Transportation Research Interdisciplinary Perspectives 34 (2025) 101636

Contents lists available at ScienceDirect

Transportation Research Interdisciplinary Perspectives

journal homepage: www.sciencedirect.com/journal/transportation-research-interdisciplinary-perspectives

Comparing econometric and machine learning models to fare evasion prediction

Benedetto Barabino, Roberto Ventura *

Department of Civil, Environmental, Architectural Engineering and Mathematics (DICATAM), University of Brescia, Via Branze, 43 – 25123 Brescia, Italy

ARTICLE INFO

Keywords:
Fare evasion
Econometric models
Machine learning methods

ABSTRACT

Fare evasion poses a significant financial threat to Transit Agencies (TAs) and Public Transport Companies (PTCs) globally, especially within Proof-of-Payment Transit Systems (POP-TSs). Understanding and estimating fare evasion frequency is crucial for developing targeted countermeasures. Traditionally, Econometric Models (EMs) have been employed for this purpose, linking fare evasion frequency to specific predictors to assess their effects and significance. However, Machine Learning Models (MLMs) have recently emerged as promising tools, offering the potential for enhanced accuracy through complex data analysis. Despite their strengths, a comprehensive comparison between EMs and MLMs for predicting fare evasion frequency has been lacking in the literature.

This study addresses this gap by developing, calibrating, and validating two alternative frequency estimation models—an EM based on a Generalised Linear Regression Model (GLRM) and an MLM based on an Artificial Neural Network Model (ANNM). Using 4,000- real-world records from an Italian mid-sized PTC, the models' performances are quantitatively assessed through regression plots, error metrics, and fare evasion event ratios. The findings indicate that ANNM slightly outperforms GLRM on the considered dataset, showing a higher correlation coefficient, reduced margin of error, and a fare evasion event ratio closer to one. Moreover, the predictor effects were explored, an area where ANNM's "black box" nature traditionally limits transparency. An overview of these effects shows that while both models identify similar key factors, each prioritises different aspects of fare evasion influences. These insights would help TAs/PTCs select models based on data, interpretability needs, and fare evasion patterns, supporting more effective, data-driven management strategies.

1. Introduction

1.1. Background

Fare evasion threatens the financial stability of Transit Authorities/Public Transport Companies (TAs/PTCs), especially in Proof-of-Payment Transit Systems (POP-TSs), where passengers must buy tickets before boarding but are not systematically checked, enabling for opportunistic evasion.[1] Currently, there is no unified strategy to face fare evasion, but TAs/PTCs primarily rely on ticket inspections and fines to deter offenders (Reddy et al., 2011; Sasaki, 2014; Guarda et al., 2016; Dai et al., 2017; Barabino et al., 2020; Alhassan et al., 2022; Wolfgram et al., 2022; Barabino et al., 2024b). These strategies could be driven by estimating the frequency of fare evasion (i.e., the total number of fare evaders or some drivers of it), which is crucial for maintaining financial sustainability, operational efficiency, and passenger satisfaction within transit systems. Specifically, measuring fare evasion frequency is beneficial for addressing various issues. Fare evasion enables TAs/PTCs to pinpoint revenue losses, optimising resource allocation for enforcement and supporting financial stability. High evasion rates can erode equity perceptions among paying customers, potentially lowering satisfaction and reducing legitimate ridership. By analysing evasion patterns, TAs/PTCs can focus inspection efforts where they are most needed, however providing equity policies in enforcement strategies. Reliable fare evasion data also inform balanced fare policies, pricing, and subsidies, minimising evasion without unfairly burdening low-income riders. Lastly, the transparent management of fare evasion fosters public and stakeholder trust, reinforcing the commitment to fair and efficient service, which is crucial for long-term support and planning.

* Corresponding author.
E-mail addresses: benedetto.barabino@unibs.it (B. Barabino), roberto.ventura@unibs.it (R. Ventura).

[1] In this paper, opportunistic evasion refers to the behaviour that occurs when individuals exploit the perceived absence or low risk of inspection to avoid payment.

https://doi.org/10.1016/j.trip.2025.101636
Received 4 November 2024; Received in revised form 3 July 2025; Accepted 13 September 2025
Available online 18 September 2025
2590-1982/

1.2. Literature review

The frequency of fare evasion is influenced by multiple factors (variables, predictors, or attributes), which are examined using various approaches, including Descriptive Models (DMs), Econometric Models (EMs), and Machine Learning Models (MLMs).

DMs focus on summarising and representing observed fare evasion data without making predictions or inferences about future trends or the overall population. They use simple statistics (e.g., mean, median, mode) to identify patterns or distributions among variables in the data. In this context, studies in the USA revealed that several factors influence the frequency of fare evasion. In San Francisco, Lee (2011) found that fare evasion in POP-TS systems varies significantly by route, time, location, and inspection level, with higher rates in the afternoon and evening; GIS mapping further highlighted evasion patterns across routes and locations. Reddy et al. (2011) observed in New York that fare evasion in the subway system is influenced by entry characteristics, time, seasonality, and income levels, with higher evasion during rush hours (but lower evasions per passenger) in low-income neighbourhoods; staff presence alone did not significantly reduce evasion, estimated at 1.3 %.

Econometric Models (EMs) aim to make assumptions or predictions about a population of passengers based on a sample of data collected through various methods (e.g., surveys, inspection logs). They use advanced statistical techniques to estimate unknown parameters, test hypotheses, or predict future events, generalising from sample data to a larger population. Usually, EMs are complementary to the former: DMs help understand the data, while EMs enable conclusions to be extended beyond the observed data. In this context, numerous studies have estimated the probability of fare evasion, while a handful of studies have directly focused on the frequency of fare evasion. Specifically, research on the probability of fare evasion has explored the motivations, behaviours, characteristics, and attitudes of potential offenders, aiming to identify determinants that profile one-size-fits-all evaders or create distinct segments. Studies using probit and logistic regression models have shown that sociodemographic attributes (e.g., gender, age, education, car availability), travel factors (e.g., trip purpose, transit frequency), and situational variables (e.g., prior violations, perception of inspection likelihood) influence fare evasion (e.g., Bucciol et al., 2013; Barabino et al., 2015; Dai et al., 2017) as well as the inspection probability and fare sensitivity (Cools et al., 2018). Moreover, Guzman et al. (2021) applied a hybrid discrete choice model and found age and evasion history as primary drivers, with satisfaction and personality traits moderating evasion.

Other studies examined passengers' groups clustered in an *a priori* and *a posteriori* way (Barabino et al., 2024b). Using a *priori segmentation*, Barabino and Salis (2020) found that young, male students and workers with past violations are more prone to fare evasion, while unemployed passengers with previous fines also tend to evade. Barabino et al. (2022) identified prior fines as the strongest predictor, with other factors varying by age and gender. Currie and Delbosc (2017) emphasised that honesty, tolerance of evasion, ticket competence, and perceived ease of evasion influence both intentional and unintentional evaders.

