Metadata of the chapter that will be visualized in SpringerLink

Book Title	Recent Advances in Computational Optimization	
Series Title		
Chapter Title	Workforce Allocation for Social Engagement Services via Stochastic Optimization	
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Abstract	Social Engagement is a novel business model whose goal is transforming final users of a service from passive components into active ones. In this framework, people are contacted by the decision-maker (generally a company) and they are asked to perform tasks in exchange for a reward. This paves the way to the interesting optimization problem of allocating the different types of workforce so as to minimize costs. Despite this problem has been investigated within the operations research community, there are no approaches that allow to solve it by explicitly and appropriately modeling the behavior of contacted candidates through consolidated concepts from the utility theory. This work aims at filling this gap, by proposing a stochastic optimization model that includes a chance constraint putting in relation, under probabilistic terms, the candidate *willingness to accept* a task and the reward actually offered by the decision-maker. The developed model aims at optimally deciding which user to contact, the amount of the reward proposed, and how many employees to use in order to minimize the total expected costs of the operations. An approximation-based solution approach is proposed to address the formulated stochastic optimization problem and its computational efficiency and effectiveness are investigated through an extensive set of computational experiments.

Workforce Allocation for Social Engagement Services via Stochastic Optimization

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Abstract. Social Engagement is a novel business model whose goal is transforming final users of a service from passive components into active ones. In this framework, people are contacted by the decision-maker (generally a company) and they are asked to perform tasks in exchange for a reward. This paves the way to the interesting optimization problem of allocating the different types of workforce so as to minimize costs. Despite this problem has been investigated within the operations research community, there are no approaches that allow to solve it by explicitly and appropriately modeling the behavior of contacted candidates through consolidated concepts from the utility theory. This work aims at filling this gap, by proposing a stochastic optimization model that includes a chance constraint putting in relation, under probabilistic terms, the candidate *willingness to accept* a task and the reward actually offered by the decision-maker. The developed model aims at optimally deciding which user to contact, the amount of the reward proposed, and how many employees to use in order to minimize the total expected costs of the operations. An approximation-based solution approach is proposed to address the formulated stochastic optimization problem and its computational efficiency and effectiveness are investigated through an extensive set of computational experiments. AQ1 AQ2

1 Introduction

Social Engagement (SE) is a new business paradigm involving the customers of a company into its operations. More precisely, people agree to perform specific

S. Fidanova (Ed.): WCO 2022, SCI 1158, pp. 1–17, 2024.
https://doi.org/10.1007/978-3-031-57320-0_5

services for a company in exchange for a reward. The SE business model has been enabled by the increase of the number of users connected on the web and technologies able to get people information [1,2]. This gives to the companies the possibility to easily communicate with *candidates* and then to propose *tasks* in exchange for a reward.

A concrete example of application of the SE paradigm is the so called *crowd-shipping* logistics, i.e., rather than building an entire network of transshipment facilities and deploying a large number of vehicles for the delivery, the companies ask the people to collect the packages to a certain location and deliver it to the final user [3]. By using crowd-shipping, the company does not only decrease its costs, but also decreases its environmental impact. In fact, people accepting the delivery usually would take advantage of travels that they have to do anyhow for other activities, thus avoiding additional trips.

Another really interesting application of SE occurs in the development of the innovative concept called Internet of Things (IoT). In the era of smart cities, IoT is playing a role of considerable importance [4]. However, its development is considerably slowed down by the difficulty and costs involved in building telecommunication networks capable of continuously transmitting large amounts of data collected by sensors, in an efficient, effective and reliable way. As an alternative to the construction of this network, in exchange for a reward, citizens use their devices (e.g., mobile phones, modems) to share the internet so that the nearby sensors can exploit it to communicate the gathered data. This is known as the opportunistic IoT (oIoT) paradigm [5].

In the light of these examples, it is clear that SE will acquire more and more importance in the future being a corner stone of both smart cities and sustainable behaviour and that a great variety of applications will soon be put in practice. In this work, therefore, we do not want to concentrate on a specific application rather on a very general SE-based setting in order to embrace all the basic characteristics of such a business model. In the rest of the paper, we will call *candidate* each person that might be contacted by the company for an operation proposal, and *task* each proposed operation.

An effective planning of operations under the SE paradigm yields an interesting optimization problem. The decision-maker must decide how much he is willing to pay to a candidate for each task, when and where to rely on employees and on candidates, which tasks to assign to the employees and for which tasks the candidates must be contacted, in order to minimize the total operational costs. It is important to note that the reward paid to a candidate is generally lower on average than the cost that the company bears for an employee. However, while an employee is obliged to accept and carry out the tasks assigned to him, there is no certainty that a candidate will accept a proposed task. This complicates the underlying optimization problem as one must correctly address this source of uncertainty.

