

Measuring Inequality: Pareto's Ambiguous Contribution

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Pareto's contribution to the theory and statistics of income has been universally recognized as a decisive step in the study of the personal distribution of income and wealth, perhaps even as the first contribution in applied econometrics. His "income law" is certainly not as universal as he thought; however, it has been shown to be a heuristic instrument of undoubted cognitive potential, which is furthermore efficient and still valid for describing the upper tail of the distribution of wealth.

On the other hand, Pareto's concept of inequality has remained in a sort of shadow zone, and particularly so the index that he had proposed for measuring it.¹ There is a double reason for this: first, the index α identified by Pareto had no universal value, being applicable only to the strictly "Paretian" distributions;² second, and paradoxically, the interpretation of

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1. Renato Cirillo ([1974] 1999, 276) holds that Pareto was the first economist "to suggest a way of measuring the equality of incomes." Effectively, in the second half of the nineteenth century, various statisticians and economists, above all the Germans, had posed questions regarding the distribution trends taking place in capitalist societies: as far as is known, however, no one before Pareto had elaborated a *concise* index of inequality for comparing the statistical series of incomes.

2. As Joseph Persky and Gilbert Bassett (2006, 81–84) have recently pointed out, Pareto's law belongs to the genre of "structural theories," in which "inequality would be identified with the second moment of an income distribution *law*."

the index gave rise to a singular and unresolved controversy, which caused scholars to speak of “confusion” over it. As Paul Samuelson (1965, 247) wrote some years ago, “Historically there has been much confusion over whether a rise in the Pareto coefficient meant greater or less ‘inequality’ of income. Even expert opinion has been divided on the question.”

There is a vast corpus of literature on the subject, which we can conventionally divide into two branches. The first includes contributions that appeared in the first half of the twentieth century, particularly in Italy, in which the idea prevails that Pareto erroneously interpreted the significance of the α index. In contributions on the second branch, from the latter half of the century, this negative judgment on Pareto is highly attenuated. Thanks principally to John Chipman (1974, 1976) and to Frank Cowell (1977), the idea that Pareto did not reason exclusively on the plane of descriptive statistics but also on that of welfare economics has been consolidated. From this viewpoint, the meaning attributed by Pareto to the index α is, at least in determinate cases, correct.

Moreover, it must not be forgotten that the confusion over the interpretation of the index α was further fed by a printing error in the *Cours* (Pareto 1896–97, 320). Some scholars, not realizing the error, came to attribute “a curious mathematical slip” to him. Even recently, much insistence has been placed on this printing error as the source of the controversy. In truth, as will be shown, the printing error had a decisive role only in the limited and isolated case of the interpretation by Allyn Young (1917),³ although it was a further element that contributed to increasing the “confusion” that surrounds the Paretian α index.

This article offers a reconstruction of the debate that involved Pareto’s α index, aimed at clarifying his thought on the question. Very recently this debate has been defined as “a minor controversy,” compared to the more general contribution made by Pareto to the statistics of income (Kleiber and Kotz 2003, 257). It is, however, a controversy that has involved a notable number of scholars and that still has many obscure aspects. In our reconstruction, a historical interpretative approach will be privileged over an analytical one, using a reading that is partially different from the prevailing interpretations. As will be argued, the attempts to defend Pareto, as well as the insistence on the error of his assumptions, have been counterproductive, at least for the aim of correctly interpreting Paretian thought. A radical shift in perspective is needed, in order for scholars to accept that

3. Among the most recent contributions that have emphasized the printing error as the source of “confusion about what Pareto actually said,” see Persky 1992 and Kirman 1987, 1998.

the author of the *Cours* discusses something very different from the problem of greater or lesser inequality. We are dealing with a recurrent problem in the field of measures of inequality, where preliminary definitions are crucial.⁴ From this point of view, Pareto was particularly rigorous and scrupulous. But this did not stop him from designating the term *lesser inequality* to mean a phenomenon that does not correspond to any of the commonly accepted meanings of the term, either from a normative or a statistical point of view. If this perspective is correct, then we should give up the idea of looking for a positive contribution to the measuring of inequality by Pareto, whether one's aim is to highlight the "errors," or to demonstrate that the analysis is correct.

The article is organized as follows. After a presentation of Pareto's ideas part 2 offers an overview of the intense debate that surrounded the α index, on a time line from the first half of the twentieth century to its end. The objective is to highlight the changing views between the first and the second halves of the century. In the first half, Pareto's position was considered to be untenable by all those who commented on it. In the second half of the century, new readings were proposed that have highly attenuated this view, and, in determinate circumstances, reevaluated the author of the *Cours*. In section 3 a particular reading of Pareto's contribution is proposed, aimed at going beyond both the view that judges Pareto's use of his index of inequality completely erroneous and the successive analytical attempt to rehabilitate him. The conclusion reached is that Pareto has not made any contribution to the criteria for measuring inequality and, as such, has to be evaluated using other parameters.

1. The α Index and Pareto's Theorem of Distribution

When, in January of 1895, Pareto expounded his celebrated law of incomes for the first time—using symbols partially different from those used in later works⁵—he only incidentally mentioned that the diminution

4. "Inequality measurement is a subject where much energy can be spent arguing about the meaning of the term. This is not a matter of taxonomy for the sake of taxonomy. The problem is that 'inequality' itself . . . is not self-defining and the definitions applied may derive from sometimes sharply contrasted intellectual positions" (Cowell 2000, 89; see also Champernowne 1974).

5. See Pareto 1895, in which the author proposes the formula $y = H/x^h$, where x represents income and y the number of families having an income between x and dx , while H and h represent constants to find through interpolation. The values of h indicated by Pareto—which

of the index h “indicates a tendency toward a lesser inequality in incomes” (Pareto [1895] 2001, 279). Exactly a year later Pareto (1896, 87) fleetingly returned to the income curve, in order to reaffirm that when that index “diminishes in value, the division of incomes is less inequal.”⁶ In neither of the two statements do we find the arguments that sustain such an assumption.

