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Rogue-wave bullets in a composite (2+1)D nonlinear medium

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Abstract: We show that nonlinear wave packets localized in two dimensions with characteristic rogue wave profiles can propagate in a third dimension with significant stability. This unique behavior makes these waves analogous to light bullets, with the additional feature that they propagate on a finite background. Bulletlike rogue-wave singlet and triplet are derived analytically from a composite (2+1)D nonlinear wave equation. The latter can be interpreted as the combination of two integrable (1+1)D models expressed in different dimensions, namely, the Hirota equation and the complex modified Korteweg–de Vries equation. Numerical simulations confirm that the generation of rogue-wave bullets can be observed in the presence of spontaneous modulation instability activated by quantum noise.

OCIS codes: (190.3100) Instabilities and chaos; (190.5530) Pulse propagation and temporal solitons; (190.4390) Nonlinear optics, integrated optics.

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1. Introduction

The intriguing fleeting existence of rogue waves (RWs) has been predicted and observed in a variety of physical settings, ranging from oceanography [1] to hydrodynamics [2, 3], and from plasma physics [4] to nonlinear optics [5–9]. When they manifest among chaotic dynamics [10], in either natural or experimental environments, these extreme waves are universally characterized by heavy-tail probability distribution functions, highlighting a more frequent occurrence than what would be anticipated from classical distributions [5, 7–9]. Mathematically, RWs are modeled as transient wavepackets localized in both space and time, to mimic the episodic giants that seemingly appear from nowhere and disappear without a trace [11]. To understand the complexity of RW manifestations in a natural environment with random initial conditions, high-order RWs should be considered too [12–15]. Indeed, RWs can be expressed by a hierarchy of rational functions that remain localized in both space and time domains.

Echoing the multidisciplinary diffusion of soliton concepts a few decades ago, RW research continues its expansion, while providing novel perspectives for the manifestation of extreme waves in a variety of nonlinear media. This requires the study of the propagation models that go beyond the scalar nonlinear Schrödinger (NLS) equation, such as extended scalar models [16–18] or coupled multicomponent (or vector) wave models [19–24].

Concerning spatiotemporal dimensionality, analytical RW investigations have been mostly confined to (1+1)D models, due to the difficulty in finding integrable models in higher dimensions. From the physical point of view, the extension of RW models to higher dimensions is essential. Oceanic RWs are manifestly (2+1)D phenomena, whereas in the context of ultrafast optics, the propagation of intense light pulses in a nonlinear slab or bulk medium entails a complex multi-dimensional dynamics with substantial spatiotemporal pulse rearrangement, where the spatial and temporal degrees of freedom can not be treated separately [25–28]. Further, the extension of nonlinear dynamics to a higher spatiotemporal dimensionality can never be considered as a trivial problem. For instance, a pure $\chi^{(3)}$ Kerr medium, which is appropriate for transverse optical confinement in the 1D case, does not provide stable confinement for higher dimensions due to wave collapse [25]. This situation explains why the practical quest of spatiotemporal solitons in higher dimensionality, also termed light bullets, has become a holy grail for nonlinear optics [29–31]. The formation of light bullets necessitates going beyond NLS models, for instance, by changing the nature of the nonlinearity [30], by including nonlinear dissipation terms [32, 33], by adding optical gain [34], or by designing a microstructured medium [35]. It is now evident that the exploration of complex spatiotemporal dynamics is particularly relevant to areas such as nonlinear microscopy, tomography, volume optical-data storage, remote sensing, and Bose-Einstein condensation (BEC) [30, 36, 37].

In this paper, we delve into the possibility of constructing analytically 3D RWs from a relevant (2+1)D integrable extension of the NLS equation, which is defined by a combination of the Hirota equation and the complex modified Korteweg–de Vries (mKdV) equation expressed in different dimensions. Although bizarre, such a combination can yield two single (2+1)D propagation equations—a newly-established extended (2+1)D NLS equation [38] and the Kadomtsev–Petviashvili I (KP-I) equation [39], respectively, hence justifying its physical origin. Furthermore, starting from our composite model, we show that the hierarchy of higher-order solutions can be presented in a simple way. Particularly, we highlight a translational invariance for the fundamental as well as for some second-order solutions. With the latter property, the solutions elude full spatiotemporal localization, and inherit the more appropriate "*rogue-wave bullets*" terminology. The robustness of these RW bullets is then confirmed numerically, in spite of the onset of modulation instability (MI).