A posteriori segmentation has leveraged clustering techniques to analyse route-specific data and categorise passenger behaviour. Delbosc and Currie (2016) identified three groups—deliberate, unintentional, and non-evaders—influenced by factors like smart card usability, inspection rates, and trip conditions. Gonzalez et al. (2019) classified passengers into paying and non-paying categories (each one clustered further), shaped by values, norms, and enforcement. Busco et al. (2021) revealed six evasion influencers, including social norms, service perception, and fear of enforcement. Barabino and Salis (2023) identified four clusters—occasional, choice-worker evaders, captive students, and chronic unemployed—where honesty negatively impacts the likelihood of frequent evasion.

As the focus shifted to the frequency of fare evasion, a handful of studies were conducted in Chile. Troncoso and de Grange (2017) found a negative correlation between unemployment and fare evasion rates using a cointegration model based on 86 months of macroeconomic data. A 10 % fare increase led to a 2 % increase in evasion, while a 10 % rise in inspection rates resulted in a 0.8 % decrease in evasion. Guarda et al. (2016) formulated a negative binomial regression model, revealing positive correlations between the frequency of fare evasion and the number of boarding (and alighting) passengers, buses with more doors, passengers boarding through the rear door, high occupancy levels, and long headways. Additionally, fare evasion was found to be more prevalent at bus stops in low-income areas during afternoon and evening periods. Similarly, Cantillo et al. (2022) developed a binomial logit model focusing on a single bus route. Their findings indicated that the incidence of fare evasion is particularly high in low-income neighbourhoods during the evening and nighttime hours. They also discovered that fare evasion rates tended to rise at bus stops situated more than 1 km from metro stations, especially those without off-board payment and ticket-selling facilities, as well as on overcrowded buses that lacked turnstiles. Furthermore, the research revealed that fare evasion was notably more frequent among young men. Recently, in Italy, Barabino et al. (2023) proposed a Generalised Linear Regression Model (GLRM) with a negative binomial regression error structure. The model showed that fare evasion frequency increased with exposure factors (carried passengers, level of occupancy) and vehicle capacity. Moreover, a higher fare evasion frequency was expected during off-peak periods, along targeted inspection paths and when inspectors were attended by checkers.

MLMs are tools that learn patterns from data to make predictions or decisions without explicit programming. These models are broadly categorised into supervised, unsupervised, and reinforcement learning. Supervised models (e.g., regression, classification) use labelled data to predict outcomes, while unsupervised models (e.g., clustering, dimensionality reduction) identify hidden patterns. Reinforcement learning models, on the other hand, learn optimal actions through trial and error (e.g., Jhaveri et al., 2022). A *plethora* of applications of MLM are found in the literature, which can be clustered in ten popular application areas (Sarker, 2021). These include: (i) Predictive Analytics and Decision-Making to analyse patterns for data-driven decisions, critical for inventory and demand management (e.g., Cao, 2017); (ii) Cybersecurity and Threat Intelligence to detect cyber threats through anomaly and malware detection using clustering and deep learning (e.g., Ślusarczyk, 2018); (iii) Smart Cities and IoT to optimize urban management, enhancing traffic, energy use, and resource allocation (e.g., Marchand & Marx, 2020); (iv) Healthcare for diagnostics and disease prediction (e.g., Fatima et al., 2017); (v) E-commerce and Product Recommendation for personalized suggestions, improving customer engagement (e.g., Marchand & Marx, 2020); (vi) Natural Language Processing (NLP) and Sentiment Analysis to interpret language, enhancing customer insights (e.g., Otter et al., 2020); (vii) Image, Speech, and Pattern Recognition for applications in social media, healthcare, and security (e.g., Fujiyoshi et al., 2019); (viii) User Behaviour Analytics and Context-Aware Applications for adaptive smartphone experiences (e.g., Sarker et al., 2021); (ix) Sustainable Agriculture to support yield prediction and disease management (e.g., Adnan et al., 2018); and (x) Transportation and traffic Prediction to forecast traffic congestion and improve urban mobility (e.g., Boukerche & Wang, 2020). For instance, in the field of bridge engineering, Ventura et al. (2024a)proposed a MLM to real-time estimating and managing the overload frequency on road bridges. However, in the specific domain of fare evasion frequency, only Barabino et al. (2024c) provided an initial attempt at modelling fare evasion risk, where the frequency is a component of it, using Artificial Neural Network Models (ANNMs). Here, exposure (i.e., passenger volume), organisational (i.e., type of inspection), and context factors (i.e., time period) showed strong importance in the predictions.

1.3. Research gaps and objective

All previous studies have provided relevant contributions to the estimation of fare evasion frequency using different modelling. Nevertheless, some main gaps remain.

First, to the best of the authors' knowledge, no literature has compared the fitting and predictive performance of GLRMs and ANNMs in predicting fare evasion events. While Barabino et al. (2024c) presented an initial application in the context of fare evasion risk, they focused primarily on model development and validation within a single predictive framework. Conversely, the study extends this research by introducing a direct, side-by-side comparison between ANNMs and GLRMs, on the frequency component.[2] Nevertheless, comparing these models is essential because each approach brings unique strengths to data analysis, enabling better insights and predictions across different contexts. GLRMs focus on understanding causal relationships, interpretability, and statistical inference, making them ideal for policy analysis and theoretical insights. Conversely, ANNMs models prioritise prediction accuracy and can handle complex, high-dimensional datasets, often discovering patterns without predefined assumptions. Thus, by this comparison, researchers and practitioners can determine the best-suited model for specific tasks, balancing interpretability with predictive power.

Second, ANNMs can capture the complex (often nonlinear) relationship between the frequency of fare evasion and its influencing factors, as well as better handle correlation issues among predictors compared to GLRMs (e.g., Chang, 2005). Moreover, they generally exhibit greater predictive performance than GLRMs (e.g., Hegde and Rokseth, 2020). However, they are often perceived as "black boxes" in contrast to GLRMs, making it more challenging to decipher the individual impact of each predictor (Zhang, 2010). The permutation feature importance can effectively rank predictors based on their importance (e.g., Strobl et al., 2007). However, to the best of our knowledge, no research has provided a method to evaluate the effect of each factor in the prediction of the fare evasion frequency by ANNM. Moreover, this study goes beyond model performance metrics by providing a systematic evaluation of the effects of each predictor, including the use of partial derivatives for interpreting nonlinear behaviour in the ANNM, which are only mentioned in Barabino et al. (2024c).

This study seeks to address these gaps by developing, calibrating, and validating two predictive models focused on fare evasion frequency, using GLRMs and ANNMs. It then performs a comprehensive evaluation of each model's global predictive performance, alongside a detailed analysis of the influence of individual factors on prediction outcomes. Real data for this experiment were obtained from the fare inspection logs available from an Italian mid-sized PTC.

1.4. Paper structure

The remainder of the paper is organised as follows: Section 2 outlines the fare evasion data and methodologies used to process it, identify predictors of frequency, set up GLRMs and ANNMs, and evaluate their performance, including the impact of each factor on fare evasion frequency. Section 3 presents and discusses the findings, while Section 4 concludes the study and suggests directions for future research. Finally, the Appendix provides a comprehensive list of all acronyms and variables used in the paper.

[2] While both studies use data from the same PTC, the scope, objectives, and analytical depth of the current work are distinct and significantly expanded.

