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Generalized Haalman tuning of PIDA controllers ⋆

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Abstract: In this paper we propose a new tuning methodology for Proportional-Integral-Derivative-Acceleration (PIDA) controllers. First, a third-order-plus-dead-time transfer function of the (high-order) process is estimated by evaluating an open-loop step response. Then, the three time constants of the controller are determined by applying a pole-zero cancellation approach and the proportional gain is finally adjusted in order to obtain a desired maximum sensitivity. The controller filters are selected in order to minimize the effects of the measurement noise and to avoid kicks in the control variable. Simulation results show the effectiveness of the methodology in comparison with PID control.

Keywords: PIDA controllers, PID controllers, Tuning, Robustness.

1. INTRODUCTION

Nowadays there is a more and more significant research effort in Proportional-Integral-Derivative-Acceleration (PIDA) controllers, also known as Proportional-Integral-Double-Derivative controller (PIDD or PIDD2) (Jung and Dorf, 1996b,a; Huba and Vrancic, 2018; Kumar and Hote, 2018, 2019; Ferrari and Visioli, 2022). This is because the addition of a control action proportional to the double derivative (acceleration) of the control error to the classic Proportional-Integral-Derivative (PID) control law can yield a relevant performance improvement for high-order systems (Milanesi et al., 2022) or for integral processes (Huba, 2019; Huba et al., 2021; Visioli and Sanchez-Moreno, 2023; Bistak et al., 2023; Huba et al., 2023a,b,c,d).

However, in order for PIDA controllers to be a valid alternative for Proportional-Integral-Derivative (PID) controllers, the same ease of use should be ensured, as the great advantange of PID controllers is that they are relatively simple and that the physical meaning of the controller parameters is well known (Ferrari and Visioli, 2022). Further, there exists a wide variety of tuning rules that facilitate their design (O'Dwyer, 2006). In particular, these tuning rules are typically based on a process model that can be obtained by means of a simple and cost-effective experiment.

With the same rationale, design methodologies for PIDA controllers have also been proposed. For example, in (Visioli and Sanchez-Moreno, 2024) an Internal Model Control (IMC) approach has been employed starting from a high-order process model estimated through the *n*-shifting technique based on a relay-feedback experiment (Sanchez et al., 2021).

In this paper we further pursue this goal by proposing a new tuning procedure for PIDA controllers that can be considered a generalized Haalman tuning. Starting from a Third-Order-Plus-Dead-Time (TOPDT) process model determined by applying the technique presented in (Sanchis and Peñarrocha-Alós, 2022), a pole-zero cancellation like in the Haalman tuning method (Åström and Hägglund, 2006) is employed in order to determine the time constants of the controller. Then, differently from the Haalman method for which the gain of the controller is found in order to achieve a fixed value of the phase margin, here the gain of the controller is determined in order to achieve a desired value of the maximum sensitivity, which is considered a better measure of the system robustness. In this way, the selected value of the maximum sensitivity is a tuning knob that allows the user to handle the trade-off between performance and robustness (and control effort). In this context, the filters in the controller are also suitably designed in order to make the controller proper, to avoid detrimental effects of the measurement noise on the actuator and to avoid the derivative and acceleration kicks.

It is worth stressing at this point that pole-zero cancellation is generally not recommended for lag-dominant processes as this results in very sluggish load disturbance step responses. However, we have to take into account that a PIDA controller is advantageous with respect to a PID controller for high-order processes and in this case the (apparent) dead time is not much smaller than the dominant time costant, making the pole-zero cancellation strategy appropriate.

The paper is organized as follows. The problem is formulated in Section 2 and the tuning procedure is explained in Section 3. Simulation results showing the performance achieved by PIDA controllers, including a comparison with PID controllers, are in Section 4. Finally, concluding remarks are in Section 5.

