THE LEANING TOWER OF PISA: KINEMATIC LEVELLING OF THE FOUNDATION PLANE IN THE PERIOD 1993-1994

Alessandra Colombo *, Carlo Monti *, Giorgio Vassena **

* Politecnico di Milano
Dip. I.I.A.R. sezione Rilevamento
Piazza Leonardo da Vinci, 32
20133 Milano

** Università degli studi di Brescia
Dipartimento di Ingegneria Civile
Via Branze, 38
25123 Brescia

ABSTRACT

The aim of this paper is to analyse kinematically the rigid tilt of the Pisa Tower foundation plane in the period 1993-1994, when a counterweight was displaced on the North side of the basement. The kinematics is studied using a kinematic levelling approach. In the first four paragraphes the kinematic levelling method and the counterweight application history are briefly described; moreover the results obtained are briefly analysed with some closing remarks.

The eight benchmarks under study are hanged to the Tower inside walls and can give a good idea of the Tower basement deformation trend. The levelling measurements have been made daily since 14/06/1993 and this fact allows to use these informations as a valuable way to check the reliability of the electronic instruments that observe continuously the Tower behaviour.

1. INTRODUCTION

The Leaning Tower of Pisa has been subjected, since its erection in 1173, to a progressive tilt towards South, reaching nowadays the alarming value of 5° 28'; in the last five years a great concern has grown on its structural integrity due to the yearly leaning increase that, at the present, is about 5-6 second of arc'.
In view of the high risk of a structural collapse, increasing with the leaning value, an International Multidisciplinary Commission has planned to stabilise the Tower and to avoid the increasing danger. The first preliminary work was planned in the way to move the Tower in safety conditions; it consisted on placing in a provisory and removable way 6000 KNewton of lead on a concrete ring built on the North side basement of the Tower. The aim of the lead displacement was to tilt the Tower towards North, in more stable conditions, permitting to work in safely conditions during the long term stabilisation works.

2. THE LEAD COUNTERWEIGHT APPLICATION

The lead counterweight was placed on the North side of the Tower basement in the period 1993-1994 in five phases. This temporary and reversible work has been made gradually keeping the Tower under constant monitoring. These five loading steps were planned and realised with a period of span stabilisation time planned after each of them.

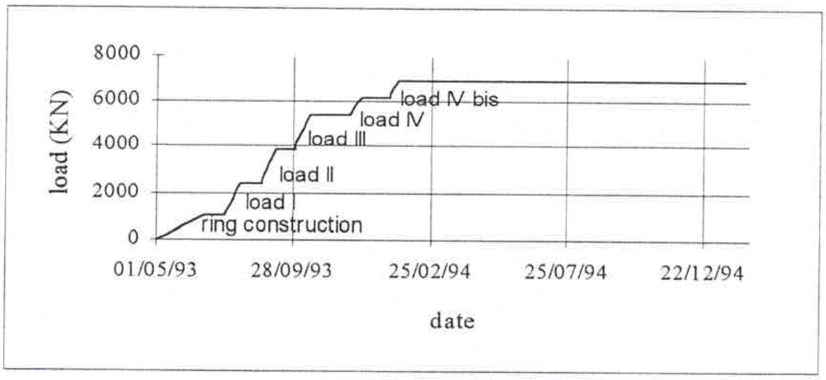

fig. 1. The counterweight displacement history

Figure 1 shows the loading history between 6/05/1993 and 21/01/1994. The counterweight was placed on a prestressed concrete platform built all around the Tower basement, in order to support the lead. After 21/01/1994 the total weight of the lead was about 6000 KN, corresponding to the 4.8% of the Tower total weight.

3. THE TOWER MOVEMENTS MONITORING

The counterweight displacement is the main origin of the deformations of the foundation plane and the main cause of the entire Tower rotation towards North in the period here studied. The Tower movements are checked using different kinds of methods; some of them control only the elevation part of the monument, some others observe only the basement deformations.

Historical studies [7] have shown that the upper part of the Tower (over the basement) is subjected to a well observable deformation due to the environmental conditions like wind, temperature changes, groundwater oscillations, sun position and so on. On the other hand the basement of the Tower is much less influenced by these effects. For this reason, in order to study only the movements due to the counterweight loading, becomes very interesting to analyse the Tower basement behaviour.