2. Data and methods

2.1. Research context, data type and data collection

This study is performed in the metropolitan area of Cagliari, Italy, which is home to approximately 0.4 M inhabitants. The transit systems in Cagliari are operated mainly by CTM, with 270 + trolleybuses and buses over 12 + Mkm yearly along 32 routes (CTM, 2023). Experimental data on fare evasion events were extracted from CTM inspection logs collected during a four-month monitoring period spanning 2013 to 2015.[3] These data were organised according to a three-level classification that included final factors of fare evasion, intermediate factors from which fare evasion can depend, and exposure data that can affect fare evasion.

Each route in the network is preliminarily divided into small, spatially homogeneous segments, each with similar characteristics but differing from one another to refer to fare evasion data for each segment.[4] Formally, let: R be the set of transit routes; $S(r)$ be the set of homogeneous segments of route $r \in R$; T be the set of time windows during the considered monitoring period; X be the set of fare evasion frequency predictors, and $x(s,r,t)$ be the value of the generic predictor $x \in X$ observed on homogeneous segment $s \in S(r)$ of route $r \in R$ during time window $t \in T$; $K \subset X$ be the set of exposure factors, and $k(s,r,t)$ be the value of the generic factor $k \in K$ observed on homogeneous segment $s \in S(r)$ of route $r \in R$ during time window $t \in T$; $freq(s,r,t)$ [events/window] be the fare evasion frequency observed on homogeneous segment $s \in S(r)$ of route $r \in R$ during time window $t \in T$.

In a POP system, passenger inspection is conducted by inspection teams and is driven by deployment (spot or patrol), location (on-board or station-based), and inspection (mass or selective) policies, resulting in various combinations that create distinct fare inspection systems (Escalona et al., 2024). According to this organisation, in this study, inspection logs were gathered through a patrolling deployment policy, where each inspection team typically consists of two people (with occasional patrols involving at least three individuals) follows a time–space path, verifying fare payment at different segments of the bus network. Inspections were performed on board vehicles to minimise service disruptions. Additionally, a selective inspection policy is implemented due to the impracticality of inspecting every ride throughout the day, given that not every ride is accompanied by inspectors. Consequently, inspectors either randomly select or specifically target certain rides for inspection, ensuring that they check the tickets of every passenger to maintain fairness in the inspection process. In the targeted strategy, inspections take place along planned routes in both directions, focusing on segments between key stops where passenger volumes are highest during peak periods. In the random approach, inspectors board vehicles at randomly chosen stops, which may result in the selection of specific routes or services. Moreover, the inspection is carried out by inspectors and checkers. While the role of inspectors is clear, checkers support inspections by collecting additional passenger data (e.g., passenger volumes and boarding/alighting data) and identifying fare evaders who validate tickets only upon seeing inspectors. They board and alight two stops before and after the inspection team, respectively. According to this policy, the experiment includes five types of inspection paths. Four are targeted inspections, labelled A through D. Paths A and B involve both checkers and inspectors, used on standard and articulated buses with high passenger volumes. Paths C and D rely

[3] We are unable to analyse more recent data due to the secrecy policy of CTM.

[4] This study adopts segmentation between consecutive bus stops because: (i) Passenger counts at each stop better capture exposure variability; (ii) PTCs commonly analyse data per consecutive stops, aligning results with established metrics; (iii) if one defines segments as "legs between two stops" avoids ambiguity from other segmentation methods.

solely on inspectors and are assigned to medium and short buses with comparatively lower ridership. In contrast, path E represents a random inspection strategy conducted only by inspectors. More details about these inspection patterns were reported in Barabino et al. (2023).

For each route segment $s \in S(r)$ investigated during a given time window $t \in T$, inspectors recorded data on passengers checked and fined. Consequently, $freq(s,r,t)$ was straightforwardly determined for each route segment $s \in S(r)$ as the number of fare evasion events[5] reported in inspection logs during the corresponding time window $t \in T$.[6]

As for the fare evasion frequency predictors (X), they encompass both intermediate factors and exposure factors. Intermediate factors pertain to the context (e.g., fine policies, temporal elements, spatial elements), organisation (e.g., system planning and design, inspection patterns), and vehicles (e.g., type, equipment, layout, capacity). Exposure factors relate to transit supply (e.g., route characteristics, service productions, patrolling time-based features), transit demand (e.g., ridership along a route), and interaction among supply and demand (e.g., passengers, level of occupancy). All these factors were derived from inspection logs and manual route-by-route ride checks.

Table 1 provides an overview of all predictors used in the modelling process, including a brief description of each variable and some descriptive statistics. Moreover, in Table 1, all variables are either categorised or converted into binary form for modelling, except for inspection path length and bus capacity, which are treated as continuous and integer variables, respectively. Additionally, dummy variables are shown in italics for ease of comparison.

A total of 4,000- real-world-raw-inspection records were adopted for this experiment.

2.2. Econometric model

Once the experimental dataset was prepared, the frequency model was first generated using a traditional EM. Both the Poisson and Negative Binomial formulations can effectively model fare evasion frequency (Guarda et al., 2016). These models were commonly utilised for non-negative discrete response variables due to their robust statistical properties. However, the frequency of fare evasion must be zero when the exposure variable is zero. Therefore, the frequency model was estimated using a GLRM with a Negative Binomial Regression (NBR) error structure, following methodologies employed in prior studies (e.g., Barabino et al., 2023).

Conceptually, the GLRM estimates the expected number of fare evasion events as a function of multiple influencing factors. The model assumes fixed effects for each predictor variable, meaning that the contribution of each factor to the outcome is additive and linear on the chosen link scale. This approach is particularly useful for identifying statistically significant relationships and quantifying the direction and magnitude of each predictor's influence.

More formally, let: TR and TE be the training and testing subsets, respectively; $\widetilde{freq}(s,r,t)$ [events/window] be the predicted value for the observed $freq(s,r,t)$; α, β_k and γ_x be the coefficients of the econometric model; p_{gl} be the global p-value associated with the model; p_x be the individual p-value associated with the predictor $x \in X$; dr be the deviance ratio of the model. The functional form of the GLRM was defined as follows:

$$\widetilde{freq}(s,r,t) = \alpha\left(\prod_{k\in K} k(s,r,t)^{\beta_k}\right) e^{\sum_{x\in X: x\notin K}\gamma_x * x(s,r,t)}; \quad \forall s \in S(r); \forall r \in R; \forall t \in T; \tag{1}$$

Before the training phase, the experimental dataset was randomly divided into TR and TE subsets. TR was used to train the model, with its parameters adjusted to fit the training data. TE remained independent of the training process, providing an objective measure of model performance. Subsequently, an automated stepwise approach was used, incorporating forward selection and/or backward elimination, to determine the most suitable set of predictors from the variable set X. The choice between backward and forward stepwise techniques was based on the highest dr and p_{gl} of the estimated model. After model fitting, the sign of each coefficient and p_x were evaluated to understand the effect of $x \in X$ on the frequency.

