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Global attractor for damped semilinear elastic beam equations with memory

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Abstract. Some nonlinear evolution problems arising in the theory of elastic beams with linear memory are considered. Under proper assumptions on the memory kernel the existence of uniform absorbing sets and of a global attractor is achieved.

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1. Introduction

This paper is concerned with long time behavior of solutions to evolution problems of viscoelastic bars. This model can be derived, for example, from a homogenization of a material with viscoelastic microstructure (see [5], [9]).

We thus consider an equation of the type

$$u_{tt} + \alpha_0 u_{xxxx} + \int_0^\infty \alpha'(s) u_{xxxx}(t-s)\, ds + \delta u_{xxxxt} - \left(\beta + \int_0^1 u_x^2(\xi, t)\, d\xi \right) u_{xx} = 0 \tag{1.1}$$

where $\alpha_0, \delta > 0$, $\alpha' \leq 0$, $\beta \in \mathbb{R}$ together with initial and boundary conditions.

Eq. (1.1) with $\delta = 0$ and $\alpha' = 0$ was proposed by Woinowsky-Krieger [13] in order to describe the transversal vibrations of an extensible beam subject to an axial internal force, and later studied, in a more general form, by Ball [1]. The problem was successively studied by Bianchi and Marzocchi in [2]-[3], but always when the memory effects are neglected.

In this work we show that the dynamical system generated by eq. (1.1) has an absorbing set in the space of solutions, *i.e.* a bounded set into which all trajectories eventually enter, and a global attractor, that is a uniformly compact

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set which attracts all bounded sets in the space of solutions. Here a strong damping term, also appearing in other physical models, appears to be useful in achieving compactness properties needed for the attractor (see for example [10] for a similar behavior and [11] for a general treatment).

Finally, the memory kernel is required to have exponential decay, as in several other results, like, *e.g.* [6], [7], [8], [9].

2. Position of the problem, preliminaries

We associate to (1.1) the following initial and boundary conditions

$$\begin{cases} u(0,t) = u_x(0,t) = u_{xx}(1,t) = u_{xxx}(1,t) = 0 \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x). \end{cases} \tag{2.1}$$

The first condition corresponds to the case when the left end point $x = 0$ is clamped and the right end point $x = 1$ is not subject to transversal force or bending torque. Of course several other conditions are possible, namely those which lead to the abstract problem given below.

We introduce in the Hilbert space $H = L^2(0,1)$ with the usual scalar product the linear operator $A : D(A) \to H$ such that $A\phi = \phi_{xxxx}$ on regular functions with

$$D(A) = V = \hat{H}^4(0,1) = \{\phi \in H^4(0,1) : \phi(0) = \phi'(0) = \phi''(1) = \phi'''(1) = 0\}.$$

It is not difficult to verify that A is self-adjoint, positive definite, with dense domain in H, and that, in weak sense,

$$\begin{aligned} A^{1/2}\phi &= -\phi_{xx} \quad D(A^{1/2}) = \overline{D(A)} \text{ in } H^2(0,1) \\ A^{1/4}\phi &= |\phi_x| \quad D(A^{1/4}) = \overline{D(A)} \text{ in } H^1(0,1). \end{aligned}$$

In this way system (1.1)–(2.1) can be seen as an abstract evolution equation in the space H whose expression becomes

$$\begin{cases} u_{tt} + \alpha_0 Au + \displaystyle\int_0^\infty \alpha'(s) Au(t-s)\,ds + \delta Au_t + \left(\beta + \|A^{1/4}u\|^2\right) A^{1/2}u = 0 \\ u(0) = u_0 \\ u_t(0) = u_1 \end{cases} \tag{2.2}$$

where the initial data u_0, u_1 belong to V and H respectively. If $\alpha' = 0$, then (2.2) reduces to the corresponding system considered in [2], [3].