In his 1896 publication, Pareto expounds his law of incomes in a more organic and systematic way, partially modifying the symbols proposed the preceding year: the coefficient h became the much better-known symbol α ,⁷ even if we still do not find explained the reasons that allowed him to interpret this parameter as the index of inequality. In order for this to be elucidated, it was necessary to await the publication of the second volume of the *Cours*, between the end of 1896 and the beginning of 1897, where Pareto clarified the logic of his index of inequality.

According to the author of the *Cours*, there is “lessening of inequality” when “the number of the poor diminishes in relation to the total number of members in society.”⁸ From this definition Pareto draws the index u_x , defined as the relation between N_x (the number of individuals with an income equal or superior to x) and N_h (the number of individuals with an income equal or superior to the minimum income h). In the Paretian meaning, u_x is properly an index of equality, being inversely correlated with inequality (inequality diminishes when the number of individuals with income superior to x increases with respect to those with income superior to h). In the case of Paretian distributions, the index u_x is inversely correlated with α , making the use of the latter as an index of inequality possible. Substituting N_x and N_h for the values obtained from the equation of incomes, $N = A/x^\alpha$, we will have

oscillate around 2.5—differ from those proposed in the successive works, when the symbol α will be substituted for h , because the incomes equation will no longer be formulated in terms of the number of taxpayers between x and dx but in terms of the number of taxpayers having at least a given income x .

6. Robert Giffen’s statistical data on the distribution of incomes in England would therefore show a lessening in inequality between 1843 and the period 1879–80, a conclusion, the Celigny economist emphasizes, that would not have been “equally sure” if the data had been “obtained by comparing averages” (Pareto 1896, 87).

7. As is known, Pareto’s equation assumed the form $\text{Log}N = \text{Log}A - \alpha\text{Log}x$, from which $N = A/x^\alpha$ (where x represents a certain income and N the number of taxpayers with an income equal or superior to x).

8. Envisioning the risk of incomprehension he was running with a similar definition, the author of the *Cours* warns that he wishes to discuss this and “nothing else” about the problem of inequality (Pareto 1896–97, 356), without, however, succeeding in his intent to avoid controversy, as we will see.

$$u_x = \frac{N_x}{N_h} = \frac{A}{x^\alpha} / \frac{A}{h^\alpha} = \frac{A}{x^\alpha} \cdot \frac{h^\alpha}{A} = \left(\frac{h}{x}\right)^\alpha,$$

with α moving in an inverse relation with respect to u_x (where $h < x$). From this comes Pareto's conclusion that inequality in incomes diminishes with the diminishing of the index α .

Thus understood, the α index is used by Pareto as an instrument for ending the long-standing discord over the distributive dynamics taking place in capitalistic societies. The author of the *Cours* held that both the socialists—for whom inequality was ineluctably destined to increase—and the “optimists” of the liberal school—according to whom economic progress meant perforce a lessening of inequality—had expressed judgments that were vague and unsupported by any rigorous criteria of measurement whatsoever. The α index instead, in Pareto's opinion, allowed for an unequivocal interpretation of the distributive dynamics: the diminution of α recorded in the most advanced countries was in particular the sign of a trend toward reduction of inequality in nineteenth-century capitalist societies.⁹ With his definition of inequality as a starting point, Pareto finally deduced what is in effect his “fundamental theorem of distribution”: increase in the minimum income or decrease in inequality (or both together) can take place only in coincidence with economic growth, or rather with the increase of the average income.¹⁰

In the Paretian distributions, the mean income \bar{x} , the minimum income h , and the index α are in fact linked by the formula $\bar{x} = h(\alpha / (\alpha - 1))$, from which arise the two propositions of the Paretian distribution theorem:

1. If the inequality remains constant, that is, if α does not change, an increase in the average income brings about an increase in the minimum income.
2. If instead the minimum income remains constant, the increase in average income generates a decrease in inequality.

9. Pareto (1896–97, 361–62) added that nothing guaranteed that this “diminution of inequality in the fortunes or in the incomes has to continue indefinitely.” In other works following the *Cours* as well as in the *Manuale di economia politica pura* (1906), Pareto, however, reaffirmed that there had been a diminution of the index α that proved the reduction of inequality (see Pareto 1896–97 and Pareto 1906, 275). Therefore the statement by Vincent J. Tarascio that “what has been completely overlooked is the fact that Pareto never used α directly as a measure of income inequality” (quoted in Chipman 1976, 118) would seem to be without any foundation.

10. “Neither an increase in the minimum income nor a diminution in the inequality of incomes can come about, except when the total income increases more rapidly than the population” (Pareto 1896–97, 2:320–21).

The implications for political economy of these propositions are evident. In the Paretian meaning of the term *inequality*, the only way to pursue a reduction in inequality consists in allowing market forces to operate, excluding any redistributive hypothesis.¹¹

2. The Confusion over Pareto's Index: A Brief History

The use of the α index did not in fact show itself to be, as Pareto had hoped, an instrument able to resolve the debate over the distributive dynamics taking place in capitalist societies: not only and not so much because it was a poor index, usable only in the presence of distributions that were strictly Paretian, but above all because the interpretation of the index itself depended on contrasting judgments. While Pareto continued to read α as an index of inequality, an ever-wider front of scholars gave α a completely opposite value, reading it as an index of equality. The debate was very intense, particularly in Italy, but soon took on an international dimension.

After Rodolfo Benini ([1897] 2001), who seems to have been the first to have expressed doubts regarding the sense in which the index α should be read,¹² Costantino Bresciani Turrone was the scholar most assiduously committed to an alternate reading of Pareto's index (see Bresciani Turrone [1905] 2001, 1907, 1910, and 1936; and Brandolini 1997). For the whole of the first half of the twentieth century, he published repeatedly on the question, defending Benini from the start.¹³ At the end of the thirties he

11. "Free competition, since it stimulates production, indirectly contributes to the increase of the lower incomes and to the diminution of inequality among incomes" (Pareto [1909] 1971, 386).

12. In 1897 Benini ([1897] 2001, 289) wrote that the index α "denotes in what measure, more or less quickly, the number of individuals in receipt of an income higher than a limit figure decreases as said limit or level rises" (see also Benini 1901, 290). The divergence from Pareto, perfectly understood by Sorel (1897, 593), would become explicit in 1905: as Benini (1905, 227) wrote, "the higher value α has, the less inequal is the sharing out of wealth: if in fact α was very large, it would be necessary to ascend only a few steps in the gradation of income to no longer find any recipients; which means precisely that the diverse layers of the population differ little between them in economic conditions" (see also Benini 1906, 187–88).