2. Composite (2+1)D model and exact RW solutions

For this purpose, let us consider an otherwise naive combination of two 1D wave equations:

$$u_z + u_{ttt} + 6|u|^2 u_t = 0, \qquad iu_x + \frac{1}{2}u_{tt} + |u|^2 u - i\varepsilon(u_{ttt} + 6|u|^2 u_t) = 0, \tag{1}$$

which are expressed in different dimensional spaces, but with a common time variable t. Here, as usual, $u(z,x,t)$ is the complex field envelope given in dimensionless variables, ε is a constant parameter characterizing the perturbation to the NLS equation (we usually let $\varepsilon > 0$ below, unless otherwise stated), and subscripts t, x and z stand for partial derivatives. It is seen that the first of Eqs. (1) is the complex mKdV equation and the second one the Hirota equation, both of which are integrable. We recall that, if considered separately, either the Hirota or the complex mKdV equation has significant implications in modeling the propagation of ultrashort optical pulses in a general medium [40, 41]. Moreover, the RW solutions for either of them were obtained recently using a Darboux dressing technique [16, 18]. Now, a natural question arises as to what physics or implications the combined Eqs. (1) could offer and as to whether they could allow any RW solutions as well, to depict the multidimensional RW patterns.

The physics may be understood in the following way. If we change variables to $Z = z - \varepsilon x$, $X = x$, and $T = t$ (with the chain rule $\frac{\partial}{\partial x} = \frac{\partial}{\partial X} - \varepsilon\frac{\partial}{\partial Z}$, $\frac{\partial}{\partial z} = \frac{\partial}{\partial Z}$, $\frac{\partial}{\partial t} = \frac{\partial}{\partial T}$), and then subtract the first of Eqs. (1) divided by 2 from the derivative of the second one with respect to T, we obtain

$$iu_Z + 2u_{XT} - 4|u|^2 u\phi_T = 0, \quad \phi = \arg(u), \tag{2}$$

which is an integrable extension of the original (2+1)D NLS equation [42]. In such a single (2+1)D equation, the parameter ε only appears in the new variable Z, and will therefore play the role of *velocity* [16]. The second term on the lefthand side, after a $\pi/4$ rotation of the coordinate axes, can become $u_{XX} - u_{TT}$, and hence entails the diffraction and normal dispersion effects [43]. The last term, also equal to $-2iu(uu_T^* - u^*u_T)$, is the Kerr nonlinearity modulated by the instantaneous frequency shift of the optical field, $-\phi_T$. Consequently, the nonlinearity will be of self-focusing nature for an upshifted frequency part of the pulse, whereas it will be defocusing for a downshifted part, whenever those situations take place. We note that Eq. (2) was first derived from the basic hydrodynamic equations in [38] and then revisited in the context of plasma physics [44], with the inclusion of the third term, $-4|u|^2u\phi_T$, being justified. As there is basically an analogy between pulse propagation in optical media and hydrodynamical wave group propagation [42, 45, 46], we anticipate that Eq. (2), and hence Eqs. (1), may as well find applications to nonlinear optics, *e.g.*, modeling the ultrashort pulse propagation in a phase-modulated hyperbolic optical system [28, 43].