Little attention has been devoted to the development of optimization models aimed at effectively scheduling companies operations that exploit SE. Just few works ([6–8]) have tried to tackle the problem and, therefore, there is a large room for improvement of existing approaches as well as for the design of more innovative and more complete ones (as claimed regarding crowd-shipping in [9]). In particular, to the best of our knowledge, there is no published optimization model that explicitly accounts for individual candidate behaviour when planning SE-based operations. As already mentioned, one characteristic that makes challenging the optimization problems deriving from the implementation of the SE paradigm is the fact that candidates are not constrained a priori to respect a contract. This means that, once contacted, the candidate may not accept the task and, if we assume a pure rational profit-maximization behavior of the candidate, the reject can happen because the proposed reward is lower than the candidate expectation. It is therefore important to integrate tools in the decision-making process that allow monitoring the individual behavior of potential candidates.

In this work, to account for individual behaviour, we rely on the candidate's *willingness to accept* a task, i.e., the minimum reward expected by a candidate to accept a task. The *willingness to accept* is a well consolidated concept in utility theory and has been used since long to explain human subject preferences in economics [10]. From the decision-maker point of view, the candidate's *willingness to accept* is not deterministically known, since it depends on some factors that are idiosyncratic of the candidates. Therefore, we consider the candidate *willingness to accept* as a random variable. Thus, the probability of acceptance for a candidate will be equal to the probability that the offered reward is greater or equal to the *willingness to accept* of the candidate. Given this knowledge, the aim of the decision maker is to decide which person to contact, the amount of the reward proposed, and how many employees to use in order to minimize the total operations cost composed expressed as sum between the costs of the employee and the expected reward for the candidates.

This paper's contribution is threefold and can be summarized as it follows:

- first, we propose a novel mathematical model for SE-based services optimization. The formulation, which includes probability constraints, results to be the first one that explicitly accounts for each individual candidates behaviour.
- second, since the complexity of the proposed model and the explicit consideration of stochastic parameters do not allow to obtain a simple solution, we derive a mixed-integer quadratic programming model that approximates the original model. This is done by making some reasonable hypothesis on the probability distribution of the *willingness to accept* of each candidate, and by exploiting the Markov inequality.
- third, we conduct several computational experiment to validate and assess the suitability of our proposed model and of the solution approach and their characteristics.

The rest of the paper is organized as it follows. Section 2 discusses the related works appeared in the literature. The considered optimization problem is defined in Sect. 3, while the related mathematical model is formulated in Sect. 4. Our solution approach is described in Sect. 5. Section 6 presents the computational experiments and analyzes the related results. Finally, Sect. 7 draws conclusions and highlights future developments.

2 Related Works

Operation research models and methods have long been leveraged for cost reduction or profit maximization by companies. With the advent of the SE business model, the operation research community has naturally focused on the question of how to effectively plan operations that rely on such a business model.

As pointed out in the introduction, one of the most interesting implementation of the SE business model is the crowd-shipping paradigm. A recent survey on operation research approaches to crowd-sourced delivery can be found in [9]. Among the open research directions listed, one regards how to determine the minimum value of the reward guaranteeing that each candidate accepts the proposed task. In fact, the decision-maker does not know a priori how many candidates will accept the proposed tasks. As a result, some papers as [11] assume that the candidates will surely accept the task proposed. However, such an assumption yields strong limitations in practice [9]. Others authors, e.g., [7,12] have proposed models that better fit practical requirements by considering the possibility that a contacted candidate may reject the tasks. In [7], the authors assume that the number of person willing to perform a task in a given area of the operational network is a random variable. In this paper, we adopt a similar perspective. However, instead of relying on a single random variable describing the number of candidates, we model each single candidate behavior through a Bernoulli random variable. The parameter of such a Bernoulli random variable, i.e. the probability that the candidate accepts the task, is not fixed but depends by the proposed reward.

To deal with any single candidate's behaviour, we consider the concept of *willingness to accept* from the utility theory [13]. This is actuality not a peculiarity of this paper since many authors have relied on utility theory concepts and tools to model individual behaviours in optimization problems. In [14] an approach to embed discrete choice models from the utility theory into mixed integer linear programming models was presented. Instead, this paper wants to be a first step toward integrating utility theory tools and concepts also in optimization models for SE-based services, so to appropriately model individual candidates behaviors.

In our optimization problem, the link between the offered reward, the minimum expectations of a candidate, and his/her probability to accept the task is expressed under probabilistic terms, naturally yielding an optimization model including chance constraints. Optimization problems involving chance constraints have been largely considered and studied in literature [15]. Although some approaches have been proposed to deal with these problems [16,17], there is a common agreement that they are in general too complex to solve directly and thus the design of ad-hoc solution approaches might be required.

3 The Social Engagement Optimization Problem

The Social Engagement optimization problem ($SEOP$) that we want to study considers a decision-maker (in general a company) whose goal is to use people, in the following called *candidates*, in addition to employees in order to perform a set of tasks. In particular, we consider a urban environment divided in several geographical areas such as mobile phone cells, neighborhoods of different market or just geographical areas. Each of these areas is characterized by a number of tasks to perform and each tasks is characterized by different workloads, thus a single task may requires more candidates to be done. This is frequently the case in oIoT applications in which the tasks can be associated to share the internet connection with more sensors in the same area, thus requiring more candidates.