2.3. Machine learning model

Next, the frequency model was built using an ML based-approach, specifically an ANNM, chosen for its effectiveness when enough data are available, even if noisy (Hegde and Rokseth, 2020). Conceptually, the ANNM mimics how the human brain processes information by learning patterns in the data through interconnected layers of artificial "neurons." Each neuron transforms its input using weighted connections and nonlinear activation functions, enabling the network to automatically capture intricate nonlinear effects and higher-order interactions between predictors without requiring explicit specification. However, this flexibility comes at the cost of interpretability, because the internal workings and parameter effects are not easily understandable, which can pose challenges for transparency and explainability in applied settings. More formally, let: TR, VA and TE be the training, validation, and testing subsets, respectively; ϕ be a mathematical function representing the ANNM; N be the set of ANNM hidden layer neurons and $n \in N$ be a generic neuron; $\overrightarrow{\lambda}$ be a generic parameter vector of the ANNM and $\overrightarrow{\lambda_0}$ be the specific vector found via the learning phase; ρ_x be the permutation feature importance associated with the predictor $x \in X$; $\varepsilon(r,s,t)$ [events/window] be the residual value for the frequency prediction on homogeneous segment $s \in S(r)$ of route $r \in R$ during time window $t \in T$; MSE be the mean squared error (acting as cost function). The functional form of the ANNM for frequency prediction was defined as follows:

$$\widetilde{freq}(s,r,t) = \phi\left(\{x(s,r,t) : \forall x \in X\}, \overrightarrow{\lambda}\right); \forall s \in S(r); \forall r \in R; \forall t \in T; \tag{2}$$

To define the function ϕ, a two-layer feed-forward network was selected for its simplicity among ANNM structures (e.g., Schmidhuber, 2015). In this architecture, information flows in a single direction, moving from input nodes through hidden nodes to output nodes. The network consists of two neuron layers: a hidden layer with $|N|$ neurons and an output layer with one neuron. The hidden layer employed a hyperbolic tangent sigmoid activation function due to its antisymmetric nonlinear mapping properties, which have been demonstrated to enhance network convergence and generalisation (e.g., Romero Reyes et al., 2013; Sramka et al., 2019). The output layer utilised a linear activation function to model fare evasion frequency as a regression problem.

Before training, the experimental dataset was randomly divided into TR, VA and TE subsets. TR and TE served the roles described in the EM, while VA was used to assess network generalisation and prevent overfitting. Training proceeded in epochs and was stopped when the model's performance on the validation set no longer improved. During training, the model parameters $\overrightarrow{\lambda}$ were iteratively optimised on TR data by minimising MSE, a cost function commonly used in regression problems. At the end of each epoch, the MSE was evaluated on VA to monitor

[5] According to CTM, fare evasion occurs when a passenger intentionally fails to purchase, validate, or obtain a ticket and/or pass for travel.

[6] To ensure comparability across observations, the monitoring period was divided into standardised time windows aligned with fare inspectors' operational hours and ridership patterns. These intervals—classified as peak (morning, midday, evening) or off-peak—provide a quasi-uniform temporal framework for calculating fare evasion frequencies. While 56.12% of observations were collected during the longer non-peak periods (covering 11 operational hours), 43.88% were gathered during shorter peak periods characterised by higher ridership and inspector presence, supporting a balanced and consistent sampling approach.

Table 1
Dataset for evaluating the frequency of fare evasion: List of variables and descriptive statistics.

Response variable	Abbreviation	Description		Min_Value	Max_Value	% Value
Number of evaders		Evaders	Total number of evaders [#]	0	76	–
Explanatory variables		**Abbreviation**	**Description**			
Exposure						
		Passenger volume	Average daily flow of passengers along each inspection path	15	19,365	–
		Level of occupancy	Ratio between the checked passengers and the vehicle capacity for each inspection path	0	1.33	–
Context						
Temporal element	Time period	Rush_Morning	Rush morning hours from 7:30 to 9:30	0	1	15.54 %
		Rush_Half-day	Rush half-day hours from 12:30 to 14:30	0	1	16.93 %
		Rush_Evening	Rush evening hours from 17:30 to 19:30	0	1	11.41 %
		Soft_During day	*The remaining hours of a day*	0	1	56.12 %
	Type of Day	Weekday	Days from Monday to Friday	0	1	84.22 %
		Weekend	*Saturday and Sunday*	*0*	1	15.78 %
Organisation						
Inspection pattern	Type	A	Planned inspection path of Type A followed from inspectors centred on maximum load section	0	1	38.01 %
		B	Planned inspection path of Type B followed from inspectors centred on maximum load section	0	1	30.77 %
		C	Planned inspection path of Type C followed from inspectors centred on maximum load section	0	1	19.01 %
		D	Planned inspection path of Type D followed from inspectors centred on maximum load section	0	1	9.96 %
		E	Random inspection path followed from inspectors ignoring the maximum load section	0	1	2.25 %
	Level of enforcement	Standard	The average number of inspectors (1 patrol inspection) is 2	0	1	43.35 %
		No standard	The average number of inspectors (1 patrol inspection) is larger than 2	0	1	56.65 %
System Planning and design	Network configuration	Path Length	Length of path [m]	100	14,650	–
Vehicle						
Layout	Vehicle design	Bus	The vehicle is a bus	0	1	90.34 %
		Trolleybus	*The vehicle is a trolleybus*	0	1	9.66 %
	Capacity	Capacity	The capacity of the adopted vehicle [seats]	*9*	*179*	–
Type	Length	Medium	The vehicle is long 10.5 *m*	0	1	15.58 %
		Standard	The vehicle is long 12 *m*	0	1	66.33 %
		Long	The vehicle is long 18 *m*	0	1	6.09 %
		Short	*The vehicle is long less than 10.5 m*	0	1	12.00 %

The same dataset was used in Barabino et al. (2023).

generalisation. Training was stopped when the validation *MSE* ceased to improve, and the final model parameters $\vec{\lambda}_0$ were chosen as those corresponding to the epoch with the lowest validation *MSE* (Eqn. (3)):

$$\vec{\lambda}_0 = \underset{\vec{\lambda}}{\operatorname{argmin}} MSE\left(\vec{\lambda}\right)_{VA} \tag{3}$$

The Levenberg–Marquardt algorithm was used to solve Eqn. (3) because it is widely recognised as one of the most effective methods for training feedforward neural networks (e.g., Ian et al., 2016). Unlike traditional algorithms that rely on a fixed learning rate, the Levenberg–Marquardt algorithm utilises an adaptive damping factor, which dynamically adjusts the step size according to training performance. This adaptive mechanism typically leads to faster convergence and greater stability compared to methods that use a fixed learning rate (Marquardt, 1963; MathWorks, 2025).

