



Motion of a Taylor bubble in a realistic shear-thinning fluid

Davide Picchi ^{a,*}, Andrea Aquino ^a, Amos Ullmann ^b, Neima Brauner ^b, Pietro Poesio ^a

^a Department of Mechanical and Industrial Engineering, Università degli Studi di Brescia, Brescia 25123, Italy

^b School of Mechanical Engineering, Tel-Aviv University, Tel-Aviv 69978, Israel

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ABSTRACT

This talk will focus on the motion of a gaseous Taylor bubble in a capillary tube. Although the dynamics of a bubble in a Newtonian liquid has been the subject of several studies since the seminal works Taylor and Bretherton, the case where the fluid exhibits a shear-thinning behaviour is much less understood. To fill this gap, we derive a lubrication model in the film region to identify the scaling laws for the bubble speed and the film thickness as a function of the Ellis number and the degree of shear-thinning. Our model identifies a universal scaling law for the effective viscosity that accounts for the interplay of the zero-shear-rate and shear-thinning effects. After discussing the features of the front and rear menisci, we present an analysis of the recirculation vortexes ahead of the bubble. In the final part of the talk, we show the results of recent numerical simulations that validate the proposed theory and suggest a strategy to account for the effect of finite capillary numbers on the scaling law for the film thickness in the case of shear-thinning fluids.

Videos to this article can be found online at <https://doi.org/10.1016/j.sctalk.2022.100080>. **Figures and tables**

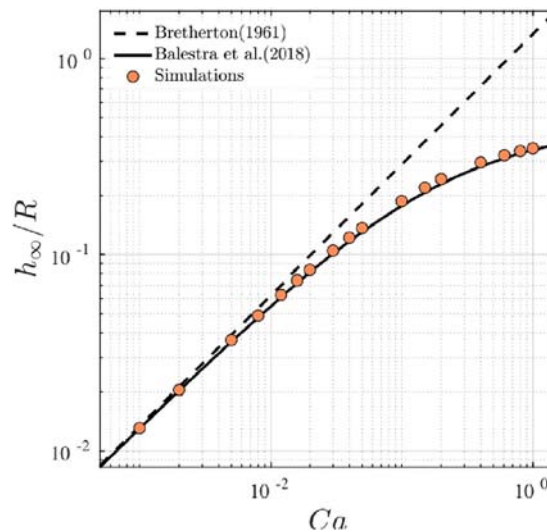


Fig. 1. The trend of the film thickness around a Taylor bubble in regimes where gravity and inertia can be neglected (Newtonian fluid). The seminal work of Bretherton [1] provides the scaling of the film thickness in the limit of small capillary numbers. The correlation by Balestra et al. [2] captures the data up to finite capillary numbers.

* Corresponding author.

E-mail address: davide.picchi@unibs.it (D. Picchi).

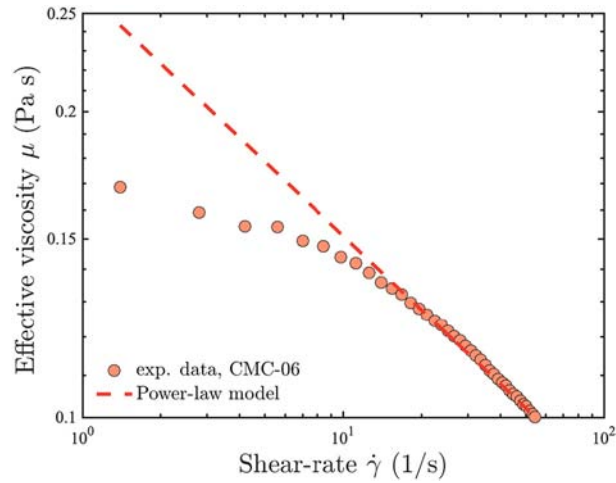


Fig. 2. Rheological characterization of a shear-thinning solution. Realistic shear thinning fluids exhibits a Newtonian behaviour at small shear-rates but the widely used power-law model does not capture this limiting behaviour.