13. "Benini's interpretation . . . I believe the more correct. In fact, the higher the concentration of recipients in the lower classes, i.e., around the average income, and the smaller the number of recipients in the higher classes, i.e., the less frequent the deviation from the average income, the higher the leveling of incomes. Conversely, the more numerous the recipients of the higher classes, the greater the dispersion of the series, i.e., the differentiation of the classes of income" (Bresciani Turrone [1905] 2001, 333).

published his best-known contributions in the *Journal of the Royal Statistical Society* (1937) and in *Econometrica* (1939), expressing severe criticism of the author of the *Cours*. The definition of inequality as proposed by Pareto was shown to be “quite inadequate as a tool of statistical analysis” (Bresciani Turrone 1937, 427; see also 1939, 112). The Paretian theory of distribution, with its relative implications for political economics, was also, according to Bresciani Turrone (1939, 132), destined to collapse, since “Pareto’s conclusions as to the relation between the average income and the degree of inequality might not be accepted.”

Corrado Gini had also contributed to dispelling doubts about the sense in which the α index was to be read. In the gestation phase of his famous concentration ratio—destined to become the canon among the measures of inequality—Gini had proposed as the measure for inequality, as an alternative to Pareto’s α index, the index δ .¹⁴ Such an index is linked to the parameter α by the formula $\delta = ((\alpha - 1) / \alpha)$, confirming further the inverse relation that links inequality and Pareto’s index α .¹⁵ The Italian statistician’s conclusion is peremptory: “Taking the terms in their etymological and current significance, we have to say that . . . Benini’s interpretation . . . corresponds better than that of Pareto to the significance that is commonly attributed to the expression ‘inequality in distribution’” (Gini [1910] 2001, 49–50). Gini’s contribution removed all ambiguity over the interpretation of the α index, as was recognized by, among others, Mortara (1911), Savorgnan (1915), and Ricci (1916).

The debate in Italy therefore left no room for doubt: The α index, if understood in Pareto’s manner, contrasted with the most elementary common sense, as even Pareto’s own closest pupils would be forced to admit.¹⁶ Besides Barone, whose significant interpretation will be discussed in our conclusion, Luigi Amoroso too had to openly distance himself from Pareto. In 1947, he would write, in fact, that the definition of inequality proposed in the *Cours*, “while being irreproachable in the algebraic aspect,” nonetheless represented an irreparable flaw in the “admirable Paretian theory of incomes,” in that it was contrary to “*all the other theoretic investigations*” carried out on the question (Amoroso 1947, 134–36).

14. As Corrado Gini ([1910] 2001, 359) writes, δ “indicates the exponent to which a certain part of the incomes should be raised to obtain the part of the rentiers who own it.”

15. On the relation between the indexes α and δ , see Amato 1948, Battara 1948, Bresciani Turrone 1936, De Vergottini 1947 and 1948, Gini 1921, Pietra 1935, and Pizzetti 1948.

16. There were very few who took Pareto’s part. Among these were Furlan (1909, 700–701), Beneduce (1909, 582–85), and Sensini (1912, 350–52).

The debate on the interpretation of Pareto's index had in the meantime taken on an international dimension. One of the most interesting cases is without any doubt that in England. Distinguished scholars like Arthur Bowley, Arthur Pigou, and Hugh Dalton showed themselves hesitant over the sense in which the α index was to be read, disoriented by the criticism against Pareto that was being made in Italy. Bowley (1915, 209–10) and Pigou (1912, 22–25), for example, had initially supported Pareto's theory, reversing their judgments only later.¹⁷ Dalton too, whose work published in 1920 is now recognized as a milestone in the literature on indexes of inequality, appears quite uncertain in evaluating the contribution made by Pareto. He prefers, however, not to take a stance on the controversy, asserting that “the question requires further study” (Dalton 1920, 359).

In America, too, the statisticians and economists who, in the wake of Max Otto Lorenz (1905), initiated an intense debate on the concept of the concentration of wealth, came to have an unequivocal reading of Pareto's index. The α parameter is not always referred to in an explicit fashion; however, the prevalent opinion was that it is a measure of inequality to be read in the opposite sense from that supposed by the author of the *Cours*.

Among the numerous points that emerged from the American debate, two appear of particular relevance. The first regards Allyn Young, who apparently was the only one who did not pick up on the printing error in the *Cours*, coming thereby to attribute an improbable “mathematical” error to Pareto. In the original edition of the *Cours* we in fact read that inequality diminishes when the number of individuals with an income inferior to x *increases* more than the number of individuals with an income greater than x . That, in fact, it is the exact opposite is proved not only by the rectification in the *Manual* (Pareto 1906, 275),¹⁸ but also by the way in which Pareto's argument proceeds. Young (1917, 476), on the other hand, was not aware of the printing error and thought that the unamended version of the *Cours* was plausible. But Young believed Pareto then made a banal error in algebra in formally translating his definition of inequality,

17. Bowley (1923, 210–11): “In the first edition this statement was put wrongly; there has been a curious difference of opinion as to whether an increase of α means more equal distribution. This is due partly to the absence of any simple measurement of inequality.” Pigou (1920, 58): “It is a matter of dispute whether the reciprocal of his measure—which of course would indicate less inequality when the measure itself indicates greater equality—is not to be preferred to that measure.” See also Pigou 1920, 695.

18. Where Pareto (1906, 275) points out the need to substitute for the wording “revenu inférieur a x augmente par rapport” the corrected “revenu inférieur a x diminue par rapporte.”

from which we have the paradoxical interpretation of the α index (476–77; see also Asso and Fiorito 2001). In truth there was no mathematical error on Pareto's part, as even Bresciani Turrone emphasizes,¹⁹ but rather a contrast that arose from the definition of inequality given by Pareto.