For example, in the crowd-shipping setting these tasks are the delivery required by customers out of the store, while in the oIoT application these tasks consist in sharing the internet connection with smart sensors in the city.

Each task can either be performed by using an employee or a candidate. Employee are more expensive, are available in a small number but they execute the tasks assigned. Instead, candidates are less expensive, their quantity is virtually unlimited (since the number of people considered for SE is far greater than the number of tasks) but they can refuse to perform a task with a given probability. We assume that the acceptance probability increases as the offered reward increase. Please notice that the employee have greater productivity than the candidates.

The goal of the decision-maker is to minimize the total operative costs while enforcing that with high probability all the tasks must be performed.

In the next sections, we will consider the following sets:

- $\mathcal{I}$: set of tasks.
- $\mathcal{M}$: set of candidates.

Moreover, we define the following parameters:

- c_i: cost of using an employee for task i.
- W_i: workload required to perform task i.
- $r > 1$: ratio between the productivity of an employee with respect to that of a candidate, i.e., the workload that a single employee can afford as compared to a candidate in the same time frame.
- α_i: required probability for tasks i to be performed.
- Δ_i^m: random variable representing the *willingness to accept* of candidates m for task i.
- B: maximum number of available employees.

4 Mathematical Model

In this section we formulate the mathematical model for the SE optimization problem. We consider the following decision variables:

- $Q_i^m \in \mathbb{R}$: reward offered to candidate m to accept task i;
- $z_i \in \mathbb{N}$: number of employee assigned to tasks i;

and the following auxiliary variables:

- $x_i^m \in [0,1]$: probability for candidate m to accept task i.
- Y_i^m are a random variables distributed according to a Bernoulli distribution of probability x_i^m, i.e.,

$$Y_i^m = \begin{cases} 1 & \text{if candidate } m \text{ accepts to perform task } i \\ 0 & \text{otherwise} \end{cases} .$$

Then, the *SEOP* can be formulated as follows:

$$\min \sum_{i \in \mathcal{I}} \sum_{m \in \mathcal{M}} Q_i^m x_i^m + \sum_{i \in \mathcal{I}} c_i z_i \tag{1}$$

subject to

$$x_i^m = \mathbb{P}[Q_i^m \geq \Delta_i^m], \quad i \in \mathcal{I}, m \in \mathcal{M} \tag{2}$$

$$\mathbb{P}[Y_i^m = a] = (x_i^m)^a (1 - x_i^m)^{(1-a)}, \quad i \in \mathcal{I}, m \in \mathcal{M} \tag{3}$$

$$a \in \{0, 1\} \tag{4}$$

$$\mathbb{P}\left[\sum_{m \in \mathcal{M}} Y_i^m + r z_i \geq W_i\right] \geq \alpha_i, \quad i \in \mathcal{I} \tag{5}$$

$$\sum_{i \in \mathcal{I}} z_i \leq B \tag{6}$$

$$\begin{aligned} & z_i \in \mathbb{N}, \quad i \in \mathcal{I}, \\ & Q_i^m \in \mathbb{R}^+, x_i^m, Y_i^m \in [0,1], \quad i \in \mathcal{I}, m \in \mathcal{M}. \end{aligned} \tag{7}$$

The objective function (1) is the total cost expressed as the summation between the expected costs offered as a reward (the reward Q_i^m is paid with probability x_i^m), and the sum of the costs from the employee. Constraints (2) define the variables x_i^m as the acceptance probability, while constraints (3) and (4) ensure Y_i^m to follow a Bernoulli distribution. Constraints (5) are chance constraints enforcing a minimum probability of doing a given task either by using employee or candidates. It is worth noting that ensuring that each task is satisfied with a given probability is less strict than requiring that all the tasks will be satisfied with a given probability. Nevertheless enforcing this second condition would lead to too conservative solutions. Finally, constraint (6) limits the number of employees.

In conclusion, it is worth noting that the *SEOP* model is quite general, embracing all the possible applications based on a SE business model.

4.1 An Analytical Solution for the Single-Candidate *SEOP*

In the particular case in which there exists a single candidate, it is possible to derive an analytical expression of the *SEOP* optimal solution. Even if it is reasonable to apply SE when the number of candidates $|\mathcal{M}|$ is large, in this section we derive such an expression for the $|\mathcal{M}| = 1$ special case since it provides some preliminary insights into how the candidate's behavior is related to the decision-maker's expectations in terms of how likely he performs the task.

For the sake of simplicity, we will assume that:

- just one task requiring a unitary effort has to be performed, i.e. $|\mathcal{I}| = 1$, $W_1 = 1$
- each employee can perform just one task, namely $r = 1$.