In fact, if we define an intensity quantity $E = |u|^2$ and perform simple manipulations on Eqs. (1), we can also obtain another (2+1)D integrable wave equation:

$$3E_{XX} - (E_Z + 3EE_T + \frac{1}{4}E_{TTT})_T = 0, \tag{3}$$

which is nothing but the well-known KP-I equation [39]. Like its older NLS cousin, the KP-I equation is also a ubiquitous nonlinear wave equation, and basically governs weakly nonlinear long waves in two dimensions (here in Z and T) with slow transverse variations (i.e., in X here). It has been obtained as a reduced model in hydrodynamics, plasma physics, ferromagnetics, BEC, and string theory [31, 47]. Particularly, it was shown that the KP-I equation can also play an important modeling role in nonlinear optics [42, 48]. This further justifies our study of the combined Eqs. (1) within optical contexts. Naturally, once the RW solutions of Eqs. (1) are obtained, they should also satisfy the above single (2+1)D wave Eqs. (2) and (3), as the latter can be directly derived from the former. The reverse may not be true, but this does not degrade the intrinsic interest of the combined model (1).

With the above considerations, the hunt for the RW solutions to the composite Hirota-mKdV model (1) becomes important and interesting. We find that it is still completely integrable and can be cast into a 2×2 linear eigenvalue problem, with the following Lax triad:

$$\mathbf{R}_t = \mathbf{UR}, \quad \mathbf{R}_z = \mathbf{VR}, \quad \mathbf{R}_x = \mathbf{WR}, \tag{4}$$

where $\mathbf{R} = [r,\ s]^T$ (T means a matrix transpose), and

$$\mathbf{U} = -i\lambda\sigma_3 + \mathbf{Q}, \quad \mathbf{V} = 4\lambda^2\mathbf{U} - 2i\lambda\sigma_3(\mathbf{Q}^2 - \mathbf{Q}_t) + \mathbf{K}, \quad \mathbf{W} = \lambda\mathbf{U} - \frac{i}{2}\sigma_3(\mathbf{Q}^2 - \mathbf{Q}_t) - \varepsilon\mathbf{V}, \tag{5}$$

with λ being the spectral parameter, and

$$\sigma_3 = \mathrm{diag}(1, -1), \quad \mathbf{Q} = \begin{bmatrix} 0 & -u \\ u^* & 0 \end{bmatrix}, \quad \mathbf{K} = \mathbf{Q}_t\mathbf{Q} - \mathbf{Q}\mathbf{Q}_t - \mathbf{Q}_{tt} + 2\mathbf{Q}^3. \tag{6}$$

As one can verify, there are three compatibility conditions on the Lax triad (4), which, when satisfied simultaneously, can elegantly reproduce Eqs. (1). As such, the standard Darboux transformation method [49] can be exploited to derive the RW solutions of Eqs. (1).

We note that the initial plane-wave seed which satisfies Eqs. (1) can be expressed as

$$u_0(z,x,t) = c\exp\left[-i\left(kt - \omega x + mz\right)\right], \tag{7}$$

where $\omega = \frac{1}{2}k^2(2\varepsilon k - 1) - c^2K$ and $m = k^3 - 6c^2k$, with $K = 6\varepsilon k - 1$. Then, after some algebra, we obtain the fundamental (first-order) RW solutions of Eqs. (1)

$$u^{[1]} = u_0\left\{1 + \frac{8ic\xi - 4}{[\tau - (k - 2c^2/k)\xi]^2 + 4c^2\xi^2 + 1}\right\}, \tag{8}$$

where u_0 is given by Eq. (7), and

$$\xi = cKx - 6ckz, \quad \tau = c(k + 2c^2/k)x + 2ct. \tag{9}$$

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Meanwhile, the general second-order RW solutions $u^{[2]}$ can also be cast into a compact form

$$u^{[2]} = u_0 \left\{ 1 - \frac{4[R_2^*(R_1 m_{22} - S_1 m_{21}) + S_2^*(S_1 m_{11} - R_1 m_{12})]}{m_{11}m_{22} - m_{12}m_{21}} \right\}, \tag{10}$$

where

$$m_{11} = |R_1|^2 + |R_2|^2, \quad m_{12} = R_1^* S_1 + R_2^* S_2 - m_{11}, \tag{11}$$

$$m_{21} = m_{12}^*, \quad m_{22} = |S_1|^2 + |S_2|^2 - m_{12} - m_{21}, \tag{12}$$