2. PROBLEM FORMULATION

The considered control scheme is the standard feedback control system of Figure 1, where P is the (high-order) process, C is the PIDA controller, r is the reference signal, y is the process

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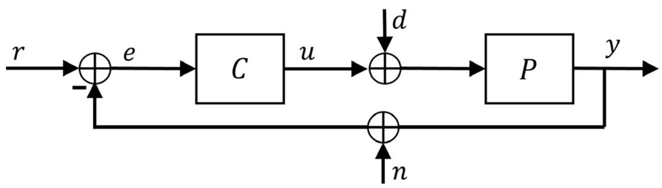

Fig. 1. The considered control scheme.

variable, d is the load disturbance signal, n is the measurement noise signal, e is the control error and u is the control variable. The controller is of PIDA type. For the purpose of tuning its parameters, the controller transfer function can be expressed as

$$C(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s + T_a s^2\right) \tag{1}$$

where K_p is the proportional gain, T_i is the integral time constant, T_d is the derivative time constant and T_a is the acceleration (double derivative) time constant.

It appears that the transfer function (1) is improper and therefore is not realizable. Thus, filters on the derivative and acceleration action can be added, yielding the following transfer function that can be realized:

$$C(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{N}s + 1} + \frac{T_a s^2}{\left(\frac{T_a}{M}s + 1\right)^2}\right) \tag{2}$$

Note that filtering the derivative and acceleration actions allows the reduction of the effects of the measurement noise on the control variable (and therefore on the actuator) and the presence of derivative and acceleration kicks, which would also be detrimental for the actuator.

The methodology proposed in this paper aims to determine all the controller parameters in order to have a satisfactory performance in both the load disturbance rejection and set-point following tasks. Further, a user-chosen maximum sensitivity, which is a measure of the control system robustness, should be obtained. It is defined as

$$M_s := \max_{\omega \in [0,+\infty)} \left| \frac{1}{1 + C(j\omega)P(j\omega)} \right|, \tag{3}$$

and typical values are in the range between 1.4 and 2.0 (Åström and Hägglund, 2006).

3. METHODOLOGY

The tuning methodology starts with the determination of a TOPDT model of the process by means of the technique proposed in (Sanchis and Peñarrocha-Alós, 2022), which consists of evaluating an open-loop step response and determining the time instants t_5, t_{35} and t_{85} when the output, respectively, achieves 5%, 35% and 85% of its final value. Then, the following model is obtained:

$$\hat{P}(s) = \frac{K}{(1+\tau s)\left(1 + \frac{(1-\alpha)L}{2}s\right)^2} e^{-\alpha L s} \tag{4}$$

where

$$\begin{aligned} L &= 1.3t_{35} - 0.29t_{85} \\ \tau &= 0.67(t_{85} - t_{35}) \\ \alpha &= 0.598 + 0.4799\frac{t_5}{L} - \frac{0.41}{\left(\frac{t_5}{\tau}\right)^{0.6}} \end{aligned} \tag{5}$$

and the gain K can be easily estimated as the ratio between the steady-state variation of the output and of that of the input. The transfer function (4) can be rewritten as

$$\hat{P}(s) = \frac{K}{a_3 s^3 + a_2 s^2 + a_1 s + 1} e^{-\alpha L s} \tag{6}$$

where

$$\begin{aligned} a_3 &= L^2 \tau \left(\frac{\alpha}{2} - \frac{1}{2}\right)^2 \\ a_2 &= L^2 \left(\frac{\alpha}{2} - \frac{1}{2}\right)^2 - 2L\tau\left(\frac{\alpha}{2} - \frac{1}{2}\right) \\ a_1 &= \tau - 2L\left(\frac{\alpha}{2} - \frac{1}{2}\right) \end{aligned} \tag{7}$$

Now, considering the controller transfer function (1), the integral, derivative and acceleration time constants can be determined by applying a pole-zero cancellation, yielding

$$\begin{aligned} T_i &= a_1 \\ T_d &= \frac{a_2}{a_1} \\ T_a &= \frac{a_3}{a_1} \end{aligned} \tag{8}$$

Then, the proportional gain K_p can be determined by considering the controller transfer function (2) with $M = N = 10$ (which are typical values for industrial controllers) and by then setting its value in order to obtain the desired maximum sensitivity. This can be achieved by means of a very simple numerical procedure, where the value of K_p can be increased until the required value of M_s is achieved.

Finally, in order to better filter the measurement noise, an additional low-pass filter is applied to the controller output (Visioli and Sanchez-Moreno, 2024). It is defined as

$$F(s) = \frac{1}{\left(\frac{0.05}{\omega_{gc}}s + 1\right)^n} \tag{9}$$

where $n = 2$ and ω_{gc} is the gain crossover frequency of the loop transfer function $C(s)\hat{P}(s)$, that is the frequency for which $|C(j\omega_{gc})P(j\omega_{gc})| = 1$. It appears that it is a second-order filter with a cut-off frequency that is 20 times larger than the system bandwidth, so that the measurement noise is filtered without introducing a significant phase lag in the control system.

