



A Comprehensive Method for Computing Suspension Elasto-kinematics With Non-linear Compliance

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Abstract. Since flexible bushings are used as the interface between the suspension arms and the chassis, the extra degrees of freedom make the design process a complex task. While the use of a multi-body model is common practice in the industry, a dedicated computational tool can be more practical and straightforward, especially when undertaking the design of a new suspension concept from the ground up. This paper presents a quasi-static method for calculating suspension compliance under the action of forces and moments, enabling real-time simulations. The algorithm proposed in this paper was devised with a threefold purpose: integrating elasto-kinematics into the kinematic design tool previously created by the authors, integrating real-time vehicle dynamics simulation, and overcoming the limitations of the traditional approach based on the superposition principle. Finally, a comparison of the proposed model with one based on the lookup-table and superposition principle is presented.

Keywords: Suspension · Kinematics · Elasto-kinematics · Bushing · Compliance · Simulation · Vehicle Dynamics

1 Introduction

Modern passenger cars require an intensive effort for the design of suspension elasto-kinematic properties because of their significant impact on ride, handling, stability, and steering feel. This importance is reflected in the vast engineering literature on the subject. The use of multi-body models, although the industry standard, is often considered demanding due to the high level of detail required. Additionally, a multi-body model is rarely suitable for real-time simulation. Instead, the development and application of relatively simple, dedicated design and simulation tools are often considered more practical, especially when designing a new suspension concept from the ground up. In fact, the proposed methodology is being implemented in the kinematics calculation tool developed by the authors [1]. This trend is evident in the related literature, where many publications describe self-developed, specific methodologies that vary in complexity

and computational approach. A multi-body model is commonly adopted for validation in these cases. Generally, however, most papers in the literature tend to neglect one or more factors of real-world design. In particular, they often overlook the non-linearity of bushings or linearize the suspension within a range of small displacements [2–4].

On the other hand, it is well known that correct modelling of elastokinematics, taking non-linearities into account, is crucial for several aspects: design [5], for handling [6] and also for ride & comfort [7]. The integration of models characterizing elastokinematics in vehicle dynamics simulations is a recurring topic, with the main challenge being the speed of computation [8].

With regard to real-time simulations, the current trend in the literature shows an approach based on modeling suspension compliance through artificial neural networks (ANN) [9]. In this way, it is also possible to consider dynamic effects as opposed to a quasi-static solver. However, a significant amount of data is needed to train the network, which can be particularly complicated or even impossible during the design phase. The purpose of this work is to provide a method for solving the suspension elastokinematic problem using a general procedure that enables the design of any layout under any combination of jounce and steering or load. Bushings can be described with real-world, non-linear stiffness curves for all six degrees of freedom. They can be located on either side of each suspension arm, i.e., on the chassis side and/or the wheel side. The axial flexibility of a track rod can also be represented by means of equivalent, non-linear bushings. The wheel bearing stiffness can be considered as well. Wheel movements, hence variations of vehicle dynamics-relevant parameters like camber, side view angle, toe, track, wheelbase, and vertical displacement, can be computed under any combination of road loads, also considering steering due to rack translation. Loads through suspension joints, components, and chassis pick-up points can also be computed.

In general, elastokinematics is implemented in vehicle dynamics simulations using look-up tables, often derived from experimental data obtained through K&C or SPMD, then applying the superposition principle [10]. This method is characterized by requiring negligible computational resources.

One of the aims of this work is to show how the superposition principle, which can work well in some cases, does not allow for a correct characterization of compliance when large lateral and longitudinal accelerations are involved, as mentioned in [10]. A comparison, through simulation, with a model based on look-up tables and the superposition principle is proposed to show how vehicle behavior can change in a test with large accelerations. The simulation is also carried out to demonstrate the possibility of using the method proposed in this work in real-time.

2 Methods

Two types of elements are considered in this model, Fig. 1 a). The first type of element is called “spring rod” and it is composed by a rod with a given axial stiffness complemented with a bushing at both ends. The second element is called “rigid element”: it is composed by a rigid body connected to any number of bushings.

The above elements can be attached to each other or to the chassis. By combining them it is possible to create any type of independent suspension. For example, in a

double wishbone both arms will consist of a rigid element connected to the chassis by two bushings and to the upright by one bushing. The upright will also be a rigid element with three bushings, two of them are connected to each wishbone and one to the steering tie rod; this one will instead be modelled using a spring rod element as in Fig. 1 b).