A second aspect worth mentioning is the graphic representation proposed by some American statisticians, which allows us to understand the sense in which the α index is to be read "at a glance." The "income pyramid" has an intuitive graphical representation reversing the axis as it is defined by Pareto. Placing the logarithm of incomes on the abscissa and the logarithm of recipients on the ordinate, Pareto's α became a measure of the slope of the income pyramid. Using such a representation, authors such as G. P. Watkins (1908) proposed reading the distributive dynamics observing the inclination of the interpolation line, considering it in its turn a measure of the inclination of the income pyramid (see figure 1).

Watkins (1908, 34) believes it to be possible, through this graphic representation, to see the minor or major inequality of the diverse distributions with the naked eye, simply observing the trend of the respective logarithmic curves: "The crucial point is their *slant*. The steeper of the two curves is the one which expresses the greater concentration."²⁰ Even without the expedient of reversing the axis, Norris O. Johnson (1935, 721) has a similar position: "The *slopes* reflect the relative equality, or inequality, in the distribution of incomes in the respective years. That is to say, the more steeply the line declines—the greater the measured slope [i.e., the more is α]—the more equally are incomes (within the range of taxation) distributed." Even without expressly stating it, Watkins and Johnson therefore use Pareto's index in its inverse form, in that the increase in slope is a sign of diminishing inequality and corresponds exactly to the increase in the α index.²¹

19. "Pareto, when explaining the significance of changes in α , made no 'curious slip,' as was contended by Professor Allyn Young" (Bresciani Turrone 1937, 424).

20. According to Warren Persons (1909, 428), the major weakness of Watkins's proposal was that "the comparison of the slopes has to be made entirely by the eye, which is a very unscientific instrument. No numerical instrument is offered." What is curious is that neither Watkins nor Persons recognizes that when the logarithmic curves are rectilinear, Pareto's and Watkins's methods coincide, since the α index (or rather its inverse) represents an estimate faithful to the inclination of the income pyramid.

21. Using this criterion, Johnson believed he could invalidate the conclusion of the Brookings Institution report, according to which "there has been a long-time tendency for income to be distributed with increasing inequality in the United States" (see Cirillo [1974] 1999, 281). Among the other contributions to the debate during the thirties, see Crum 1935 and Tucker 1938. For an account of the debate, see Asso and Fiorito 2002. Quite different was the conclusion of Mary Bowman (1945, 612), according to whom the use of the "Pareto ' α '—the slope of the curve—as a measure of degree of inequality for an entire income distribution" was a fallacy.

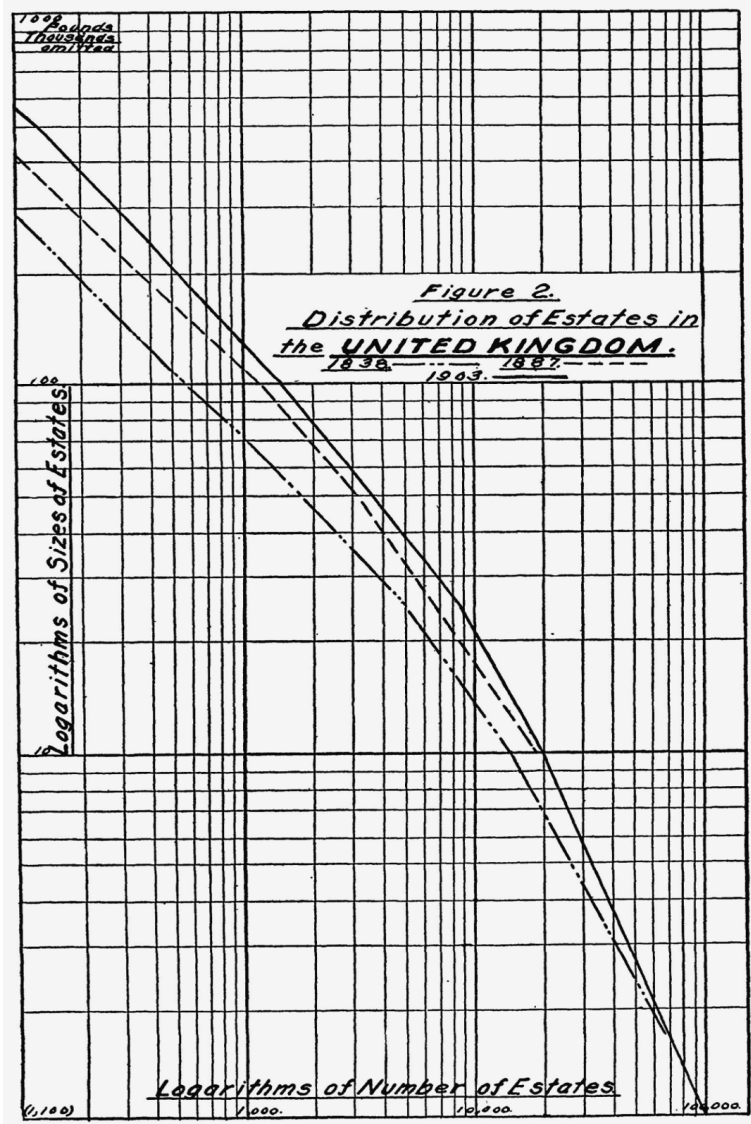


Figure 1 Income pyramid. Source: Watkins 1908, 43.

In the second half of the twentieth century, the debate on Pareto's index reduced in intensity, but we nonetheless witness a singular fact: in the few contributions that deal directly with the problem of the interpretation of the α index, there is a prevailing tendency to try to partially rehabilitate Pareto. There are in fact at least two important analytical revisitations that profoundly modify the way of reading the controversy.

