Event-Triggered Control of Port-Hamiltonian Systems Under Time-Delay Communication

Ernesto Aranda-Escolástico, Leonardo J. Colombo, María Guinaldo, and Antonio Visioli

***Abstract*—We study the problem of periodic event-triggered control of interconnected port-Hamiltonian systems subject to time-varying delays in their communication. In particular, we design a threshold parameter for the event-triggering condition, a sampling period, and a maximum allowable delay such that interconnected port-Hamiltonian control systems with periodic event-triggering mechanism under a time-delayed communication are able to achieve asymptotically stable behaviour. Simulation results are presented to validate the theory.**

***Index Terms*—Event-triggered control, port-Hamiltonian systems, time-delay, Lyapunov stability.**

I. Introduction

THE PORT-HAMILTONIAN framework appears as an alternative to Euler-Lagrange formalism to model nonlinear systems with dissipation [1], [2], [3], which makes it a natural candidate to describe many physical systems such as robots [4], quadrotors [5], spacecrafts [6], or microgrids [7]. Essentially, port-Hamiltonian systems are based on a geometric structure that empathizes the importance of the total energy, the interconnection pattern, and the dissipation of the system. A key factor in control is this idea of interconnection since many control laws are implemented from an external device through external port variables [8], [9], [10]. This achieves a special relevance with the development of networked control systems [11] since, despite the obvious benefits of the communication networks, several challenges arise such as network delays, limited bandwidth, or loss of information [12]. In this regard, several solutions have been proposed to deal with time-delays in port-Hamiltonian frameworks.

In [13], time-delays appearing in a skew-symmetric form are studied. These results are improved in [14] using the Wirtinger inequality. A larger class of delays that are not constrained to be skew-symmetric is studied in [7], [15]. In [16], the delays are considered in the context of time-varying port-Hamiltonian systems. Recently, several works have focused on the effect of input delays when there exists an actuator saturation [17], [18]. Despite these developments, the potential benefits of techniques specifically designed for networked control systems have not been studied for port-Hamiltonian systems. For this reason, in this letter, we present, to the best of our knowledge, the first event-triggered strategy for port-Hamiltonian frameworks.

Event-triggered control [19], [20], [21] is based on transmitting information only in the instants of time that the physical system demands it. In this way, communication resources can be saved and more efficiently used reducing congestion and delays. The traditional concept of event-triggered control involves an event-triggering condition that determines when the information is sent through the network. Usually, this condition is continuously evaluated to obtain the exact triggering instant. However, this might yield several problems. On the one hand, it is necessary to guarantee a minimum inter-event time in the theoretical design to avoid Zeno behavior. On the other hand, the implementation of the continuous event-triggering condition might be problematic in digital platforms. Periodic event-triggered frameworks [22], [23], [24], [25], [26] have been proposed with the aim to evaluate the event-triggering condition only in prefixed instants of time, combining advantages of event-triggered control and periodic control. This approach is adopted in this letter to port-Hamiltonian systems. Therefore, the main contribution of this letter is the design of the first periodic event-triggered control framework for port-Hamiltonian systems. Additionally, it is combined with the study of time-varying delays. This strategy emerges as a less demanding solution in terms of communication resources than previous approaches focused only on time-delays.

The remainder of this letter is organized as follows. Port-Hamiltonian systems and the design of a periodic

Manuscript received 21 October 2023; revised 29 December 2023; accepted 12 January 2024. Date of publication 18 January 2024; date of current version 2 February 2024. This work was supported in part by the Agencia Estatal de Investigación under Project PID2020-112658RB-I00/AEI/10.13039/501100011033; in part by UNED under Project 2021V/-TAJOV/001, Project 2021V/PUNED/008, and Project 2022V/PUNED/007; and in part by the Mobility Stay "José Castillejo" under Grant CAS21/00108. The work of Leonardo J. Colombo was supported by MCIN/AEI/ 10.13039/501100011033 under Grant PID2022-137909NB-C21. Recommended by Senior Editor C. Prieur. *(Corresponding author: Leonardo J. Colombo.)*

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Digital Object Identifier 10.1109/LCSYS.2024.3355811

event-triggered port-Hamiltonian interconnection are introduced in Section II. In Section III, the main results about the stability of the new framework are obtained. In Section IV, the results are applied to a normalized pendulum and several simulations are obtained to show the benefits of the strategy.

