THE CONNECTIVITY HOUGH TRANSFORM AND ITS FAST IMPLEMENTATION *

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Abstract This work addresses the problem of curve detection in images using a novel transform that takes advantage of the connectivity of curves and the spatial integration ability of the human visual system. This new transform differs from the traditional Hough transform as it introduces a connectivity measure to strengthen those points that are connected to each other in the image space. Besides, for each point in the image, the response is integrated over a small neighborhood along the curve. Simulation results for the case of straight line detection are presented to show that this transform is extremely robust in the presence of noise and can outperform significantly the conventional Hough transform. In addition to the performance issues of the Connectivity Hough transform, this work discusses as well the computational complexity issues.

INTRODUCTION

Since its introduction in 1962 [1], the Hough transform (HT) has found considerable support in image processing and computer vision applications. A recent survey [2] provides more than 130 references to work related to the HT. In principle, the HT is used to facilitate the recognition of complex patterns of points in multidimensional spaces. Given a parametric representation of the pattern of interest (e.g., an ellipse), the pattern analysis task is performed in the parameter space in which occurrences of the pattern result in dense clusters, making the recognition task simpler. The HT is effective even when the signal in the original space (e.g., image space) is affected by noise or when the occurrence of the pattern presents gaps (e.g., occlusions of objects). However, when comparing the performances of the HT to the human visual system for applications such as line detection, the latter remains superior, suggesting that additional processing is performed in the visual system. In designing the Connectivity Hough transform (CHT), we have tried to incorporate some of these processings. First, a connectivity measure along the direction of the curve has been introduced in the CHT to strengthen the effect of those connected segments that match a certain curve parameterization. Second, the information is integrated for each point in the original space within a certain neighborhood. This reduces noise sensitivity and takes advantage of the focus of attention present in foveal vision. The CHT outperforms the conventional HT significantly by eliminating the "background noise" inherent in the HT. Lower signal-to-noise ratios could still give good recognition results, at the expense of some extra computational effort.

Section 2 gives a survey of the HT and presents a possible mathematical formalism. In Section 3, the CHT is introduced in general terms, then compared to the standard HT for line detection in images. In Section 4, computational issues are addressed: pyramidal partitioning of the parameter space and multi-resolution analysis of the original and parameter space are considered. Finally Section 5 gives insight into further research effort, and the use of the CHT for more complex pattern matching tasks.

For simplicity, we shall call the original space, in which the pattern or curve of interest is searched, the image space denoted by D. The parameter space often called the Hough space will be denoted by Ω.

HOUGH TRANSFORM

As mentioned earlier, the Hough transform converts a difficult pattern recognition or parameter estimation task in the image space D into the simple search of clusters of points in the parameter space Ω. For clarity, let us consider the following example (see Figure 1). Consider a straight line equation given by

$$f(x, y, a, b) = ax + by - 1 = 0 \quad (1)$$

or using a vector notation

$$f(\mathbf{x}, \omega) = \mathbf{x}^T \omega - 1 = 0 \quad (2)$$

where $\mathbf{x}^T = (x\ y)$ and $\omega^T = (a\ b)$. For each point $\mathbf{x}$ in the image space D, the mapped set of points in the parameter

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space Ω defines a straight line which is the solution to (1). In other words, there is a one-to-one correspondence between lines in Ω and points in D. Any two points $\mathbf{x}_1$ and $\mathbf{x}_2$ of D will generate two corresponding lines in Ω which intersect at a certain parameter point $\omega_0^T = (a_0\ b_0)$. This point represents the characteristic parameter value of the line connecting $\mathbf{x}_1$ and $\mathbf{x}_2$. More colinear points in D to $\mathbf{x}_1$ and $\mathbf{x}_2$ will generate lines in Ω that will all join at ω_0. Moreover, nearly colinear points in D will transform into dense clusters of intersecting lines in the parameter space. The line detection problem is reduced into finding these points of concentration of crossing lines. Instead of studying the line intersection resulting from any two pair of points in D, it is easier to initially quantize the Hough space. The usual approach starts by partitioning the Hough space Ω into contiguous cells [3,4]. The size of each cell is chosen according to the coarseness with which a line is to be located in the image space. These cells define an accumulator array. For each point in the image space D, the related line in Ω will increment each element of the accumulator array that it crosses. After all image points have been treated, the array is inspected to find cells with highest counts. The corresponding cells locate sets of nearly colinear points in D.