Exploiting an idea of Dafermos [4], introduce the new variable

$$w(t,s) = u(t) - u(t-s), \qquad s \geq 0.$$

Differentiation of the above relation yields

$$w_t(t,s) = u_t(t) - w_s(t,s),$$

and setting now for simplicity $\mu(s) = -\alpha'(s)$ we obtain

$$\int_0^\infty \alpha'(s) Au(t-s)\, ds = Au \int_0^\infty \alpha'(s)\, ds + \int_0^\infty \mu(s) Aw(t,s)\, ds$$

so that the problem in the two variables u, w becomes

$$\begin{cases} u_{tt} + \delta A u_t + \alpha A u + \displaystyle\int_0^\infty \mu(s) A w(s)\, ds + \left(\beta + \|A^{1/4}u\|^2\right) A^{1/2} u = 0 \\ w_t = u_t - w_s \end{cases} \tag{2.3}$$

where

$$\alpha = \alpha_0 + \int_0^\infty \alpha'(s)\, ds \quad (^1)$$

and the initial conditions are translated into

$$\begin{cases} u(0) = u_0 \\ u_t(0) = u_1 \\ w(0,s) = w_0(s) \ (s \geq 0) \end{cases} \tag{2.4}$$

and where we have assumed the initial history

$$u(-s) = u_0 + w_0(s) \ (s \geq 0)$$

to be given.

Equations (2.3)–(2.4) can also be seen as

$$z_t = \mathcal{A} z + \mathcal{F}(z)$$

where $z = (u, v, w)$ and

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 \\ -\alpha A & -\delta A & -\displaystyle\int_0^\infty \mu(s) A(\cdot)\, ds \\ 0 & 1 & -\dfrac{\partial}{\partial s} \end{bmatrix}, \quad \mathcal{F} = \begin{bmatrix} 0 \\ -(\beta + \|A^{1/4}u\|^2) A^{1/2} \\ 0 \end{bmatrix}$$

(1) The condition for the integral to be convergent is accomplished in the requirement $(h1)$ on the memory kernel.

We will throughout the paper suppose that the memory kernel satisfies the following hypotheses:

$$(h1)\qquad \mu \in C^1(\mathbb{R}^+) \cap L^1(\mathbb{R}^+), \quad \mu(s) \geq 0, \quad \mu'(s) \leq 0 \quad \forall s \geq 0$$

$$(h2)\qquad \exists \gamma > 0: \ \mu'(s) + \gamma\mu(s) \leq 0 \quad \forall s \geq 0.$$

Classical existence and uniqueness theory for the system (2.3) consists in looking for a triple (u, v, w) solution of the system

$$\begin{cases} u_t = v \\ v_t = -\delta A v - \alpha A u - \int_0^\infty \mu(s) A w(s)\, ds - \left(\beta + \|A^{1/4}u\|^2\right) A^{1/2} u \\ w_t = v - w_s \end{cases}$$

belonging to the space

$$\mathcal{H} := L^\infty(0, T; H_0^2) \times L^\infty(0, T; L^2) \cap L^2(0, T; H_0^2) \times L^2(0, T; L^2_\mu(\mathbb{R}^+; H_0^2)),$$

where $M := L^2_\mu(\mathbb{R}^+; H_0^2)$ is just the Hilbert space of functions with values in H_0^2 with weighted scalar product

$$< \phi, \psi >_\mu = \int_0^\infty \mu(s)(\phi, \psi)_{H_0^2}\, ds.$$

This can be done exploiting a Faedo-Galerkin scheme following *e.g.* the lines of [1] and [6], the key ingredient being the energy inequality that we are going to prove in the following section. We omit the details for the sake of brevity.

Furthermore, if $u_0 \in H_0^2 \cap H^4 := K$, $u_1 \in H_0^2$ and $w_0 \in L^2_\mu(K)$, then the solution is regular in the sense that $u \in L^\infty(0, T; K)$, $u_t \in L^\infty(0, T; H_0^2) \cap L^2(0, T; K)$, $\frac{\partial^2 u}{\partial t^2} \in L^2(0, T; L^2)$ and finally $w \in L^2(0, T; L^2_\mu(\mathbb{R}^+; K))$.

