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Experimental Comparison of The Two Most Used Vehicle Sideslip Angle Estimation Methods for Model-Based Design Approach

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Abstract. Vehicle sideslip angle estimation is still one of the most challenging research topics in the automotive industry. Many papers can be found on this topic, where authors propose varied methods to reach the goal. Which is the most effective? After an extensive literature review, two very different methods have been identified as the most used: Extended Kalman Filter with dynamic model and Artificial Neural Network. In this work a comparison among these methods is presented. A fully instrumented car has been used to gather typical vehicle dynamics data and feed the models required for a model-based design approach. Results showed that each method has either positive aspects or drawbacks.

1. Introduction

Vehicle sideslip angle (VSA) estimation has been a big challenge since the introduction of the very first on-board active systems controlling vehicle stability, such as the electronic stability control (ESC, aka ESP) in the early 1990s [1]. It can be used to investigate the cornering behavior in terms of understeer/oversteer and stability, which are related to active safety and drivability, for either passenger cars or high-performance vehicles [2]. Moreover, active safety systems such as ESC can be used in conjunction with VSA estimation in order to extend the vehicle performance envelope and/or enhance driver perception and comfort [3] as the average driver feels uncomfortable when large VSA values occur, during emergency maneuvers for instance [4]. It is also worth mentioning that vehicle drivability is strictly related to VSA: whenever it is large, the authority of the steering wheel angle in generating a yaw moment becomes significantly reduced. An accurate, reliable and effective VSA estimation can also be used for autonomous driving strategies and advanced motion control of high-performance electric vehicles, thus improving active safety and efficiency which are considered key factors in the automotive market [5].

However, devices required to obtain a direct measure of the vehicle sideslip angle are not robust to adverse environmental conditions and very expensive as well, therefore they are suitable for R&D rather than for production vehicles. The VSA hence requires estimation on the basis of conventional measurements such as lateral/longitudinal acceleration, yaw rate, and steer angle. An estimate of vehicle speed is also required, to be based in turn on wheel angular speeds.



All the above calls for an innovative approach where traditional mechanical engineering skills for chassis design are integrated with digital knowledge, and the use of innovative tools makes for a research framework where subjective factors are also taken into account, thus making it also suitable for the development of Advanced Driver Assistance Systems (ADAS). On top of that, such an approach recalls what is usually defined as model-based design, where digital models are the key to product definition. In this case the models related to VSA estimation strategies can be used for functional simulations at the same time.

VSA estimation is usually classified as Kalman Filter-based and Neural Network-based. According to [6] however none of the proposed methods succeeded as the most effective, and the problem is not yet fully resolved. The aim of this paper is to further investigate estimation performances of both methods through a direct comparison with real-world data.

2. Methods

2.1. Kalman Filter-based estimation

Some It is well known that the Kalman filter (KF) provides optimal state estimation of linear systems. When dealing with nonlinear systems, as it is often the case in practice, a possible solution to the state estimation problem is offered by the Extended Kalman Filter (EKF). The EKF is based on a recursive linearization of the system model around the estimated state so that the KF equations can be applied. A complete dissertation about the EKF is beyond the scope of this work and the reader may conveniently refer to [7]. Nevertheless, in order to make the present work more self-contained, the EKF equations required to estimate the state of a given nonlinear system are reported. Generally speaking, KF (and its variants) addresses the general problem of trying to estimate the state vector $\mathbf{x} \in \mathfrak{R}^n$ of a discrete-time controlled process that is governed by the generic set of equations [6]. The following system can be considered:

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}, \mathbf{w}_{k-1}), \quad \mathbf{z}_k = h(\mathbf{x}_k, \mathbf{v}_k) \quad (1)$$

where \mathbf{x} is the state vector, \mathbf{u} the system input, \mathbf{z} the system output (the measured variables) and \mathbf{v} , \mathbf{w} given Gaussian white noise. \mathbf{Q} and \mathbf{R} are defined as the system noise \mathbf{w} covariance matrix and the measurement noise \mathbf{v} covariance matrix, respectively. Now if the system is linear, (1) can be written in the form:

$$\mathbf{x}_k = A\mathbf{x}_{k-1} + F\mathbf{u}_{k-1} + \mathbf{w}_{k-1}, \quad \mathbf{z}_k = H\mathbf{x}_k + \mathbf{v}_k \quad (2)$$

