A column generation approach for the Single Batch-Processing Machine Scheduling Problem, with Non-identical jobs, Compatible families and Sequence-dependent setups

Castelletti Annalisa[1], Druetto Alessandro[2], Mansini Renata[1], Rener Elena[3]

[1] Dept. of Information Engineering, University of Brescia, Via Branze 38, 25123 Brescia, Italy
[2] Dept. of Computer Science, University of Torino, Via Pessinetto 12, 10149 Torino, Italy
[3] Dept. of Business Administration, University of Bern, Engehaldenstrasse 4, 3012 Bern, Switzerland

Abstract: The Single Batch-Processing Machine Scheduling Problem, with Non-identical Jobs, Compatible Families and Sequence-dependent Setups (SBPM-NCS), was recently introduced by [1] as a challenging extension of the classical single-machine parallel-batching problem [3]. Jobs can be processed simultaneously in batches, the processing time of each batch is determined by its longest job, and the objective is to minimize the makespan in presence of family compatibility constraints and sequence-dependent setup times. The mixed-integer linear formulation proposed in earlier work captures these features exactly, but becomes difficult to solve on medium- and large-size instances. To address this issue, we propose a column generation–based approach relying on an arc-flow formulation, building upon the frameworks of [2]. Feasible solutions are represented as paths in a directed multigraph, where nodes encode both the position in the sequence and the dominant family of the current processed batch. Each arc corresponds to a feasible batch and is associated with a cost accounting for both batch processing times and sequence-dependent setup contributions. The formulation involves an exponential number of variables and is solved via a Restricted Master Problem, which is iteratively enriched through column generation. The pricing problem consists in identifying batches (columns) with negative reduced cost. We derive its structure and show that it can be reformulated, for fixed positions, as a cardinality-constrained knapsack problem, where item profits are given by dual variables, while weights correspond to job sizes, and feasibility is enforced through capacity and cardinality constraints, together with family constraints. The resulting pricing problem is solved to optimality via a pseudo-polynomial dynamic programming algorithm. Preliminary computational results indicate an improvement in the initial lower bound and competitive solution quality within reasonable computational times.

Keywords: Combinatorial optimization; scheduling; parallel-batching; column generation.

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Annalisa Castelletti is a PhD student in the Department of Information Engineering at the University of Brescia. Her research focuses on combinatorial optimization for scheduling problems, including machine scheduling, batch scheduling, and dynamic rescheduling, with emphasis on exact and decomposition-based approaches, particularly column generation and MILP formulations, for large-scale and computationally complex optimization problems. During her PhD, she has been collaborating with industry on real-world scheduling problems, contributing to bridging the gap between theory and practice.