Then, the resulting model is

$$\min_{z,Q} QF_{\Delta}(Q) + cz \tag{8}$$

subject to

$$\mathbb{P}[Y = a] = (F_{\Delta}(Q))^a (1 - F_{\Delta}(Q))^{(1-a)} \tag{9}$$

$$a \in \{0, 1\} \tag{10}$$

$$\mathbb{P}[Y \geq 1 - z] \geq \alpha \tag{11}$$

$$z \in \{0, 1\}, Q \in \mathbb{R}^+, x \in [0, 1]. \tag{12}$$

where F_{Δ} denotes the cumulative probability distribution (cdf) of the *willingness to accept* (i.e. Δ). Note that, to obtain Model (8)–(12) from *SEOP*, we rearranged the terms in constraint (11), we set $z \in \{0, 1\}$ due to the characteristic of the instance and $x = F_{\Delta}(Q)$ due to constraint (2). Moreover, the index i has been dropped for ease of notation.

Since Y in Eq. (11) is a Bernoulli random variable and $1 - z \in \{0, 1\}$, we have that

$$\begin{cases} \text{if } z = 1, \quad \mathbb{P}[Y \geq 0] \geq \alpha \implies 1 \geq \alpha \\ \text{if } z = 0, \quad \mathbb{P}[Y \geq 1] \geq \alpha \implies F_{\Delta}(Q) \ \geq \alpha. \end{cases}$$

Hence, if $z = 1$ constraint (11) is always satisfied, else it must hold that $F_{\Delta}(Q) \ \geq \alpha$ or, equivalently that $Q \geq F_{\Delta}^{-1}(\alpha)$. Since the goal of the problem is to minimize the total costs, the optimal solution will consider $Q = F_{\Delta}^{-1}(\alpha)$. Hence, the optimal solution to the problem can be obtained by considering the relation between the cost c (the cost paid to the employee) and $\alpha F_{\Delta}^{-1}(\alpha)$ (the expected costs for the candidate's reward). In particular, if $c \leq \alpha F_{\Delta}^{-1}(\alpha)$, the optimal solution is $z = 1, Q = 0$ with value c, otherwise it is $z = 0, Q = F_{\Delta}^{-1}(\alpha)$, with value of $\alpha F_{\Delta}^{-1}(\alpha)$.

In other words, in this simple example the company uses SE if $\alpha F_{\Delta}^{-1}(\alpha) \leq c$. Hence, the higher is α, the more convenient are the employees since $F_{\Delta}^{-1}(\alpha)$ is increasing. This claim is also verified in the real field in which SE is used for tasks that are not safety critical.

Unfortunately, the insights obtained for *SEOP* in the single-candidate case cannot be generalized in more complex scenarios involving more candidates, since the chance constraint is coupling the related variables. In fact, considering two candidates ($|M| = 2$), a task requiring $W_1 = 2$, and considering two employee ($B = 2$), we have that

$$\begin{cases} \text{if } z = 0 \ \ \mathbb{P}[Y_1 + Y_2 \geq 2] \geq \alpha \implies F_{\Delta_1}(Q^1) F_{\Delta_2}(Q^2) \geq \alpha \\ \text{if } z = 1 \ \ \mathbb{P}[Y_1 + Y_2 \geq 1] \geq \alpha \implies \\ F_{\Delta_1}(Q^1)(1 - F_{\Delta_2}(Q^2)) + (1 - F_{\Delta_1}(Q^1)) F_{\Delta_2}(Q^2) \geq \alpha \\ \text{if } z = 2 \ \ \mathbb{P}[Y \geq 0] \geq \alpha \implies 1 \geq \alpha. \end{cases}$$

This means that, even for $|\mathcal{M}| = 2$, we lose an easy interpretation and, in turn, an easy solution for the problem.

5 Approximation-Based Solution Approach

The optimization problem (1)-(6) is difficult to solve due to the definition of x_i^m in constraints (2), of Y_i^m in constraints (3) and (4), and the chance constraints (5). Hence, we approximate these constraints in order to get a model which can be readily solved with off-the-shelf solvers.

Constraints (2) involve the cdf of the random variable Δ_i^m. We approximate it by means of a piece-wise linear function with J breakpoints. In particular, instead of constraints (2) we add a set of constraints of the form

$$x_i^m \leq k_j Q_i^m + q_j, \quad j = 1, \dots, J, i \in \mathcal{I}, m \in \mathcal{M}, \tag{13}$$

where k_j and q_j are obtained by imposing proper conditions (e.g. the passage in J points of the cdf). This choice is equivalent to enforce $x_i^m \leq \min[1, m_1 Q_i^m + q_1, \dots, m_J Q_i^m + q_J]$, where the first term of the minimum comes from the definition of x_i^m. Since the approximation proposed in Eq. (14) just lead to concave functions (being the pointwise minimum of affine functions) and since the a general cdf may be convex in some portion of the domain, the proposed approximation is not guarantee to converge to the cdf for all the distributions. In the following, for the sake of simplicity, we consider just $J = 1$ and we impose the passage for the point $(0, 0)$ meaning that with 0 reward the probability that the candidate will perform the task is 0, and for the point $(\bar{Q}_i^m, 1)$ where $\bar{Q}_i^m$ is reward for which the candidate m is willing to perform the task with a probability that we may approximate to be 1. By making this choice, the obtained final approximation of Constraints (2) is:

$$x_i^m \leq Q_i^m / \bar{Q}_i^m, \quad i \in \mathcal{I}, m \in \mathcal{M}. \tag{14}$$

This choice leads to the approximation of the cdf depicted in red in Fig. 1. Considering Eq. (14) is equivalent to suppose that Δ_i^m is distributed according to a uniform distribution. Thus, adding more functions is equivalent to approximate the density function with a function that is piece-wise constant.