For generality, the Hough transform problem can be stated as follows. Consider an image $I(\mathbf{x})$, which can be either a binary line-drawing image or a gray level gradient image, and a pattern Γ described by a functional over the coordinate vector variable $\mathbf{x}$ and the parameter vector variable ω as

$$\Gamma : f(\mathbf{x}, \omega) = 0 \tag{3}$$

Note that the image space and parameter space dimensions may be arbitrary with the implicit practical constraint that $dim(\Omega)$ remains small. The continuous Hough transform associated with $I(\mathbf{x})$ is given by

$$H_c(\omega) = \sum_{\mathbf{x}} I(\mathbf{x})\ \delta(f(\mathbf{x}, \omega)) \tag{4}$$

where $\delta(x)$ is the Dirac impulse defined as

$$\delta(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

When using (4), however, this means that any point in D that does not fall exactly on the pattern Γ associated to a quantized parameter value ω_0 has no contribution to the corresponding continuous Hough response $H_c(\omega_o)$.

In a discrete implementation of the Hough transform using an accumulator array, it is convenient for computational reasons to force the contribution of each $\mathbf{x}$ to one discrete parameter value only, i.e., the one producing the nearest neighbor pattern. In this case, the Hough transform is expressed by

$$H(\omega) = \int_{\omega}^{\omega + \Delta\omega} \sum_{\mathbf{x}} I(\mathbf{x})\ \delta(f(\mathbf{x}, \omega'))\ d\omega' \tag{6}$$

where $\Delta\omega$ represents the width of each cell in the quantized parameter space.

If a soft contribution to each quantized parameter value is expected, an approximation of the Dirac impulse in (4) can be used. Its argument should match the distance $d(\mathbf{x}, \Gamma)$ between the actual point $\mathbf{x}$ in D space and the pattern Γ. In other words, the Hough transform can be rewritten as

$$H(\omega) = \sum_{\mathbf{x}} I(\mathbf{x})\ \hat{\delta}(d(\mathbf{x}, \Gamma)) \tag{7}$$

A weighted average of the continuous expression described by (4) can be obtained with the use of a normal distribution

$$\hat{\delta}(x) = exp(-x^2/2\sigma^2) \tag{8}$$

In this case, the standard deviation σ must be set according to the quantization of the Ω space. The computational cost is increased with respect to (6) as there is for each point $\mathbf{x}$ in the image space, a region of activity around the surface represented by the solution of (3) in the parameter space. The extent of this region is a function of σ. Implementations according to both (6) and (7) have been performed, but only simulations based on (6) will be presented here.

A considerable effort has been placed in modeling analytically the statistical and quantization noise affecting the HT [5], from a signal detection point of view. Due to the discrete nature of both image and Hough spaces, the HT images are often blurred and no sharp peaks can be detected.

Brown [6] has explained the deficiencies of the HT by the presence of a background noise that results from "side-lobe" effects. As an example, any combination of 2 points creates a background Hough response of 1 in the Hough image of a line detection problem, even if the points are very far apart. When increasing the noise level in the image space D or the dimension of the parameter space Ω, the search for peaks in the parameter space becomes very cumbersome. To illustrate this phenomenon, we present in Figure 3 the Hough images obtained from the cross pattern of Figure 2. Even though only horizontal and vertical lines are present in the cross pattern, fairly strong peaks appears also in the Hough image at those positions corresponding to the 45° and 135°. When the cross pattern is perturbated by noise as shown in Figure 6, the true peaks of the "Hough image" at the 0° and 90° are submerged in the background noise, and are actually lower than the false peaks at 45° and the 135°, see Figure 7. Brown [6] suggested the use of complementary (negative) votes to cancel off-peak positive votes in parameter space, i.e., background noise. When applying the HT in the context of curve detection problems, it must be kept in mind that additional constraints can be inserted in the computation of the image to parameter transform. These are not supposed to reduce the "noise" present when working in a parameter space. They instead should take advantage of relevant assumptions that are related to curve properties that the human eye uses in its detection process.