Remark 1. Although the presence of the filters makes the pole-zero cancellation not exact, the selected values of M and N, which place the filters at high frequency, do not impair the soundness of the approach. In fact, the exactness of the pole-zero cancellation is sacrificed in order to achieve the desired robustness.

Remark 2. The Haalman method for PID controller consists in determining the integral and derivative time constants by applying a pole-zero cancellation to a SOPDT process transfer function and then in selecting the proportional gain in order to obtain a phase margin equal to 52 degrees. Considering a desired value of the maximum sensitivity instead of a fixed value of the phase margin can therefore be seen as a generalization of the methodology.

4. SIMULATION RESULTS

In the following illustrative examples we evaluate the proposed PIDA tuning procedure by considering the control system response to a unit step set-point signal and to a unit step load disturbance signal. In both cases measurement noise has been added to the feedback signal (see Figure 1). In particular, the signal n consists of a random value in the range [-0.1,0.1], thus equal to the 10% of the step amplitude. The performance of the PIDA controller has been compared with that obtained by a PID controller with a transfer function

$$C(s) = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{N}s + 1}\right). \tag{10}$$

In order to provide a fair comparison, the tuning of the PID controller has been performed by applying an approach that is analogous to that applied for the PIDA controller. In particular, a method based on the evaluation of the open-loop step response is employed in order to estimate a Second-Order-Plus-Dead-Time (SOPDT) transfer function (Veronesi et al., 2024). Then, pole-zero cancellation (by neglecting the filter on the derivative action) is used to determine the values of T_i and T_d. Subsequently, $N = 10$ is fixed and the value of the proportional gain is found in order to achieve the desired maximum sensitivity. Finally, in order to filter the measurement noise, the low-pass filter (9) with $n = 1$ is applied to the PID output. As for the PIDA controller, the cut-off frequency is selected 20 times higher than the gain crossover frequency of the loop transfer function.

4.1 Example 1

As a first illustrative example we consider the fourth-order process

$$P_1(s) = \frac{1}{(s+1)^4} \tag{11}$$

The evaluation of the open-loop step response yields the TOPDT model ($K = 1$, $L = 2.159$, $\alpha = 0.382$, $\tau = 2.037$)

$$\hat{P}_1(s) = \frac{1}{0.905s^3 + 3.159s^2 + 3.37s + 1} e^{-0.826s} \tag{12}$$

Based on (12), the three time constants of the PIDA controller results as $T_i = 3.37$, $T_d = 0.938$, and $T_a = 0.269$. Then, after having fixed the desired maximum sensitivity as $M_s = 1.4$ and after having fixed $M = N = 10$, the proportional gain is determined as $K_p = 1.31$. The gain crossover frequency of the loop transfer function results to be $\omega_{gc} = 0.39$, so that we have

$$F(s) = \frac{1}{(0.13s+1)^2} \tag{13}$$

The results related to the unit set-point step response are shown in Figure 2 (left) where the process variable and the control variable are plotted. Note that the control variable is plotted with a different time scale in order to better highlight the transient response. In the same figure, the results obtained with a PID controller are also shown. In this case the SOPDT model is

$$\hat{P}_1(s) = \frac{1}{2.062s^2 + 2.875s + 1} e^{-1.12s} \tag{14}$$

which yields $T_i = 2.88$, $T_d = 0.72$, and $K_p = 0.91$ (note that M_s is again equal to 1.4). Then, $N = 10$, $\omega_{gc} = 0.39$ and

$$F(s) = \frac{1}{0.158s+1} \tag{15}$$

It appears that the PIDA controller provides a better performance. This is confirmed by the values of the integrated absolute error (IAE), defined as

