Cross-Polarization Modulation Domain Wall Solitons for WDM Signals in Birefringent Optical Fibers

Stefan Wabnitz, *Member, IEEE*

***Abstract*—Nonlinear cross-polarization modulation between two wavelength multiplexed channels propagating in an elliptically birefringent optical fiber leads to polarization domain wall solitons. These solitons represent a switching of the polarization of both waves between two stable orthogonal states. The potential application of polarization domains to nonlinear loss-free polarizers is described.**

***Index Terms*—Nonlinear optics, optical fiber polarization, optical propagation in nonlinear media, optical solitons, wavelength-division multiplexing (WDM).**

I. Introduction

THE instantaneous change of the state of polarization of an optical signal due to its cross-interaction with adjacent wavelength channels may have a significant impact on wavelength-division-multiplexed (WDM) transmissions. Indeed, cross-polarization modulation (CPM) degrades the performance of polarization-multiplexed WDM systems [1], [2] and polarization-mode dispersion compensators [3]. CPM was earlier theoretically and experimentally investigated for two and three continuous-wave (CW) or intensity modulated channels propagating in randomly birefringent fibers [4], [5].

In this letter, we study CPM among two WDM channels in fibers with elliptical birefringence, and obtain novel polarization domain wall soliton solutions [6]. These solitons describe temporal switching of both waves among orthogonal, stable polarization states, in analogy with domain walls along the longitudinal axis of the fiber from counterpropagating waves [7], [8]. An interesting and novel application of these temporal polarization domains is the lossless polarization attraction [9] of an initially depolarized probe beam to the same polarization state of a copropagating pump wave.

II. Theory

Let us express two optical waves at nearby frequencies ω_a and ω_b, copropagating in a spun birefringent optical fiber, in terms of the normal modes $\mathbf{e}_x$ and $\mathbf{e}_y$. These linearly polarized modes are aligned with the local axes of birefringence, and rotate along the fiber with a rate τ (rad/m). The mode amplitudes $\mathbf{E}^{a,b}(Z,T) = E_x^{a,b}(Z,T)\mathbf{e}_x + E_y^{a,b}(Z,T)\mathbf{e}_y$ obey the coupled equations

$$(\partial_Z \pm \Delta\,\partial_T)\,E_{x,y}^{a,b} = i\frac{\delta H^{a,b}}{\delta E_{x,y}^{a,b^*}} \qquad (1)$$

where Z is the distance along the fiber, T is a retarded time in a reference frame traveling with the average speed $V_{\text{ave}} = (V_a + V_b)/2$, $\Delta = (1/V_a - 1/V_b)/2$, and $V_{a,b}$ are the group velocities of pulses at ω_a and ω_b. We neglect the effect of stimulated Raman scattering, polarization-mode dispersion, and intrachannel group-velocity dispersion (GVD); i.e., we suppose that the walk-off distance between two pulses at frequencies ω_a and ω_b is much shorter than the dispersion distance. The total Hamiltonian in (1) is $H^{a,b} = H_L^{a,b} + H_S^{a,b} + H_X$, where H_L, H_S, and H_X describe the action of linear birefringence, nonlinear self-polarization modulation, and CPM, respectively [6], [10]. The H_X contribution (for H_L and H_S, see [10]) reads as

$$\begin{aligned} H_X = R\int dT\big[&2\big|\mathbf{E}^a\big|^2\big|\mathbf{E}^b\big|^2 \\ &+ \gamma_2\big(\big|E_x^a\big|^2\big|E_y^b\big|^2 + \big|E_y^a\big|^2\big|E_x^b\big|^2\big) \\ &+ \gamma_3\big(E_x^a E_y^{a^*} E_x^b E_y^{b^*} + c.c.\big) \\ &+ \gamma_4\big(E_x^a E_y^{a^*} E_x^{b^*} E_y^b + c.c.\big)\big] \end{aligned} \qquad (2)$$