Next, the importance of each predictor was assessed to determine its impact on the estimated frequency. To accomplish this, the permutation feature importance (ρ_x) was employed. It is defined as the reduction in model performance when the value of a single predictor is randomly shuffled (Breiman, 2001). Specifically, let:

- B be the set of random permutation occurrences.
- $\overline{\overline{INP_{b,x}}} \in \mathbb{R}^{|B|,|X|}$ be the "broken" input matrix at the permutation $b \in B$, produced by randomly reordering the column $x \in X$ of the uncorrupted input matrix $\overline{\overline{INP}}$.
- $MSE\left(\vec{\lambda}\right)_{TE}$ be the cost function of the trained ANNM, evaluated on TE considering the "unbroken" input matrix $\overline{\overline{INP}}$.
- $MSE\left(\vec{\lambda}\right)_{TEb,x}$ be the cost function of the trained ANNM, evaluated on TE by considering the "broken" input matrix $\overline{\overline{INP_{b,x}}}$.
- $s = -MSE\left(\vec{\lambda}\right)_{TE}$ be the reference score of the trained ANNM on the "unbroken" input matrix $\overline{\overline{INP}}$.

- $s_{b,x} = -MSE\left(\overrightarrow{\lambda}\right)_{TEb,x}$ be the score of the trained ANNM on the "broken" input matrix $\overline{\overline{INP_{b,x}}}$.

Therefore, many permutation occurrences ($|B|$) are performed for each $x \in X$ and the associated ρ_x is computed by the average of the scores according to eqn. (4).

$$\rho_x = s - \frac{1}{|B|}\sum_{b \in B} s_{b,x}; \forall x \in X; \tag{4}$$

The higher ρ_x, the higher the importance of $x \in X$. This technique disrupts the nexus between the predictor and the target variable, and the reduction in model performance indicates the extent to which the model relies on the predictor.

Although ρ_x is a highly relevant indicator for evaluating MLMs, it cannot directly capture the effect of each factor on fare evasion frequency. Therefore, to accurately compare EMs and MLMs, it is essential to evaluate these effects. One approach could involve computing partial derivatives for each inspection record and analysing their relative frequency distribution. This distribution is particularly informative for ANNM, which is expected to exhibit nonlinearity. In nonlinear models, such as ANNM, a predictor's effect on fare evasion frequency varies across records depending on other input factors. By contrast, in a linear model, the partial derivative (representing a factor's marginal effect) remains constant across records due to the additive nature of the relationship. Hence, the distribution's shape provides insight into the strength of nonlinearity in the ANNM. A higher dispersion in the partial derivative distribution for a given variable implies greater nonlinearity, as it indicates that the relationship between the predictor and the predicted outcome varies depending on the values of other features. In contrast, lower dispersion—zero in the case of a perfect linear model—indicates more consistent effects across records. Specifically, let:

- $\frac{\partial \phi}{\partial \overline{x}}(s,r,t)$ be the partial derivative of the function ϕ with respect to the predictor $\overline{x} \in X$, computed for a homogeneous segment $s \in S(r)$ of route $r \in R$ during the time window $t \in T$.
- $\mu\left(\frac{\partial \phi}{\partial \overline{x}}\right)$, $\sigma\left(\frac{\partial \phi}{\partial \overline{x}}\right)$, and $CoV\left(\frac{\partial \phi}{\partial \overline{x}}\right)$ be the mean, the standard deviation and the coefficient of variation of the $\frac{\partial \phi}{\partial \overline{x}}(s,r,t)$ values computed across the dataset, respectively.
- $P\left(\frac{\partial \phi}{\partial \overline{x}}\right)^+$ be the probability that $\frac{\partial \phi}{\partial \overline{x}}(s,r,t) \geq 0$ within the dataset.
- $\Delta\overline{x} \approx 0$ be a marginal increase in the value of predictor $\overline{x} \in X$.

Thus, $\frac{\partial \phi}{\partial \overline{x}}(s,r,t)$, $\mu\left(\frac{\partial \phi}{\partial \overline{x}}\right)$, $\sigma\left(\frac{\partial \phi}{\partial \overline{x}}\right)$, $CoV\left(\frac{\partial \phi}{\partial \overline{x}}\right)$ and $P\left(\frac{\partial \phi}{\partial \overline{x}}\right)^+$ are numerically estimated as follows:

$$\frac{\partial \phi}{\partial \overline{x}}(s,r,t) \cong \frac{\phi\left(\overline{x}(s,r,t) + \Delta\overline{x}, \{x(s,r,t) : \forall x \in X, x \neq \overline{x}\}, \overrightarrow{\lambda_0}\right) - \phi\left(\{x(s,r,t) : \forall x \in X\}, \overrightarrow{\lambda_0}\right)}{\Delta\overline{x}}; \tag{5}$$
$$\forall \overline{x} \in X; \forall s \in S(r); \forall r \in R; \forall t \in T;$$

$$\mu\left(\frac{\partial \phi}{\partial \overline{x}}\right) = \frac{\sum_{t \in T}\sum_{r \in R}\sum_{s \in S(r)}\frac{\partial \phi}{\partial \overline{x}}(s,r,t)}{|T| \bullet \sum_{r \in R}|S(r)|}; \forall \overline{x} \in X; \tag{6}$$

$$\sigma\left(\frac{\partial \phi}{\partial \overline{x}}\right) = \sqrt{\frac{\sum_{t \in T}\sum_{r \in R}\sum_{s \in S(r)}\left(\frac{\partial \phi}{\partial \overline{x}}(s,r,t) - \mu\left(\frac{\partial \phi}{\partial \overline{x}}\right)\right)^2}{|T| \bullet \sum_{r \in R}|S(r)|}}; \forall \overline{x} \in X; \tag{7}$$

$$CoV\left(\frac{\partial \phi}{\partial \overline{x}}\right) = \frac{\sigma\left(\frac{\partial \phi}{\partial \overline{x}}\right)}{\left|\mu\left(\frac{\partial \phi}{\partial \overline{x}}\right)\right|}; \forall \overline{x} \in X; \tag{8}$$

$$P\left(\frac{\partial \phi}{\partial \overline{x}}\right)^+ = \frac{\left|\left\{\frac{\partial \phi}{\partial \overline{x}}(s,r,t) \geq 0 : \forall s \in S(r), \forall r \in R, \forall t \in T\right\}\right|}{|T| \bullet \sum_{r \in R}|S(r)|}; \forall \overline{x} \in X; \tag{9}$$

A positive value of $\mu\left(\frac{\partial \phi}{\partial \overline{x}}\right)$ suggests that, on average, a rise in $\overline{x} \in X$ prompts to a rise in the frequency forecast. Similarly, a $P\left(\frac{\partial \phi}{\partial \overline{x}}\right)^+$ greater than 0.5 indicates that positive effects are more frequent than negative effects. Conversely, a negative $\mu\left(\frac{\partial \phi}{\partial \overline{x}}\right)$, or a $P\left(\frac{\partial \phi}{\partial \overline{x}}\right)^+$ less than 0.5, suggests an overall tendency toward a negative effect.

While these metrics may align, they offer complementary insights. The mean $\mu\left(\frac{\partial \phi}{\partial \overline{x}}\right)$ captures the average effect size, whereas $P\left(\frac{\partial \phi}{\partial \overline{x}}\right)^+$ provides the recurrence of positive effects, making both critical for a full understanding of the predictor's impact. Furthermore, a higher $CoV\left(\frac{\partial \phi}{\partial \overline{x}}\right)$ indicates a stronger ANNM nonlinearity concerning $\overline{x} \in X$, signaling greater variability in the predictor's effect across records.