$$IAE = \int_0^\infty |e(t)|dt, \tag{16}$$

which results to be $IAE = 3.31$ for the PIDA controller and $IAE = 3.88$ for the PID controller. The improvement on the IAE is paid with a higher maximum value of the control variable, but it can be observed that there are no proportional, derivative and acceleration kicks and the noise is filtered very effectively. It is worth noting that the filter on the PID controller is also essential to filter the noise.
The load disturbance step response is also shown in Figure 2 (right). Also in this case the PIDA controller performs better than the PID controller, without a significant increment of the control effort and by keeping the noise level in the control signal at a reasonable level. The IAE results to be $IAE = 3.01$ for the PIDA controller and $IAE = 3.59$ for the PID controller.
The case with $M_s = 2.0$ is then evaluated. The proportional gains result to be modified as $K_p = 2.48$ for the PIDA controller and $K_p = 1.69$ for the PID controller. These values yields a gain crossover frequency $\omega_{gc} = 0.78$ for the PIDA controller and $\omega_{gc} = 0.60$ for the PID controller. Results are shown in Figure 3, in the left part for the set-point step response (note that, again, the control variable has been plotted in a different time scale) and in the right part for the load disturbance step response. As expected, the PIDA controller is more aggressive than in the previous case, which implies that higher values of the control variable and of the noise are obtained. However, also in this case the PIDA controller reduces the IAE with respect to the PID controller (from $IAE = 3.36$ to $IAE = 2.79$ for the set-point following task and from $IAE = 2.39$ to $IAE = 1.96$ for the load disturbance rejection task). It is therefore confirmed that the desired value of M_s represents a tuning knob that allows the user to manage the trade-off between performance and control effort (as well as between aggressiveness and robustness).

4.2 Example 2

In the second example we consider a eighth-order process (again with multiple poles)

$$P_2(s) = \frac{1}{(s+1)^8}, \tag{17}$$

which is approximated by the following TOPDT system ($K = 1$, $L = 5.51$, $\alpha = 0.61$, $\tau = 2.87$)

$$\hat{P}_2(s) = \frac{1}{3.304s^3 + 7.304s^2 + 5.01s + 1} e^{-3.36s}. \tag{18}$$

The pole-zero cancellation approach yields $T_i = 5.01$, $T_d = 1.46$, and $T_a = 0.659$. If the desired maximum sensitivity is $M_s = 1.4$, the resulting value of the proportional gain is $K_p = 0.55$. This yields a gain crossover frequency $\omega_{gc} = 0.11$ and therefore the output filter $F(s)$ is selected as

$$F(s) = \frac{1}{(0.46s+1)^2} \tag{19}$$

Process and control variables related to set-point and load disturbance step signals are shown in Figure 4, where a comparison with a PID controller is also performed. The PID controller is tuned starting from the approximated model

$$\hat{P}_2(s) = \frac{1}{4.231s^2 + 4.115s + 1} e^{-3.88s}. \tag{20}$$

We have $T_i = 4.12$, $T_d = 1.03$, $K_p = 0.40$, $N = 10$, $\omega_{gc} = 0.099$ and

$$F(s) = \frac{1}{0.51s+1} \tag{21}$$

The resulting integrated absolute values for the set-point step response are $IAE = 10.31$ for the PIDA controller and $IAE =$