Denoting with $\widehat{\mathbf{x}}_k^-$ the state estimate at step k according to the dynamic evolution of the system (first equation in (2)), the KF estimation is:

$$\widehat{\mathbf{x}}_k = \widehat{\mathbf{x}}_k^- + K(\mathbf{z}_k - H\widehat{\mathbf{x}}_k^-) \quad (3)$$

One form of K is given in [6]: it depends upon \mathbf{Q} and \mathbf{R} . For example, if \mathbf{R} approaches zero i.e. for an extremely reliable measurement, then $K \approx H^{-1}$ hence $\widehat{\mathbf{x}}_k \approx H^{-1}\mathbf{z}_k$. If the system is not linear i.e. it can be described only in the general form (1), it can be linearized and written in a form similar to (2). Such an approach is known as an EKF. The vehicle model provides a detailed description of its dynamic behavior, as it is based on the equilibrium equations. Considering the vehicle as a rigid body, the following is a generic set of equilibrium equations:

$$\begin{aligned} m a_x &= m(\dot{u} - vr) = \sum_{i=1}^{p_x} F_{x,i} \\ m a_y &= m(\dot{v} + ur) = \sum_{i=1}^{p_y} F_{y,i} \\ J_z \dot{r} &= \sum_{i=1}^{p_z} M_i \end{aligned} \quad (4)$$

where m is the vehicle mass, J_z is the polar moment of inertia, $F_{x,i}$ and $F_{y,i}$ are the generic force contributions along the longitudinal and lateral direction respectively, and M_i is the generic yaw moment contribution. A dynamic model can feature different levels of detail and complexity and adopt various assumptions, all these factors affecting the estimation accuracy. For the purpose of this work, a standard four-wheel model has been used (see Figure 1), while the three equilibrium equations can be found in [6].

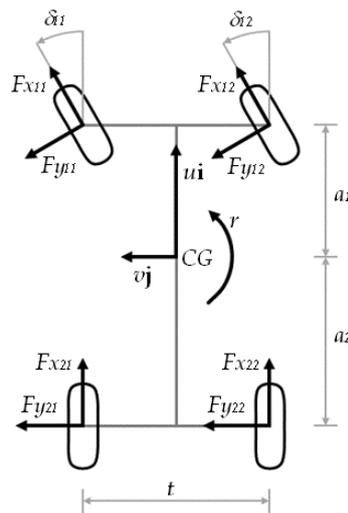


Figure 1. Dynamic model of a vehicle with road-tyre interaction forces.

2.2. *Neural Network-based estimation*

The Artificial Neural Network (ANN) approach is the same adopted by Chindamo et al. in [8], using the Levenberg-Marquardt back-propagation (BP) algorithm along with the mean squared error (MSE) performance index, as seen in [9]. This BP optimization algorithm updates weights and biases of the network as presented in [10]. See Figure 2 for the high-level schematics.

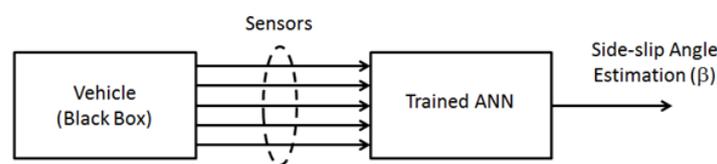


Figure 2. ANN estimation high-level schematics.

The ANN performance is then determined by the MSE which is:

$$MSE = \frac{1}{N} \sum_{i=1}^N (y_i - y_k)^2 \tag{5}$$

where y_i and y_k are the predicted and target value of the i -th pattern respectively and N is the number of patterns. The real-world dataset required to train and test the ANN is composed of input data patterns together with corresponding targets. In this case 90% of the dataset has been used to train the network while the remaining 10% has been used to test the capability of the network. This is a common procedure as reported in [11]. The mathematical background along with testing and training procedures can be found in [10, 11]. For this application inputs (which determine the ANN's input layer neuron number) are vehicle speed (V_x), steering angle (δ), lateral acceleration (a_y), longitudinal acceleration (a_x) and yaw rate (r). The output is the VSA (β), hence the ANN has only one neuron in

its output layer. Real-time estimation capabilities of ANNs strictly depend on the number of neurons in the hidden layer, to be carefully chosen: 10 in this case.

The most challenging part of this work was the identification of an appropriate set of maneuvers to be used for training. In order to be effective, the training data set should be able to represent the system and its dynamics in any possible conditions. When this criterion is met, the neuron number of the hidden layer can be minimized, thus resulting in a simple and light ANN structure [6, 7].