The first of these rereadings is that of Samuelson (1965). Generalizing Pareto's equation within the compass of the so-called Pareto-Lévy distributions made famous by Mandelbrot's contributions in 1960 and 1963, the α parameter is revealed to be an improper index of inequality, such as to be totally insensitive to forms of transfer that move all incomes in the direction of the average income (248). Samuelson's conclusion is therefore that Pareto's coefficient is not a "measure of true changes in inequality," thus delegitimizing the bustle that had been created around the interpretation of the index (250).²²

The second analytic rereading belongs to Chipman (1974, 1976), in whom the attempt to rehabilitate Pareto is more evident. Chipman holds that Pareto was reasoning not only on a statistical descriptive plane but also upon a normative one. The essential question was not only the course of inequality but also the dynamics of social welfare, measured in terms of the average income and the minimum income. Evaluating the effects of the changes in the α parameter in the Paretian-type distribution, Chipman emphasizes two cases:

1. From a comparison of the distributions with the same mean income, the diminishing of the α parameter would in effect be, contrary to what Pareto had claimed, "a bad thing," thus justifying the criticisms that had been aimed at the Lausanne economist.
2. However, in the case in which it is the minimum income that remains constant (and supposing that the parameter A of the "income equation" also remains constant), then a bettering in welfare would be associated with lower values of α (Chipman 1974, 279).

22. The problem, which is historically not irrelevant, is that "Pareto wrote before Paul Lévy," not dealing with what would only at a later date become known as "Pareto-Lévy distributions." Samuelson (1965, 250) himself, at the end of his contribution, is forced to recognize that "Pareto and his original commentators worked only with the truncated tail of the income distribution and not with the full Pareto-Lévy distribution." From here came the bitter admission that "this left enough ambiguity in the problem and its relevant definitions to make it possible for scholars to remain confused." On Samuelson's explanation, see Cirillo [1974] 1999, 284, according to which no debate would have emerged if the improper nature of the coefficient α as a measure of inequality had been understood.

Chipman (1976) has given us what is still the most systematic and informed reconstruction of the dispute over Pareto's α index. He reaffirms that Pareto had in mind the more complex plane of the relation between α , minimum income, and average income when he formulated his index. As Chipman writes, "Welfare can be increased either by an increase in the minimum income m , or by a decrease in Pareto's measure of inequality α , or both. . . . Pareto's intuition is completely vindicated." In Chipman's opinion, "The question of whether Pareto's coefficient should be considered as a measure of 'equality' or 'inequality' is thus resolved" (126–27).

Chipman's theory was taken up a few years later by Cowell in his classic work on the measures of inequality. The decrease in the average income that in some cases accompanies the increase in α , in Cowell's ([1977] 1995, 84) opinion too, cannot in fact be interpreted as a loss of welfare, justifying thus the use of α as proposed by Pareto (also see Corsi 1994, 7).

The most recent contributions on the question refer back to this interpretation, also emphasizing the printing error present in the *Cours* as the source of the controversy over the α index. Kirman (1987, 1998) and Persky (1992), for example, on the one hand emphasize how, following the "printer's error in the *Cours* . . . there has since been considerable confusion about what Pareto actually said" (Kirman 1987, 807; see also Kirman 1998, 31; and Persky 1992, 185); on the other they use the explanations by Chipman and Cowell to clarify how in determinate cases "Pareto's view is the appropriate one" (Persky 1992, 185; see also Kirman 1998, 31).

3. A Question of Words

As we have seen, the idea advanced by Pareto of interpreting the α index as a measure of inequality fed a debate that we can divide into two conventional periods: the first covers the first decades of the twentieth century, when the idea prevails that the position of the author of the *Cours* is unsustainable in a net fashion; the second period covers the latter half of the century, when an attempt to attenuate the contrast between Pareto and his critics emerges.

The interpretation advanced in this article partially diverges from both of the above perspectives. Pareto's definition of *inequality* is totally different from the commonly accepted definition of the term (a state that diverges from a hypothetical situation of perfect equality). This hypothesis will be supported using, on the one hand, Pareto's own remarks, and on the other, some diverse findings by Wicksell and Barone.

One often-neglected aspect of the critical literature on the argument is that Pareto, in introducing the distribution problem, divides the meanings of “lesser inequality” into two distinct parts: first, the diminishing of higher incomes, and second, the increase in lower incomes. Pareto (1896–97, 354–55) observes that these are two phenomena, distinct and irreducible one from the other, and he says that economics, if it were “a positive science in which facts are all and words do not count for anything,” would “designate [them] with different terms.” The use of a term taken from common speech instead impedes precise differentiation between the two phenomena, which are erroneously joined together. In defining the concept of lesser inequality Pareto therefore states his desire to scrupulously hold only to the second meaning—that of increasing lower incomes.

This distinction has consequences of a certain importance. Having discarded the sense that common sense attributes to the term *lesser inequality*—decreasing higher incomes—Pareto peremptorily denies that any redistributive hypotheses whatsoever of transference of income from the “richer” to the “poorer” can be defined as “lesser inequality.” Similarly, progressive taxation does not generate, in the Paretian sense of the term, a more egalitarian distribution *after tax*. There are no doubts therefore that Pareto is traveling in a different direction than were other economists of his time, not to mention economists who came after him.

Let us now examine the second meaning of the term *lesser inequality*: the progressive increase in lower incomes. This phenomenon, which is the only one, in Pareto’s opinion, that is properly termed *lesser inequality*, can show itself in at least two ways:

1. Through an increase in lower incomes that is proportionately higher than the increase in upper incomes. For example, lower incomes might grow by 15 percent, and the higher ones by 10 percent.
2. Through an increase in lower incomes that is proportionately lower than the increase in upper incomes. Lower incomes might grow by 10 percent while higher incomes grow by 15.

Taking up the visual representation of the social pyramid as suggested by Watkins, we could thus describe the two phenomena: in the first case, through a shift toward the top of the lower end of the “curve of incomes,” there is a lessening of the slant of the curve itself; in the second case, there is still a shift toward the top of the lower end, but with an increase in the slant of the curve (that is, a lengthening of the social pyramid that

corresponds, as may easily be imagined, to a situation in which it is the higher incomes that increase proportionally more in respect of the lower incomes). If the first case—with a corresponding increase in the α parameter—fits perfectly into the significance commonly attributed to the idea of lesser inequality, the second—in which the α parameter diminishes—does not. We must, however, emphasize that it was precisely this second case that Pareto was thinking of in defining his concept of “lesser inequality.”