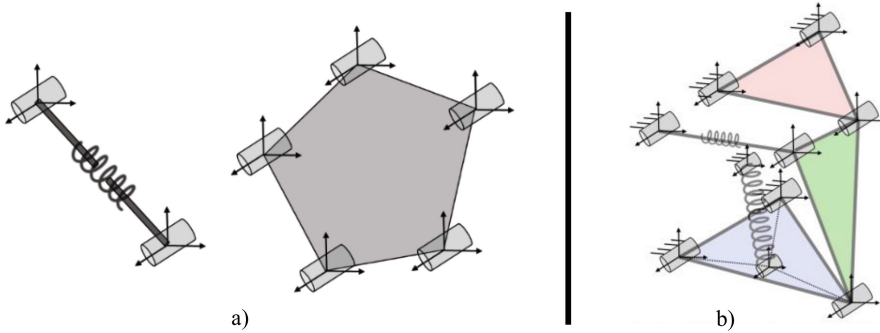


Fig. 1. a) General “spring rod” and “rigid element” of the suspension model. b) Example of a Double Wishbone suspension built using the general elements.

Each bushing is defined with its position, orientation, and the “Reaction Forces Vector” containing six functions that correspond to the three reaction forces and moments generated by the bushing as a function of the six deformations along or around its local reference system. A ball joint can be represented by means of a bushing with very high stiffness values. The suspension model is therefore composed of nonlinear equations representing the balance of forces and moments for each element as a function of bushing deformations. The number of degrees of freedom hence equations is a function of the type of elements used to compose the suspension: 6 degrees of freedom for each “rigid element” and 7 for each “spring rod”. The model, solved through the Newton-Raphson algorithm, enables the computation of the deformed configuration of the suspension as a function of forces and moments applied at any point on the wheel. It has been fully validated in [11] (currently under review).

A generic model of a double wishbone suspension was created as in Fig. 1 b), featuring nonlinear, force-displacement bushing characteristics defined with a fifth-degree polynomial to recreate their typically progressive, stiffening behavior.

The suspension model was implemented as a Matlab® function and transferred to Simulink®. This second function was then linked to Vi-CarRealTime®. A dynamic simulation has been carried out as a case study. The simulation is a corner-braking test where the vehicle starts from a speed of 108 km/h and enters in a cornering with a radius of 75 m. A virtual driver model controls the steering. After 2 s the driver brakes following a target deceleration of $10 \frac{m}{s^2}$.

The purpose is the comparison of the above model with a simpler one based on the superposition principle, where the effect of each force or moment on the suspension is considered as orthogonally decomposed. The suspension deflection is calculated as a simple summation of all effects of each external action.

A basic vehicle model was therefore used. Using the proposed algorithm, a series of 3-dimension look-up tables were generated in as a function of wheel travel, steer and external force, each describing the effect that each external force has on the suspension's 6 degrees of freedom.

The simulation has been carried out in the Vi-CarRealTime® environment with Simulink® co-simulation, with an integration step of 0.001 s. Only the compliance of the front axle was considered. To ensure that the simulation worked in real time, the algorithm calculating the elastokinematics has been compiled in C and has been run at 500 Hz, assigning the calculation of each wheel in parallel to one core of an Intel® Xeon Gold 6134 CPU.