In order to deal with constraints (5) we notice that by splitting the random variable from the deterministic components, the probability that we aim to compute is

$$\mathbb{P}[\sum_{m \in \mathcal{M}} Y_i^m \geq W_i - r z_i] \geq \alpha_i, \quad i \in \mathcal{I}. \tag{15}$$

By using Markov inequality (i.e., the fact that if X is a nonnegative random variable and $a > 0$, it holds that $\mathbb{P}[X \geq a] \leq \frac{\mathbb{E}[X]}{a}$), we can write:

$$\frac{\mathbb{E}[\sum_{m \in \mathcal{M}} Y_i^m]}{W_i - r z_i} \geq \mathbb{P}\left[\sum_{m \in \mathcal{M}} Y_i^m \geq W_i - r z_i\right] \geq \alpha_i, \quad i \in \mathcal{I}. \tag{16}$$

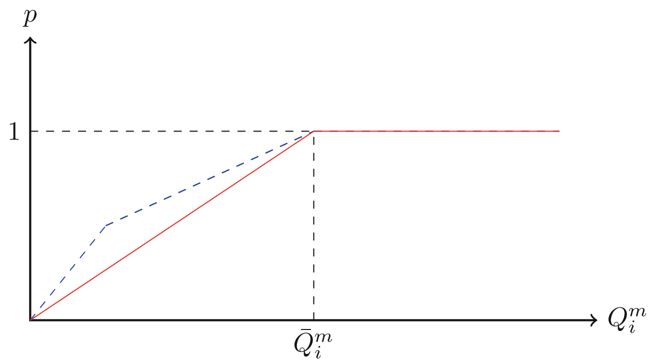

Fig. 1. Eq. (2) approximation In red simple piece-wise approximation, in dashed blue a more finer approximation.

Hence, since

$$\mathbb{E}\left[\sum_{m\in\mathcal{M}} Y_i^m\right] = \sum_{m\in\mathcal{M}} \mathbb{E}[Y_i^m] = \sum_{m\in\mathcal{M}} x_i^m,$$

Equation (16) leads to the following constraint:

$$\sum_{m\in\mathcal{M}} x_i^m \geq \alpha_i(W_i - rz_i), \quad i \in \mathcal{I}. \tag{17}$$

Equation (17) is enforcing that the expected workload form the candidates must be greater than the α_i percent of the people needed. Moreover, by considering the bound provided by Eq. (17), we are reducing the feasible set, thus the condition in (15) will be satisfied for greater value of α_i.

Then, the resulting approximation of the Social Engagement Optimization Problem ($SEOP_{ap}$) is the following Mixed-Integer Quadratic Programming model:

$$\min \sum_{i\in\mathcal{I}} \sum_{m\in\mathcal{M}} Q_i^m x_i^m + \sum_{i\in\mathcal{I}} c_i z_i \tag{18}$$

subject to

$$x_i^m \leq \frac{Q_i^m}{\bar{Q}_i^m}, \quad i \in \mathcal{I}, m \in \mathcal{M} \tag{19}$$

$$\sum_{m\in\mathcal{M}} x_i^m \geq \alpha_i(W_i - rz_i), \quad i \in \mathcal{I} \tag{20}$$

$$\sum_{i\in\mathcal{I}} z_i \leq B \tag{21}$$

$$\begin{aligned} &z_i \in \mathbb{N}, \quad i \in \mathcal{I}, \\ &Q_i^m \in \mathbb{R}^+, x_i^m \in [0,1], \quad i \in \mathcal{I}, m \in \mathcal{M}. \end{aligned} \tag{22}$$

It is worth noting that, although theoretically still hard to solve, the above model can be tackled in an exact way quite efficiently by using standard algorithms embedded in off-the-shelf solvers such as Gurobi.

6 Experimental Results and Analyses

In this section, we present some computational experiments regarding the problem addressed and the quality of our approximation.

6.1 Data Generation

Since SE is a relatively new topic of research and very few data related to instances generation is available. Moreover, different applications are characterized by different magnitude of the parameters describing the instances. Since crowd-shipping and oIoT are the domains of application characterized by more data available, we start by some available data for these two settings to define general benchmark instances.

In crowd-shipping applications, the number of tasks $|\mathcal{I}|$ is generally considered between 5 and 20 and $W_i = 1, \forall i \in \mathcal{I}$, since each task is a single delivery [3]. Instead, in oIoT applications, it is reasonable to consider much greater number for $|\mathcal{I}|$ and W_i. Within a use case related to the collection of data from smart waste collection sensors [7], the authors consider $|\mathcal{I}| = 100$, with $W_i = 10, \forall i \in \mathcal{I}$. We, therefore, will consider $|\mathcal{I}| = \{5, 10, 20, 50, 100\}$.