We recall that Poincaré inequality

$$\|u\|^2 \leq \lambda_0 \|\nabla u\|^2 \tag{2.5}$$

holds in our case, as well as

$$\|\nabla u\|^2 \leq \lambda_1 \|\Delta u\|^2 \tag{2.6}$$

and analogous inequalities for higher derivatives.

We finish this section with a simple lemma.

(2.7) **Lemma.** *Suppose $z : \mathbb{R}^+ \to \mathbb{R}^+$ is a positive integrable function and $C, t^* \geq 0$ are two constants such that for $t \geq t^*$ the inequality $z(t) \leq C$ holds. Suppose moreover that $y : \mathbb{R}^+ \to \mathbb{R}^+$ verifies*

$$y'(t) + \varepsilon y(t) \leq z(t).$$

Then for every $\eta > 0$ there exists $t_\eta \geq 0$ such that

$$y(t) \leq \frac{C}{\varepsilon} + \eta.$$

Proof. Let $t \geq t^*$ and integrate; it is easy to deduce

$$y(t) \leq \left(y(0) + \int_0^{t^*} z(s)\, e^{\varepsilon s}\, ds \right) e^{-\varepsilon t} + e^{-\varepsilon t} \int_{t^*}^{t} z(s)\, e^{\varepsilon s}\, ds$$

whence it follows

$$y(t) \leq \left(y(0) + \int_0^{t^*} z(s)\, e^{\varepsilon s}\, ds \right) e^{-\varepsilon t} + \frac{C}{\varepsilon} e^{-\varepsilon t^*}.$$

Since now the quantity in brackets is bounded and independent on t, the claim follows. □

3. Uniform estimates

In this section we establish the existence of a so-called absorbing set for the flow generated by problem (2.3).

(3.1) **Theorem.** *There exists $R > 0$ such that for every initial condition (u_0, u_1, w_0) in a bounded set of $H \times V \times L^2_\mu$ there is a $T \geq 0$ with the property that*

$$\forall t \geq T: \qquad u(t) \in B_H(0, R)$$

where $B(0, R)$ denotes the unit ball in H.

Proof. Set $v = u_t + \varepsilon u$, where $\varepsilon > 0$ is to be determined. Then

$$v_t = u_{tt} + \varepsilon u_t = u_{tt} + \varepsilon(v - \varepsilon u)$$

and using equation (1.1) we get

$$\begin{gathered} v_t + \delta A(v - \varepsilon u) + \alpha A u - \varepsilon(v - \varepsilon u) + \\ + \int_0^\infty \mu(s) A w(t,s)\, ds + \left(\beta + \|A^{1/4} u\|^2\right) A^{1/2} u = 0 \\ w_t = u_t - w_s. \end{gathered} \tag{3.2}$$

Multiplying the first equation by v in H, the second by w in L^2_μ and adding the results, it is not difficult to get

$$\begin{aligned} &\frac{1}{2}\frac{d}{dt}\left[\|v\|^2 + (\alpha - \delta\varepsilon)\|A^{1/2}u\|^2 + \varepsilon^2\|u\|^2 + \|A^{1/2}w\|^2_\mu\right] + \\ &+ \varepsilon\Big[\frac{\delta}{\varepsilon}\|A^{1/2}v\|^2 + (\alpha - \delta\varepsilon)\|A^{1/2}u\|^2 \\ &- \|v\|^2 + \varepsilon^2\|u\|^2 + \langle Aw, u\rangle_\mu + \frac{1}{\varepsilon}\langle w_s, Aw\rangle_\mu\Big] + \\ &+ \frac{1}{2}(\beta + \|A^{1/4}u\|^2)\frac{d}{dt}\|A^{1/4}u\|^2 + \varepsilon(\beta + \|A^{1/4}u\|^2)\|A^{1/4}u\|^2 = 0. \end{aligned} \tag{3.3}$$