II. Port-Hamiltonian Systems and Periodic Event-Triggered Interconnection

Consider two (parallel, i.e., no coupling) input-to-state port-Hamiltonian systems Σ_1 and Σ_2 such that

$$\dot{x}_i(t) = (J_i(x_i) - R_i(x_i))[\nabla H_i(x_i(t))] + G_i(x_i)u_i(t)$$
$$y_i(t) = G_i^\top(x_i)[\nabla H_i(x_i(t))], \text{ for } i = 1, 2. \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the state vector, $u_i \in \mathbb{R}^m$ is the control input, and $y_i \in \mathbb{R}^m$ is the system output. $J_i \in \mathbb{R}^{n\times n}$ is a skew-symmetric matrix, called the structure matrix and corresponds to the internal power-conserving structure of physical systems, such as kinematic constraints, oscillation between potential and kinetic energy, transformers, Kirchhoff's laws, etc. $R_i \in \mathbb{R}^{n\times n}$ is a positive matrix in presence of energy dissipation (due, for instance, to damping, viscosity, resistance, etc.), and $R_i = 0$ in the lossless energy case in the system i, with $i = 1, 2$. $G_i \in \mathbb{R}^{n\times m}$ is the input force matrix, so $G_i(x_i(t))u_i(t)$ describes the generalized forces resulting from the control input u_i.

Finally, H_i represents the energy of system i and it satisfies the passivity condition as described by the energy balance

$$H_i(x_i(t)) = H_i(x_i(t_0)) + \int_{t_0}^{t} y_i^\top(s)u_i(s)ds - \int_{t_0}^{t} [\nabla H_i(x_i(s))]^\top R_j[\nabla H_i(x_i(s))]ds.$$

The uncontrolled system is $\dot{x}_i = B_i(x_i)\nabla H_i(x_i)$ with $B_i(x_i) := J_i(x_i) - R_i(x_i)$, while an interconnected system Σ_{12} can be written as

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}B_1(x_1(t)) & 0\\ 0 & B_2(x_2(t))\end{bmatrix}\begin{bmatrix}\nabla H_1(x_1(t))\\ \nabla H_2(x_2(t))\end{bmatrix} + \begin{bmatrix}G_1(x_1(t)) & 0\\ 0 & G_2(x_2(t))\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}. \tag{2}$$

Let us describe the feedback interconnection, as drawn in Figure 1, between the plant to be controlled Σ_1 and the controller Σ_2 designed following an emulation-based approach. In this scheme, we assume that the output y_1 is sampled with period $h > 0$, i.e., the output sampling sequence is described by the set $S_1 = \{0, h, 2h, \dots, \ell h\}$ for $\ell \in \mathbb{N}$. An event-triggering mechanism at the output of the port-Hamiltonian system Σ_1 determines if the data should be transmitted over a communication network. Then, the transmission sequence from Σ_1 to Σ_2 is described by $S_2 = \{t_1, t_2, \dots, t_k\} \subseteq S_1$ for $k \in \mathbb{N}$. We also consider that the communication network induces a time-varying delay $\tau(t)$ which satisfies $0 < \tau_m \leq \tau(t) \leq \tau_M$, where τ_m and τ_M are the minimum and the maximum delay, respectively. Finally, the inputs u_1 and u_2 are generated by a zero-order-hold (ZOH) with the holding time $t \in [t_k + \tau(t_k), t_{k+1} + \tau(t_{k+1}))$. Consequently, denoting by $\mathcal{I}$ the identity matrix, we can write

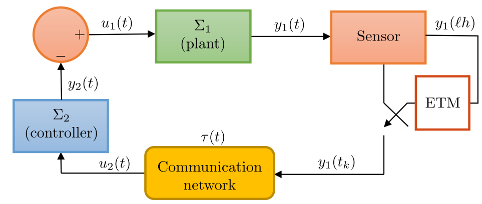

Fig. 1. Block diagram of the event-triggered interconnected port-Hamiltonian system.

$$\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix} = \begin{bmatrix}0 & -\mathcal{I}\\ \mathcal{I} & 0\end{bmatrix}\begin{bmatrix}\hat{y}_1(t)\\ y_2(t)\end{bmatrix}, \tag{3}$$

where $\hat{y}_1(s) := y_1(t_k)$ for $s \in [t_k + \tau(t_k), t_{k+1} + \tau(t_{k+1}))$.

Remark 1: The proposed scheme in Figure 1 considers a sampler and an event-triggering mechanism at the output of the subsystem Σ_1. However, the procedure in [27] can be followed to extend this idea and obtain a dual-rate framework with two samplers, two event-triggering mechanisms and their respective delays. ■

Now, let us express the transmission instants such as

$$t_k = \inf\{\ell h : \ell \in \mathbb{N}, \ell h > t_{k-1}, \mathcal{C}(e(\ell h), y_1(\ell h)) \geq 0\}, \tag{4}$$

where $e(s)$ is the error vector $e(s) = \hat{y}_1(s) - y_1(s)$ and $\mathcal{C}(e(s), y_1(s)) = e^\top(s)\Omega e(s) - \sigma y_1^\top(s)\ \Omega y_1(s)$, being $\Omega \in \mathbb{R}^{m\times m}$ a positive definite weighting matrix to be designed and $\sigma \geq 0$ is a given parameter that regulates the number of events. Since the triggering condition in (4) is only verified periodically with sampling period h, then the Zeno behaviour is naturally avoided and the minimum inter-event time is fixed by h.