A comprehensive set of demanding maneuvers was designed according to the authors' experience and also with an eye on ISO vehicle test standards, and performed as reported in Section 2.3: Step steer from 30° to 120° steering wheel angle with speed ranging from 60 to 150 kph; 2Hz constant frequency sine steering (amplitude 60° and 90°) at 60 and 90 kph; 60m and 90m radius steering pad, with acceleration rate of 5 kph/s. Such data set was employed to train/feed the ANN thus meeting all the above-mentioned criteria. Therefore, the network can be considered ready for testing with a generic data set, as shown in Section 3.

2.3. Real-world test and procedures

The car selected for this test was a high-performance road car. All the tests were performed with a help of a professional test driver at the Nardò proving ground on a flat and dry tarmac surface with a consistent friction coefficient. Every maneuver was repeated at least three times for statistical purposes. Both EKF- and ANN-based methods are fed with the basic but critical data set related to vehicle dynamics, as mentioned in Section 2.2. In order to prove the effectiveness of each method, a measurement of the VSA (β) is required as well, to be used as a reference and for the purpose of ANN training. Therefore, a custom set of sensors was designed (see Table 1) and used in conjunction with a motorsport data logger.

Table1. List of sensors.

Sensor	Type	Range	Accuracy
GPS	MoTeC L10	n.a.	0.1 m/s
Accelerometer	Bosch AM600	±5 G	0,01 G
Yaw rate sensor	YRS3	±210 deg/s	0,1 deg/s
Steering (rotary pot.)	Gefran PZR20	±720 deg	0,1 deg
VSA sensor	Correxit S-400	±60 deg	0,1 deg

3. Results and discussion

This Section reports the comparison between the two VSA estimation methods. Each Figure refers to a different maneuver and includes the time histories of real-world VSA measured with the optical sensor (red) and VSA estimation with ANN (blue) and with EKF (green). Figures 3 and 4 represent two of the so-called steering pad maneuvers in steady state and with different final speed, while Figures from 5 to 8 represent four different step-steer maneuvers with different combinations of speed and target steering wheel angle.

First of all, as far as the EKF estimation method is concerned, the overall VSA estimation is considerably accurate. Potential drawbacks are related to the fact that tire/road force interactions should be modelled through accurate experimental data processed with Pacejka's well-known magic formula [6], but unfortunately data available from tire manufacturers might not be accurate enough quite often. Moreover, the computational effort required by the EKF is fairly high.

Furthermore, it is worth to point out that EKF estimation is quite good in lateral steady state conditions, while the error in transient conditions is larger. This is related to a state-space vehicle model composed of four state variables only, in order to limit the computational burden required: yaw rate, lateral acceleration, lateral forces on front and rear axles [6]. Hence transient maneuvers where combined lateral and longitudinal forces act on the vehicle at the same time are not fully modelled.

Finally, the ANN method appears to be the most accurate on both steady-state and transient maneuvers. This method is also lighter in computational terms since it is based on simple math operations only. However, as mentioned in Section 2, ANN estimation performances are strictly related to training quality, which in turn might be highly time and cost demanding. Moreover, the ANN is not able to consider heavy environmental and system changes such as tires wear and abrupt transition from dry to wet road surface if they are not explicitly included in the training.

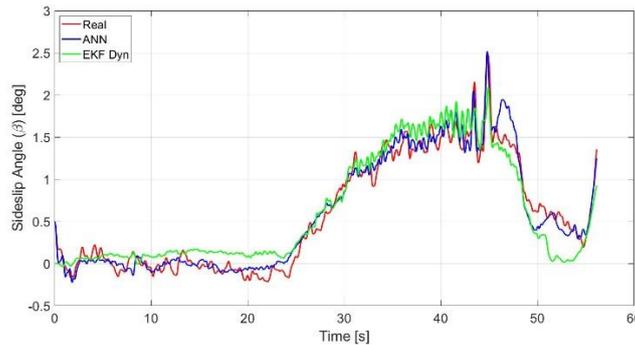


Figure 3. 90m radius steering pad manoeuvre, speed increasing from 60 to 120 kph.

