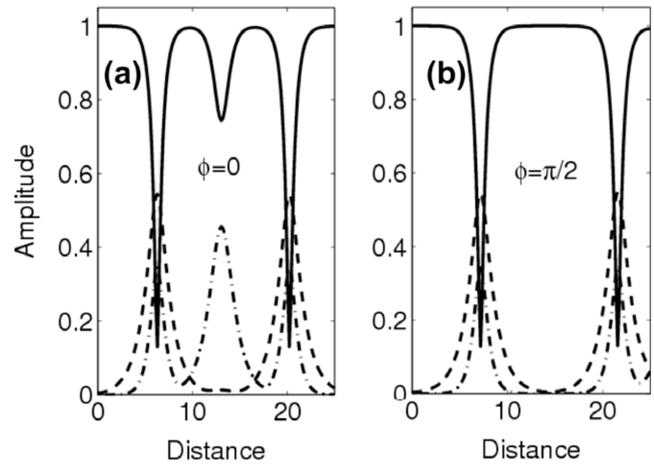

Fig. 2. Evolution with distance z of the amplitude of the CW pump (solid curve), of the initial modulation sideband at frequency detuning $\Omega = 0.8718$ (dashed curve), and of the 2Ω or second harmonic (dot–dashed curve), for (a) AM or (b) FM perturbations as in Fig. 1.

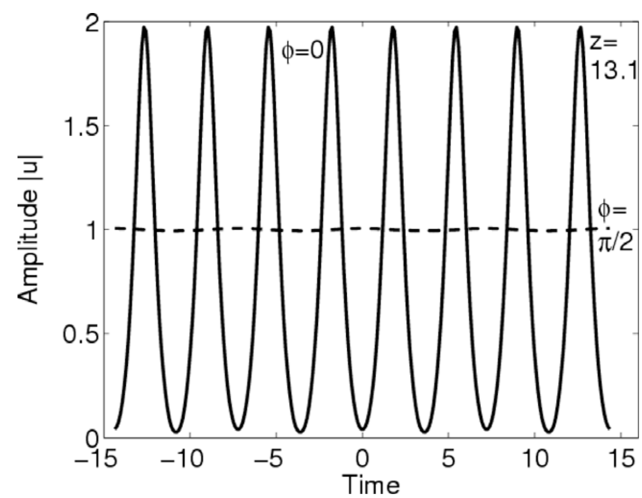

Fig. 3. Field amplitude $|u|$ at $z = 13.1$ with either AM as in Fig. 1a (solid curve) or FM as in Fig. 1b (dashed curve).

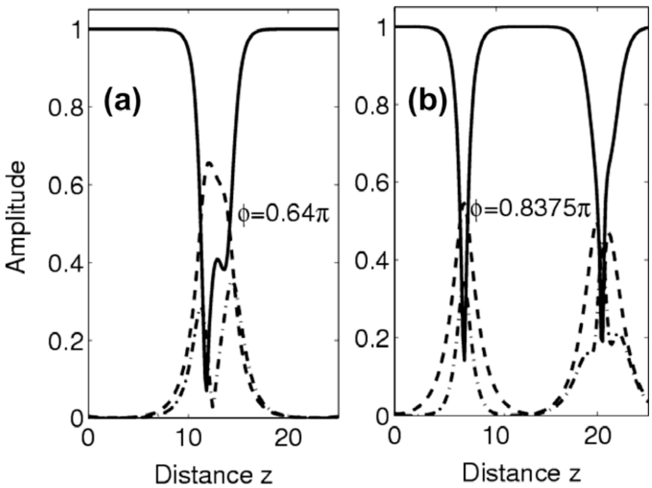

Fig. 4. Same as Fig. 2, with (a) $\phi = 0.64\pi$ or (b) $\phi = 0.8375\pi$.