Now, using Poincaré inequality (2.6) and choosing ε such that

$$\varepsilon < \frac{\delta}{3\lambda_0\lambda_1}$$

we find

$$\delta\|A^{1/2}v\|^2 - \varepsilon\|v\|^2 \geq \frac{\delta}{2}\|A^{1/2}v\|^2 + \frac{\varepsilon}{2}\|v\|^2;$$

next, using $(h2)$ after an integration by parts, we get

$$\frac{1}{\varepsilon}\left\langle w_s, Aw\right\rangle_\mu = -\frac{1}{2\varepsilon}\int_0^\infty \mu'(s)\|A^{1/2}w\|^2\,ds \geq \frac{\gamma}{2\varepsilon}\|A^{1/2}w\|_\mu^2.$$

Finally, by virtue of $(h1)$, we set

$$\mu_0 = \int_0^\infty \mu(s)\,ds$$

and we find with the help of Young inequality

$$\left\langle Aw, u\right\rangle_\mu \geq -\frac{\alpha}{4}\|A^{1/2}u\|^2 - \frac{2\mu_0^2}{\alpha}\|A^{1/2}w\|_\mu^2.$$

Using the previous inequalities, we infer from (3.3)

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left[\|v\|^2 + (\alpha-\delta\varepsilon)\|A^{1/2}u\|^2 + \varepsilon^2\|u\|^2 + \|A^{1/2}w\|_\mu^2 + \frac{1}{2}(\beta + \|A^{1/4}u\|^2)^2\right]\\&+\varepsilon\Big[\frac{\delta}{2\varepsilon}\|A^{1/2}v\|^2 + \left(\frac{3}{4}\alpha - \delta\varepsilon\right)\|A^{1/2}u\|^2 + \frac{1}{2}\|v\|^2 + \varepsilon^2\|u\|^2\\&+\left(\frac{\gamma}{2\varepsilon} - \frac{2\mu_0^2}{\alpha}\right)\|A^{1/2}w\|_\mu^2\Big] +\\&+\frac{\varepsilon}{2}\left(\beta + \|A^{1/4}u\|^2\right)^2 \leq \frac{\beta^2}{2}.\end{aligned} \tag{3.4}$$

At this point we set

$$\begin{aligned}F(t) = \|v(t)\|^2 &+ (\alpha-\delta\varepsilon)\|A^{1/2}u(t)\|^2 + \varepsilon^2\|u(t)\|^2\\&+ \|A^{1/2}w(t)\|_\mu^2 + \frac{1}{2}(\beta + \|A^{1/4}u(t)\|^2)^2\end{aligned}$$

and taking

$$\varepsilon \leq \min\left\{\frac{\alpha}{2\delta}, \frac{\delta}{3\lambda_1}, \frac{\gamma}{1 + 4\mu_0^2/\alpha}\right\}$$

we find

$$\frac{d}{dt}F(t) + \varepsilon F(t) \le \frac{\beta^2}{2}.$$

Remembering that F is positive with the given choice of ε (also when $\beta < 0$) and using the Gronwall lemma it follows that

$$F(t) \le F(0)e^{-\varepsilon t} + \frac{\beta^2}{2\varepsilon}.$$

From this it is clear that every ball $B_H(0, R)$ with $R > \beta^2/2\varepsilon$ verifies the statement of the theorem. Finally, this result is true for every initial condition in $D(\mathcal{A})$ and, by the density of this set in $\mathcal{H}$ and the continuous dependence on the initial data, it holds for every initial condition in $\mathcal{H}$. □

(3.5) **Remark.** From the expression of F, it is also apparent that

$$\limsup_{t\to+\infty} |u_t|(t) < +\infty.$$

(3.6) **Remark.** In the linear case, it is immediate to see that (3.4) holds with $\beta = 0$, and so u tends to zero strongly in H.

(3.7) **Remark.** A similar result holds true also when the nonlinear term is of the type

$$g(A^{1/4}u)A^{1/2}u$$

and g is subjected to suitable growth conditions (see [2]).

4. Construction of the attractor

In this section we prove the existence of a global attractor in the space H for the solution u. First, we need a preliminary lemma.