where $\gamma_2 = -(1+B)$, $\gamma_3 = 2B$, $\gamma_4 = (1-B)$, $R = 2\pi n_2/\lambda A_{\text{eff}}$, $\lambda = (\lambda_a + \lambda_b)/2$, n_2 is the intensity-dependent refractive index, A_{eff} is the fiber mode effective area, and B depends upon the nonlinearity; e.g., $B = 0$ for electrostriction, or $B = 1/3$ for electronic distortion. Since $\omega_a \cong \omega_b$, henceforth we shall suppose that $\beta_{x,y}^a \cong \beta_{x,y}^b$, where β_x and β_y are the linear propagation constants of the modes $\mathbf{e}_x$ and $\mathbf{e}_y$. Let us consider the elliptically polarized (with ellipticity θ) eigenstates of H_L, i.e., $\mathbf{e}_1 = (\mathbf{e}_x + it\mathbf{e}_y)/\sqrt{1+t^2}$ and $\mathbf{e}_2 = (t\mathbf{e}_x - i\mathbf{e}_y)/\sqrt{1+t^2}$, where $t \equiv (\beta_x - \beta_y)/2\tau + \sqrt{(\beta_x - \beta_y)^2/4\tau^2 + 1} = \tan(\theta/2)$ [10], [11]. We assume $\beta_y > \beta_x$, so that $0 \le t \le 1$. One obtains

$$\mathbf{E}^{a,b}(Z,T) = \sum_{j=1}^{2} A_j^{a,b}(Z,T)\exp(i\beta_j Z)\,\mathbf{e}_j \qquad (3)$$

where $\beta_{1,2} = (\beta_x + \beta_y)/2 \pm \sqrt{(\beta_x - \beta_y)^2/4 + \tau^2}$. Typically, the linear beat length of a birefringent fiber $L_b = 2\pi/(\beta_y - \beta_x) \ll L_{nl}$, where the nonlinear length $L_{nl} = 1/(RP)$, and P is the wave power. As a result, nonlinear self-polarization modulation and CPMs lead to $A_{1,2}^{\pm}(Z,T)$ that are slowly varying in Z with respect to L_b. By expressing $\mathbf{E}^{a,b}$ in terms of $\mathbf{e}_1$ and $\mathbf{e}_2$, one has $H_L = 0$ since the two elliptical eigenmodes propagate uncoupled in the absence of nonlinearity. Whereas the terms proportional to γ_3 in H_X (2) contain the rapidly oscillating factors $\exp[\pm 2i(\beta_1 - \beta_2)Z]$. When considering the nonlinear polarization changes of the beams over the relatively long scale L_{nl}, we

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The author is with the Dipartimento di Elettronica per l'Automazione, Università di Brescia, 25123 Brescia, Italy (e-mail: stefano.wabnitz@ ing.unibs.it).

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may set $\langle\gamma_3\rangle = 0$, where brackets denote Z-average. Additionally, we obtain $\langle\gamma_2\rangle = -[(1+B)(1+t^2)^2 - 16Bt^2]/(1+t^2)^2$ and $\langle\gamma_4\rangle = [(1-B)t^4 + 2t^2(1+3B) + (1-B)]/(1+t^2)^2$. Let us express the nonlinear polarization evolution of the two waves at frequencies ω_a and ω_b by means of their respective Stokes parameters, defined as

$$S_0^{a,b} = \left|\mathbf{E}^{a,b}\right|^2, \; S_1^{a,b} = A_1^{a,b^*} A_2^{a,b} + c.c.$$
$$S_2^{a,b} = iA_1^{a,b^*} A_2^{a,b} + c.c., \; S_3^{a,b} = \left|A_1^{a,b}\right|^2 - \left|A_2^{a,b}\right|^2.$$