CONNECTIVITY HOUGH TRANSFORM

Using the mathematical formalism introduced in the previous section, this part of the paper describes two features that have been added to the usual HT in constructing the CHT. Only the continuous expression of the CHT, $CHT_c(\omega)$, corresponding to the continuous HT, $HT_c(\omega)$, shown in (4) is introduced. In a discretized Ω space, the results can be extended using the same approach of the previous section for expressions such as (6) and (7), but they shall be omitted here for lack of space.

When performing visual pattern recognition tasks, the human eye uses in its processing edge elements or connected segments of contours instead of separate points. Therefore, it seems reasonable that any pattern matching algorithm should include a connectivity measure to strengthen edge points that define continuous segments. To include this parameter in the Hough transform expression, equation (4) should become

$$H_c'(\omega) = \sum_{\mathbf{x}} I(\mathbf{x})\, C(\mathbf{x},\omega)\, \delta(\mathbf{f}(\mathbf{x},\omega)) \tag{9}$$

Where $C(\mathbf{x},\omega)$ is a connectivity measure at the point $\mathbf{x}$ along the contour defined by ω. The connectivity measure is a function of ω since it should be associated to the pattern shape Γ. In other words, in a line detection problem, the connectivity is used along linear paths whereas for a circle recognition task, the connectivity is searched along a circular path. The circle or line orientation is specified by the particular parameter value of interest. Two points are considered connected if they are adjacent in image space (e.g., 8-connected [7]) and their intensity values satisfy certain similarity criterion, say their difference is less than a certain threshold. Let $C(\mathbf{x})$ represent the connectivity of a given point $\mathbf{x}$ in D defined by the number of points that are connected to $\mathbf{x}$. $C(\mathbf{x},\omega)$ is defined as the projection of $C(\mathbf{x})$ onto the tangent of the contour at $\mathbf{x}$. Specifically, let us call $T_\Gamma(\mathbf{x})$, the tangent to the pattern Γ at $\mathbf{x}$ for a specific parameter value ω, and $T_{\mathbf{x}}$ the tangent at $\mathbf{x}$ to the set of connected points at that location. we define the connectivity along the contour $C(\mathbf{x},\omega)$ as the connectivity $C(\mathbf{x})$ weighted by the cosine of the angle between these two tangents, provided they are not separated by more than 45°.

$$C(\mathbf{x},\omega) = \begin{cases} C(\mathbf{x}) \cos(\angle(T_{\mathbf{x}}, T_\Gamma)) & if\ \angle(T_{\mathbf{x}}, T_\Gamma) < 45^\circ \\ 0 & otherwise \end{cases} \tag{10}$$

In particular, in the case of line detection with the parameterization of the line described as

$$x\cos\theta + y\sin\theta = \rho \tag{11}$$

the connectivity along the line (ρ,θ) is determined as

$$C(x,y,\rho,\theta) = | \, dx\sin\theta - dy\cos\theta \, | \tag{12}$$

Where dx and dy are the projections of the connected segment associated with the point $(x\ y)$ along the x and y directions, respectively.

A second feature that is used to improve the performance of the Hough transform involves the focus of attention present in the foveal vision. When trying to recognize curves or shapes, the human eye will match its model of the pattern to the actual portion of the picture on which it can focus its attention, an area of about 2° to 5° of angle, that will correspond to the portion of the image that is formed on the fovea. This second type of processing suggests to increase the contribution to the Hough image of points inside the neighborhood of the point of focus (point that is at the center of the focus of attention). It is reasonable to provide it by integrating the information of interest within the neighborhood along the contour direction, somehow to match the integration ability of the human eye. From (9), the Connectivity Hough Transform is then expressed as

$$CHT_c(\omega) = \sum_{\mathbf{x}} I(\mathbf{x})\delta(f(\mathbf{x},\omega)) \left[\sum_{\mathbf{x}'\in N(\mathbf{x},\omega)} I(\mathbf{x}')C(\mathbf{x}',\omega)\delta(\mathbf{f}(\mathbf{x}',\omega))\right] \tag{13}$$

where $N(\mathbf{x},\omega)$ represents the neighborhood around every point $\mathbf{x}$ along the curve Γ.

With the above two new features added to the traditional HT, not only the sensitivity to noise in the image space D are reduced, the effect of "background noise", inherent to the initial formulation of the Hough transform is also minimized.