(4.1) **Lemma.** *Let B be a bounded set in $D(\mathcal{A})$. Then, for any initial data in B there exists a finite constant K_B such that*

$$\limsup_{t\to+\infty} \|A^{3/4}u\|^2(t) \le K_B.$$

Proof. Multiply the first equation in (3.2) with $A^{1/2}v$ in H and the second with

$A^{3/2}w$ in M. In this way we easily come to the inequality

$$\frac{1}{2}\frac{d}{dt}\Big[\|A^{1/4}v\|^2+(\alpha-\delta\varepsilon)\|A^{3/4}u\|^2+\varepsilon^2\|A^{1/4}u\|^2$$
$$+\|A^{1/4}u\|^2\|A^{1/2}u\|^2+\|A^{3/4}w\|^2_\mu\Big]+$$
$$+\varepsilon\Big[(\alpha-\delta\varepsilon)\|A^{3/4}u\|^2+\varepsilon^2\|A^{1/4}u\|^2+\frac{\delta}{\varepsilon}\|A^{3/4}v\|^2-\|A^{1/4}v\|^2+$$
$$+\|A^{1/4}u\|^2\|A^{1/2}u\|^2+\Big\langle Aw,A^{1/2}u\Big\rangle_\mu+\frac{1}{\varepsilon}\Big\langle w_s,A^{3/2}w\Big\rangle_\mu\Big]$$
$$=-\beta\Big\langle A^{1/2}u,A^{1/2}v\Big\rangle+\|A^{1/2}u\|^2\Big\langle A^{1/4}u,A^{1/4}u_t\Big\rangle.$$

By Cauchy inequality the right-hand side is bounded by

$$\frac{\delta}{2}\|A^{3/4}v\|^2+\frac{\beta^2}{2\delta}\|A^{1/4}u\|^2+\|A^{1/2}u\|^2\Big\langle A^{1/4}u,A^{1/4}u_t\Big\rangle.$$

With an appropriate choice of ε we can have

$$\delta\|A^{3/4}v\|^2-\varepsilon\|A^{1/4}v\|^2\geq\frac{\delta}{2}\|A^{3/4}v\|^2+\frac{\varepsilon}{2}\|A^{1/4}v\|^2$$

so that, if the initial data belong to a certain ball B in H, then from Poincaré inequality, Theorem (3.1) and Remark (3.5) it follows that there exist a t^* such that for $t\geq t^*$

$$q(t):=\frac{\beta^2}{2\delta}\|A^{1/4}u\|^2+\|A^{1/2}u\|^2|\Big\langle A^{1/4}u,A^{1/4}u_t\Big\rangle|$$
$$\leq\frac{\beta^2}{2\delta}\|A^{1/4}u\|^2+\|A^{1/2}u\|^3|u_t|\leq k_1^B$$

where k_1^B is a constant dependent on B. We can now proceed similarly as in Theorem (3.1), putting

$$Z(t)=\|A^{1/4}v\|^2+(\alpha-\delta\varepsilon)\|A^{3/4}u\|^2$$
$$+\varepsilon^2\|A^{1/4}u\|^2+\|A^{1/4}u\|^2\|A^{1/2}u\|^2+\|A^{3/4}w\|^2_\mu$$

and getting

$$\dot{\mathrm{Z}}(\mathrm{t})t+\varepsilon Z(t)\leq q(t)$$

by which, using Lemma (2.7) the desired estimate for $Z(t)$, that is for $\|A^{3/4}u(t)\|^2$.
□

Remark. This kind of procedure is slightly different from the case of the uniform estimates since β may be large and negative. In this way the presence of the damping term is necessary in our estimates.