In 1911 fifteen levelling benchmarks were placed on the external base of the Tower; nowadays they are a part of a wider high precision spirit levelling net placed all around the Tower and on the surrounding Square. In 1993 nine new benchmarks, shown in figure 2, were placed in the inside part of the Tower and fixed to the stone walls. This small levelling net, composed by 30 centimetres long invar rods, can be easily measured also several times a day. For this reason it was decided to measure this network at least ones a day, at 9.00 a.m.. During the loading periods these measurements were fitted reaching the number of 7-8 per day. Due to their position, these benchmarks are practically not effected by environmental changes and in this way they are suitable to describe the evolution of the Tower basement rigid tilt.

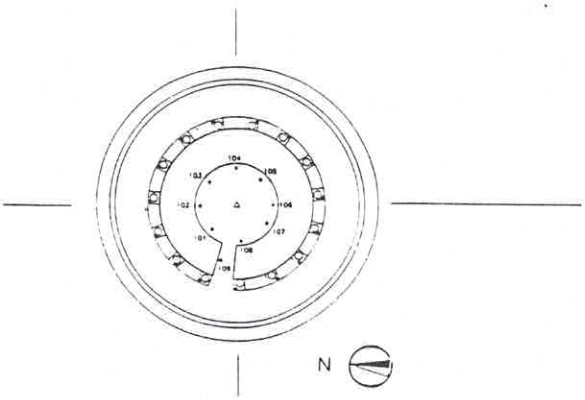

fig. 2. The position, inside the Tower, of the levelling net benchmarks.

4. KINEMATIC LEVELLING METHOD

In order to better understand the behaviour of the Tower basement it was decided to analyse also the kinematic trend of the levelling benchmarks. This study can easily show the different deformation speeds and movements of the different benchmarks after each lead displacement.

The kinematic levelling is a method used to obtain kinematic informations about vertical movements, from the measurements of height difference done during several levelling campaigns of the same network. By a kinematic model that links, in a motion equation for each benchmark, high and motion, it is possible to make a reconstruction of the vertical movements for the entire net. When the number of time of measurement doesn't permit to make a time series analysis, it is possible to built a functional model by a polynomial, function of opportune degree of time t, to describe the vertical shift of the benchmarks. After the adjustment, the parameters of the model can be analysed as samples from a realisation of a stationary stochastic process. If the vertical shifts of the network are regular they can be modelled by a polynomial function of time like this:

$$H_i = H_0 + c_1 \cdot \Delta t + \frac{1}{2!} c_2 \cdot \Delta t^2 + \frac{1}{3!} c_3 \cdot \Delta t^3 + \dots \qquad (4.1)$$

where the symbols meaning is:

H_i = height at time t_i $\quad$ H_0 = height at time t_0 $\quad$ Δt_i = spantime
c_1 = speed at time t_0 $\quad$ c_2 = acceleration at time t_0

The parameters c_i with $i > 2$, have not physical meaning but can be interesting in a statistic analysis.

By this model, done for each benchmark, the height difference between a couple of benchmarks (A and B) at time i is modelled in such a way:

$$\Delta H_{A,B;i} = H_{B;i} - H_{A;i} = H_{B;0} - H_{A;0} + \frac{1}{j!}\sum_{j=1}^{m_B} c_{jB} \cdot \Delta t_i^j - \frac{1}{j!}\sum_{j=1}^{m_A} c_{jA} \cdot \Delta t_i^j \qquad (4.2)$$

The kinematic levelling, that makes a model for the movements of the net by a polynomial, works in the right way only during spans time without counterweight changes when vertical shifts can be considered regular. If, during the span time, there are some changes in the lead displacement or weight, it is necessary to split the total gap in several spans.

All the height difference measurements in each span time are processed in only one adjustment; in this way all the parameters are estimated only ones for each span. The estimation of H_0, H_i and c_i for each benchmark and in each span time is done by a least squares adjustment using this *FUNCTIONAL MODEL*:

$$l_i + v_i = A_{h,i} \cdot h_i + A_{1,i} \cdot c_1 + A_{2,i} \cdot c_2 + \dots + A_{J,i} \cdot c_j \qquad (4.3)$$

and this *STOCHASTIC MODEL:*

$$\Sigma_{ll,i} = \sigma_0^2 \cdot Q_{ll,i} \qquad (4.4)$$

where the symbol meaning is:

l_i = observation vector v_i = residue vector $A_{....i}$ = design matrix h_i = height vector
c_i = parameter vector $\Sigma_{ll,i}$ =covariance matrix $Q_{ll,i}$ = cofactor matrix σ_0^2 = variance factor

The reference system is chosen taking at the start time zero, at least one constrained benchmark (at high arbitrary chosen); this is like taking its height constant and the others motion parameters equal to zero during the span time.