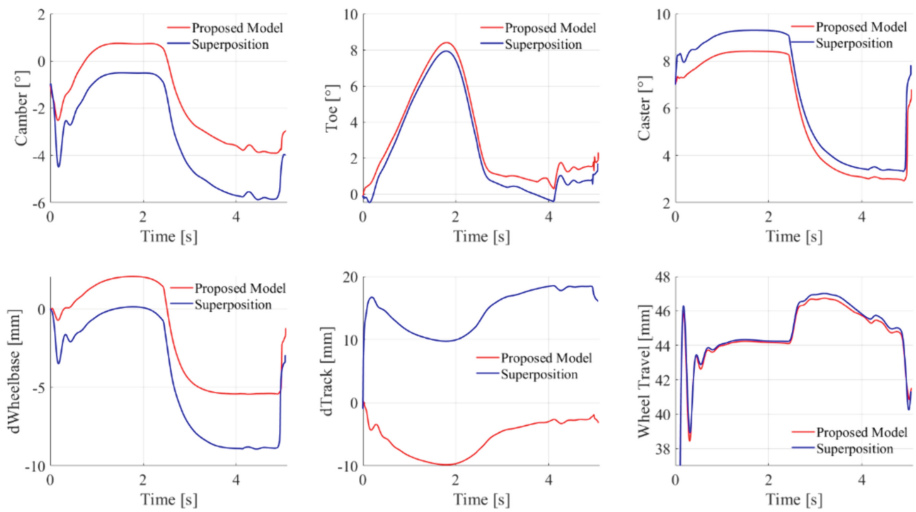


Fig. 2. Simulation results for left-hand wheel movements: camber, toe, caster, wheelbase variation and wheel travel. In red the comprehensive proposed model, in blue the model based on lookup-table and superposition.

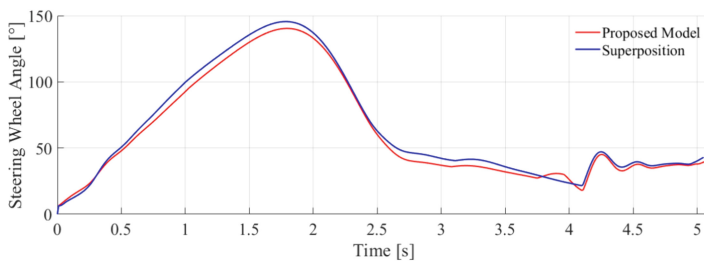


Fig. 3. Steering wheel angle over time in the simulation. In red the comprehensive proposed model, in blue the model based on lookup-table and superposition.

Figure 2 shows the simulation results for the movements of the left wheel (i.e. the most loaded as it is a right-hand bend): camber, toe, caster, wheelbase variation and wheel travel, comparing the two models.

Figure 3 represents the steering wheel angle over time. This is different between the two simulations as the test is in closed loop, i.e. virtual driver maintains an imposed trajectory and target deceleration.

3 Conclusion and Discussion

By using typical, strongly nonlinear force-displacement curves for the bushings, the difference between the two models becomes particularly apparent, as previously seen in Fig. 2, where large differences in track variation can be observed, even with opposite signs. The differences seen in camber, caster, and toe can significantly influence the handling and performance of the vehicle. This is also reflected in the varying values of steering angle, which alters the feedback for the driver. The difference in toe angle is opposite to the difference in steering angle, indicating a different contribution of elastokinematics in the two models.

Separating the effects of the various actions and combining them through the superposition principle does not lead to an accurate solution when large deformations occur, and large forces act on the wheel. In normal driving, the differences between the two models tend to be negligible. Even when examining the inner wheel during a turn, the differences are less significant.

For performance applications, the need arises for an appropriate solver to accurately calculate compliance effects, enabling an effective design and simulation process without limitations. Elasto-kinematic properties can be computed for any combination of wheel jounce and steering rack position, with the flexibility to change hardpoints or bushing stiffness curves at any time.

Unlike a multi-body model, the proposed model is a quasi-static solver that is well-suited for real-time applications, such as in a driving simulator. In this context, each solution of the problem is close to that of the previous instant, allowing the Newton-Raphson algorithm to converge in negligible time.

However, compared to a multi-body solver, dynamic features of rubber bushings, such as damping and hysteresis effects, are not accounted for. Another limitation of this model is the lack of an anti-roll bar model, which can transfer forces onto the suspension system and affect its elastokinematics.

In this work, the proposed model has been implemented in the simulation with its calculation frequency limited to 500 Hz, so that the vehicle model could operate at 1000 Hz, which is considered the minimum frequency for real-time. Lowering the update frequency of the elastokinematics does not significantly influence the vehicle model, as the dynamic effects of the suspension are not accounted for here. It is expected that with more powerful hardware, the suspension model frequency can also be increased to 1000 Hz.

In the author's opinion, an analytical approach suitable for both design and real-time operation is the best solution in the initial design phase of a vehicle with its suspension and steering systems. Additionally, with the proposed model, it is possible to calculate

the constraint reactions on each individual bushing, which is useful for FEM analyses, for example something that cannot be done with a model based on look-up tables or an artificial neural network.

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