Remark 2: Note that the smaller the σ, the more sensitive the event-triggering mechanism (4) becomes to the output change. Hence, the measurements are transmitted more frequently, which makes the controller (3) approach to a periodic controller. Note also that in the limit case, i.e., $\sigma \to 0$, the system is transformed into a sampled-data system. On the contrary, if σ is enlarged, less events are triggered, but properties such as stability might be compromised.

For a detailed timing analysis, we divide the holding interval $[t_k + \tau(t_k), t_{k+1} + \tau(t_{k+1}))$ into sampling intervals $[\ell h + \tau(\ell h), (\ell+1)h + \tau((\ell+1)h))$ and define a piece-wise function $\delta(t) = t - \ell h$ with $0 < \tau_m \leq \delta(t) \leq \tau_M + h = \delta_M$.

Since the event-triggering condition is evaluated in each $\ell h = t - \delta(t)$, it is convenient to write

$$y_1(t_k) = y_1(t - \delta(t)) + e(t - \delta(t)). \tag{5}$$

Combining now (2) and (3) with (5), we obtain

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}B_1(x_1(t)) & 0\\ 0 & B_2(x_2(t))\end{bmatrix}\begin{bmatrix}\nabla H_1(x_1(t))\\ \nabla H_2(x_2(t))\end{bmatrix} + \begin{bmatrix}0 & -G_1(x_1(t))\\ G_2(x_2(t)) & 0\end{bmatrix} \times \begin{bmatrix}y_1(t - \delta(t)) + e(t - \delta(t))\\ y_2(t)\end{bmatrix} \tag{6}$$

for $t \in [t_k + \tau(t_k), t_{k+1} + \tau(t_{k+1}))$.

Next, if we replace the output using (1) in (6), then

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}B_1(x_1(t)) & 0\\ 0 & B_2(x_2(t))\end{bmatrix}\begin{bmatrix}\nabla H_1(x_1(t))\\ \nabla H_2(x_2(t))\end{bmatrix} + \begin{bmatrix}0 & -G_1(x_1(t))G_2^\top(x_2(t))\\ 0 & 0\end{bmatrix}\begin{bmatrix}[\nabla H_1(x_1(t))]\\ [\nabla H_2(x_2(t))]\end{bmatrix} + \begin{bmatrix}0 & 0\\ G_2(x_2(t))G_1^\top(x_1(t)) & 0\end{bmatrix}\begin{bmatrix}[\nabla H_1(x_1(t-\delta(t)))]\\ [\nabla H_2(x_2(t-\delta(t)))]\end{bmatrix} + \begin{bmatrix}0\\ G_2(x_2(t))\end{bmatrix}e(t-\delta(t)). \tag{7}$$

Denote the state of Σ_{12} by $\xi = \begin{bmatrix}x_1 & x_2\end{bmatrix}^\top$ and its total energy by $\mathcal{H}(\xi) = H_1(x_1) + H_2(x_2)$. For simplicity, we refer in the following by $\mathcal{H}_t$ to indicate that $\mathcal{H}$ is taken at time t, i.e., $\mathcal{H}(\xi(t)) = \mathcal{H}_t$. Thus, (7) can be written as

$$\dot{\xi}(t) = A\nabla\mathcal{H}_t + A_d\nabla\mathcal{H}_{t-\delta(t)} + A_e e(t-\delta(t)), \tag{8}$$

where $A = \begin{bmatrix}B_1(x_1(t)) & -G_1(x_1(t))G_2^\top(x_2(t))\\ 0 & B_2(x_2(t))\end{bmatrix}$, $A_d = \begin{bmatrix}0 & 0\\ G_2(x_2(t))G_1^\top(x_1(t)) & 0\end{bmatrix}$ and $A_e = \begin{bmatrix}0\\ G_2(x_2(t))\end{bmatrix}$.

Note that (8) describes a system formed by two port-Hamiltionan subsystems with a periodic event-triggered interconnection and perturbed with time-delays. However, the port-Hamiltonian structure is no longer preserved due to the presence of sampled data and delays in $\nabla\mathcal{H}_{t-\delta(t)}$, and the stability cannot be concluded only from the properties of the Hamiltonian $\mathcal{H}$. Instead of that, we consider (8) as an *interconnected time-delayed port-Hamiltonian system*, i.e., a nonlinear time-delayed system in a perturbed Hamiltonian form, and use Lyapunov-Krasovskii theory (see [28] for instance) to take into account the periodic event-triggered scheme and the delays. For the further analysis, two assumptions over the system (8) are also necessary:

Assumption 1: The time-delayed port-Hamiltonian system (8) posses an equilibrium point $\xi_e = 0$.