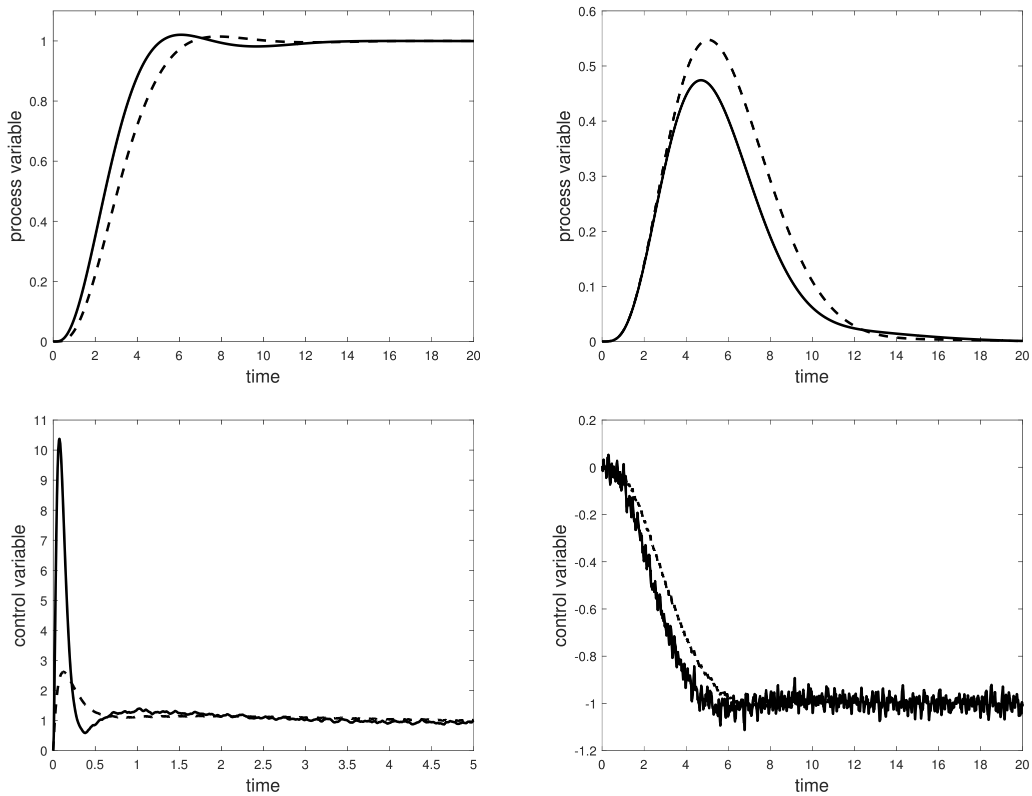

Fig. 2. Set-point (left) and load disturbance (right) step response for Example 1 with $M_s = 1.4$. Solid line: PIDA controller. Dashed line: PID controller.

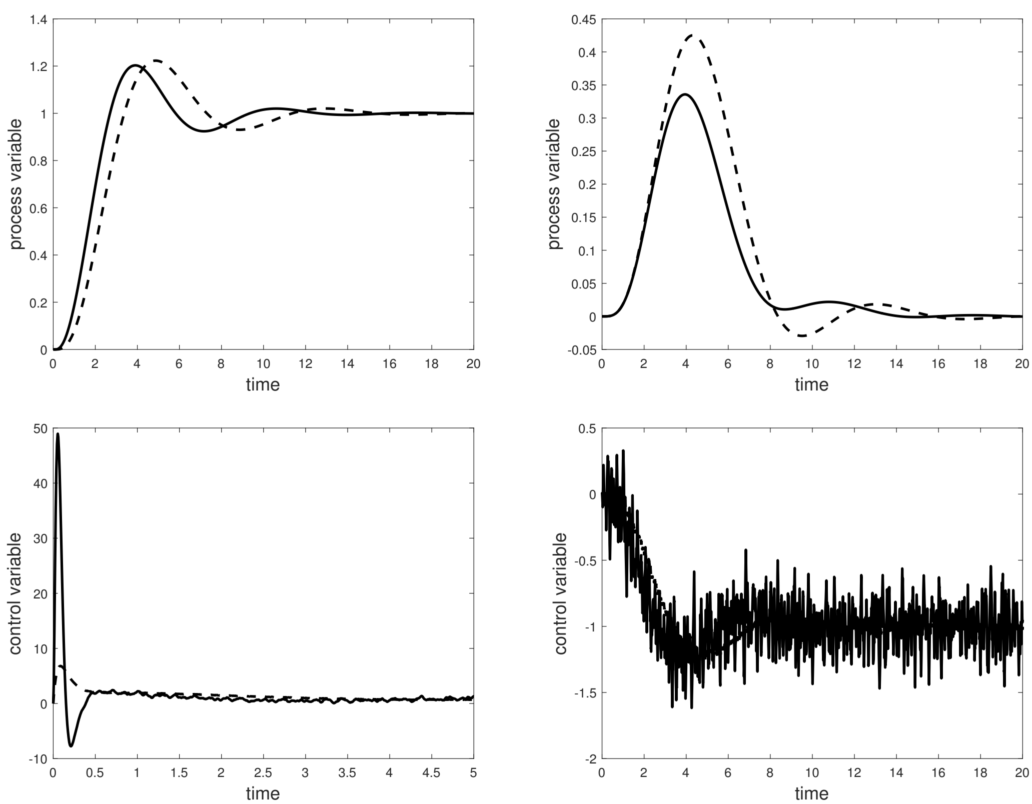

Fig. 3. Set-point (right) and load disturbance (left) step response for Example 1 with $M_s = 2.0$. Solid line: PIDA controller. Dashed line: PID controller.