The number of candidates is the most difficult parameter to tune since it may be arbitrarily large: we can assume it to be equal to the number of people that has agreed to receive information related to SE possibilities. Nevertheless, we suppose that to be $|\mathcal{M}| = 4|\mathcal{I}|$ because the goal of the paper is to present a model and its properties rather then solution methods able to solve big instances.

Concerning the parameters, we set $w_i = W * (0.8 + 0.4U)$, where W is proportional to the hourly wage, and U is a uniform random variable between $[0, 1]$. We set $W = 7$ € (being 14 € the average hourly wage for a non skilled worker) [18]. By making this assumption we are the implicit assuming that the time unit for the work effort will be a half an hour. In the crowd-shipping application this means that we assume that one employee will take half an hour to bring $r = 1$ parcels and to make deliveries, while in the oIoT we consider that to be the time for going in one area and to collect the data from $r = 10$ sensors. Given a number of task r that each employee can do, we consider the maximum number of employee to be $B = 20/r$ in the crowd-shipping application and $B = 20/r$ in the oIoT one. With this choice, we assume that the instances have no feasibility problems being the employee enough to perform all the tasks.

The required probability for the task i to be performed is α_i. In general, we expect that to be different for all the tasks. For example, in crowd-shipping applications, an higher probability would be set for more important customers while in the oIoT application higher probability must be associated with sensors which information is more valuable. Since setting the α_i would be totally arbitrarily, we suppose that to be equal for all the tasks, i.e., $\alpha_1 = \alpha_2 = \cdots = \alpha_{|\mathcal{I}|} := \alpha$. Since the application is not safety critical and it is always possible to perform all the task with the employee, the values of α are considered in the interval $[0.6, 0.9]$.

Finally, Δ_i^m is the *willingness to accept*, i.e.,the minimum monetary quantity that a person is willing to accept to sell a good or service, or to bear a negative externality. In our case it is the minimum amount of money for which a candidate is willing to accept to make a task. While no study are available in the oIoT framework, in the crowd-shipping applications this quantity has been shown to depend by the size of the parcel, the sex of the person and by the time of the day [19–21]. Since modelling all these aspects require ad-hoc work, we postpone this topic in future study and we consider Δ_i^m to be a simple random variable. Since Δ_i^m can be assumed to be an unobserved utility, we consider it to be distributed according to a Gumbel (or *Extreme Value distribution type I*) [22]. The cdf of this distribution is $F(s) = e^{-e^{-s}}$ and by twice differentiating it is possible to observe that it is a concave function, thus leading the approximation proposed in Sect. 5 to converge to the exact cdf. Another hypothesis done in utility theory is that unobserved utility are normally distributed, we left this assumption for future work since it does not have the same concave property of the Gumbel cdf. For calibrating the distribution, we consider the average reward for crowd-shipping operations. In particular, in [23] it is reported that reward are usually in the interval between 5–7\$ equivalent to 4 to 6€. Thus, we fix the average of the Gumbel to be equal to 5€, with a standard deviation of 1€. By using this distribution we then compute $\bar{Q}_i^m$ to be $\bar{Q}_i^m = F^{-1}_{\Delta_i^m}(0.99)$.

6.2 Results and Discussion

All the experiments presented hereafter were performed on a *Intel(R) Core(TM)* i7-5500U CPU@2.40GHz computer with 16GB of RAM and running *Ubuntu* v20.04. The exact solver used is Gurobi v9.1.1 via its Python3 APIs.

Computational Results. In the following, we study the CPU time with respect to the dimension of the problem of $SEOP_{ap}$. In particular, the following measures are evaluated versus the growth of $|\mathcal{I}|$ and $|\mathcal{M}|$:

- the CPU time (sec).
- the time-to-best (sec): it is the number of seconds from the start of the execution of Gurobi to the time in which it founds the best solution of the run. This is an interesting parameter since often exact solvers find really fast the optimal solution, then they spend a lot of time for proving optimality. Thus, by observing this parameter is possible to better understand the performance of the exact solver.
- the MIP gap (%): it is computed as the percentage difference between the lower and upper objective bound. In particular, we consider the least gap value that Gurobi has to reach before stopping its execution.

The average (Avg) and standard deviation (Std Dev) of the above indicators calculated over 10 instances are shown in Table 1. In all the runs we set the solver time limit to 1 h. We chose this threshold because the decision-maker may, in principle, run the model several time a day and allowing for longer CPU times to deepen its usability.

Table 1. Average and standard deviations of the CPU time, time to best, and MIP gap for different values of $\mathcal{I}$, and $\mathcal{M}$.