5. KINEMATIC LEVELLING OF INSIDE NETWORK MEASUREMENTS

The period analysed, from 14/06/1993 to 03/02/1995, is composed by 588 measurement dates of the levelling network placed inside the Tower. The measurements, taken every day with a Zeiss Ni002 level and composed by 8 level measurements connecting 8 benchmarks, have been already adjusted using a classical least squares approach obtaining a 0,01 mm medium r.m.s. The Kinematic adjustment is carried on using the program *Level* (Politecnico di Milano) [4], [5], [6] taking the benchmark 101 constrained. Program *Level* asks as input two files, the first with the height observations and the second with the benchmark of the network and the steering parameters. The first file contains, for each observed height difference: the two benchmark names, the observed height difference, the value of σ_0^2 , the length of the levelling segment, the time of measurement and the group to which the observation belongs (when variance factors have to be estimated for group of measurements).

The second file contains: the benchmark name, a parameter to define if the benchmark is a constrained one or not, the value of preliminary height and the degree of the polynomial. This program gives in output point by point and for each span time: the values of the height, of the motion parameters and of their r.m.s..

Using the program it is necessary at first to choose the right degree of the polynomial equation, as described in par. 4.

The second step consists on dividing the whole time studied in several spans of time; in each of these the deformations must be considerable as regular. These choices can be done processing the measurements in different combinations and looking for the less r.m.s. of the parameters. For these reasons several tests were carried on, before the data processing of the measurements coming from the Leaning Tower inside levelling network. In this way, it was defined that the best choice was the use of 1st degree polynomials. Therefore it was decided to divide the total measurements time in 11 spans time, so to make the model of the Tower behaviour in each loading step and in each break between two loadings. The final 1st degree polynomial model used to describe the motion of each benchmark *j* in the span time *i* appears as follows:

$$H_{j,i} = H_{j,0} + a_{j,i} \cdot (t_i - t_0) \tag{4.5}$$

where the parameter $a_{j,i}$, constant during the all span time *i*, is the speed of the benchmark *j*.

The second step of work is to process the entire data base with the combination work above described to estimate all the parameters.

6. RESULTS OF KINEMATIC LEVELLING

The Kinematic study of the Tower behaviour during the counterweight loadings is extremely important for the link between the counterweight displacement and the Tower motions.
The results of the kinematic analysis are shown in table 1, span time by span time, for the benchmarks 102 and 106 that are considered the most interesting ones for their position. 102 is placed in the North side and 106 in the South, that means along the direction on the Tower leaning.
Looking at table 1 it is necessary to remember that the speed is taken positive if the benchmark sink and that the movements and the speed are referred to point 101, taken as constrained benchmark, having for this reason speed and movements always equal to zero. The difference of level between 102 and 106 it is obtained as a result of the kinematic levelling analysis. The results show that after the first loads the Tower basement has immediately a quick reaction. After the first load the benchmarks are characterised by a non regular speed; from LOAD II the benchmarks speed is always more regular, that means a more coherent reaction of the ground to the lead displacement.

BENCHMARK	*H_{102}-H_{106} (mm)*	*SPEED ($mm10^{-2}/day$)*	*SPAN TIME*	*PERIOD*
102	106,44	0,07	BREAK 0	from 14/06/1993 to 13/07/1993
106		0,50		
102	106,32	0,18	LOAD I	from 14/07/1993 to 30/07/1993
106		1,06		
102	106,01	0,04	BREAK I	from 31/07/1993 to 23/08/1993
106		0,13		
102	106,05	-0,13	LOAD II	from 24/08/1993 to 08/09/1993
106		1,50		
102	105,73	-0,14	BREAK II	from 09/09/1993 to 29/09/1993
106		0,10		
102	105,71	0,00	LOAD III	from 30/09/1993 to 15/10/1993
106		0,44		
102	105,59	-0,05	BREAK III	from 16/10/1993 to 26/11/1993
106		0,02		
102	105,55	-0,50	LOAD IV	from 29/11/1993 to 10/12/1993
106		0,33		
102	105,40	0,00	BREAK IV	from 11/12/1993 to 10/01/1994
106		0,00		
102	105,36	0,10	LOAD IV BIS	from 11/01/1994 to 20/01/1994
106		1,00		
102	104,75	0,09	BREAK IV BIS	from 21/01/1994 to 03/02/1995
106		-0,43		