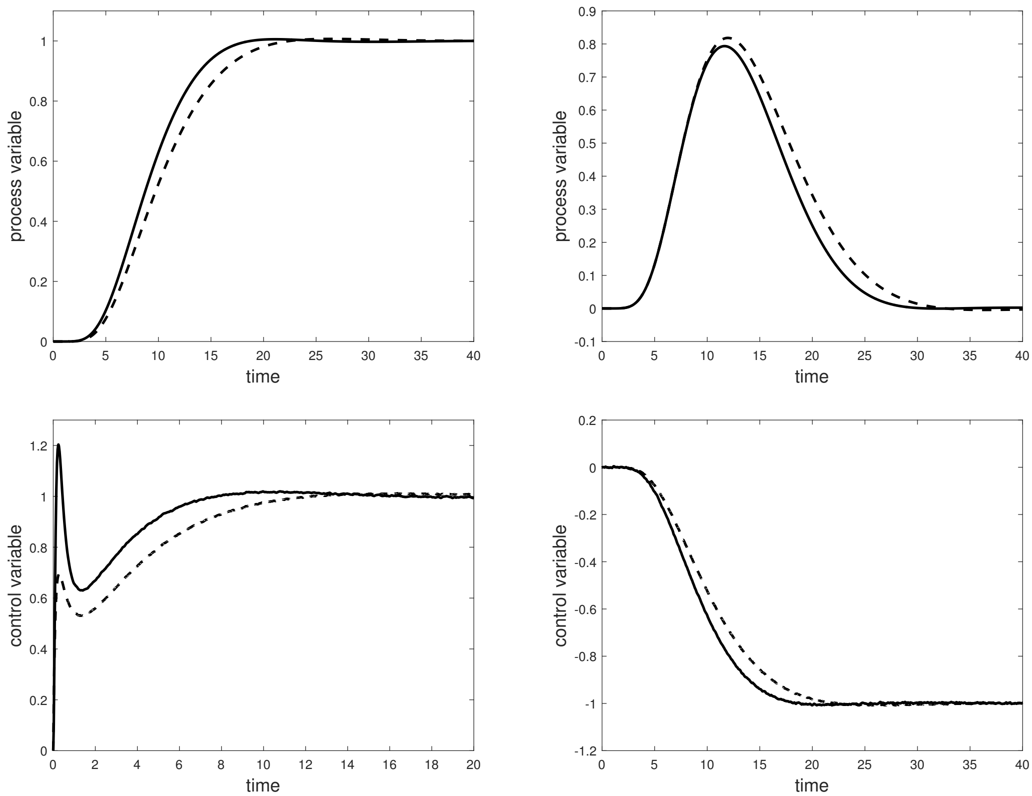

Fig. 4. Set-point (left) and load disturbance (right) step response for Example 2 with $M_s = 1.4$. Solid line: PIDA controller. Dashed line: PID controller.