2.4. Econometric vs. machine learning performance comparison

Following the estimation of both GLRMs and ANNMs, the models were compared to determine which one fits and predicts fare evasion frequency more effectively. The evaluation relied on four approaches, selected considering the regression nature of the task, i.e., estimating a continuous or integer-valued outcome (e.g., Botchkarev, 2018).

I. Regression plots (RPs): RPs provided a visual assessment of the model's fitting and forecasting performance. These plots displayed pairs of target frequency versus predicted frequency on a Cartesian plane. Specifically, for each experimental subset (*TR*, *VA* and *TE*) and model (GLRM and ANNM), RPs were generated by plotting the pairs ($freq(s,r,t)$, $\widetilde{freq}(s,r,t)$) and calculating the associated correlation coefficient (denoted as R). The analysis of RPs associated with GLRM and ANNM was performed by considering two aspects: (i) The proximity of points ($freq(s,r,t)$, $\widetilde{freq}(s,r,t)$) to the first quadrant bisector indicated superior performance as it moved toward a perfect fitting or prediction (i.e., $\widetilde{freq}(s,r,t) = freq(s,r,t)$); (ii) A R coefficient closer to 1 signified greater model performance, as it denoted an exact correlation between observed and predicted values

II. Measures of errors (MoEs): Various MoEs were assessed as indicators to quantify the disparities between model-predicted and actual observed values on each subset (*TR*, *VA* and *TE*). MoEs included Mean

Absolute Error, Root Mean Squared Error, and coefficient of Variation, denoted as *MAE*, *RMSE* and *CoV*, respectively. Particularly: (i) *MAE* [events/window] measured the average magnitude of errors in predictions, disregarding their direction, by computing the arithmetic mean of absolute errors; (ii) *RMSE* [events/window], a quadratic scoring rule, provided relatively greater weight to large errors, making it particularly useful when large errors were undesirable and; (iii) *CoV* [-] measured model fit by comparing the magnitude of the squared errors to the observed values of the response variable, providing a normalized interpretation of the *RMSE* metric

III. Precited *vs* observed fare evasion events ratio: The ratio between the whole amount of model-predicted and actual-observed fare evasion events was computed on each subset (*TR*, *VA* and *TE*). The evaluation of this indicator (denoted as π) was based on the principle that the closer π is to 1, the better the fitting performance, as $\pi = 1$ signifies perfect alignment between the total predicted events and the observed occurrences

IV. Importance and Effect of the Factors: For each predictor $x \in X$, importance was assessed by the individual p-value p_x for the GLRMs and the permutation feature importance ρ_x for the ANNMs. Predictors were then ranked in ascending order of p_x for the GLRMs and in descending order of ρ_x for the ANNMs, resulting in a prioritised list of the most important predictors for each model. The GLRM list reflects decreasing statistical significance (higher p-values), while the ANNM list reflects decreasing feature importance (lower ρ_x values). Moreover, the direction of the effect was evaluated by the sign of coefficient estimate for GLRM, and by $P\left(\frac{\partial\phi}{\partial x}\right)^+$ for the ANNMs. If both models run in the same direction, the effects were assumed equal.

3. Results and discussion

The GLRM was specified as outlined in eqn. (1). A 70 % to 30 % splitting ratio was chosen to delineate *TR* and *TE* subsets, respectively, according to, e.g., Ferlitsch, 2021; Ventura et al., 2024b. Table 2 presents the results for the best-fit GLRM obtained through forward selection. In detail, it provides coefficient estimates and associated p-values for each factor, with the latter highlighted in bold font when < 0.001. The ending portion of Table 2 depicts summary statistics.

The model exhibited a robust data fit, with p_{gl} consistently less than 0.001, providing substantial evidence for a regression effect (i.e., not all β_k and γ_x were zero). This section discusses only the statistical significance of the predictors; effects are addressed later, in comparison with the ANNM.

Among the exposure-related variables, both passenger volume and occupancy level were highly statistically significant ($p_k < 0.001$), highlighting the importance of including flow-related factors in fare evasion models.

Regarding intermediate context variables, the significance of time of day varied. Rush Morning and Rush Half-Day periods were significant relative to the reference Off-Peak period ($p_x < 0.001$ and $p_x = 0.003$, respectively). In contrast, Rush Evening did not reach the conventional 0.05 level of significance ($p_x = 0.067$). Although this result does not allow for definitive conclusions, its marginal significance may still be considered in exploratory analyses and may hint at underlying behavioural trends requiring further investigation.[7]

Intermediate organisational factors showed distinct patterns. Inspection paths A, B, and C were significant ($p_x = 0.021$, 0.006, and 0.019, respectively), indicating their association with fare evasion rates. Path D was near significance ($p_x = 0.065$), and Enforcement Level (i.e., inspection team configuration) was highly significant ($p < 0.001$).

Regarding intermediate vehicle characteristics, both Medium and Long categories were significant versus the Short baseline ($p_x = 0.046$ and $p_x = 0.027$, respectively), and vehicle Capacity was highly significant ($p_x < 0.001$), confirming the role of vehicle configuration in fare evasion incidence.

The frequency model was subsequently fitted using an MLM based on ANNM according to eqns. (2) and (3). To delineate *TR*, *VA* and *TE* a splitting ratio of 70 %, 15 %, and 15 % was utilized (e.g., Flach, 2012; Ventura et al., 2024b). After experimenting with different network sizes, a hidden layer comprising 10 neurons was chosen as a trade-off between data fitting capability and computational demand. While the default maximum number of epochs was 1000, training was stopped early at epoch 13, when the validation MSE reached its minimum. Table 3 provides the results in terms of permutation feature importance, mean partial derivative, coefficient of variation, and the associated probability of positive partial derivatives for each predictor (i.e., ρ_x, $\mu\left(\frac{\partial\phi}{\partial x}\right)$, $CoV\left(\frac{\partial\phi}{\partial x}\right)$ and $P\left(\frac{\partial\phi}{\partial x}\right)^+$, respectively), computed using eqns. from (4) to (9).

Moreover, Fig. 1 shows the relative frequency distribution of the partial derivative for each factor in the ANNM. Fig. 1 is self-explicative.

This section examines only the PFIs, which reflect each input's relative importance in the ANNM. Effects are discussed later in comparison with the GLRM.

Passenger volume is by far the most influential factor (PFI of 70.281), indicating the model's strong reliance on this variable. In contrast, occupancy level plays a minor role (PFI = 0.731).

For intermediate context features, time of day has a modest influence. Rush Morning and Rush Half-Day show PFIs of 1.506 and 1.296, while Rush Evening is slightly lower at 0.978—suggesting limited temporal impact on predictions, particularly when compared to exposure and organisational factors.

Inspection type stands out among organisational variables. Type B is the second most important feature (PFI = 24.214), followed by Type A (PFI = 16.062). Types C and D contribute less (PFIs = 5.772 and 2.672), indicating a gradient in inspection effectiveness. The enforcement level (two-inspector teams) has a moderate influence (PFI = 2.131).

Regarding vehicle features, long vehicles show a slightly stronger effect (PFI = 1.023) than medium ones (PFI = 0.615). Vehicle capacity is similarly modest (PFI = 0.650). While not primary drivers, these variables enhance model accuracy when combined with other inputs.