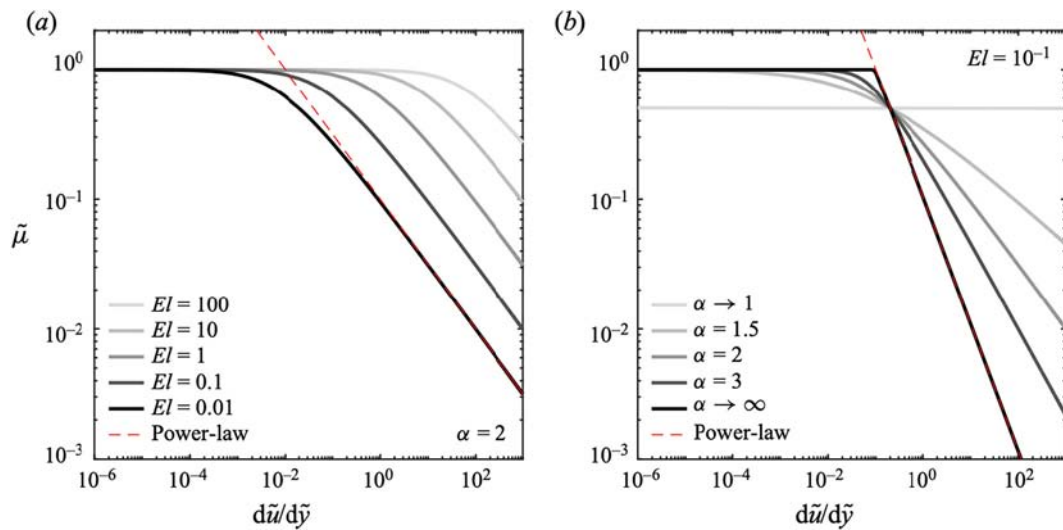


Fig. 3. The Ellis viscosity model. Evolution of the dimensionless effective viscosity as a function of the shear rate and the rheological parameters of the Ellis viscosity model. The power-law limit is plotted as a red-dashed line. The picture has been taken from Picchi et al. [3], with permission from Cambridge University Press. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

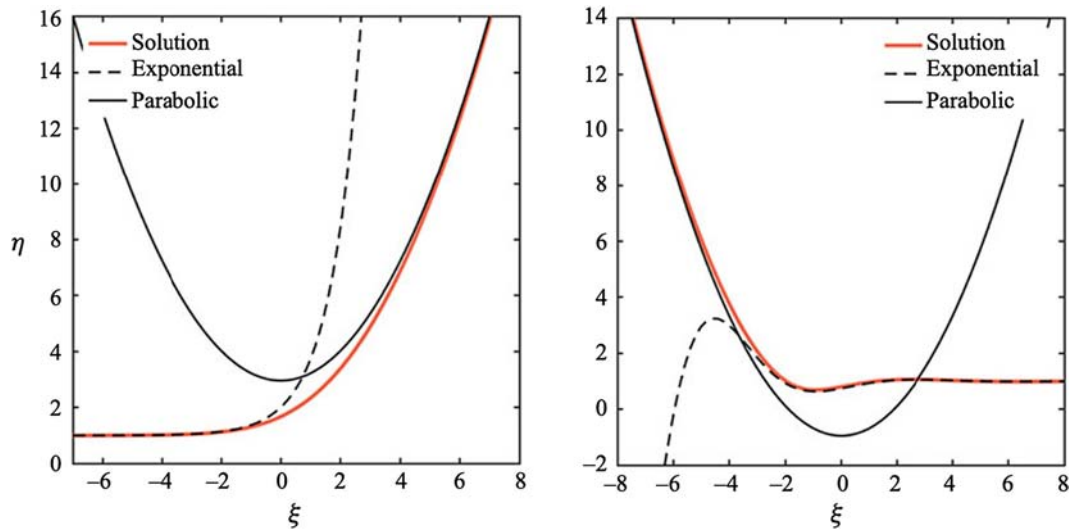


Fig. 4. Numerical solution of the film equation for an Ellis fluid for both the front and the bubble rear. Close to the uniform film region the profile follows an exponential behaviour while, far from it, it follows a parabolic trend. The picture has been taken from Picchi et al. [3] with permission from Cambridge University Press.