Table 1

A similar analysis can be done in the direction East-West using benchmarks 104 and 108 (104 on East side and 108 on West side). Table 2 shows the results:

BENCHMARK	H_{104}--H_{108} *(mm)*	*SPEED* *(mm10⁻² /day)*	*SPAN TIME*	*PERIOD*
104	13,19	0,33	BREAK 0	from 14/06/1993 to 13/07/1993
108		0,30		
104	13,22	-0,12	LOAD I	from 14/07/1993 to 30/07/1993
108		0,53		
104	13,35	0,08	BREAK I	from 31/07/1993 to 23/08/1993
108		0,50		
104	13,24	0,25	LOAD II	from 24/08/1993 to 08/09/1993
108		0,50		
104	13,17	-0,10	BREAK II	from 09/09/1993 to 29/09/1993
108		-0,14		
104	13,16	0,13	LOAD III	from 30/09/1993 to 15/10/1993
108		0,13		
104	13,21	0,05	BREAK III	from 16/10/1993 to 26/11/1993
108		0,02		
104	13,14	0,33	LOAD IV	from 29/11/1993 to 10/12/1993
108		0,25		
104	13,06	0,03	BREAK IV	from 11/12/1993 to 10/01/1994
108		0,00		
104	13,06	0,70	LOAD IV BIS	from 11/01/1994 to 20/01/1994
108		0,30		
104	13,07	-0,18	BREAK IV BIS	from 21/01/1994 to 03/02/1995
108		-0,24		

Table 2

7. THE LEANING TOWER BEHAVIOUR

Taking as true the hypothesis of the Tower basement as rigid body, it is possible to study the tilt movements from the vertical speed informations and to analyse the movements coming from the benchmarks placed in the directions North-South and East-West. In such a way the analysis of the tilt speed is done separately for the two components North-South and East-West. The data are taken from the vertical speeds of the couples of opposite benchmark (N-S) and (E-W) and from their distances. By using of the vertical speeds of benchmarks 106 and 102 and of their distance, the tilt speed of the Tower basement in the North-South direction is obtained as follow:

$$\vartheta = \arctan(\frac{a_{106} - a_{102}}{\text{distance}})(7.1)$$

In table 3 the Tower basement tilt speeds towards North and East are shown.

SPAN TIME	*SPEED OF TOWER RIGID TILT TOWARD NORTH ("/day)*	*SPEED OF TOWER RIGID TILT TOWARD EAST("/day)*	*LOADING COUNTERWEIGHT (KN)*
BREAK 0	0,12	0,01	1020
LOAD I	0,25	0,08	2374
BREAK I	0,04	-0,18	2374
LOAD II	0,45	-0,07	3855
BREAK II	0,07	0,01	3855
LOAD III	0,12	0,02	5366
BREAK III	0,02	-0,05	5366
LOAD IV	0,23	-0,16	5997
BREAK IV	0,00	0,01	5997
LOAD IV BIS	0,25	0,11	6896
BREAK IV BIS	-0,14	0,02	6896

Table 3

The figures 3 and 4 show how the two components of speed (respectively in N-S and E-W direction) of the Tower change increasing the total applied counterweight.