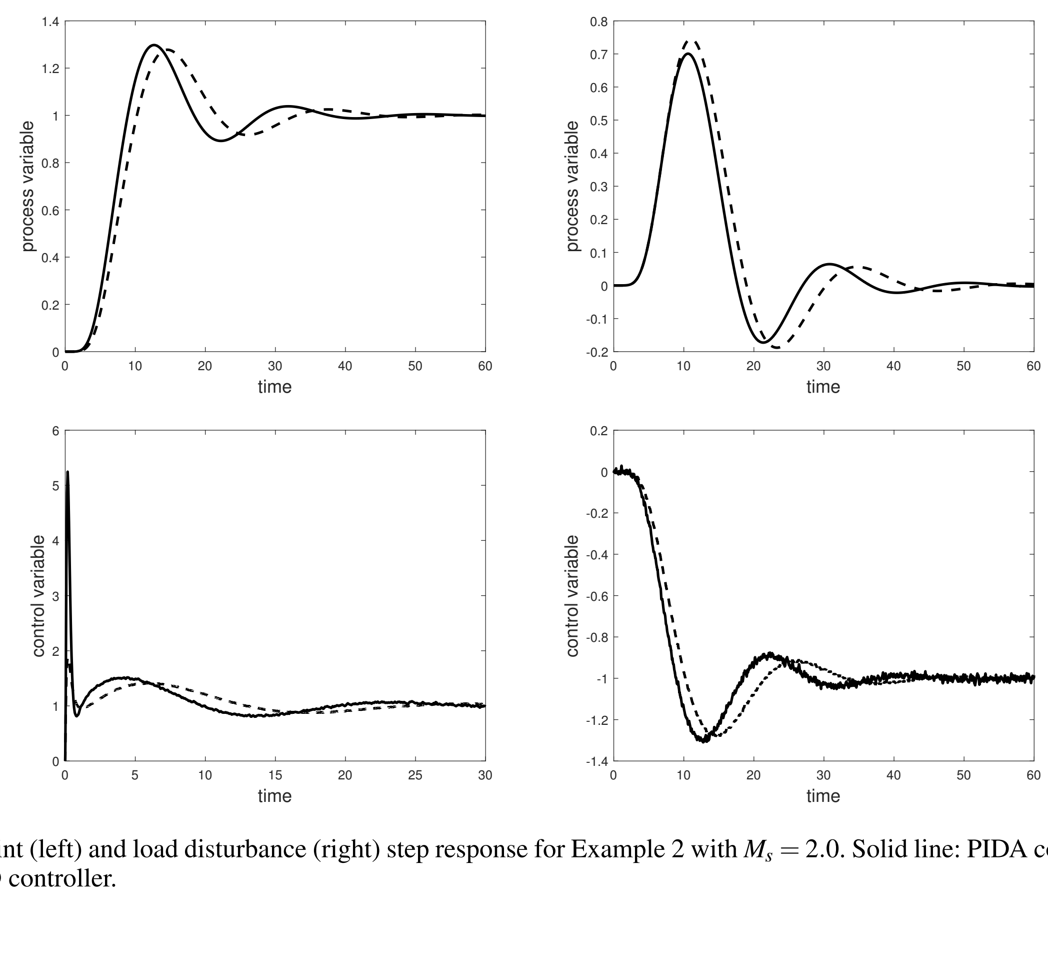

Fig. 5. Set-point (left) and load disturbance (right) step response for Example 2 with $M_s = 2.0$. Solid line: PIDA controller. Dashed line: PID controller.

11.33, while those for the load disturbance step response are $IAE = 9.97$ for the PIDA controller and $IAE = 11.02$ for the PID controller. These results confirm the effectiveness of the tuning methodology for the PIDA controllers, by also taking into account that the noise filtering is effective and the increment of the control effort is reasonable.

If the value $M_s = 2.0$ is chosen as the desired maximum sensitivity, the proportional gain for the PIDA controller becomes $K_p = 1.02$, with a consequent increment of the gain crossover frequency to $\omega_{gc} = 0.20$. On the other side, for the PID controller, we have $K_p = 0.74$ and $\omega_{gc} = 0.18$. The setpoint step response is shown in the left part of Figure 5 (note again the different time scale of the control variable), while the load disturbance step response is shown in the right part. The corresponding values of the integrated absolute errors are $IAE = 10.38$ for the PIDA controller and $IAE = 11.16$, while those for the load disturbance step response are $IAE = 8.65$ for the PIDA controller and $IAE = 9.70$ for the PID controller. Also in this case the capability of the PIDA controller to improve the performance is confirmed, as well as the use of the desired value of M_s to handle the trade-off between performance and control effort.

5. CONCLUSIONS

In this paper a simple new tuning methodology for PIDA controllers has been presented. The main idea is to generalize the Haalman method (based on pole-zero cancellation) by including the acceleration action and by selecting the controller gain in order to achieve a predefined maximum sensitivity. This is a desired feature as the trade-off between aggressiveness and robustness and between performance and control effort can be easily handled. The challenge of avoiding the amplification of the measurement noise and kicks in the control variable is faced by suitably designing low-pass filters in the controller. It turns out that the PIDA controller can be considered as a valid alternative to PID controller for high-order processes where an improvement of the performance is required.

Future work will include an analysis of robustness of the methodology when measurement noise is present in the open-loop step response used for the determination of the TOPDT process model.

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