MILP Formulations for Single Batch-Processing with Non-Identical Jobs, Compatible Families, and Sequence-Dependent Setups

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Motivated by heat treatment in metalworking, this study investigates a new scheduling problem involving a single batch-processing machine (industrial oven) with limited capacity, where jobs are semi-finished products to be treated. The non-identical jobs must be partitioned into batches for simultaneous processing, minimizing the overall makespan. All jobs in a batch start and end their processing together, with the processing time of the batch being determined by the longest job. Moreover, jobs are grouped into compatible families, each with a specific minimum processing temperature. Within a batch, the dominant family is the one with the highest temperature requirement. An innovative aspect of our work is the inclusion of sequence-dependent setup times between consecutive batches, determined by the dominant families of each batch. Following Graham et al. (1979) classification, we address the $1 \mid s_j, p-batch, SU^k_{fg} \mid C_{max}$ problem. We call it the Single Batch-Processing Machine problem with Non-identical job sizes, Compatible families, and Sequence-dependent setup times.

We develop two mixed-integer linear programming models for the problem, and enforce them with different valid inequalities and lower bounds on both the number of non-empty batches and the makespan. The first model extends the mathematical formulations of the problem proposed by Melouk et al. (2004) and Trindade et al. (2018), incorporating new constraints to capture family dominance and family sequence-dependent setup times. The second adopts a different set of decision variables, providing an alternative representation of the scheduling decisions. Computational experiments validate the effectiveness of both formulations and provide a performance comparison between them.

Keywords: Batch-Scheduling, Sequence-Dependent Setups, Non-identical jobs, Mixed Integer Linear Programming

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