$$R_1 = 2\gamma_1 - 2\gamma_2 \vartheta, \qquad R_2 = R_1 - 4\gamma_2, \tag{13}$$

$$S_1 = \gamma_1 \vartheta^2 - \gamma_2 \left[\frac{\vartheta^3}{3} + \vartheta - \frac{32c^3 x}{3k} + 8c(i - \frac{4c}{3k})\xi \right] + 8\gamma_3 - 2\gamma_4 \vartheta, \tag{14}$$

$$S_2 = S_1 - 2\gamma_2 \vartheta^2 + 4(\gamma_1 - \gamma_2)\vartheta + 2(2\gamma_1 - \gamma_2 - 2\gamma_4), \tag{15}$$

$$\vartheta = (k - 2c^2/k + 2ic)\xi - \tau - 2. \tag{16}$$

Here γ_j ($j = 1,2,3,4$) are arbitrary complex constants defining the RW structures. We note that, apart from its propagation factor, the fundamental solution (8) can be expressed as a Peregrine soliton form [3,6] in terms of the combined variables ξ and τ, while the second-order solution (10) depends also on x besides ξ and τ [see Eq. (14)]. However, it should be stressed that these RW solutions can not be factorized into a trivial product of two RWs obtained from the Hirota and the complex mKdV equations separately; they are totally inseparable. As one can verify, both RW solutions, Eqs. (8) and (10), can exactly satisfy Eqs. (2) and (3) after a change of variables. Obviously, our results obtained above generalize the analytical findings in [38,50].

3. RW bullet dynamics and numerical simulations

The spatiotemporal dynamics of the fundamental and second-order RW solutions are demonstrated in Fig. 1, each plotted in $(t,\ z)$, $(t,\ x)$, and $(x,\ z)$, respectively. It is shown that the fundamental RW in either plane takes the form of Peregrine soliton, featuring a doubly localized hump with a maximum fixedly three times the significant wave height [see Figs. 1(a)–1(c)]. More interestingly, as it involves a ϑ-dependent polynomial of six order when $\gamma_2 \neq 0$, the second-order RW could take a triplet structure [13,14] making up of three Peregrine solitons, each of which can reach a peak amplitude of $3c$ [see Figs. 1(e)–1(g)].

We also provide their isosurface plots at $|u| = 2$ in Figs. 1(d) and 1(h), respectively. It is revealed that both the fundamental and second-order RWs have a directional preference or a *bullet nature* by which we mean they can propagate in a certain direction ζ with transverse double localization [see Fig. 1(d)]. For this reason, we prefer to term such a special 3D RW behavior a *rogue-wave bullet*. Basically, the propagation direction ζ of the fundamental RWs can be determined by the spatial line

$$\frac{6z}{K} = \frac{x}{k} = -\frac{2t}{k^2 + 2c^2}, \tag{17}$$

which passes through the origin, as indicated by $\xi = 0$ and $\tau = 0$ in Eq. (9). So is the case with the RW triplet, for which its three RW components can propagate almost along the direction vector $(\frac{K}{6},\ k,\ -\frac{k^2+2c^2}{2})$ of the spatial line (17), as seen in Fig. 1(h). However, it may become involved for other second-order RW states, as their directional preference depends also on the choice of four structural parameters γ_j. Figures 2(a) and 2(b) illustrate the spatiotemporal distributions of the composite RWs obtained with two different sets of structural parameters, both of which can be thought of as the collisions of three fundamental RW bullets.