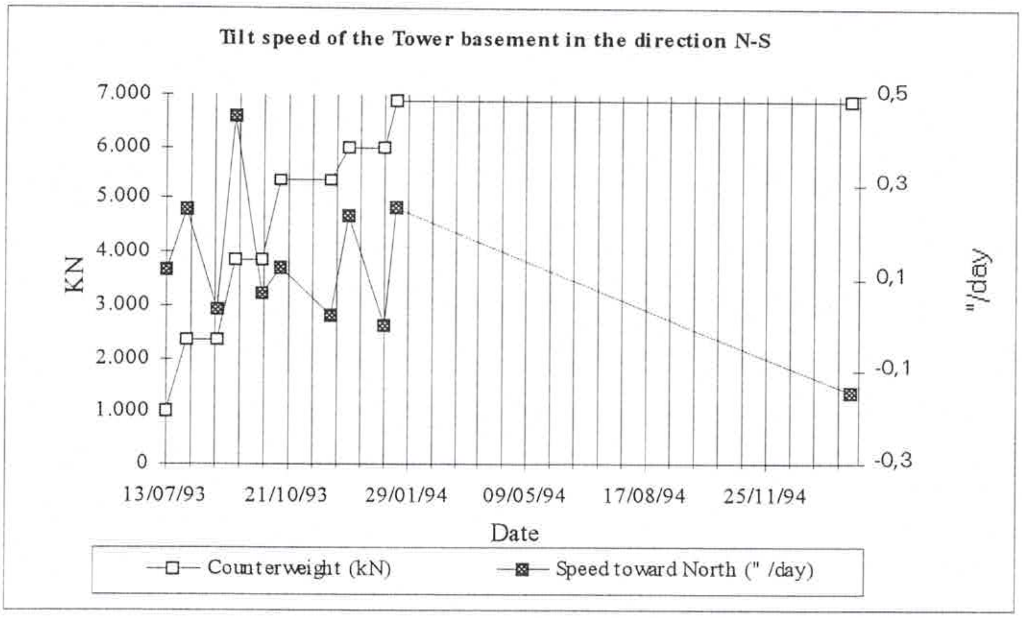

fig. 3. The speed of tilt towards North and the counterweight displacement history.

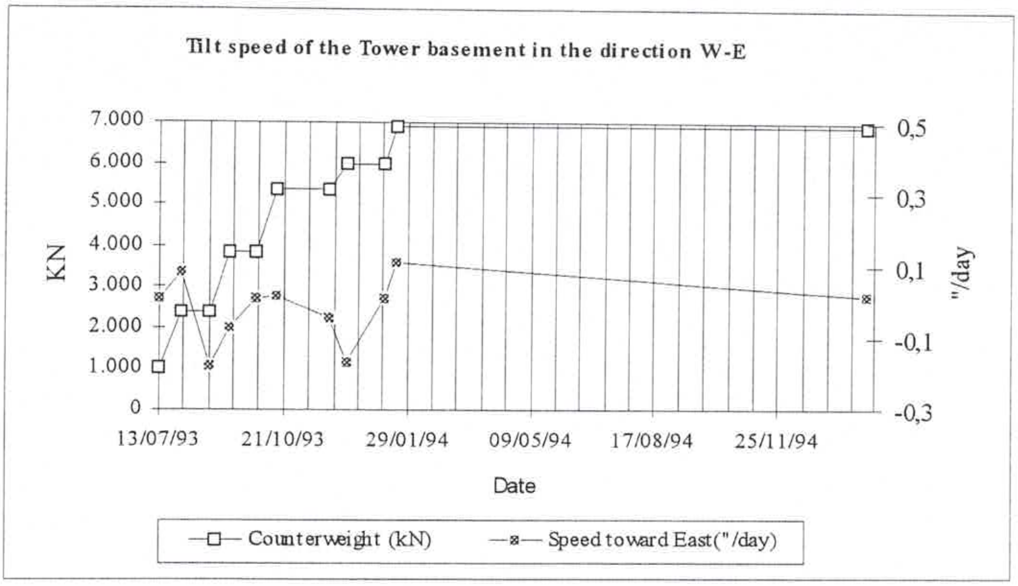

fig.4. The speed of tilt towards East and the counterweight displacement history.

The Tower behaviour in the two different directions N-S and W-E is quite different. In the N-S direction the speeds are much higher and this is a natural consequence of the load positioning on the North side. While in the direction N-S there is an important movement towards North, that decreases the original Tower leaning component (that is about 5° 28' towards South), in the direction E-W the Tower swings between East and West. In E-W direction the motion is like an oscillation and it doesn't change the little originally E-W Tower leaning. For all these reasons (important tilt movements and original mean direction of the Tower leaning) the N-S direction seems at the moment much more interesting than the W-E one. In the N-S direction each loading step involves important increases of tilt speed, whereas during the breaks span time important decreases of speed are present.

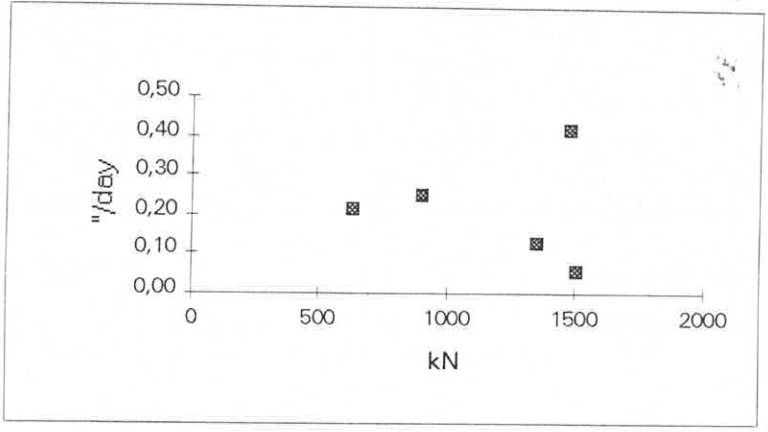

fig. 5. The relation between counterweight and tilt speed increases.