This is most paradoxical, since the author of the *Cours* had declared that he was inspired, in his own formulation of the problem, by the work of Paul Leroy-Beaulieu (1881), for whom the centuries-old dynamic of the developed societies was shifting in the direction of a bettering of the lower classes that was proportionally greater than that of the middle and upper classes.²³ As is also emphasized by Chipman (1976), Pareto, instead, in the end comes to exclude this possibility, defining as lesser inequality only that which generates a shrinking of the social pyramid. The paradox is clearly declared by Wicksell ([1897] 1999, 56), in a Swedish review of the *Cours* to which it would seem none of those who have dealt with Pareto’s index have ever made reference:

Closer inspection reveals that Pareto’s definition does not at all coincide with what Leroy-Beaulieu intended, nor what is customarily understood by the equalisation of incomes. Leroy-Beaulieu refers to an improvement of the economic situation of the lower classes during which the position of the upper classes remains relatively unchanged. This is equivalent to an alteration of the social pyramid, in which the apex remains relatively fixed, while the base is pushed upwards, producing a compression or flattening of the structure. Pareto’s definition implies rather that for every level of incomes, the part of the social pyramid below the limit in question becomes relatively smaller than before, and the part above relatively larger. It can be seen without mathematics that this need not always correspond to a compression of the social pyramid, but may equally well correspond to an elongation in certain circumstances (i.e. to increased inequality of incomes), according to whether

23. “The progress in the welfare of the lower classes of the population are, and above all, will be in the near future more rapid than that of the middle and upper classes. Without reaching a leveling out of the condition, which is impossible[,] . . . the current economic shift leads to a major approximation of the social conditions, to a lesser inequality among the fortunes” (quoted in Pareto 1896–97, 356).

the apex or the base remains relatively unchanged. Pareto's general formula gives us no information about this question at all.

To sum up, lower incomes can rise and thus increase welfare in three ways: they can grow at the expense of higher incomes, as in the case of redistribution from the rich to the poor; they can grow proportionately more than higher incomes; and they can grow proportionately less than higher incomes. Pareto leaves only the last standing. And it is only to this last phenomenon that he, counter to prevalent opinion, gives the label "lesser inequality."

The problem in effect is resolved by Pareto, appealing against the conventional nature of the labels and definitions. The author of the *Cours* has identified a phenomenon to which he has chosen to give a label—or a name—that up to then had designated (and still designates today) a different phenomenon. That the problem has a fundamental linguistic dimension is proved by the way in which Pareto takes the criticisms of his interpretation of the α index. He does not seem at all perturbed by these comments; on the contrary, he reaffirms the validity of his own position.²⁴ In the second French edition of the *Manual* (Pareto [1909] 1971, 289) he expresses in fact his regret at not having found a better expression for defining the phenomenon under discussion and for having made use of a term—"decrease in income inequality"—that has "given rise to ambiguity."²⁵ The debate over the α index therefore seemed to be attributable to the fact that Pareto's critics overlooked the ambiguity that surrounds the words "decrease in income inequality." Rather, they relied on the vague sense of the term sanctioned by the common meaning of the words, not understanding that they needed to define the term in a rigorous fashion. In order to avoid further misunderstandings, Pareto declared that he wanted to substitute for the term *income inequality*, as used in

24. See apropos this topic the testimony of Bresciani Turrone (1937, 425): "Pareto's interpretation of the changes in the value of α is the logical consequence of his definition of inequality. . . . This standpoint was always taken by Pareto himself, also in private conversation with the present writer. He never took into consideration his critics' objections against his interpretation, because, as he said, they proceeded from a definition of inequality which differed from his own; and, in his opinion, it was futile to argue about definition, since definitions are arbitrary."

25. As Pareto himself confessed privately to Sorel (1897, 586), the choice was an infelicitous one and it would have been better to introduce a new term to denominate the phenomenon, were it not that a similar decision would have once more exposed him to the same accusations he had been the object of when he had proposed substituting the term *utility* with *ophelimity*.

the *Cours*, a newly coined phrase—“decrease in the inequality of the *proportion* of incomes”—which he considered more consonant with his own idea. Given this punctilious precision of meaning, Pareto can therefore allow himself to repeat that the changes that came about in the distribution of incomes in the course of the nineteenth century, documented by the diminishing of the α index, were going “in the direction of a decrease in the proportion of inequality of incomes” (290).

It seems that no one took any notice of Pareto’s terminological clarification, which might have served to make it plain that the author of the *Cours* was reasoning on a plane that was totally different from that which was traditional. The terminological revision in the second edition of the *Manual* appears, however, rather weak: if in truth the problem was one of a linguistic nature, Pareto would have done better to radically change the term used, so as to exclude every reference to the idea of greater or lesser inequality.

Not by chance, this is the path followed by Enrico Barone, one of the most authoritative of Pareto’s disciples, whose interpretation of the controversial question seems to confirm the reading discussed above. Aware of the fact that as a measure of inequality there is no way of using the α index understood in the manner of Pareto, except at the cost of serious misunderstandings (Barone 1912, 53), Barone (1936, 264) resolves the problem with a radical lexical revolution: “to the exponent α we will give the name of *convergence*,” so that the changes in its value are no longer interpreted as lesser or greater inequality but as lesser or greater “convergence of incomes.”²⁶ The two propositions that form the backbone of the fundamental theorem of Pareto’s distribution are therefore reformulated by Barone, excluding any judgment on the lessening of inequality and substituting for it instead the expression “lesser convergence of the income curve,”²⁷ an expression considered more “precise and

26. “This phenomenon, which shows itself in connection with the variations in the mean income, will not be given the name of ‘major or minor inequality of conditions’: such expressions lend themselves to diverse interpretations, because that concept of ‘major or minor inequality of conditions’ is extremely indeterminate” (Barone 1912, 338–39).