After estimating the GLRM and ANNM, their fitting and predictive performances were evaluated to identify the most effective modelling approach.

In terms of the I comparison approach, the resulting predictions are illustrated in Fig. 2, and the correlation coefficient (R) between the actual and predicted frequencies is computed (see Table 3). Fig. 2 might suggest conflicting outcomes. Specifically, points generated by the GLRM tended to cluster closer to the first quadrant bisector than those from the ANNM on the *TR* dataset, despite a similar *R*. Thus, at first glance, the GLRM might appear superior to the ANNM as it exhibited a better fit to the *TR* dataset in terms of RPs. However, the apparent advantage of the GLRM may stem from a slight overfitting, which in turn reduces its generalizability. This is evidenced by its lower performance on the *TE* dataset, where several points deviate significantly from the bisector, indicating less accurate predictions on unseen data. Contrarywise, the ANNM maintained superior generalisation performance on the *TE* dataset, as confirmed by points closer to the bisector and a greater *R*.

As for the II comparison approach, the results for MoEs are summarised in Table 4.

These results demonstrate that the ANNM surpassed the GLRM in

[7] Some researchers advocate for the cautious inclusion of variables with marginal significance (i.e., p-values between 0.05 and 0.10), particularly for exploratory or interpretive purposes in complex models. This approach can help uncover suggestive patterns or guide the refinement of future hypotheses (e.g., Greenland et al., 2016; Barabino et al., 2024a).

Table 2
Results of the best-fit GLRM. For dummy variables, the contrast variable is indicated in italic font in brackets.

Level		Factor (x, k)	Coefficient Estimate (α, β_k and γ_x)		p-val (p_x)	
		Constant (natural log)	−1.857		<.001	
Exposure		Passenger volume	0.4159		**<.001**	
		Level of occupancy	0.6129		**<.001**	
Intermediate context		Time period (*Off-peak*)				
		Rush Morning	−0.457		**<.001**	
		Rush Half-Day	−0.345		0.003	
		Rush Evening	−0.211		0.067	
Intermediate organisational		Inspection Type (*Random*)				
		A[+]	0.476		0.021	
		B[+]	0.568		0.006	
		C[+]	0.503		0.019	
		D[+]	0.420		0.065	
		Level of enforcement (*Non-standard*)				
		Standard[++]	−0.4301		**<.001**	
Intermediate vehicle		Length (*Short*)				
		Medium	0.230		0.046	
		Long	−0.502		0.027	
		Capacity	0.0113		**<.001**	
Summary statistics	Source	Deg. of freed.	Deviance	Mean deviance	*dr*	39.50
	Regression	13	513.4	39.4961	p_{gl}	0<.001
	Residual	773	712.3	0.9214		
	Total	786	1225.7	1.5594		

[+] – Planned inspection path of Type A (B, C or D) followed from inspectors centred on maximum load section of a route to maximize the number of checked passengers- A and B paths differ from C and D ones owing to the use of checkers in the former.
[++] The average number of inspectors, for one inspection team, is 2. For further details refer to Barabino et al. (2024c).

Table 3
Results of the best-fit ANNM. For dummy variables, the contrast variable is indicated in italic font in brackets.

Level	Factor	PFI (ρ_x)	$\mu\left(\frac{\partial\phi}{\partial x}\right)$	$CoV\left(\frac{\partial\phi}{\partial x}\right)$	$P\left(\frac{\partial\phi}{\partial x}\right)^{+}$
	Constant (natural log)	–	–		
Exposure	Passenger volume	70.281	0.001	0.453	99.91 %
	Level of occupancy	0.731	4.481	0.596	100.00 %
Intermediate context	Time period (*Off-peak*)				
	Rush Morning	1.506	−1.276	1.885	30.23 %
	Rush Half-Day	1.296	−1.417	1.669	24.38 %
	Rush Evening	0.978	0.551	3.709	64.98 %
Intermediate organisational	Inspection Type (*Random*)				
	A	16.062	4.916	0.880	86.17 %
	B	24.214	6.574	0.672	90.60 %
	C	5.772	4.390	0.690	88.83 %
	D	2.672	4.652	0.769	88.30 %
	Level of enforcement (*Non-standard*)				
	Standard	2.131	−1.561	1.326	21.72 %
Intermediate vehicle	Length (*Short*)				
	Medium	0.615	2.017	1.475	78.01 %
	Long	1.023	−0.074	31.384	51.24 %
	Capacity	0.650	0.039	1.402	90.34 %

both fitting and prediction activities. In particular, the ANNM demonstrated an enhanced capability in fitting the training data, as reflected by lower error rates on the *TR* subset. Likewise, for the forecasting task, the ANNM achieved fewer errors on the *TE* subset, highlighting its superior ability to predict the frequency of previously unseen fare evasion incidents compared to the GLRM.

Concerning the III comparison approach, the results for the π ratio are summarised in Table 4. They show that the GLRM underestimated the total number of fare evasion events by a proportion ranging from approximately 1.6 % to 5.9 %. Contrariwise, the ANNM shows the opposite effect. Specifically, it somewhat overestimated the amount of fare evasion events by approximately 1.1 % and 3.2 % on the *TR* and *VA* subsets, respectively, while achieving nearly perfect prediction on the *TE* subset. Overall, these findings indicate that the ANNM exhibited a bit better fitting and predictive performance compared to the GLRM. Moreover, a moderate overestimation in the number of detected events by the ANNM can be deemed a conservative approach.

Regarding the IV comparison approach, Table 5 provides a detailed overview of how fare evasion factors are ranked and interpreted by two modelling approaches.

Focusing on the ranking, the models show strong alignment on some key predictors, particularly in the exposure category. Both GLRM and ANNM rank “Passenger volume” as the most influential variable (1st), highlighting the consistent role of demand in driving fare evasion.

Beyond this, rankings begin to diverge. For example, “Level of occupancy” is ranked 2nd by GLRM but 11th by ANNM. This may be due to redundancy with other variables, such as passenger volume and time period. GLRM assigns separate importance to each included predictor, while ANNM can implicitly account for overlapping information, downweighting less distinctive factors and emphasising those with unique or interaction-driven contributions.