It is not easy to define a model that links vertical speed and counterweight displacement.

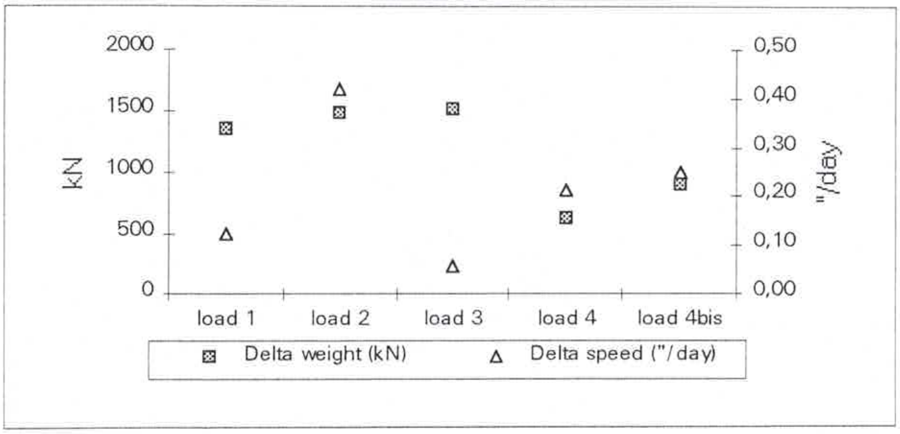

fig. 6. The link between weight and speed increases, loading step by loading step.

Figure 6 shows how changes, step by step, the link between weight and speed increases. At the moment it isn't possible to define any kind of relationship between these two parameters, but in order to find one vertical speeds of all benchmarks are compared span time by spine time, looking for a congruence between all the values.

During loading steps the speeds are quite high but they change regularly along the Tower basement and this means that there are not important deformations in the basement in the period studied but only a tilt towards North of the entire basement (see figure 7). Only the first loading step involves some deformations inside the basement, in fact benchmark

103 has vertical speed very different from benchmarks 104 and 102 next to it. The irregular values for the vertical speed are probably due to a local movement of the stones of the inside wall of the Tower where the invar rods are hanged and to a first redistribution of the Tower basement loads on the foundation soil. Anyway during the loading steps there is an important component of tilt rowards North of the entire basement, while the component in the W-E direction is very small.

