

A new approach to equations with memory

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Abstract

We discuss a novel approach to the mathematical analysis of equations with memory, based on the notion of a *state*. This is the initial configuration of the system at time $t = 0$ which can be unambiguously determined by the knowledge of the dynamics for positive times. As a model, for a nonincreasing convex function $G : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ such that

$$G(0) = \lim_{s \rightarrow 0} G(s) > \lim_{s \rightarrow \infty} G(s) > 0$$

we consider an abstract version of the evolution equation

$$\partial_{tt}\mathbf{u}(\mathbf{x}, t) - \Delta \left[G(0)\mathbf{u}(\mathbf{x}, t) + \int_0^\infty G'(s)\mathbf{u}(\mathbf{x}, t - s)ds \right] = 0$$

arising from linear viscoelasticity.