Assumption 2: The Hamiltonian $\mathcal{H}$ is regular and positive definite around ξ_e (i.e., $\mathcal{H} > 0$, $\nabla\mathcal{H}(\xi_e) = 0$ and $\nabla\mathcal{H}(\xi) \neq 0$ for $\xi \neq \xi_e$ around ξ_e).

Assumption 2 implies that the port-Hamiltonian system is locally asymptotically stable in absence of event-triggered transmissions and time-delays with Lyapunov function $\mathcal{H}$. Finally, the following problem is established:

Problem Statement: Given the interconnected time-delayed port-Hamiltonian system (8) under Assumptions 1-2, design the sampling period h, the maximum allowable delay τ_M and the threshold parameter σ, such that (8) with periodic event-triggering mechanism (4) is locally asymptotically stable, and under appropriate circumstances, globally asymptotically stable.

III. Problem Solution

In this section, we study the interconnected time-delayed port-Hamiltonian system (8) with event-triggered mechanism (4) and under time-varying delays. First, the local asymptotic stability is proved. Then, the conditions of global asymptotic stability are proposed and the particular case of linear port-Hamiltonian systems is studied.

Theorem 1: For given positive scalars σ, h and τ_M, the interconnected delayed port-Hamiltonian system (8) with event-triggered mechanism (4) and under Assumptions 1-2 is locally asymptotically stable, if there exist matrices P, Q and Ω - of appropriate dimensions - such that

$$(i)\ P \succ 0,\ (ii)\ Q \succ 0,\ (iii)\ \Omega \succ 0 \tag{9}$$

$$\Xi = \begin{bmatrix}\Xi_{11} & \star & \star & \star\\ \Xi_{21} & \Xi_{22} & \star & \star\\ 6Q & 6Q & -12Q & \star\\ \Xi_{41} & \Xi_{42} & 0 & \Xi_{44}\end{bmatrix} \prec 0, \tag{10}$$

is feasible around a neighborhood of $\xi_e = 0$ with $\Xi_{11} = \frac{1}{2}(A + A^\top) + A^\top\nabla^2\mathcal{H}_t^\top P + P\nabla^2\mathcal{H}_t A + \delta_M^2 A^\top\nabla^2\mathcal{H}_t^\top Q\nabla^2\mathcal{H}_t A - 4Q$, $\Xi_{21} = A_d^\top\nabla^2\mathcal{H}_t^\top P + \frac{A_d^\top}{2} + \delta_M^2 A_d^\top\nabla^2\mathcal{H}_t^\top Q\nabla^2\mathcal{H}_t A - 2Q$, $\Xi_{22} = \delta_M^2 A_d^\top\nabla^2\mathcal{H}_t^\top Q\nabla^2\mathcal{H}_t A_d - 4Q + \sigma\mathcal{G}^\top\Omega\mathcal{G}$, $\Xi_{41} = A_e^\top\nabla^2\mathcal{H}_t^\top P + \frac{A_e^\top}{2} + \delta_M^2 A_e^\top\nabla^2\mathcal{H}_t^\top Q\nabla^2\mathcal{H}_t A$, $\Xi_{42} = \delta_M^2 A_e^\top\nabla^2\mathcal{H}_t^\top Q\nabla^2\mathcal{H}_t A_d$, $\Xi_{44} = \delta_M^2 A_e^\top\nabla^2\mathcal{H}_t^\top Q\nabla^2\mathcal{H}_t A_e - \Omega$, $\mathcal{G} = \begin{bmatrix}G_1^\top & 0\end{bmatrix}$. Besides, if (9)–(10) is feasible for all $\xi \in \mathbb{R}^{n_1\times n_2}$, then the system is globally asymptotically stable.

Proof: Consider the functional

$$V = V_1 + V_2, \tag{11}$$

where $V_1 = \mathcal{H}_t + \nabla\mathcal{H}_t^\top P\nabla\mathcal{H}_t$,

$$V_2 = \delta_M\int_{-\delta_M}^{0}\int_{t+s}^{t}\frac{d}{dr}\left(\nabla\mathcal{H}_r^\top\right)Q\frac{d}{dr}(\nabla\mathcal{H}_r)drds.$$