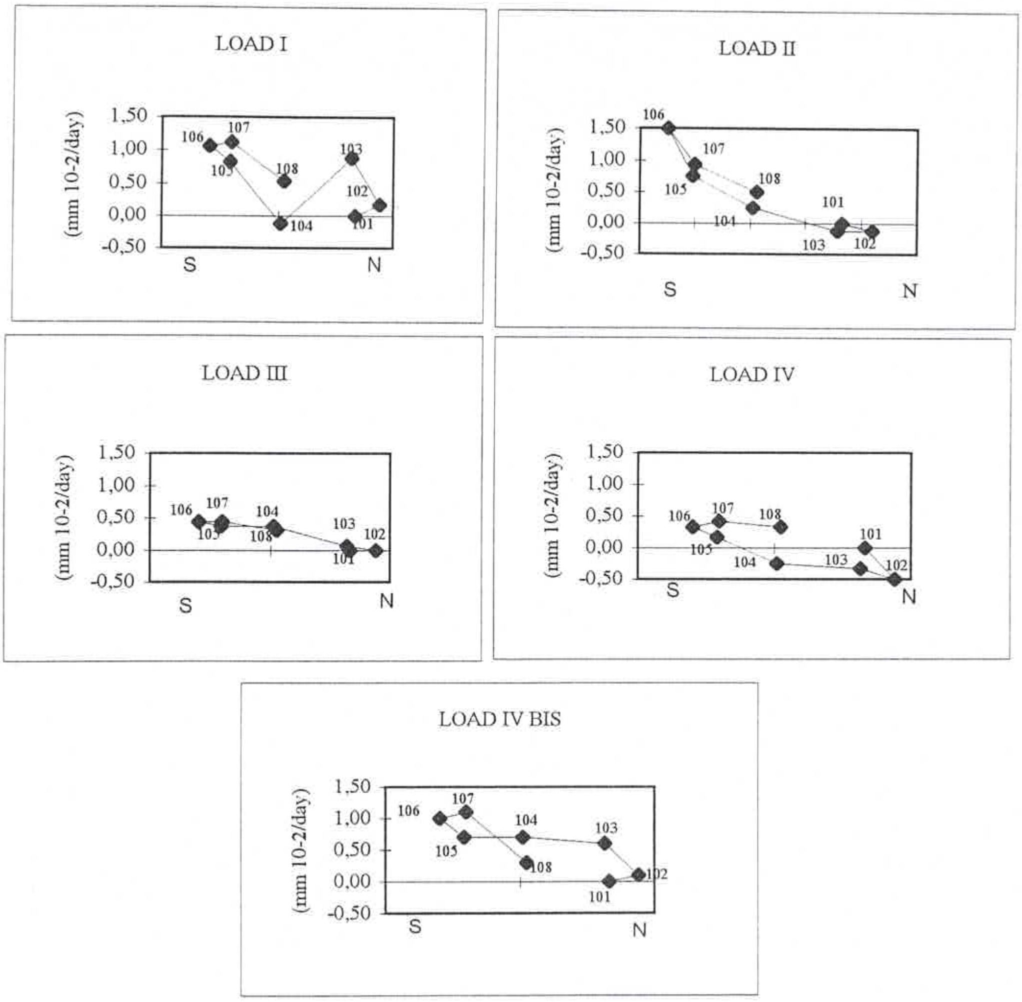

fig. 7. Vertical speeds during loading steps

On the contrary, during break steps, vertical speeds are very small (see figure 8) and this means that the component of movement due to the increases of the counterweight applied go quickly to zero; all these little vertical speeds are settlement movements of the basement due to the previous counterweight increase. It is very important to notice that during breaks all the vertical speeds are quite regular and this means that there are not important deformations inside the basement during break times (except break 0) when benchmark 108 has a vertical speed very different from benchmarks 107 and 101 next to it.

During the breaks span time the W-E component of the Tower tilt is similar to the value in N-S direction.

This analysis shows that the hypothesis of rigid body for the Tower basement in the period 14/06/1993 -03/02/1995 is quite right at least disregarding few local movements.