Substitution in (1) leads to $\partial_\xi S_0^a = 0, \; \partial_\eta S_0^b = 0$, and

$$\partial_\xi \mathbf{S}^a = \mathbf{S}^a \times \left(J_S \mathbf{S}^a + J_X \mathbf{S}^b\right)$$
$$\partial_\eta \mathbf{S}^b = \mathbf{S}^b \times \left(J_S \mathbf{S}^b + J_X \mathbf{S}^a\right) \quad (4)$$

where $\mathbf{S}^{a,b} = (S_1^{a,b}, S_2^{a,b}, S_3^{a,b})$, $J_S = \mathrm{diag}\,(0,0,J_{s3})$, and $J_X = \mathrm{diag}\,(-R\langle\gamma_4\rangle, -R\langle\gamma_4\rangle, R\langle\gamma_2\rangle) \equiv \mathrm{diag}\,(J_{x1}, J_{x1}, J_{x3})$ are the self-polarization modulation and CPM tensors (note that a different J_X is obtained with counterpropagating beams [10]). Moreover, $J_{s3} = -RB[(1-t^2)^2 - 8t^2]/(1+t^2)^2$, and we introduced the characteristic coordinates $\xi = (Z + T/\Delta)/2$, $\eta = (Z - T/\Delta)/2$. Note that for $t = \hat{t} \equiv \sqrt{3} - \sqrt{2}$, or $\theta = \tan^{-1}(1/\sqrt{2}) \cong 35^\circ$, the self-induced polarization tensor vanishes [10], [11]. Such a condition is particularly significant, as it corresponds to the case of a long, randomly birefringent optical fiber. Indeed, by setting $2\tau = (\beta_y - \beta_x)\sin\psi$, where ψ is a uniformly distributed random angle, one obtains $\langle\tau^2\rangle = (\beta_y - \beta_x)^2/8$ and $t(\langle\tau^2\rangle^{1/2}) = \hat{t}$, so that (4) reduces to the principal chiral field equations [6]

$$\partial_\xi \mathbf{S}^a = J\,\mathbf{S}^a \times \mathbf{S}^b, \quad \partial_\eta \mathbf{S}^b = J\,\mathbf{S}^b \times \mathbf{S}^a \quad (5)$$

where $J = -8R/9$, and we have set $B = 1/3$ as electronic distortion is the dominant nonlinear index mechanism in fibers. Analytical polarization soliton solutions in the special case of (5) were discussed by Tratnik and Sipe in 1987 [12].

We shall consider here the general case of (4), and restrict our analysis for simplicity to the case of equal power waves; i.e., we set $S_0^a = S_0^b = P$. To find the stationary (i.e., such that $\partial_Z = 0$) solutions of (4), we set $\partial_\xi = -\partial_\eta = \partial_{T'}$, where $T' \equiv T/\Delta$. Based on the symmetries of (4), we find that their solutions admit the two conserved quantities

$$K \equiv S_3^a - S_3^b$$
$$\Gamma \equiv J_{x1}\left(S_1^b S_1^a + S_2^b S_2^b\right) + (J_{x3} + J_{s3})\,S_3^a S_3^b \quad (6)$$

which, in addition to the two conserved wave powers, permit for the integration of (4) by quadratures. Let us consider the special case where $\mathbf{s}^a \equiv \mathbf{S}^a/S_0^a = (s_1^a, s_2^a, s_3^a) = (s_1^b, -s_2^b, s_3^b)$ and $\mathbf{s}^b \equiv \mathbf{S}^b/S_0^b$, so that $K = 0$ and $\partial_{T'} s_3^{a,b} \neq 0$. The time evolution of $\mathbf{s}^a$ may be expressed in terms of Jacobian elliptic functions, and is represented on the Poincaré sphere as illustrated in Fig. 1, where we show the two extreme cases with either $t = 0, \theta = 0$ (unspun highly birefringent fiber), or $t = 1$, $\theta = 90^\circ$ (highly spun fiber). As can be seen in Fig. 1, the nonlinear eigenpolarizations of (4) (i.e., the Stokes vectors such that

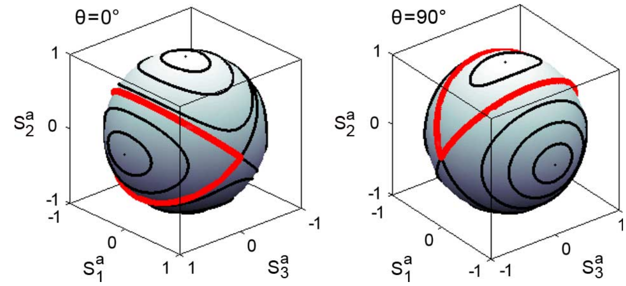

Fig. 1. Trajectories of Stokes vector $\mathbf{s}^a$ on the Poincaré sphere representing stationary solutions for the self-polarization modulation and CPM in a linearly ($\theta = 0$) and circularly ($\theta = 90^\circ$) birefringent fiber. Thick red curves represent polarization kink solitons.