Since $\mathcal{H}$ is positive definite around the equilibrium point by Assumption 2 and considering (9) (i) and (ii), then (11) is an admissible Lyapunov-Krasovskii Function satisfying [28, Th. 1]. The derivative of V_1 along the trajectories (8)

$$\begin{aligned}\dot{V}_1 &= \nabla\mathcal{H}_t^\top\dot{\xi}(t) + \nabla\mathcal{H}_t^\top P\nabla^2\mathcal{H}_t\dot{\xi}(t) + \dot{\xi}^\top(t)\nabla^2\mathcal{H}_t P\nabla\mathcal{H}_t\\ &= \nabla\mathcal{H}_t^\top\left(\frac{1}{2}\left(A + A^\top\right) + A^\top\nabla^2\mathcal{H}_t^\top P + P\nabla^2\mathcal{H}_t A\right)\nabla\mathcal{H}_t\\ &\quad + \nabla\mathcal{H}_{t-\delta(t)}^\top\left(\frac{A_d^\top}{2} + A_d^\top\nabla^2\mathcal{H}_t^\top P\right)\nabla\mathcal{H}_t\\ &\quad + e^\top(t-\delta(t))\left(\frac{A_e^\top}{2} + A_e^\top\nabla^2\mathcal{H}_t^\top P\right)\nabla\mathcal{H}_t\\ &\quad + \nabla\mathcal{H}_t^\top\left(\frac{A_d}{2} + P\nabla^2\mathcal{H}_t A_d\right)\nabla\mathcal{H}_{t-\delta(t)}\\ &\quad + \nabla\mathcal{H}_t^\top\left(\frac{A_e}{2} + P\nabla^2\mathcal{H}_t A_e\right)e(t-\delta(t)),\end{aligned} \tag{12}$$

while the derivative of V_2 along (8) is

$$\dot{V}_2 = \delta_M^2\dot{\xi}^\top\nabla^2\mathcal{H}_t^\top Q\nabla^2\mathcal{H}_t\dot{\xi} - \delta_M\int_{t-\delta_M}^{t}\frac{d}{ds}\nabla\mathcal{H}_s^\top Q\frac{d}{ds}\nabla\mathcal{H}_s ds. \tag{13}$$

Applying now the Wirtinger inequality for time-delay systems [29, Corollary 4], the integral term (13) is bounded as

$$-\delta_M \in t_{t-\delta_M}^{t}\frac{d}{ds}\nabla\mathcal{H}_s^\top Q\frac{d}{ds}\nabla\mathcal{H}_s ds \leq \begin{bmatrix}\nabla\mathcal{H}_t - \nabla\mathcal{H}_{t-\delta(t)}\\ \nabla\mathcal{H}_t + \nabla\mathcal{H}_{t-\delta(t)} - \frac{2}{\delta(t)}\int_{t-\delta(t)}^{t}\nabla\mathcal{H}_s ds\end{bmatrix}^\top\begin{bmatrix}Q & 0\\ 0 & 3Q\end{bmatrix}$$

$$\times\begin{bmatrix}\nabla\mathcal{H}_t-\nabla\mathcal{H}_{t-\delta(t)}\\ \nabla\mathcal{H}_t+\nabla\mathcal{H}_{t-\delta(t)}-\frac{2}{\delta(t)}\int_{t-\delta(t)}^{t}\nabla\mathcal{H}_s ds\end{bmatrix}. \tag{14}$$

Now, equations (12), (13) and (14) are combined, and the S-procedure [30] is used to take into account the knowledge about the event-triggering mechanism (4), i.e., the quadratic form $-e^\top(t)\Omega e(t)+\sigma\nabla\mathcal{H}_{t-\delta(t)}^\top\mathcal{G}^\top\Omega\mathcal{G}\nabla\mathcal{H}_{t-\delta(t)}\leq 0$, with Ω satisfying (9) (*iii*), is added to the right of $\dot{V}$ such that $\dot{V}=\dot{V}_1+\dot{V}_2\leq\begin{bmatrix}\nabla\mathcal{H}_t\\ \nabla\mathcal{H}_{t-\delta(t)}\\ \frac{1}{\delta(t)}\int_{t-\delta(t)}^{t}\nabla\mathcal{H}_s ds\\ e(t-\delta(t))\end{bmatrix}^\top\Xi\begin{bmatrix}\nabla\mathcal{H}_t\\ \nabla\mathcal{H}_{t-\delta(t)}\\ \frac{1}{\delta(t)}\int_{t-\delta(t)}^{t}\nabla\mathcal{H}_s ds\\ e(t-\delta(t))\end{bmatrix}=\psi^\top(t)\Xi\psi(t)$. Since $\Xi<0$ by (10), $c>0$ exists such that $\dot{V}\leq -c\|\psi(t)\|\leq -c\|\nabla\mathcal{H}_t\|$. Since H is regular around ξ_e by Assumption 2, $\|\nabla\mathcal{H}_t\|$ is a continuous positive definite function in a neighborhood of the origin. By the comparison lemma [31, Lemma IV.1], a class $\mathcal{K}_\infty$ function κ exists with $\kappa(\|\xi(t)\|)\leq\|\nabla\mathcal{H}_t\|$ in a neighborhood of the ξ_e. Thus, conditions in [28, Th. 1] are satisfied, and (8) is asymptotically stable. ∎

Note that (10) is generally state-dependent due to the term $\nabla^2\mathcal{H}_t$ and the matrices A, A_d and A_e. Consequently, the feasibility of (10) might be difficult to prove. To overcome this issue, a polytopic approach can be followed similarly as in [7]. The objective is to transform (9)–(10) into a set of linear matrix inequalities (LMIs), which can be efficiently solved. Previously, two assumptions are necessary.