27. “The variations in the dynamics of the distribution of incomes are intimately connected with the variations of the average income. That is: (1) Growth in the average income (that is to say the total of the incomes growing more rapidly than the population . . .) gives rise to a minor convergence of the income curve. (2) A minor convergence of the income curve cannot take place unless the total of the incomes grows more rapidly than the population—that is, if there is no growth in the average income” (Barone 1936, 264–65).

neutral" (Steve [1974] 1997, 329). Barone explains also that this "lesser convergence" is associated with an increase in the value of the α index and that it presents positive aspects, above all a major social mobility. It is principally a situation in which the "rise of a certain number of individuals from the lower income group to the higher income bracket" becomes possible, so that "the percentage of the lower incomes decreases and that of the higher incomes grows" (Barone 1936, 269). Reasoning in figurative terms that correspond, as has already been observed, to the social pyramid becoming acute, with a shrinking of its base and an increase in its height, Barone emphasizes moreover how this major economic social dynamism does not bring about any worsening conditions for the lower classes: "for those who are able to progress," Barone writes, there are "greater and more ample prospects . . . of the betterment of their condition," but without this causing, "in the absolute sense," a worsening of the condition of those who are further down the scale (272). If this is "a sufficiently exact image of the phenomenon" that happens following the increase of the average income and the diminishing of the convergence between incomes, Barone believes that the confusion resulting from the diverse interpretations of the distributive dynamics taking place in capitalistic societies, which divided scholars into "pessimists" on one side and "optimists" on the other, could be cleared up once and for all.

Without any doubt, an increase in the average income produces something very different from the growing proletarianization that was predicted by Marx: the lower classes, instead of increasing, are thinned out, and the middle class tends to be augmented notably. But it is also a fact that while in the two classes mentioned the average class income tends not to increase significantly, a notable increase does take place in the income of the higher class, making the condition of the middle and lower classes not worse in the absolute sense, but making the discontent arising from comparison more blatant. (273)

Barone's is probably the best rendering of the distributive dynamics as envisaged by the author of the *Cours* when he spoke "improperly" of the "lessening in inequality" associated with the diminishing of α . Barone's intuition of qualifying this phenomenon as a diminishing of the "convergence of incomes" would certainly have resolved the misunderstanding, precisely because it cleared up once and for all that it was not any eventual "lesser inequality" that was under discussion. The core of

Pareto's and Barone's reasoning focused on the problem of capitalistic dynamics and on the possibility that such a dynamic was accompanied by a general betterment in economic welfare.²⁸ The diminishing of the α parameter, in this view, is the sign of a social structure that has become more fluid and diversified—where precisely the process of “circulation of the elite” acts with greater efficacy—and where there are fewer people with low incomes. This has little to do with the decreasing of inequality, which on the contrary, being a “relative” concept, may well be increased (as in fact Barone implicitly recognizes in revealing the growing “discontent coming from comparisons” between those at the top and those at the bottom).

Unfortunately Pareto did not have the opportunity of seeing Barone's reformulation, and so we do not know what his opinion of it would be. However, this seems to be the correct way to interpret the meaning attributed by Pareto to the α index.

If, however, this interpretation is correct, then there are still some questions that remain unanswered. Why did Pareto continue to consider his own definition of inequality as valid? Why did he not feel, as Barone had, that it was necessary to modify the terminology? And why, finally, did he remain faithful to a definition from which certain forms of inequality were categorically excluded?

It is probable that underlying Pareto's choice were motives of a political nature. Let us not forget that Pareto intended his inequality index to settle the heated debate over the distributive dynamics in capitalist societies. This debate, while centered on the “facts,” had an undeniable political substratum, which probably conditioned Pareto himself in the choice of his particular index of inequality. If up to now this article has illustrated *how* the author of the *Cours* elaborated his index and *how* he came to attribute to it a wholly particular meaning, from this point on it will try to advance some hypothesis as to *why* he made those choices.

Unfortunately the evidence is somewhat scarce. As has been said, Pareto avoided taking part in the debate involving his inequality index, limiting himself to a few ambiguous comments regarding terminology.

28. As Sergio Steve ([1974] 1997, 329) has pointed out, for both Barone and Pareto “the diminishing of the convergence was to be interpreted as a bettering of the distribution,” despite the fact that “the decrease in the number of the poor in respect to the number of the rich” also led to “an increase in the distance between rich and poor. It leads, that is, to two phenomena to which it is possible to give opposing evaluations, positive for the first, negative for the second, on the basis of the value premises most common to the distribution of incomes.”

In the second edition of the *Manual* he introduced some clarification that cast light on two aspects:

1. He continued to consider his own interpretation as legitimate and to consider the criticism unfounded; and
2. he considered the problem to be exclusively one of terminology.

This thesis was also vigorously repeated in the *Trattato di sociologia generale* (translated in 1935 as *The Mind and Society*), although within the more general context of the Paretian theory of “sentiments” which may perhaps shed light on the stance he took.

4. Inequality and the “Logic of Sentiments”

In the *Trattato*, Pareto ([1916] 1935, 1:42–43) writes that when the *Cours* was published, “it was an article of faith with many people that social evolution was in the direction of the rich growing richer and the poor, poorer.” This type of reasoning, in Pareto’s opinion, was based on “sentiment” and not on the “logical-experimental” method. It was in fact an egalitarian sentiment that led many observers to see the polarization of the social classes, understood as the proof of an increasing inequality in incomes, as behind the dynamics of history. It is not necessary to dwell here on how Pareto painted these “egalitarian sentiments,” that is, as veiled demands for new privileges.²⁹ The fundamental point is that, according to the author of the *Cours*, the egalitarian sentiment impeded reading, free of value judgments, the phenomenon linked to the distributive dynamics. After the French Revolution, equality had become a positive political ideal, which went hand in hand with the “emotive” discredit of social and economic inequality. From this ideal came the widespread tendency to assign a negative value to increasing inequality.

In Pareto’s eyes, therefore, the problem of the measurement of inequality was strongly distorted by the so-called logic of sentiments. The aversion to inequality was based, on the one hand, on a sentimental political prejudice, and on the other, the absence of a rigorous terminology: “Every

29. “The sentiment that is very inappropriately named equality is fresh, strong, alert, precisely because it is not, in fact, a sentiment of equality and is not related to any abstraction, as a few naïve ‘intellectuals’ still believe; but because it is related to the direct interests of individuals who are bent on escaping certain inequalities not in their favor, and setting up new inequalities that will be in their favor, this latter being their chief concern” (Pareto [1916] 1935, 2:735–36).

argument based on sentiment, as all metaphysical arguments are, must of necessity use terms lacking in exactness, since sentiments are indefinite and the name cannot be more definite than the thing. Such arguments, besides, actually rely on the lack of exactness in everyday language to mask their defects in logic and carry conviction" (Pareto [1916] 1935, 57).