(4.2) **Lemma.** *There exists $R > 0$ such that for every initial condition (u_0, u_1, w_0) in a bounded set of $D(\mathcal{A})$ there is a $T \geq 0$ with the property that*

$$\forall t \geq T : \qquad z(t) \in B_{D(\mathcal{A})}(0, R)$$

Proof. Take this time the scalar product of the first equation in (3.2) with Av in H and the second with $A^2 w$ in M. We obtain, in a similar way to the previous lemma,

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\Big[\|A^{1/2}v\|^2 + (\alpha - \delta\varepsilon)\|Au\|^2 + \varepsilon^2\|A^{1/2}u\|^2 \\
&+\|Aw\|_\mu^2 + \|A^{1/4}u\|^2\|A^{3/4}u\|^2\Big] + \\
&+\varepsilon\left[(\alpha - \delta\varepsilon)\|Au\|^2 + \varepsilon^2\|A^{1/2}u\|^2 + \frac{\delta}{\varepsilon}\|Av\|^2 - \|A^{1/2}v\|^2 + \right. \\
&\left.\|A^{1/4}u\|^2\|A^{3/4}u\|^2 + \langle Aw, Au\rangle_\mu + \frac{1}{\varepsilon}\langle Aw, Aw_s\rangle_\mu\right] \\
&= -\beta\left\langle A^{1/2}u, Av\right\rangle + \|A^{3/4}u\|^2\left\langle A^{1/4}u, A^{1/4}u_t\right\rangle.
\end{aligned}$$

Now, the right-hand side is bounded by

$$\frac{\delta}{2}\|Av\|^2 + \frac{\beta^2}{2\delta}\|A^{1/2}u\|^2 + \|A^{3/4}u\|^2\left\langle A^{1/4}u, A^{1/4}u_t\right\rangle$$

and, thanks to the previous lemma, the last two terms are bounded for t large. From now on the proof parallels that of Lemma (4.1). □

Now we are in position to prove our main result.

(4.3) **Theorem.** *The dynamical system generated by (2.3) has a global attractor in the solution space.*

Proof. We decompose $(u, w) = (\hat{u}, \hat{w}) + (\tilde{u}, \tilde{w})$ where

$$\begin{cases}
\hat{u}_{tt} + \delta A\hat{u}_t + \alpha A\hat{u} + \displaystyle\int_0^\infty \mu(s)A\hat{w}(s)\,ds = 0 \\
\hat{w}_t = \hat{u}_t - \hat{w}_s \\
\hat{u}(0) = u_0, \quad \hat{u}_t(0) = u_1, \quad \hat{w}^0(s) = w_0(s), \ s \geq 0
\end{cases}$$

and

$$\begin{cases}
\tilde{u}_{tt} + \delta A\tilde{u}_t + \alpha A\tilde{u} + \displaystyle\int_0^\infty \mu(s)A\tilde{w}(s)\,ds = \varphi(t) \\
\tilde{w}_t = \tilde{u}_t - \tilde{w}_s \\
\tilde{u}(0) = 0, \quad \tilde{u}_t(0) = 0, \quad \tilde{w}^0(s) = 0
\end{cases} \tag{4.4}$$

with $\varphi(t) = -(\beta + \|A^{1/4}u\|^2)A^{1/2}u$. It is clear that $(\hat{u}, \hat{w})$ tends to zero uniformly in $\mathcal{H}$. Now set $\tilde{v} = \tilde{u}_t + \varepsilon\tilde{u}$, multiply $(4.4)_1$ with $A\tilde{v}$ in H and $(4.4)_2$ with $A^2\tilde{w}$ in M. From lemma (4.2) we have then

$$|(\beta + \|A^{1/4}u\|^2)\|A^{1/2}u\|| \leq k_2^B$$

so that

$$|(\beta + \|A^{1/4}u\|^2)\left\langle A^{1/2}u, A\tilde{v}\right\rangle| \leq \frac{1}{2\eta}k_2^B + \frac{\eta}{2}\|A^{1/2}\tilde{v}\|^2$$

with an arbitrary fixed positive η. Now, following the same reasoning as in lemma (4.2) we find that $(\tilde{u}, \tilde{w})$ eventually enter in a bounded set of $D(\mathcal{A})$, which, by the Rellich compactness embedding, is relatively compact in $\mathcal{H}$. This corresponds to say that the semigroup associated with $(\tilde{u}, \tilde{w})$ is uniformly compact for t large. It is now sufficient to apply [12, Theorem I.1.1] to get the desired result. □

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