Figure 4 shows the "Hough image" associated to the cross image of Figure 2 using the CHT. The false peaks at the 45° and 135° due to the "crosstalk" are completely eliminated. Figure 8 shows the "Hough image" associated to the noisy cross image of Figure 6. The true peaks appear much sharper than in Figure 7 and can easily be detected by the use of a single threshold. To evaluate the effect of neighborhood integration, Figures 5 and 9 show the "Hough images" obtained without neighborhood integration, i.e., through the use of the formula (9). Obviously, the neighborhood integration increases the ability of eliminating the "sidelobe" effect.

COMPUTATIONAL ISSUES

In the discrete implementation of a Hough type transform, the computational complexity grows exponentially with the dimensionality N of Ω. Let the desired number of parameters in each direction of the parameter space be L_i, and the number of samples in each direction of the M-dimensional image space be K_i, one should compute

$\prod_{i=1}^{M} K_i \times \prod_{i=1}^{N} L_i$ distances $d(\mathbf{x},\Gamma)$. This becomes untractable when N is large or K_i is large. As the architecture of the HT is perfectly parallel, several authors have suggested to reduce the computational load with parallel pipeline projection engines [8]. An even more efficient way of implementation was suggested by Li, *et. al.* with the so called Fast Hough Transform (FHT) [9]. which iteratively divides the parameter space into hypercubes of various sizes. At each step, the Hough transform is further computed on those hypercubes for which the Hough measure computed over the parent hypercubes exceeded a certain threshold. This pyramidal approach leads to a significant reduction of both the computation and storage.

The maximal precision achievable in the parameter space depends on the sampling resolution of the image space, and vice versa. In order to further reduce the computational complexity, we suggest a combined image and parameter space multi-resolution approach. The idea is to limit the resolution in both spaces in a first step and get a coarse estimation of the peak locations in the reduced size Hough image. In a second step, the full resolution are used to refine the initial estimate. Suppose that in the first step, the initial image is down-sampled by a certain factor r after adequate low-pass filtering to avoid aliasing artifacts, the maximal precision in parameter space is also reduced by r, which allows the partition of the Ω space with an r times larger quantization step. Once P most significant peaks are detected in the reduced size Hough image, $P \times (r)^N$ higher resolution parameter values are computed but using the original image data $I(\mathbf{x})$. The method can be iterated several times to build a multi-layer pyramid. Assuming that we start with only K_0 samples in each dimension of the image space and L_0 samples in each dimension of the parameter space. We increase the number of samples by a factor of r in each dimension in each higher level of the pyramid and stop until the number of samples in the image space reaches the original sampling rate, say, K in each dimension, and the number of samples in the parameter space reaches the desired parameter sampling rate, say, L in each dimension. The computation required would be reduced to from $L^N K^M$ to $O(K_0^M L_0^N + PK^M r^N)$, independent of L.

CONCLUSION

In this work, we have tried to incorporate certain processing steps of the human visual system in the traditional Hough Transform by defining the so called Connectivity Hough Transform. These include a connectivity measure and the integration of the image signal along the direction of the curve or pattern to be detected. The first one may be related to the particular detection ability of connected segments of contours that is performed by the complex cells in the visual cortex. The second one takes advantage of the integration ability of the retina present in foveal vision. The efficiency of the recognition capacity of this new transform seems very promising for line detection problems and will probably extend to more complex curve shapes such as circles and ellipses. By becoming more robust with respect to the "background noise" inherent in the Hough transform approach, it should be even more efficient for the detection of curves that have large dimension parameter spaces associated with them. It can certainly achieve better performances than the HT in any problem that involves visual pattern recognition, such as three dimensional object detection, motion estimation, etc. We are currently investigating the power of the CHT in this direction and continuing our efforts to make its implementation simpler.