Assumption 3: J_i, R_i and G_i are constant matrices for $i=1,2$.

Assumption 4: The Hessian $\nabla^2\mathcal{H}_t$ is embedded into a polytope $\mathbb{P}$.

Theorem 2: Under Assumptions 3-4, the asymptotic stability conditions (9)–(10) are satisfied if they are satisfied on the set of vertices of the polytope $[0,\delta_M]\times\mathbb{P}$.

Proof: First, let us show that (10) is affine with respect to $\nabla^2\mathcal{H}_t$. Applying the Schur complement over (10), it is obtained that $\Xi\prec 0$ if and only if $\Theta\prec 0$ where

$$\Theta=\begin{bmatrix}\Theta_{11} & \star & \star & \star & \star\\ \Theta_{21} & \Theta_{22} & \star & \star & \star\\ 6Q & 6Q & -12Q & \star & \star\\ \Theta_{41} & 0 & 0 & -\Omega & \star\\ \nabla^2\mathcal{H}_t A & \nabla^2\mathcal{H}_t A_d & 0 & \nabla^2\mathcal{H}_t A_e & -\frac{Q^{-1}}{\delta_M}\end{bmatrix} \tag{15}$$

and $\Theta_{11}=\frac{1}{2}A+A^\top+A^\top\nabla^2\mathcal{H}_t^\top P+P\nabla^2\mathcal{H}_t A-4Q$, $\Theta_{21}=A_d^\top\nabla^2\mathcal{H}_t^\top P+\frac{A_d^\top}{2}-2Q$, $\Theta_{22}=-4Q+\sigma\mathcal{G}^\top\Omega\mathcal{G}$, $\Theta_{41}=A_e^\top\nabla^2\mathcal{H}_t^\top P+\frac{A_e^\top}{2}$. Besides, Assumption 3 implies that A, A_d and A_e are constant matrices. Then, because of Assumption 4, constant matrices T_j with $j=1,\ldots,N$ exist such that for all ξ in $\mathbb{P}$ constants $0\leq\lambda_j\leq 1$ with $\sum_{j=1}^{N}\lambda_j=1$ such that

$$\nabla^2\mathcal{H}_t=\sum_{j=1}^{N}\lambda_j T_j. \tag{16}$$

Note that $\nabla^2\mathcal{H}_t$ is affine with respect to the vertices of the polytope and, therefore, (14) is also affine with respect to the vertices. Thus, $\Theta\prec 0$ is satisfied if it is satisfied in the vertices of the polytope and the corollary is proved. ∎

Remark 3: Note that the conditions in Theorem 2 are state independent, but they are not a set of LMIs since (14) depends on Q^{-1}. An approach to convert it into LMIs is to add the constraint $Q\succ\alpha\mathcal{I}$, and consequently $Q^{-1}\prec\frac{\mathcal{I}}{\alpha}$.

Besides, from Theorem 1, it is possible to easily derive conditions for interconnected linear port-Hamiltonian systems with delays. For linear port-Hamiltonian systems, Assumption 3 is inherently satisfied, while the total energy of the interconnected system is $\mathcal{H}=\frac{1}{2}\xi^\top M\xi$ with $M\succ 0$ to fulfill Assumption 2. Consequently, $\nabla\mathcal{H}_t=M\xi(t)$, and (8) is transformed into

$$\dot{\xi}(t)=AM\xi(t)+A_dM\xi(t-\delta(t))+A_e e(t-\delta(t)) \tag{17}$$

Then, the following corollary is stated.

Corollary 1: For given positive scalars σ, h and τ_M, the interconnected delayed port-Hamiltonian system (17) with event-triggered mechanism (4) and under Assumptions 1-2 is globally asymptotically stable, if there exist matrices P, Q and Ω - of appropriate dimensions - such that the LMIs $P\succ 0$, $Q\succ 0$, $\Omega\succ 0$, and

$$\Xi_l=\begin{bmatrix}\Xi_{l11} & \star & \star & \star\\ \Xi_{l21} & \Xi_{l22} & \star & \star\\ 6Q & 6Q & -12Q & \star\\ \Xi_{l41} & \Xi_{l42} & 0 & \Xi_{l44}\end{bmatrix}\prec 0, \tag{18}$$

are feasible with $\Xi_{l11}=\frac{1}{2}(A+A^\top)+A^\top MP+PMA+\delta_M^2A^\top MQMA-4Q$, $\Xi_{l21}=A_d^\top MP+\frac{A_d^\top}{2}+\delta_M^2A_d^\top MQMA-2Q$, $\Xi_{l22}=\delta_M^2A_d^\top MQMA_d-4Q+\sigma\mathcal{G}^\top\Omega\mathcal{G}$, $\Xi_{l41}=A_e^\top MP+\frac{A_e^\top}{2}+\delta_M^2A_e^\top MQMA$, $\Xi_{l42}=\delta_M^2A_e^\top MQ\nabla^2\mathcal{H}_tA_d$, $\Xi_{l44}=\delta_M^2A_e^\top MQMA_e-\Omega$.