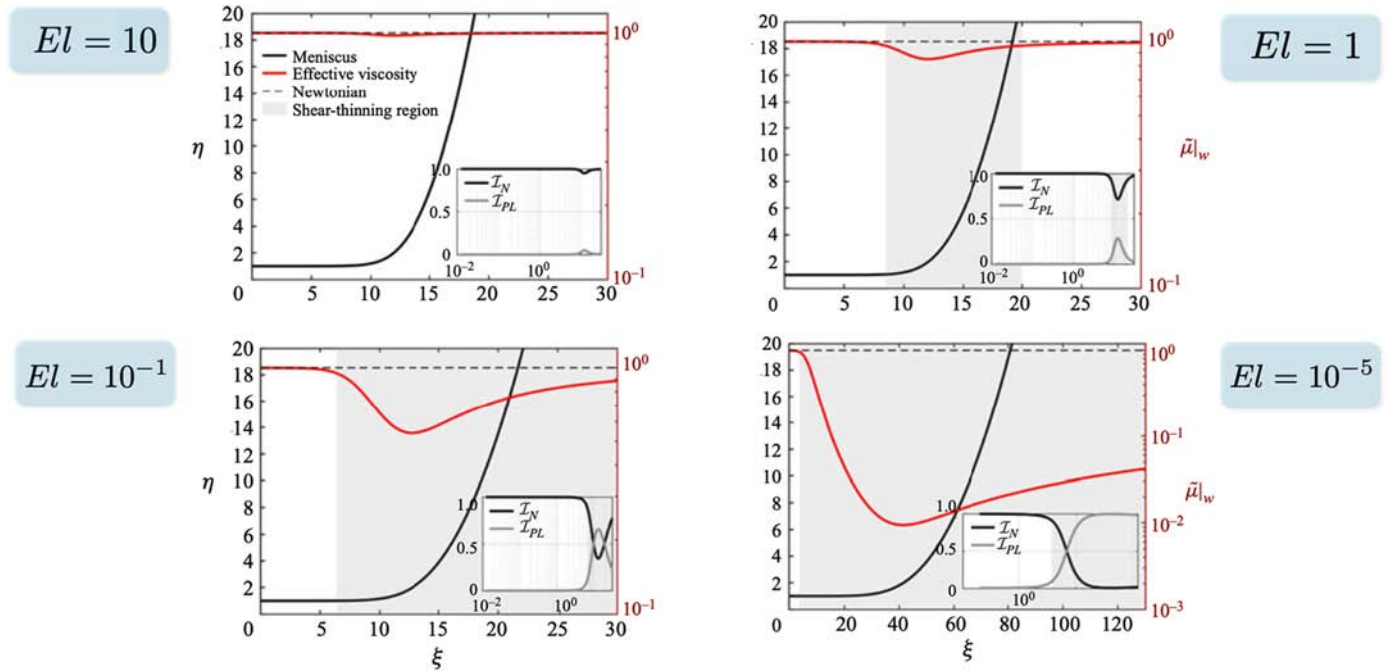


Fig. 5. Identification of the Newtonian and the shear-thinning regions in the bubble front as a function of the Ellis number. The insets show the weight of the Newtonian and the shear-thinning terms in the film equation. The pictures have been taken from Picchi et al. [3] with permission from Cambridge University Press.

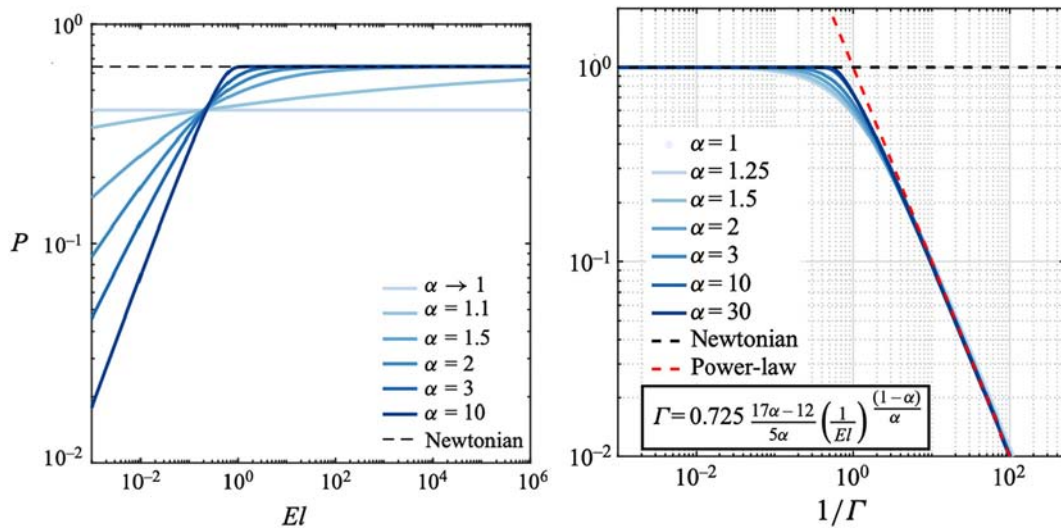


Fig. 6. Master curve for the effective viscosity of the system. (Left) Evolution of the coefficient which characterizes the curvature of the film thickness in the parabolic region of the bubble front. (Right) Collapsing of all the viscosity curves around the master curve for the effective viscosity. The pictures have been taken from Picchi et al. [3] with permission from Cambridge University Press.