It was precisely to counter this sentimental opinion that Pareto had formulated in the *Cours* his own theory of distribution, repeated without any variation in the *Trattato* and referring, for the definition of the concept of lesser inequality, to the second edition of the *Manual*. Pareto had therefore no hesitation in restating that from 1897 to 1911 the increase in average income had produced both an increase in the minimum income and a decrease in inequality (naturally within the Paretian definition of the term) (42).

In the light of what he wrote in the *Trattato*, Pareto's strategy, with its aim of reaching two objectives, one scientific and one political, becomes even clearer. From the scientific point of view, Pareto wanted above all to address the debate over the distributive dynamics, sticking to the facts and to scientific rigor, shedding the indeterminacy that had marked it up to that moment. "Logico-experimental arguments, being based . . . on objective observations, tend to use words strictly to designate things and therefore to choose them in such a way as to avoid ambiguities and have terms as exact as possible. Moreover, they eventually equip themselves with a special, technical language and so escape the indefiniteness of common parlance" (Pareto [1916] 1935, 1:57). In Pareto's opinion, this scientific rigor was lacking in the debate on the problem of inequality at the end of the nineteenth century. In the *Trattato* too, Pareto thus appeals to terminological precision³⁰ and uses the term *lesser inequality* according to the narrow definition he gave it in the *Cours*.

From the political point of view, Pareto's strategy appears less clear-cut, though of equal importance. Although not explicit, an attempt by the author of the *Cours* to put into practice the same persuasive strategy that he opposed in his critics emerges between the lines. The problem is that of the negative evaluation assigned to "inequality." Such an assignment may appear forced, but it is not improbable that Pareto wanted to lay his cards on the table. As he wrote in the *Trattato*, many phenomena "are considered on the basis of sentiments" that are elicited by the words that represent them, and therefore it is to the advantage of those phenom-

30. As Pareto in fact observes, following the scientific method means that "first one examines the thing and then hunts up a name to give it" (Pareto [1916] 1935, 1:62–63).

ena “to have a name that awakens favorable sentiments and to its disadvantage to have a name inspiring unfavourable sentiments” (Pareto [1916] 1935, 1:60). The phenomenon linked to the decrease of the α index—that is, the proportional increase in incomes equal to or higher than x with respect to the number of incomes superior to minimum income h (for every x higher than h)—may evoke blame or approval if defined as “greater inequality” rather than as “lesser inequality.” By privileging the use of the latter definition, Pareto was trying perhaps to second the common sentiment that judged lesser inequality “positively.”

Pareto's rhetorical strategy is thus not without contradictions. The anti-egalitarian sentiments of the author of the *Cours* are widely known. In more than one circumstance he had described not only the inevitability of social and economic inequalities but also the positive elements that they entailed. The problem of social choice, for example, is illustrated by Pareto through the hypothesis of “a collective” that had to choose between being “a very wealthy community with great inequalities in income among its members” or being “a poor community with approximately equal incomes” (Pareto [1916] 1935, 4:1471). The way in which the alternatives are posed leaves little doubt as to Pareto's preference. But this case also makes it evident that it is the acceptance of greater or lesser inequality in the common sense of the terms that Pareto intends to attack. Applying his own index of inequality to the letter, Pareto ought to have said instead that the choice would have been between a rich and egalitarian collective and a poor and unequal one, thus overcoming every type of trade-off between efficiency and inequality. Is this perhaps the objective that Pareto set himself?

The author of the *Cours* does not appear to be completely aware of having used two different linguistic registers: while one kept the common meaning of the terms *equality* and *inequality*, the other gave these same terms a different sense. Every form of ambiguity would have been avoided if Pareto had had the same daring as Gary Becker and Kevin Murphy (2006) in stating explicitly that Pareto's social-economic theory allowed for reading the increase in inequality in a positive sense, overturning in this fashion the prevalent sentiment of disapproval linked to such a phenomenon.

5. Conclusion

This article has attempted to document the ambiguity of Pareto's contribution to the criteria of measurement of inequality. The “confusion”

highlighted by Paul Samuelson (1965) is principally due to the lack of recognition by numerous interpreters of the fact that Pareto dealt with the problem of inequality in terms that are not absolutely comparable with those that are traditional. Pareto chose to use the words “lesser inequality” to describe a very specific phenomenon that many economists in Pareto’s time, as well as of today, would not describe as “lesser inequality.”

The solution advanced by Chipman (1974, 1976), and successively supported by Cowell (1977) and by Kirman (1987, 1998), of dividing the meaning of α into two parts—one for distributions having the same minimum income and the other for distributions of the same average income—seems counterproductive, or at least a purely scholastic exercise. Through the use of the α index, Pareto intended to express a judgment, in his opinion objective, on the distributive dynamics taking place in the capitalist societies in which both the minimum income and the average income may have the tendency to increase. In this case, interpreting the reduction of the α index as a sign of lesser inequality made reference to a phenomenon³¹—which was perfectly clear to him—that had nothing to do with what is commonly understood as the lessening of inequality.

In the current debate over the indexes of inequality, attention is focused primarily on the problem of the ordering of the distributions, with an approach that integrates the purely descriptive dimension of the diverse measurement criteria with a normative logic. The objective is naturally that of identifying analytical tools that permit comparison of diverse distributions, on the basis both of relative inequality and of average income. As is pointed out by Anthony F. Shorrocks (1983, 4), major equity in the distribution of income could be accompanied by a lesser average income, making problematic the statement “that the welfare of that country has improved.” An adequate function of social welfare would therefore be to define the increase in the average income needed to compensate for an eventual increase in inequality, the latter increase varying according to a preference for efficiency or for equality.³² The temptation to read Pareto’s contribution in this perspective could be very strong, provided that it is not forgotten that, in the Paretian meaning, there is no trade-off between efficiency and equality.

31. In substance, the phenomenon of an economic growth that involves all the income classes, but where the better-off classes become progressively richer with respect to the lower orders.

32. See the discussion in Tam and Zhang 1996.

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