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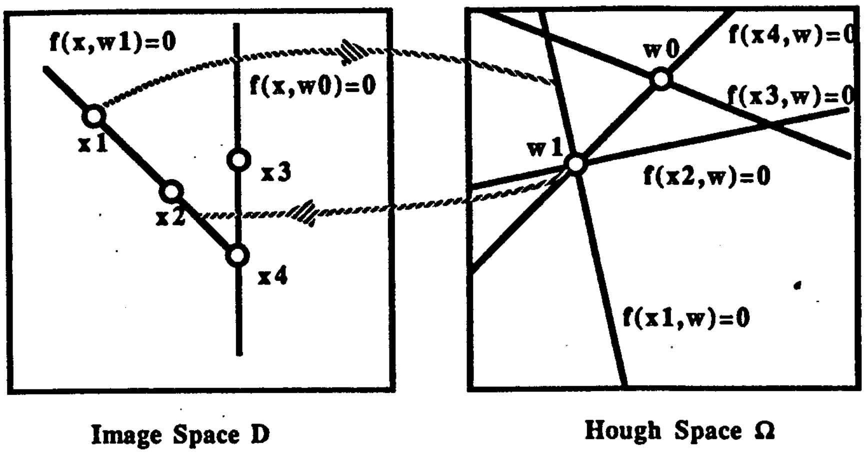

$$f(x,w) = ax+by-1=0$$

Figure 1: One-to-one mapping between image space D and Hough space Ωfor a straight line detection

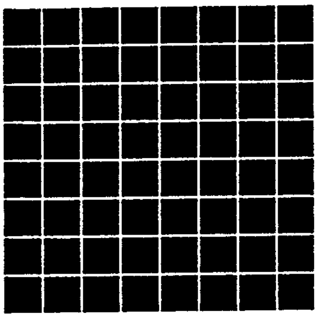

Figure 2.

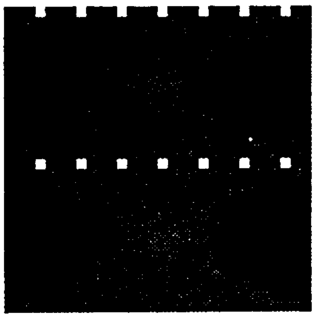

Figure 3.

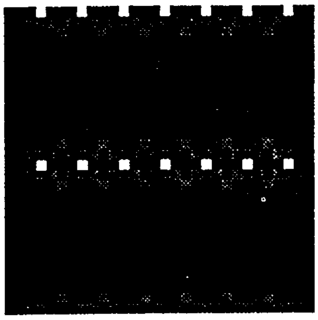

Figure 4.

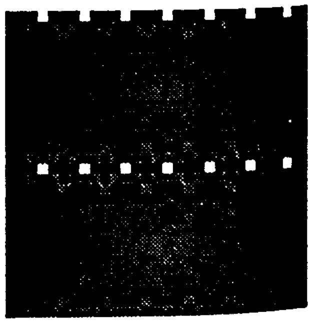

Figure 5.

Figure 2: Original cross image; Figure 3-5: "Hough image" obtained by HT (3), CHT(4), and CHT without neighborhood integration (5); The horizontal direction represents distance, while the vertical direction represents orientation.

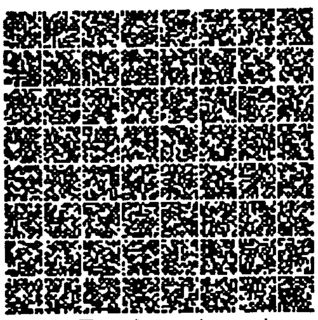

Figure 6.

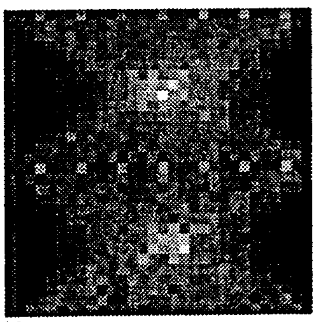

Figure 7.

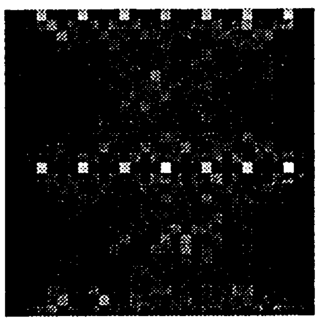

Figure 8.

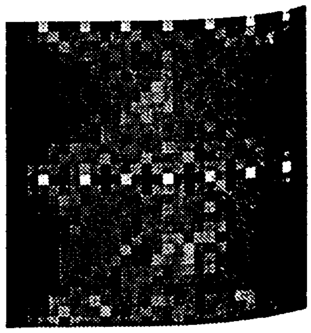

Figure 9.

Figure 6: Impulse noise corrupted cross image; Figure 7-9: "Hough image" obtained by HT (7), CHT(8), and CHT without neighborhood integration (9).