Instance		CPU time (sec)		time-to-best (sec)		MIP gap (%)	
$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{M}\rvert$	Avg	Std Dev	Avg	Std Dev	Avg	Std Dev
5	20	0.09	0.01	0.09	0.01	0	0
10	40	1569.56	175.83	614.31	235.43	0	0
20	80	2780.84	573.26	2634.94	897.98	0	0
50	200	3600.00	0.00	1054.31	562.25	56	47
100	400	3600.00	0.00	1679.57	720.77	100	0

As the reader can notice, the instances with $|\mathcal{I}| = 5$, and $|\mathcal{M}| = 20$ are solved almost instantaneously with 0% of MIP gap. The time-to-best is equal to the CPU time since the difference are below the hundredths of a second.

For instances with $|\mathcal{I}| = 10, |\mathcal{M}| = 40$, and $|\mathcal{I}| = 20, |\mathcal{M}| = 80$, the CPU time increases, but the solver is still able to find the optimal solution inside the time limit. For the instances with $|\mathcal{I}| = 10, |\mathcal{M}| = 40$ the time to best is near one half of the total CPU time but solutions with gap below the 5% are found by the solver already in the first minutes of the run. Instead, for the instance with $|\mathcal{I}| = 20, |\mathcal{M}| = 80$, the time-to-best is close to the whole CPU time and no solution with gap below the 5% is found in the first minute of the run. For instances of greater dimensions, the exact solver is not able to find the optimal solution within the given time limit. For this reason the CPU time is equal to 3600 s with a null standard deviation. Nevertheless, for instances with $|\mathcal{I}| = 50$, and $|\mathcal{M}| = 200$, several times, the final gaps are smaller than the 10%, while for instances with $|\mathcal{I}| = 100$, and $|\mathcal{M}| = 400$, the solver is not able to find a good bound in the allocated CPU time (hence, a 100% MIP gap with null standard deviation is reported).

These results enable us to claim that $SEOP_{ap}$ can be effectively used in the real setting when considering crowd-shipping, due to the low number of delivery and people considered in such applications, while for the oIoT applications heuristic solution methods are definitely required.

Approximation Analysis. We now analyze the goodness of the $SEOP_{ap}$ approximation. Since Δ_i^m is distributed according to a Gumbel distribution with concave cdf, the approximation proposed converges to the exact function and several techniques for developing good piece-wise approximation are available, e.g. the Concave Adaptive Value Estimation [24]. Thus, we are interested in how much conservative is the Markov inequality with respect to Eq. (5).

In order to quantify this difference we compute the optimal solution of $SEOP_{ap}$ and we use it to compute $\hat{\alpha} := \mathbb{P}[\sum_{m\in\mathcal{M}} Y_i^m + rz_i \geq W_i]$. It is possible to easily compute this quantity by noting that the Y_i^m are independent with respect to the index m since the knowledge about candidate m performing a task does not provide any information related to the execution of the same task by other candidates. Thus, $\sum_m Y_i^m$ is a sum of independent random variable distributed according to a Bernoulli distribution of parameter x_i^m. Central Limit Theorems for non identically distributed random variables are available and, in particular, by applying the Lyapunov's variant of the Central Limit Theorem (see [25]) it is possible to prove that, for large values of $|\mathcal{M}|$ (in practice $|\mathcal{M}| \geq 30$), it holds:

$$\sum_{m\in\mathcal{M}} Y_i^m \sim \mathcal{N}\left(\sum_{m\in\mathcal{M}} x_i^m, \sum_{m\in\mathcal{M}} x_i^m(1-x_i^m)\right), \quad i \in \mathcal{I}. \tag{23}$$

Then, by using (23), we can compute α by solving:

$$\alpha_i = 1 - \Phi\left(\frac{W_i - rz_i - \sum_{m\in\mathcal{M}} x_i^m}{\sqrt{\sum_{m\in\mathcal{M}} x_i^m(1-x_i^m)}}\right), \quad i \in \mathcal{I}, \tag{24}$$

where Φ is the cdf of a standard normal distribution.

In Fig. 2, we report the value of the $\alpha \in [0.5, 1]$ in $SEOP_{ap}$ versus the $\hat{\alpha}$ computed with Eq. (24). All the results are averaged over 10 runs and the standard deviation of the observation is represented as a shaded area around the average values (in orange). We compute the results for $|\mathcal{I}| = 10$ and $|\mathcal{M}| = 40$ since we are able to get the optimal solution in a reasonable amount of time. Moreover, with $|\mathcal{M}| = 40$ there are enough candidates to apply results in (23)–(24).

The orange curve is always above the line $\hat{\alpha} = \alpha$, thus justifying the use of the Markov inequality. Nevertheless, the results are close to the exact value being on average 10% higher than the α set in the model. Hence, in the real field, the decision-maker may lower by 10% the values of the αs and get a solution compliant with the wanted probability of execution.

In conclusion, it is worth noting that using the result of Eq. (23) for Eq. (5) lead to the following equation

$$W_i - rz_i - \sum_{m\in\mathcal{M}} x_i^m \geq z_{1-\alpha}\sqrt{\sum_{m\in\mathcal{M}} x_i^m(1-x_i^m)}, \quad i \in \mathcal{I} \tag{25}$$

where $z_{1-\alpha}$ is the $1-\alpha$ standard normal quantile. Nevertheless, due to the negative square term on the right hand side of Eq. (25), it is not a second order cone constraint, preventing its usage to simplify $SEOP$.