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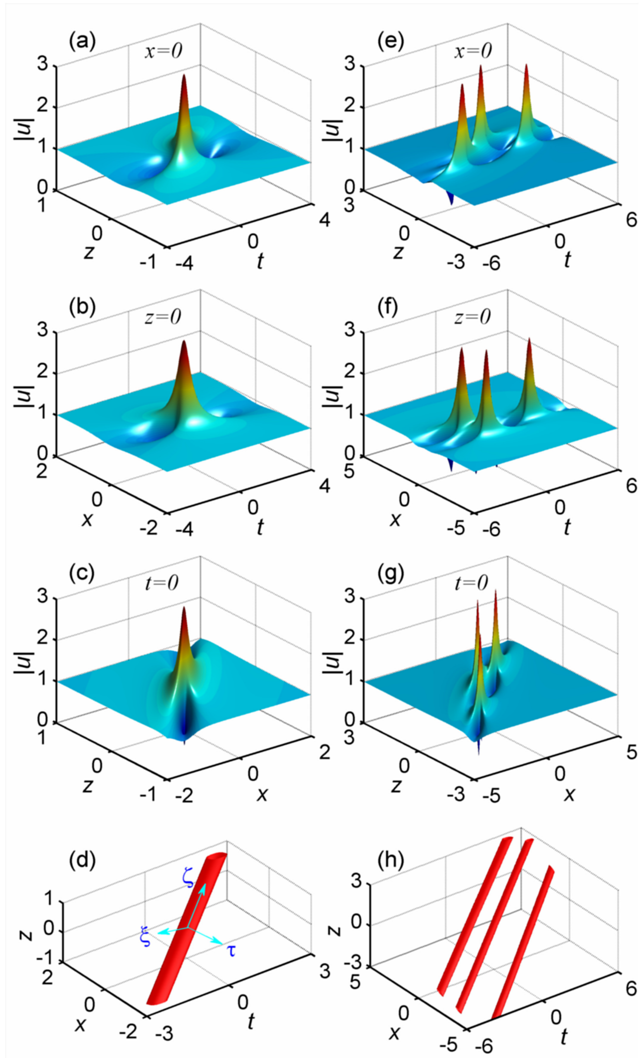

Fig. 1. (a)–(c) Fundamental and (e)–(g) second-order RWs formed at $k=-1$, $c=1$, and $\varepsilon=1/2$, plotted in $(t,\ z)$, $(t,\ x)$, and $(x,\ z)$, respectively. The four structural parameters in (e)–(g) are $\gamma_1=4i$, $\gamma_2=1$, $\gamma_3=20$, and $\gamma_4=-10$. (d) and (h) show the isosurface plots of the corresponding RW states, both given at $|u|=2$.

Let us further remark on the MI of the background fields [15, 23, 51, 52] for such a (2+1)D nonlinear system. We add small-amplitude Fourier modes to the plane-wave solution (7), and express it as $u=u_0\{1+p\exp[-i\Omega(\mu x-t+\kappa z)]+q^*\exp[i\Omega(\mu^* x-t+\kappa^* z)]\}$, where p and q are small amplitudes of the Fourier modes, Ω accounts for the modulation frequency ($\Omega\geq 0$), and the propagation parameters μ and κ are assumed to be complex. A substitution of such a perturbed plane-wave solution into Eqs. (1) followed by linearization yields two systems of coupled linear equations for p and q. These systems have a nontrivial solution only when μ, κ, and Ω satisfy simultaneously two dispersion relations, from which the MI growth rates, defined

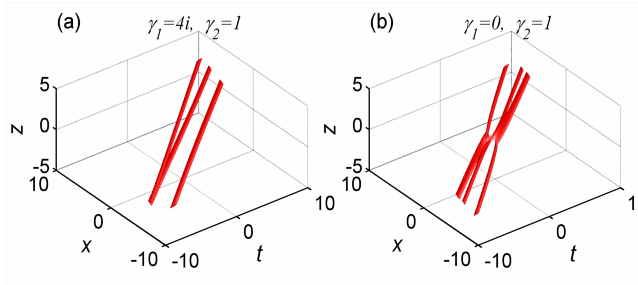

Fig. 2. Isosurface plots at $|u| = 2$ for the second-order RWs for $k = -1$, $c = 1$, and $\varepsilon = 1/2$ but with different structural parameters: (a) $\gamma_1 = 4i$, $\gamma_2 = 1$; (b) $\gamma_1 = 0$, $\gamma_2 = 1$. The other two structural parameters are given by $\gamma_3 = \gamma_4 = 0$.