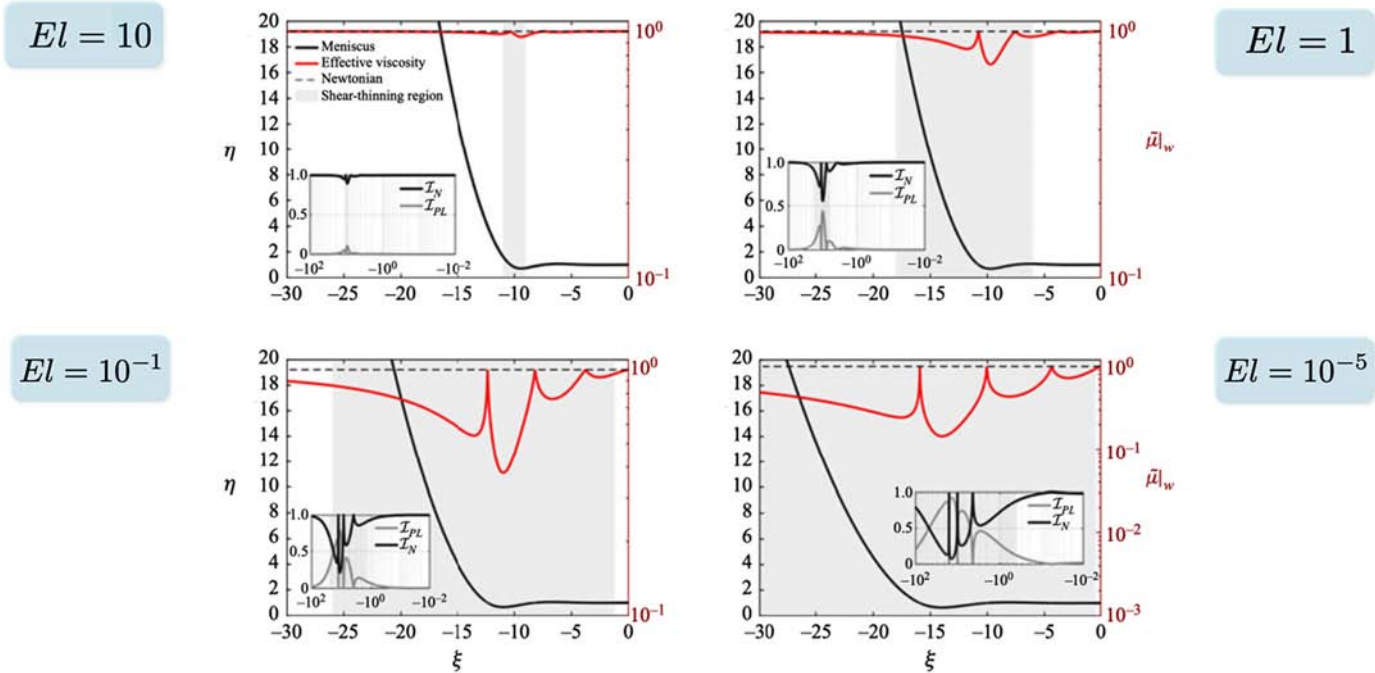


Fig. 7. Identification of the Newtonian and the shear-thinning regions in the bubble rear as a function of the Ellis number. The insets show the weight of the Newtonian and the shear-thinning terms in the film equation. The pictures have been taken from Picchi et al. [3] with permission from Cambridge University Press.