Sensitivity Analysis and Managerial Insights. Since for values of $|\mathcal{I}| = 10$ and $|\mathcal{M}| = 40$, the optimal solution can be computed in a reasonable amount of

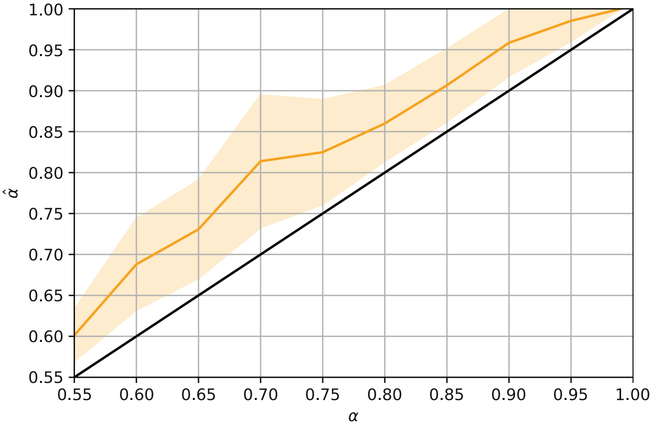

Fig. 2. Values of α set in the model vs real value of α.

time, we study how the solution characteristics change in different settings. In particular, we will evaluate the following measures versus different values of α in the range $[0.5, 1]$:

- ρ_E: the expected percentage costs for the employees' services, calculated as

$$\rho_E = \frac{\sum_{i \in \mathcal{I}} c_i z_i}{\sum_{i \in \mathcal{I}} \sum_{m \in \mathcal{M}} Q_i^m x_i^m + \sum_{i \in \mathcal{I}} c_i z_i};$$

- ω_E: the percentage of services performed by employee, calculated as

$$\omega_E = r \frac{\sum_{i \in \mathcal{I}} z_i}{\sum_{i \in \mathcal{I}} W_i}.$$

The average results over 10 instances for ρ_E and ω_E are shown in Fig. 3.

Note that we do not show values of α lower than 0.5 since for those values the solution uses only candidates.

As α increases, the solutions use more employees and less candidates. It is interesting to notice that even for $\alpha = 1$, a percentage of the candidates it is still selected since, by offering to several of them a small Q_i^m, the corresponding sum of the x_i^m satisfies constraint (20). Nevertheless, it is not a important drawback since the domain of usage of the model will consider lower values of α.

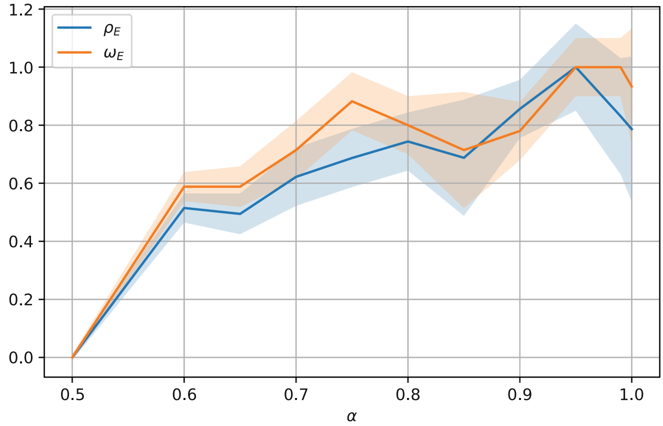

Fig. 3. Values of ρ_E and ω_E for different values of α.

7 Conclusions and Future Works

In this study, we proposed a new probabilistic model for SE-based services optimization encompassing the *willingness to accept* of the candidate involved in the business model. We prove, by means of computational experiments that, despite the difficult formulation, the model can be approximated and converted into a nice tractable form able to provide timely solution, at least for crowd-shipping applications.

Being the SE a very seminal topic within the optimization field, several improvements can be sketched:

- first, it is needed a full-fledged experimental design to explore all the solution characteristics. Some questions to answer are related to the performance of the method in the case in which non-concave distributions for the *willingness to accept* are considered or how the solutions of the model are related to the number of breakpoints used by the piece-wise linear cumulative distribution approximation of the *willingness to accept*.
- second, the problem should be faced by means of other techniques, such as dynamic programming, or other paradigms to treat the uncertainty, such as stochastic programming or robust optimization. In particular, we believe that another promising paradigm could be the distributionally robust optimization, since it relaxes the assumption of knowing the real characteristics of the *willingness to accept* distribution.
- finally, we highlight the need for ad-hoc methodologies in order to get, in a reasonable time, good solutions for oIoT applications (i.e., those having the larger number of tasks and candidates). This may be achieved by introducing new approximation methodologies or by developing heuristics for the proposed Mixed-Integer Non-Linear Programming model obtained for the $SEOP$.

We expect to cover some of these aspects in future studies.

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Chapter 5

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