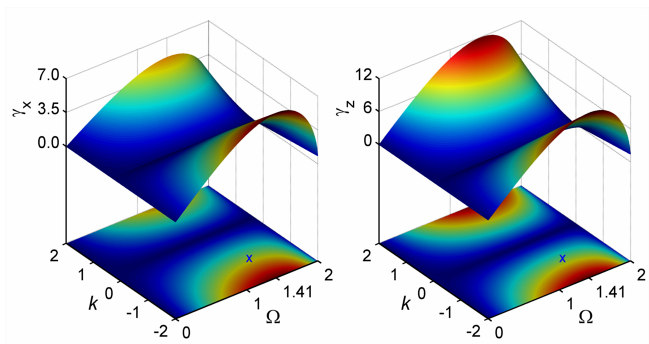

Fig. 3. 3D and contour plots showing the MI gain maps versus (Ω, k) for $c = 1$ and $\varepsilon = 1/2$. The blue crosses indicate the maximums of the growth rates γ_x and γ_z for $k = -1$, both of which occur at the modulation frequency $\Omega = \sqrt{2} \simeq 1.41$.

by $\gamma_x = \Omega|\mathrm{Im}(\mu)|$ and $\gamma_z = \Omega|\mathrm{Im}(\kappa)|$, can be found to be

$$\gamma_x = \frac{\Omega}{2}\sqrt{K^2(4c^2 - \Omega^2)}, \quad \gamma_z = 3\Omega\sqrt{k^2(4c^2 - \Omega^2)}. \tag{18}$$

Figure 3 displays the MI gain maps versus Ω and k. It is suggested that the RW states would occur in the whole domain of k. Specially, as $k = (6\varepsilon)^{-1}$ or 0, they can be manifested as the line RW forms in the plane $(t,\ x)$ or $(t,\ z)$. It follows from Eq. (18) that the growth rates, γ_x and γ_z, have the same evolution with Ω and reach their maximums simultaneously as $\Omega = \sqrt{2}c$. The ratio of γ_x to γ_z, which is equal to $|\varepsilon - (6k)^{-1}|$, depends solely on the values of ε and k. If $\gamma_x/\gamma_z > 1$, the MI in the x dimension may develop faster than in z, and vice versa.

Besides, we numerically study the stability of these RW bullets against small perturbations, based on the split-step Fourier method [10]. Here we only provide the numerical versions of the fundamental RW solution shown in Figs. 1(a)–1(c), by solving Eqs. (1) consecutively under otherwise identical initial conditions. Shown in Figs. 4(a)–4(c) are the numerical solutions without any perturbations, exhibiting an excellent agreement with our analytical solutions. We then perturbed the initial profiles by small amounts of white noise and demonstrate the simulation results in the right column by one-to-one correspondence. As done in previous works [20, 23, 24], the white noise is added into the initial analytical solution $u(z_0, x_0, t)$ [that is imperceptible at the initial position, which in our simulations was taken to be $(x_0, z_0) = (-2, -1)$] by multiplying