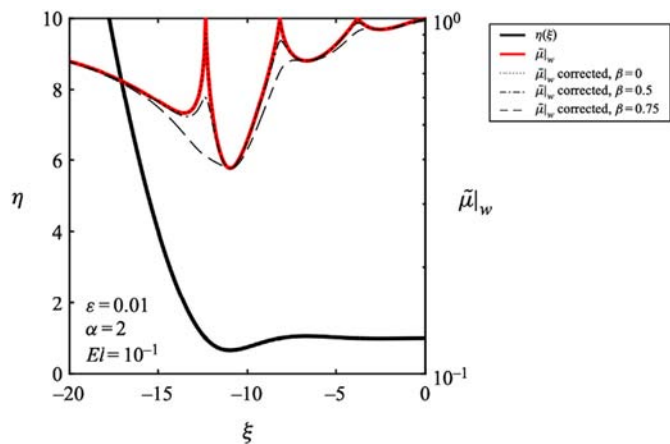


Fig. 8. Spikes of the wall-effective viscosity predicted by the film equation from Picchi et al. [3]. The picture has been taken from Picchi et al. [3] with permission from Cambridge University Press.

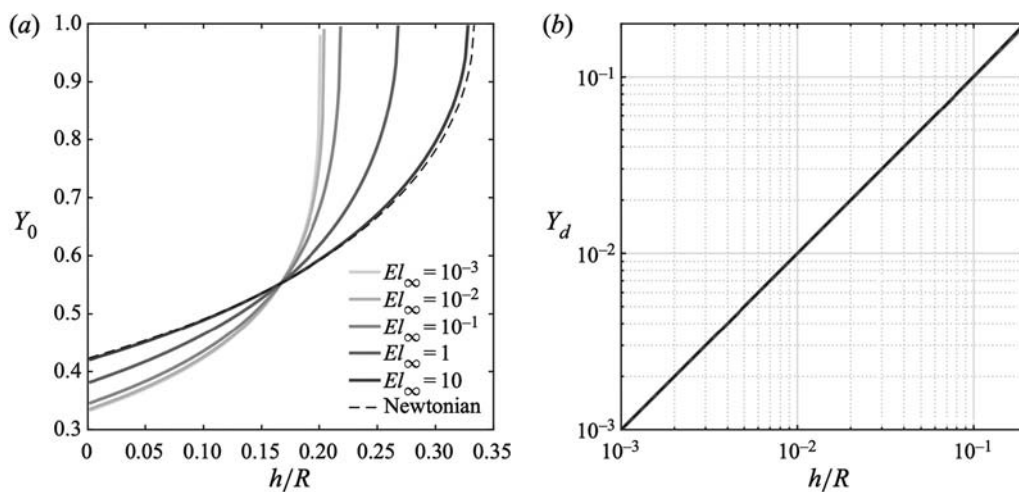


Fig. 9. Recirculation vortices ahead of the Taylor bubble. (a) Location of the center of the recirculating vortices ahead of the bubble as a function of the Ellis number. (b) Location of the dividing streamline as a function of Ellis number. The picture has been taken from Picchi et al. [3] with permission from Cambridge University Press.

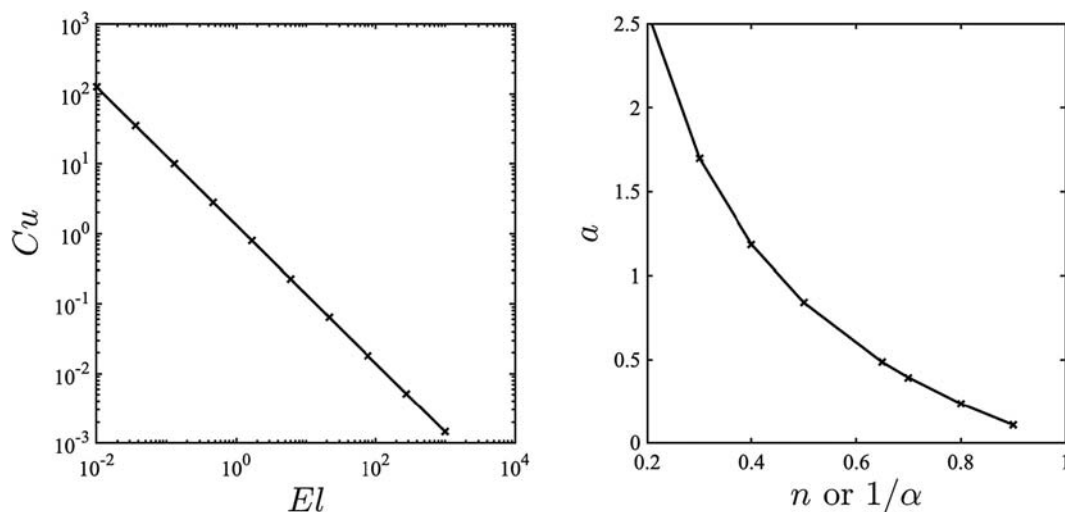


Fig. 10. Correspondence between the Ellis and the Carreau viscosity models.