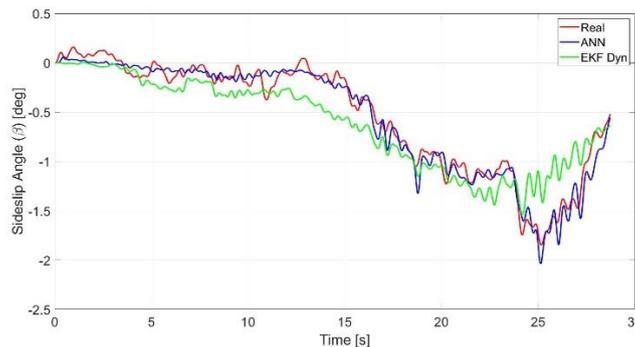


Figure 4. 90m radius steering pad manoeuvre, speed increasing from 60 to 100 kph.

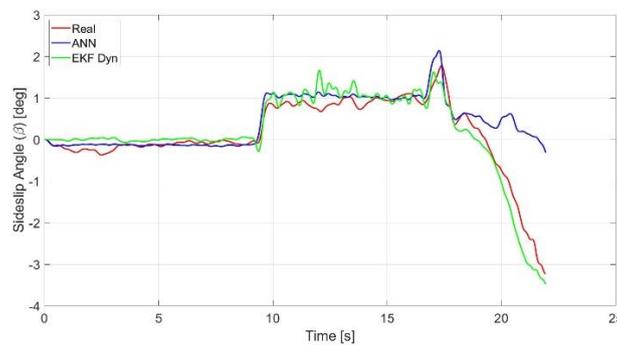


Figure 5. 70° step steer manoeuvre @ 125 kph.

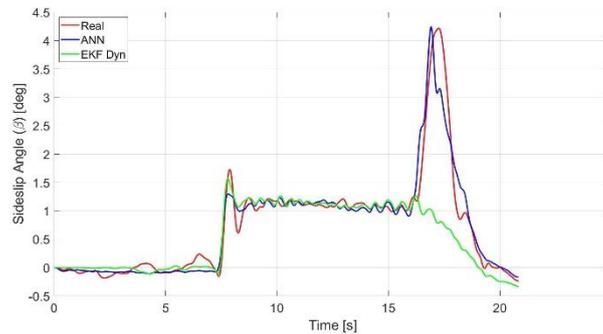


Figure 6. 90° step steer manoeuvre @ 125 kph.

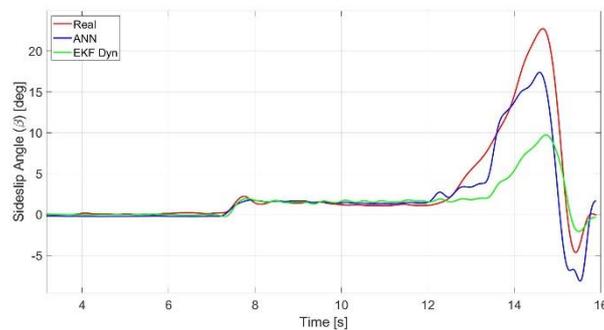


Figure 7. 70° step steer manoeuvre @ 125 kph.

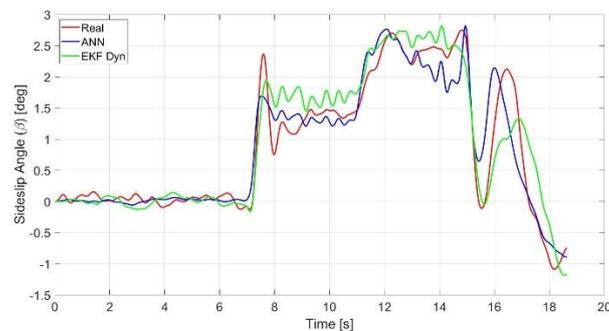


Figure 8. 90° step steer manoeuvre @ 150 kph.

4. Conclusion

This paper presents a comparison between two different VSA estimation methods, that are the most popular in vehicle dynamics literature: Kalman Filter and Artificial Neural Network.

All of them have positive aspects and drawbacks: the ANN method can give the best overall estimation with a low computational burden but it needs a very demanding training procedure, moreover it is not able to deal with changes of the system and boundary conditions that were not included in the training. The EKF itself is able to give a good VSA estimation as well but the computational burden increases with the accuracy of the vehicle model, which requires in turn many parameters (i.e. tire data) that might be difficult to get. This makes it difficult to use in key applications such as the typical model-based design approach required by real time operations in state-of-the-art driving simulators [12].

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