Proof: The proof is equivalent to the proof of Theorem 1 but replacing $\nabla^2\mathcal{H}_t$ by M. Since A, A_d, A_e and M are constant matrices by the definition of linear port-Hamiltonian system, then (18) is a LMI and the proof is completed. ∎

IV. Case Study

Let us consider a dumped normalized pendulum as described in [2]. The equations of the system are

$$\ddot{q}+\sin(q)+\zeta\dot{q}=u\quad y_1=\dot{q}, \tag{19}$$

where q is the angle described by the pendulum and $\zeta>0$ a damping constant. Considering the total energy of the system $H_1(q,\dot{q})=\frac{1}{2}\dot{q}^2+(1-\cos(q))$, (19) can be written in the port-Hamiltonian form (1) with $J_1=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$, $R_1=\begin{bmatrix}0 & 0\\ 0 & \zeta\end{bmatrix}$, $G_1=\begin{bmatrix}0\\ 1\end{bmatrix}$. Note that the origin is a stable equilibrium point, but not asymptotically stable. Therefore, we consider a controller with Hamiltonian $H_2(x_2)=\frac{K}{2}x_2^2$ such that $J_2=0$, $R_2=d_c$ and $G_2=1$, with $K>0$ a feedback gain $\zeta_c>$ a damping constant for the controller to be chosen. So, the whole system can be written in the form of (8) with $\mathcal{H}(\xi)=\frac{1}{2}\xi_2^2+(1-\cos(\xi_1))+\frac{1}{2}\xi_3^2$ and $A=\begin{bmatrix}0 & 1 & 0\\ -1 & -\zeta & -1\\ 0 & 0 & -\zeta_c\end{bmatrix}$, $A_d=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$, $A_e=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}$. Assumption 1-3 can be easily verified. So, to apply Theorem 2,

TABLE I
MAXIMUM VALUES FOR δ_M AND σ WHICH SOLVE (9)-(10) FOR NORMALIZED PENDULUM WITH $\zeta = 0.1$, $\zeta_c = 1$, $K = 3$

δ_M	0.1	0.2	0.3	0.4	0.5	0.6	0.7
σ	2.19	1.41	0.88	0.54	0.30	0.13	0.008

TABLE II
AIET AND IAE FOR $h = 0.3$ AND DIFFERENT VALUES OF σ

σ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
AIET	0.49	0.58	0.67	0.75	0.87	0.97	1.09	1.27
IAE	2.97	2.99	3.02	3.06	3.13	3.26	3.40	3.65

TABLE III
AIET AND IAE FOR $\delta_M = 0.5$

Transmission scheme	Periodic ($h = \delta_M - \tau_M$)			Even-triggered ($h = \delta_M - \tau_M$, $\sigma = 0.2$)		
τ_M	0	0.2	0.4	0	0.2	0.4
AIET	0.50	0.30	0.10	0.85	0.67	0.37
IAE	2.89	2.91	2.93	3.01	3.02	3.05

we compute the Hessian of the total Hamiltonian $\nabla^2\mathcal{H}(\xi) = \mathrm{diag}(\cos(\xi_1), 1, K)$. Clearly, $-1 \leq \cos(\xi_1) \leq 1$ and $\nabla^2\mathcal{H}(\xi)$ can be written in the form of (16) with vertices $\mathcal{H}_1 = \mathrm{diag}(-1, 1, K)$ and $\mathcal{H}_2 = \mathrm{diag}(1, 1, K)$, so Assumption 4 is verified. Then, using Theorem 2, the problem (9)–(10) is solved using LMI solvers. Let us fix $\zeta = 0.1$, and $\zeta_c = 1$ and $K = 3$ for the controller. Note that these values can be adjusted or tuned with an emulation-based approach. Then, the solution of (9)–(10) depends on σ and on $\delta_M = h + \tau_M$, i.e., the flexibility to trigger the events and the sum of the sampling period and the maximum admissible delay. Hence, a trade-off between both quantities is obtained and summarized in Table I.