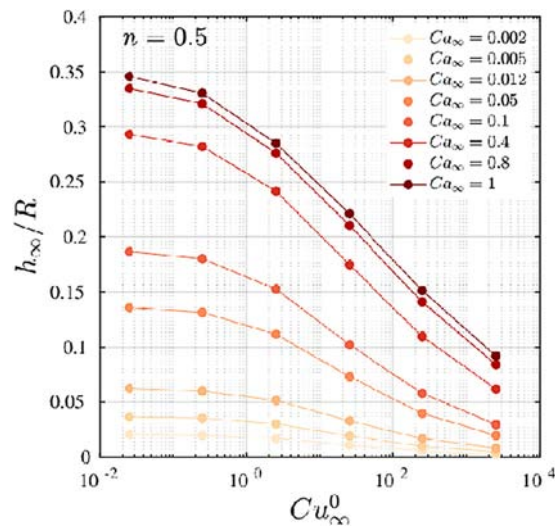


Fig. 11. Uniform film thickness as a function of the capillary and the Carreau numbers. The data are obtained from numerical simulations in a planar geometry.

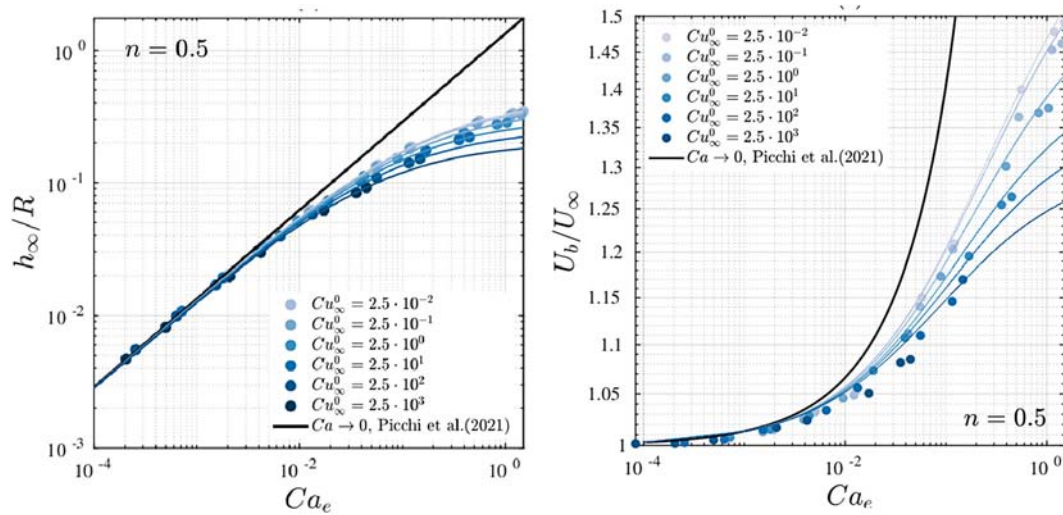


Fig. 12. Validation of the theoretical results with CFD data. (Left) Dimensionless film thickness as a function of the effective capillary number. (Right) Bubble speed ratio as a function of the effective capillary number. The lubrication theory by Picchi et al. [3] captures the evolution of both the film thickness and the bubble speed ratio in the small capillary number limit. An adaptation of the Aussilous and Quéré [4] correlation captures the behaviour up to finite capillary number for the case of shear-thinning fluids. Specifically, the functional dependence is preserved when the data are rescaled in terms of the effective capillary number.