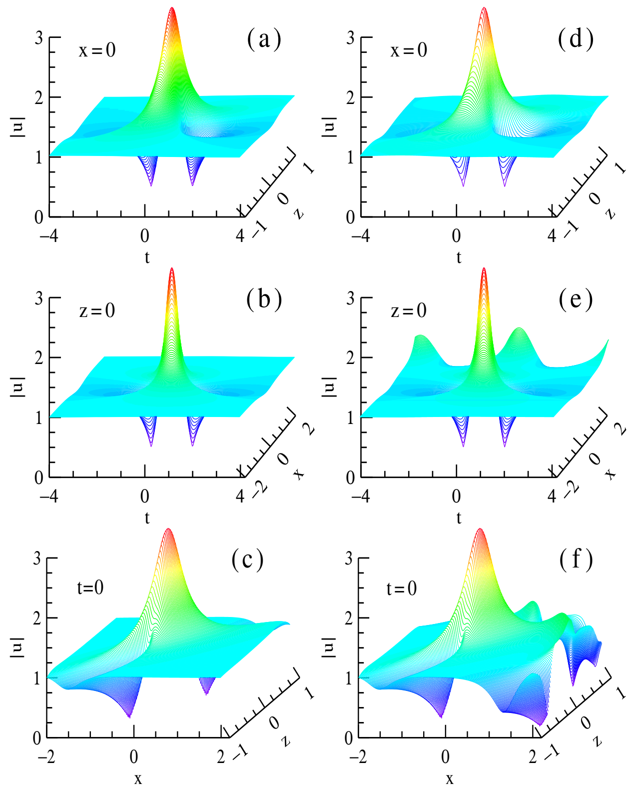

Fig. 4. Simulation results of the 3D fundamental RWs under otherwise identical initial conditions as in Figs. 1(a)–1(c). (a)–(c): the unperturbed solutions; (d)–(f): the perturbed solutions obtained with the noise level $\eta = 2\times10^{-4}$.

their real and imaginary parts with a factor $(1+\eta r_i)$ $(i = 1,2)$, where $r_{1,2}(t)$ are two uncorrelated random functions uniformly distributed in the interval $[-1,\ 1]$ and η is a small parameter defining the noise level (here we use $\eta = 2\times10^{-4}$). As seen, with this tiny perturbation, the above RW profiles can maintain very well for a rather long distance till the MI of the background fields grows up. Here we need to point out that the MI wave in Fig. 4(d) will be much weaker because of the shorter propagation distance, compared with those shown in Figs. 4(e) and 4(f). In addition, as one can see from Fig. 4(e), the peak number of the MI wave is nearly 2 within the interval of 8 in t axis, very consistent with the analytical value $4\sqrt{2}/\pi \simeq 1.8$ obtained from our MI analysis above (see blue crosses in Fig. 3).

As a final remark, we would like to propose a possible experimental setting for realization of these RW bullets in the context of optics. Inspired from the optical relevance of the KP-I Eq. (3) [42,48], we may consider the small-amplitude pulse propagation with slow transverse variations in a planar glass waveguide (*e.g.*, see the experimental setup in [27]), or in a quadratic lithium niobate planar waveguide, in the regime of high phase mismatch, which mimics an effective Kerr nonlinear regime (*e.g.*, see the experimental setup in [53]). Also, the extended (2+1)D NLS Eq. (2) could provide a clue to realize such RW bullets, for instance, in a planar AlGaAs waveguide that exhibits normal dispersion [43]. However, to mimic the phase-modulated Kerr nonlinearity requires more advanced material fabricating technique, although earlier works suggested that such kind of nonlinearity may be achieved in engineered waveguide arrays or photonic crystal structures [54].

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4. Conclusion

In conclusion, we have found analytically a novel type of light bullets, which take the shape of RWs and travel on a finite (2+1)D space-time background, by virtue of a combination of the Hirota and complex mKdV equations. Fully inseparable explicit 3D RW solutions up to the second order were unveiled, exhibiting intriguing cluster dynamics. We confirmed numerically the robustness of these RW bullets in spite of the onset of spontaneous MI. We may draw an analogy between these unusual RW-shaped bullets and the peculiar X-shaped light bullets that were discovered in 2-component quadratic media [55]. Besides departing from a conventional bell-shaped pulse, in both cases, the localization of the light bullet requires feeding from a substantial background field. Moreover, the combination of a RW and a bullet-like propagation may also echo the question arose whether one could launch a RW onto a target [56].

In higher-dimensional media, the experimental complexity multiplies in both material fabrication and pulse characterization aspects. However, experimental advances have recently provided a strong incentive in the area of RW investigation in complex media. For instance, the deterministic dark RW dynamics predicted for a (1+1)D vector NLS equation [19] has just been observed experimentally [57]. Leonetti and Conti [58] recently reported in a linear spatial system the realization of 3D RWs that is caused by inhomogeneity [46]. In the wake of these developments, we anticipate that our prediction of RW bullets may spark significant experimental research on the generation of 3D RWs in nonlinear optical systems.

Acknowledgments

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