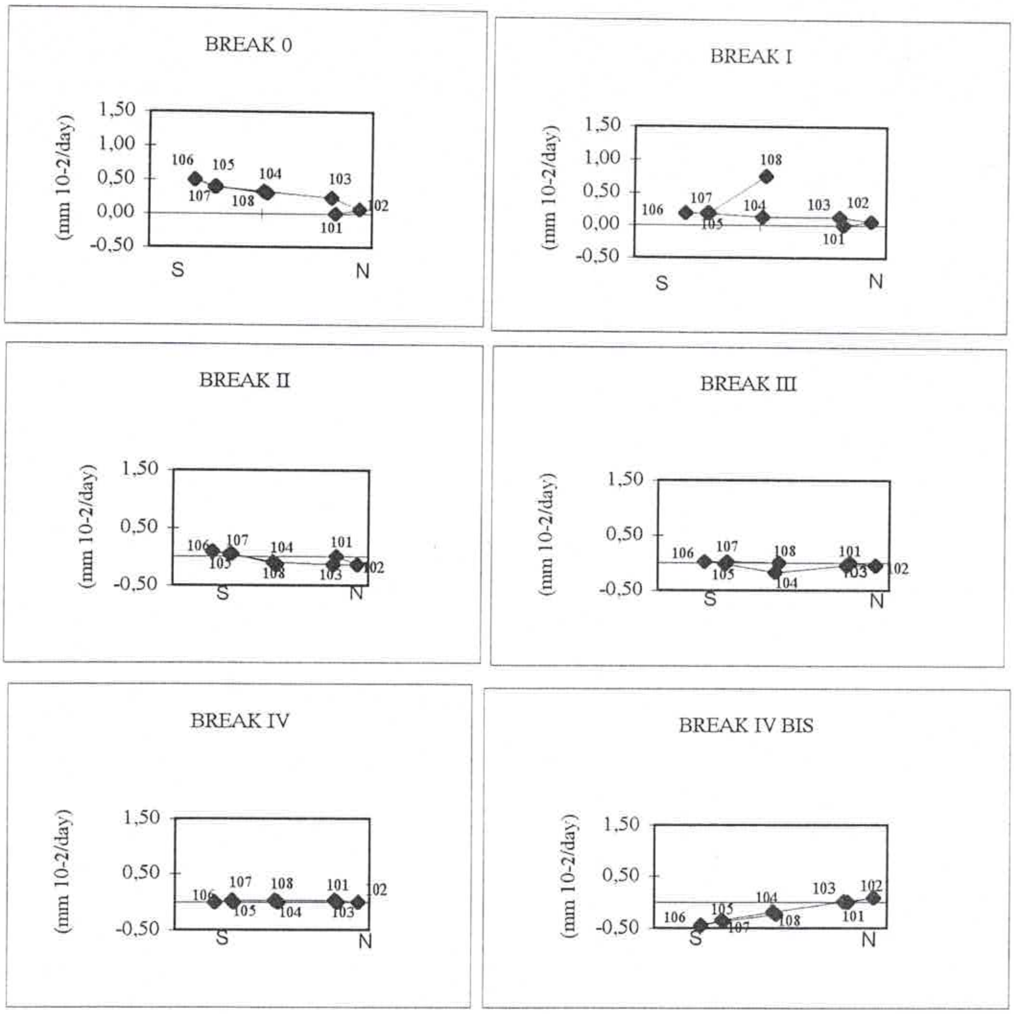

fig. 8. Vertical speeds during break steps.

7. CLOSING REMARKS

By a kinematic levelling it is possible to make a very simple model of the Tower behaviour.

This model is easy to make, to use and describes in a right way the Tower motions under the counterweight loading. Since a soil geotechnical model is very difficult to do, this model even if doesn't explain the geotechnical problems, is very important and useful to understand what happens to the Tower under the counterweight displacements and to plan next works. In particular important informations are obtained about the soil reaction to the stabilisation works; additionally these data can be later used for geothecnical soil analysis.

A result of the kinematic levelling method, is the observation that the Tower basement, after a first not regular reaction (LOAD 0), show a quite regular deformation speed. This means that until now aren't present dangerous discontinuites in the deformation process. This analysis show that the hypotesis to consider the counterweight displacement just as a temporary and not resolutive remedy is almoust right. In fact the total counterweight produced at the beginning a decrease of the Tower leaning but it doesn't changes the Tower characteristic trend (regular increase of leaning toward South); the results show that the counterweight causes rapid tilt towards North only during the short period after the loading displacement. Later the tilt speed decreases until zero and then Tower begins again to move with its characteristic trend of low tilt speed towards South.

REFERENCES

[1] A. Colombo: "Analisi delle deformazioni della Torre di Pisa dalla data della sua fondazione a oggi", Tesi di laurea in Ingegneria Civile per la Difesa del Suolo e la Pianificazione del Territorio, Politecnico di Milano, facoltà di Ingegneria, anno accademico 1993/1994.

[2] L. Colombo, G. Fangi, L. Mussio, F. Radicioni: "Kinematic processing and spatial analysis of levelling control data of the Ancona '82 landslide", Proceeding of the symposium on height determination and recent crustal movements in Western Europe, Hannover, 15-17 September 1986.

[3] D. Costanzo, M. Jamiolkowski, R. Lancellotta, M.C. Pepe: "Leaning Tower of Pisa: Description of the behaviour", Invited Lecture-Settlement '94, Texas A&M University, 1994.

[4] A. De Haan, G. Forlani: "The system of Program LEVEL for Kinematic Levelling Adjustments", Bollettino di Geodesia e Scienze affini, year XLVIII, n° 4, Ottobre-Novembre- Dicembre, 1989.

[5] A. De Haan, G. Forlani: " Kinematic Levelling Adjustments with Polynomials and Polynomial Splines", Bollettino di Geodesia e Scienze Affini, year XLVIII, n° 3, Luglio- Agosto- Settembre, 1989.

[6] G. Forlani, L. Mussio. "The Kinematic Adjustment of levelling networks", Bologna, ed. Unguendoli, 1989.

[7] C. Monti, G. Vassena: "The leaning tower of Pisa: the geodetic approach to the control of the deformations", Ist Turkish International Symposium on Deformations, 5-9 September 1994, Istanbul, 1994.

[8] Politecnico di Milano - Dipartimento di Ingegneria Idraulica, Ambientale e del Rilevamento, Comitato per gli intervento di consolidamento e restauro della Torre di Pisa. "Supervisione ed interpretazione dei rilievi geodetici - topografici nella Piazza dei Miracoli e sulla Torre di Pisa", Milano, 1994.