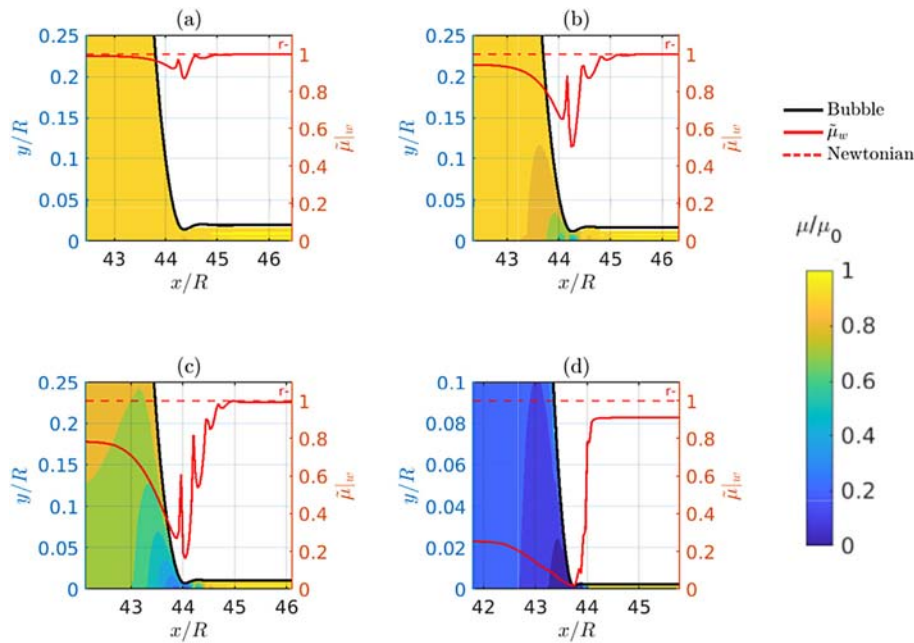


Fig. 13. Spatial distribution of the effective viscosity in the bubble rear. The cases depicted corresponds to the ones in Fig. 10. In all the cases the uniform film is the less-sheared region.

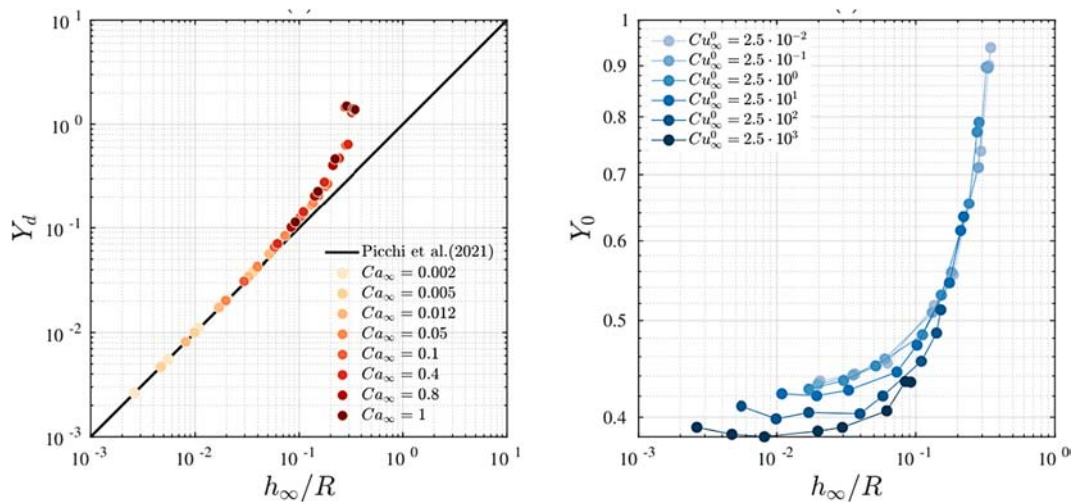


Fig. 14. Validation of the recirculation characteristics with CFD data. (Left) Location of the dividing streamline as a function of the capillary number; The prediction by Picchi et al. [3] capture the evolution in the limit of small capillary numbers. (Right) Heights of the center of the recirculating vortices computed from the numerical simulations.

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CRedit authorship contribution statement

Davide Picchi: Writing – original draft, Formal analysis, Validation, Conceptualization, Methodology. **Andrea Aquino:** Writing – review & editing, Software. **Amos Ullmann:** Writing – review & editing, Supervision, Conceptualization. **Neima Brauner:** Writing – review & editing, Supervision, Conceptualization. **Pietro Poesio:** Writing – review & editing, Supervision, Conceptualization.