$\partial_\xi \mathbf{S}^a = \partial_\eta \mathbf{S}^b = 0$) are directed along the principal axes of J_X, i.e., s_1, s_2, or s_3 axis. For $t < \hat{t}$, so that $J_{x1}^2 < J_{x3}^2$, the vectors $\mathbf{s}^{a,b} = (\pm 1, 0, 0)$ are saddle points emanating separatrix trajectories (thick red curves) that represent the kink solitons

$$\mathbf{s}^a(T) = \Bigg(\mp\tanh\left[C\,(T - T_0)\right], \pm\frac{a}{\cosh\left[C\,(T-T_0)\right]}, \frac{b}{\cosh\left[C\,(T-T_0)\right]}\Bigg) \quad (7)$$

with $a = \sqrt{(J_{x3} - J_{x1} + J_{s3})/(J_{x3} + J_{x1} + J_{s3})}$, $b = \sqrt{2J_{x1}/(J_{x3} + J_{x1} + J_{s3})}$, $C = \sqrt{2J_{x1}(J_{x3} - J_{x1} + J_{s3})P/\Delta}$, and T_0 is an arbitrary time shift.

The polarization domain wall (7) describes a switching of polarization between the two stable (with respect to spatio-temporal perturbations) parallel polarization arrangements $\mathbf{s}^{a,b} = (1,0,0)$ and $\mathbf{s}^{a,b} = (-1,0,0)$. Fig. 1 also shows that for $t > \hat{t}$, so that $J_{x1}^2 > J_{x3}^2$, the saddles coincide with $\mathbf{s}^{a,b} = (0,0,\pm 1)$. The associated CPM solitons read as

$$\mathbf{s}^a(T) = \Bigg(\mp\frac{a'}{\cosh\left[C'\,(T-T_0)\right]}, \frac{b'}{\cosh\left[C'\,(T-T_0)\right]}, \mp\tanh\left[C'\,(T - T_0)\right]\Bigg) \quad (8)$$

with $a' = \sqrt{(J_{x3} + J_{x1} + J_{s3})/(2J_{x1})}$, $b' = \sqrt{(J_{x1} - J_{x3} - J_{s3})/(2J_{x1})}$, and $C' = \sqrt{(J_{x1} + J_{x3} + J_{s3})\,(J_{x1} - J_{x3} - J_{s3})P/\Delta}$. Note that for $t = \hat{t}$ [equation (5)], all $\mathbf{s}^a$ trajectories reduce to circles along the parallels and the steady-state solutions may be expressed in terms of elementary trigonometric functions [12].

III. Simulations

With reference to a specific example, we demonstrate by numerical integration of (4) the propagation stability of the polarization domain wall solutions (7), whose Z-invariant time profiles are shown in Fig. 2 for the case with negative s_2^a. We considered two 1-mW beams at $\lambda = 1550$ nm spaced by 50 GHz, copropagating in a linearly birefingent fiber ($t = \theta = 0$) with $n_2 = 3.2 \times 10^{-20} (\mathrm{mW})^{-1}$, $A_{\mathrm{eff}} = 10\ \mu\mathrm{m}^2$, and the GVD of $D = 1$ ps/(nm km). Note that $L_{nl} = 77$ km, and the walk-off distance L_w of the hyperbolic secant pulse components in Fig. 2 is $L_w \cong 15$ km. Fig. 2 shows that full switching between two

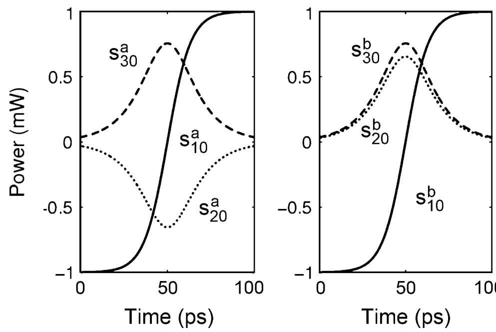

Fig. 2. Time profile of polarization domain wall soliton (7) components in waves at frequency ω_a and ω_b for a linearly birefringent fiber ($t = 0$).