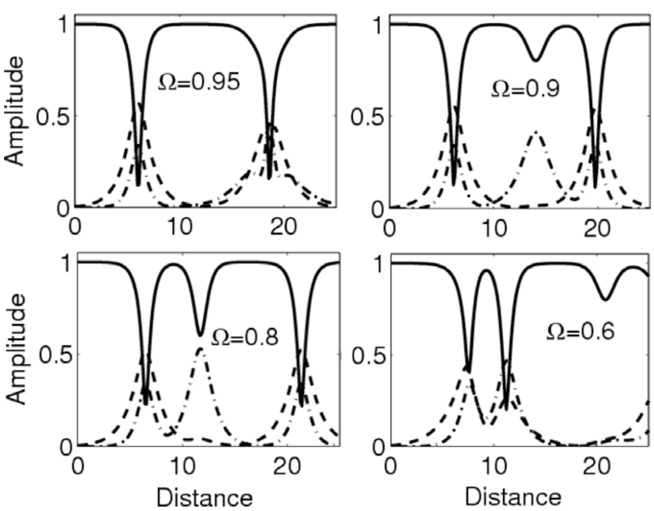

Fig. 5. Same as Fig. 2a, with $\Omega = 0.95$, 0.9, 0.8, and 0.6.

larger until its position has merged with that of the second occurrence of the primary FPU modulation at $z \cong 20$.

So far we have considered a single value of the IMI frequency Ω. However, a similar situation is observed for most frequencies Ω such that $0.5 < \Omega < 1$. In fact, Fig. 4 shows the amplitude of the pump and its first two sidebands at $+\Omega$ and $+2\Omega$ with the same input conditions as in Fig. 2a, and $\Omega = 0.95$, 0.9, 0.8, and 0.6, respectively. As it can be seen, for $\Omega = 0.95$ both the second occurrence of the primary FPR modulation at Ω and the first occurrence of the secondary modulation at 2Ω coincide at the same spatial position $z \cong 18$. Whereas when Ω decreases from 0.9 to 0.6 the secondary modulation is observed, with a progressively reduced spatial period, thus approaching the primary FPU modulation (see Fig. 5). Additionally, Fig. 5 shows that for $\Omega = 0.6$ both the first and the second modulations that develop at $z \cong 8$ and $z \cong 11$, respectively, are composed by a large number of synchronous harmonics besides the second one.

High sensitivity of the wave evolution to the phase of the initial modulation is the consequence of the instability of the second harmonic of the modulation. The growth rate of this instability could be lower or higher than the growth rate of the first harmonic (see e.g. Fig. 1 of Ref. [13]). As the amplitude of the modulation is finite rather than infinitesimal, from the beginning of the evolution there is a nonzero component of the second harmonic which independently grows competing with the fundamental frequency Ω. The two components comprise a nonlinear superposition, with maxima of their contribution at independent points along the z axis [14]. This understanding explains our results of the previous paragraph. Controlling both the frequency of the modulation within the instability band and its amplitude would allow us to reach maximum flexibility in the shape of the output signal.

It is interesting to envisage, as a potential application of the present analysis, the possibility of obtaining a high-extinction ratio modulation of a CW laser at frequency 2Ω by seeding its propagation in a nonlinear optical fiber with a weak modulation at frequency Ω. In real units, the pump power $P = (\gamma Z_c)^{-1}$, γ is the fiber nonlinear coefficient, $Z_c = t_c^2/|\beta_2|$, $t_c = \Omega/(2\pi\Delta\nu)$, where β_2 is the chromatic dispersion and $\Delta\nu$ is the actual input sideband shift. Taking the typical highly nonlinear fiber value $\gamma = 13.1\ \mathrm{W^{-1}km^{-1}}$ and $P = 1$ W, one has that the distance $z = 13.1$ as in Fig. 3 corresponds to the actual fiber length $Z = 1$ km. With $\beta_2 = -2\ \mathrm{ps^2/km}$, $t_c = 0.39$ ps, so that $\Delta\nu = 355$ GHz for $\Omega = 0.8718$. Therefore, one may produce a fully modulated periodic pulse train at the repetition rate of 710 GHz, that is well beyond the capabilities of electro-optical or electro-absorption modulators, by weakly modulating a 1 W CW laser at 355 GHz.

In conclusion, we have pointed out that if the frequency, amplitude and phase of the initial weak periodic modulation to the unstable CW pump are properly chosen, a full transfer of energy from the pump into the second harmonic of the initial modulation may be observed at a certain propagation distance.

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