Data availability

No data was used for the research described in the article.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

- [1] F.P. Bretherton, The motion of long bubbles in tubes, *J. Fluid Mech.* 10 (1961) 166, <https://doi.org/10.1017/S0022112061000160>.
- [2] G. Balestra, L. Zhu, F. Gallaire, Viscous Taylor droplets in axisymmetric and planar tubes: from Bretherton’s theory to empirical models, *Microfluid. Nanofluid.* 22 (2018) 1–27, <https://doi.org/10.1007/s10404-018-2084-y>.
- [3] D. Picchi, A. Ullmann, N. Brauner, P. Poesio, Motion of a confined bubble in a shear-thinning liquid, *J. Fluid Mech.* 918 (2021) A7, <https://doi.org/10.1017/jfm.2021.321>.
- [4] P. Aussillous, D. Quéré, Quick deposition of a fluid on the wall of a tube, *Phys. Fluids* 12 (2000) 2367–2371, <https://doi.org/10.1063/1.1289396>.

Further reading

- [1] L. Gamet, M. Scala, J. Roenby, H. Scheufler, J.L. Pierson, Validation of volume-of-fluid OpenFOAM® isoAdvector solvers using single bubble benchmarks, *Comput. Fluids* 213 (2020), 104722. <https://doi.org/10.1016/j.compfluid.2020.104722>.
- [2] H. Scheufler, J. Roenby, Accurate and efficient surface reconstruction from volume fraction data on general meshes, *J. Comput. Phys.* 383 (2019) 1–23, <https://doi.org/10.1016/j.jcp.2019.01.009>.
- [4] F. Kamişli, M.E. Ryan, Perturbation method in gas-assisted power-law fluid displacement in a circular tube and a rectangular channel. *Chemical Engineering Journal* 75 (3) (1999) 167–176, [https://doi.org/10.1016/S1385-8947\(99\)00088-1](https://doi.org/10.1016/S1385-8947(99)00088-1).



Davide Picchi is an Assistant Professor in the Dept. of Mechanical and Industrial Engineering of the University of Brescia. He joined UniBS after a BSc, MSc, and PhD in Mechanical Engineering at the University of Brescia, followed by two postdocs, the first at Tel-Aviv University and the second at Stanford University. His background is in thermal-fluid sciences and his research integrates theory development and experiments to investigate transport processes of multiphase flow and complex fluids in confined environments. Davide is also interested in multi-scale and multi-phase dynamics of flow in porous materials.



Andrea Aquino is an R&D project manager at the Research and Innovation Office of the University of Brescia. He earned an MEng in Building Engineering and a Ph.D. in Energy and sustainable development at the University of Perugia, followed by a postdoctoral position at the University of Brescia. His research focuses on numerical analysis for fluid dynamics and energy systems oriented toward heat and mass transfer optimization. Andrea is also interested in innovation and sustainable development policies, and he is serving as a UN volunteer to support the roadmap toward the 17SDGs.



Amos Ullmann is a Professor of Mechanical Engineering at the Tel Aviv University, Israel. He received the B.Sc. and the M.Sc. degrees in Mechanical Engineering from Ben-Gurion University of the Negev, Beer Sheva, Israel in 1984 and 1987 respectively, and the Ph.D. degree from the City University of New York, USA in 1992. Currently, he founded the Environmental Engineering program at the Faculty of Engineering, Tel-Aviv University and served as the head of the program for 22 years. His research interests include transport phenomena, heat and mass transfer processes, separation processes, soil/sediments remediation, multiphase flows, energy storage, and micro-pumps.



Neima Brauner is a Professor of Mechanical Engineering at Tel-Aviv University. She received her B.Sc. and M.Sc. in Chemical Engineering from the Technion in Haifa, and a Ph.D. in Mechanical Engineering from Tel-Aviv University in 1983. She is the editor of *Reviews in Chemical Engineering*, Associate editor of *Heat Transfer Engineering* and a member of several Editorial Boards, including *International Journal of Multiphase Flow*, *Multiphase Science and Technology*, *Experimental Thermal and Fluid Science*. She served as the President of the Israel Institute of Chemical Engineers (IICChE) and an honorary fellow of the IICChE. Her research interests include multiphase flow and transport phenomena, environmental engineering and data analysis.



Pietro Poesio is Professor of Applied Thermal and Fluid Sciences at the Università degli Studi di Brescia, where has been a faculty member since 2004. He graduated cum laude at the Università degli Studi di Brescia and he got a PhD cum laude in fluid Mechanics at Delft University of Technology. His research interests are in multiphase flows, heat transfer, and applied thermodynamics with particular emphasis on their industrial applications.