To check the results, we have performed several simulations. First, we consider the case without delay, i.e., $h = \delta_M$, and set $h = 0.3$. In Table II, the Average Inter-Event Time (AIET) for different values of σ and the Integral Absolute Error (IAE) of H_1 are summarized for a set of 1000 simulations with random initial conditions. It is shown that, in general, a larger σ implies less triggered events, but also some performance's degradation since the input signal is not updated so often. In Figure 2, the position and velocity of the pendulum, for the different values of σ are depicted for initial conditions $\xi(0) = [1\ 1\ 0]$. As suggested by Table II, a lower σ is related to a faster convergence to the equilibrium, but this is not a necessary consequence if avoiding updating the input signal implies that the pendulum is pushed in the correct direction with a larger torque for a larger time interval.

Finally, we test the system under random time-varying delays $0 < \tau(t) \leq \tau_M = 0.5$ s and with h and σ obtained from Table I. For the test, we simulate the system with different maximum delays again for 1000 random initial conditions. The AIET and the average IAE are summarized in Table III. One the one hand, there is a logical correlation between the delay and the performance, which is more affected the larger the delay is. On the other hand, we can observe that

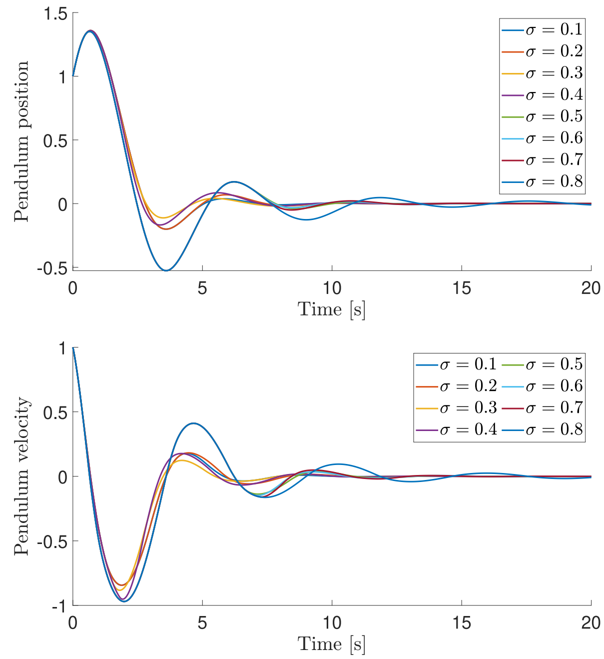

Fig. 2. Pendulum position (top) and velocity (bottom) for $h = 0.3$ and several σ.

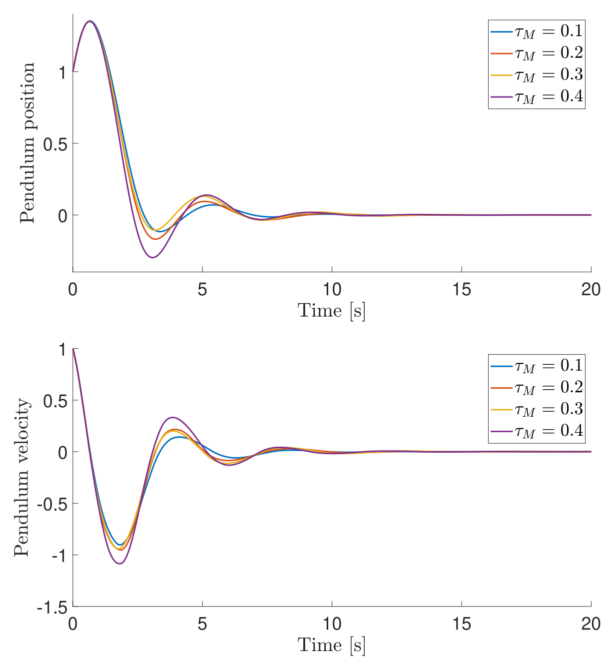

Fig. 3. Pendulum position (top) and velocity (bottom) for $h = 0.2$, $\sigma = 0.2$. Several τ_M.

the periodic event-triggering mechanism enable us to obtain inter-event times beyond the maximum theoretical sampling period while dealing with the delays. Naturally, a larger delay implies also more triggered events to compensate the effect. In Figure 3, we depict the temporal evolution of the system with initial conditions $\xi(0) = [1\ \ 1\ \ 0]$, which might be considerably affected by the delay. For example, the overshoot in pendulum position and velocity in Figure 3, respectively, is clearly enlarged when the maximum delay is larger.

V. Future Work

The application of event-triggered control to port-Hamiltonian systems opens the possibility of new research lines. On the one hand, the method can be extended to a larger class of systems, e.g., port-Hamiltonian systems with intrinsic time-delay, i.e., when the delays appear in the port-Hamiltonian system itself and not due to the communication network. Besides, once an event-triggered control framework is established, new triggering conditions, such as dynamic ones, can be tested to improve the transmission rate. Finally, the co-design of the sampling and triggering parameters and the controller is a natural extension of this framework, specially under specific constraints and/or assumptions, for instance, systems with quadratic Hamiltonian or linear port-Hamiltonian systems.

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