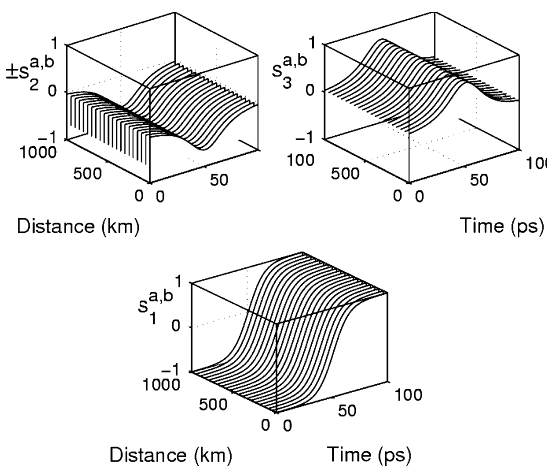

Fig. 3. Stable propagation of the cross-polarization soliton of Fig. 2 over 1000 km of linearly birefringent fiber.

pairs of orthogonal polarization arrangements (these are initially corotating circularly polarized waves, since for $T = 0$ and $Z = 0$ one has $A_1 = E_x$ and $A_2 = iE_y$) occurs on a timescale of about 60 ps. Fig. 3, which shows the propagation the cross-polarization soliton of Fig. 2 over 1000 km, was obtained by numerically solving (4) along the characteristics ξ and η, and it confirms the spatio-temporal stability of the domain wall solutions (7). Fig. 3 shows that the nonlinear CPM fully compensates for the GVD induced temporal walk-off between the two channels. Clearly, at low powers the two pulses walk away from each other after L_w.

The existence of stable domains of parallel mutual polarization arrangements for two wavelength channels in birefringent fibers leads to possibility of implementing a nonlinear, loss-free polarizer in a copropagating configuration. A numerical proof of this principle is provided by the result of Fig. 4: in the left we show the power in the two orthogonal polarization components (defined as $P_{\pm} = (S^a_0 \pm S^a_1)/2$) of an initially randomly polarized in time, 1-mW CW probe wave. This probe copropagates in a short fiber loop (so that resonance effects can be neglected) containing linearly birefringent fiber (with parameters as given above) along with a 50-GHz-spaced circularly polarized CW pump ($S^b_0 \equiv S^b_1 = 3$ mW). As shown in Fig. 4, right, after recirculating for 50 ms, the probe virtually acquires the same polarization state of the pump.

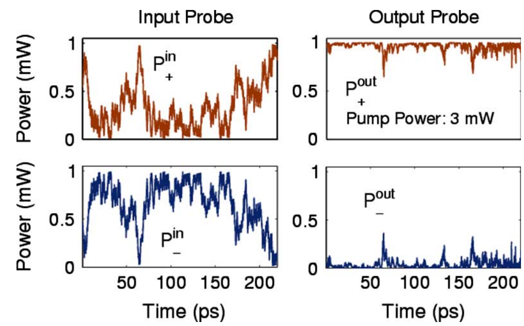

Fig. 4. Power in the two orthogonal polarization components of an initially depolarized 1-mW probe, at the fiber input and after copropagation with a polarized 3-mW pump.

IV. Conclusion

We theoretically described CPM between two WDM channels in optical fibers with elliptical birefringence. We obtained analytical domain wall soliton solutions that represent the locked temporal switching of the state of polarization of both beams. Lossless polarization attraction in a copropagating geometry was also predicted.

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