In the organisational domain, a notable contrast emerges. While GLRM ranks inspection types (A–D) from 7th to 12th, ANNM places them much higher (2nd–5th). This suggests that ANNM captures more nuanced effects of targeted inspections, potentially influenced by contextual interactions—such as occupancy levels or time of day—that

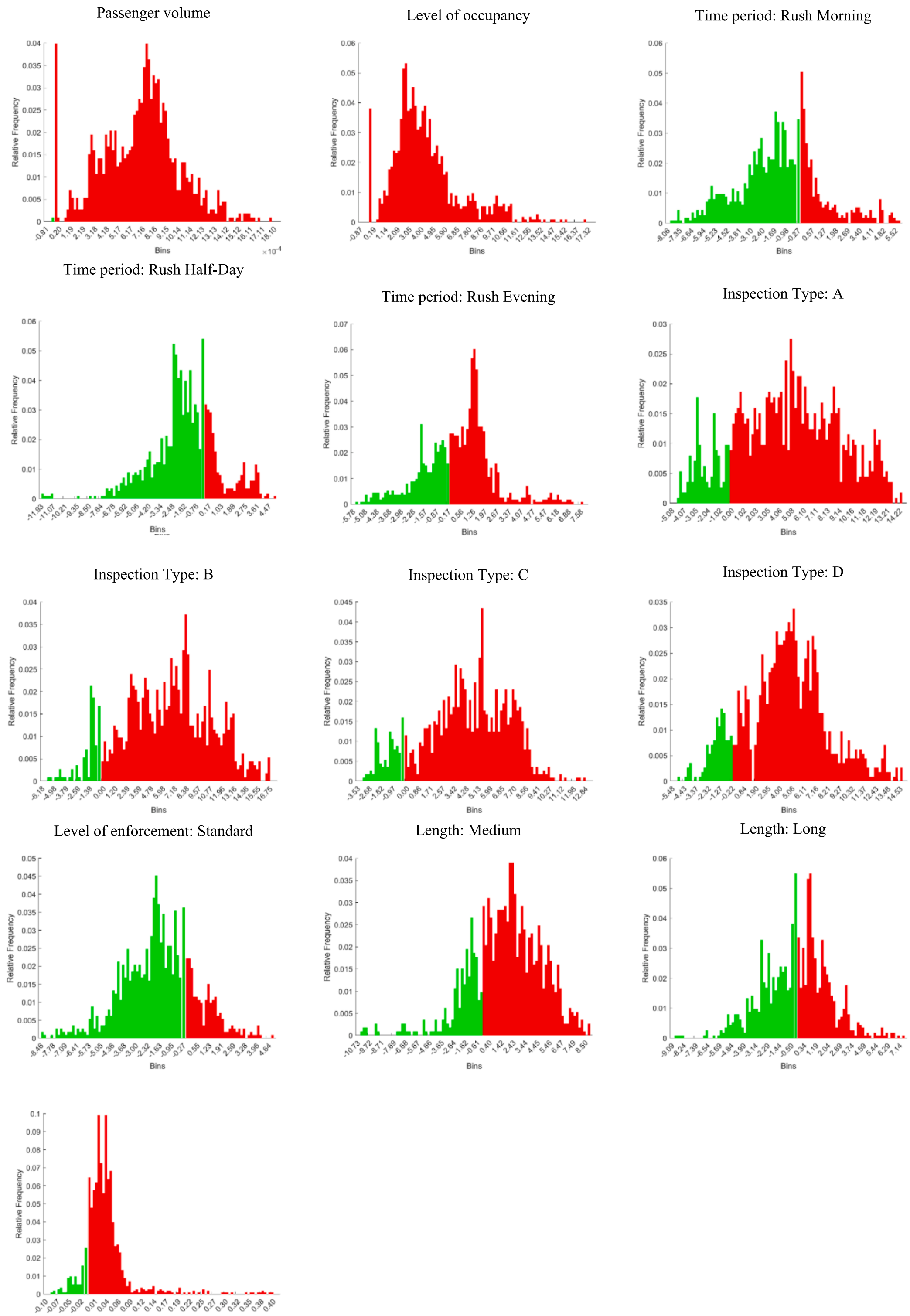

Fig. 1. Relative frequency distribution of the partial derivative for each factor in the ANNM. Red colour means that the sign of partial derivative is positive (i.e., the variable contributes to the increasing of the frequency of fare evasion); green colour assumes the opposite meaning. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

GLRM treats as uniform and additive.

Regarding vehicle-related factors, such as "Capacity," the models exhibit some disagreement: GLRM ranks it 4th, while ANNM lowers its priority to 12th. This may reflect the ANNM's ability to model interactions and account for redundancy with related variables, such as vehicle length or occupation level, whereas GLRM treats each predictor independently.

When comparing the effect directions produced by the GLRM and the ANNM, the two models exhibit a strong overall convergence, reinforcing the robustness of the findings across both linear and nonlinear modelling paradigms. Almost all factors demonstrate the same influence on fare evasion frequency in both models (▲ in Table 5). Notably, the exposure

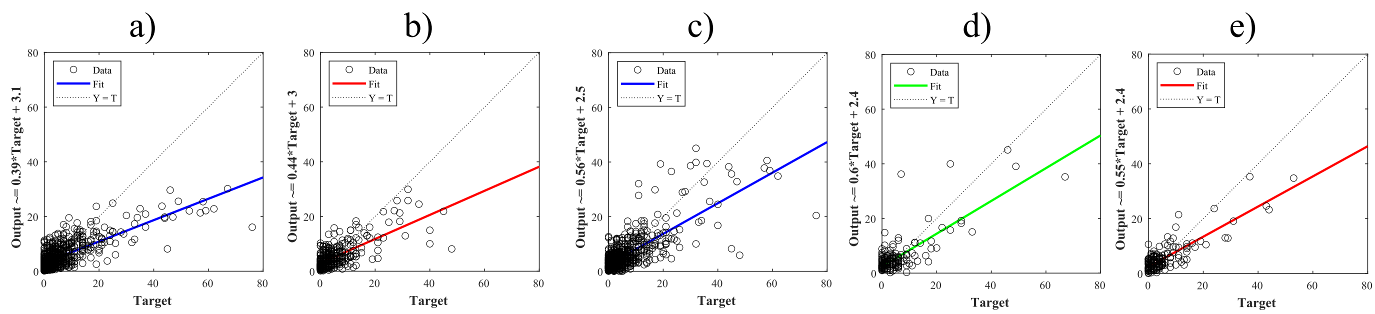

Fig. 2. Regression Plot (RP) for a) GLRM on TR dataset; b) GLRM on TE dataset; c) ANNM on TR dataset; d) ANNM on VA dataset; e) ANNM on TE dataset. The GLRM utilises only two datasets, resulting in the presentation of just two graphs.

Table 4
Comparison between the fitting and prediction performances of GLRM and ANNM.

Parameter	GLRM		ANNM		
	Training (TR)	Test (TE)	Training (TR)	Validation (VA)	Test (TE)
I comparison approach					
Correlation coefficient related to RPs (*R*) [-]	0.76	0.71	0.76	0.77	0.83
II comparison approach					
Mean Absolute Error (*MAE*) [*events/timeslot*]	3.50	3.39	3.39	3.78	3.33
Root Mean Squared Error (*RMSE*) [*events/timeslot*]	6.19	5.31	5.49	5.67	4.80
Coefficient of Variation (*CoV*) [-]	1.185	0.977	0.994	0.977	0.892
III comparison approach					
Ratio between predicted and observed fare evasion events (π) [-]	0.941	0.984	1.011	1.032	1.000

Table 5
Comparison between the ranking and effects returned by GLRM and ANNM.

Level	Factor	GLRM – P-val Ranking	ANNM – PFI Ranking	Effect Direction
	Constant (natural log)	–-	–-	
Exposure	Passenger volume	1	1	▲
	Level of occupancy	2	11	▲
Intermediate context	Time period (*Off-peak*)			
	Rush Morning	5	7	▲
	Rush Half-Day	6	8	▲
	Rush Evening	13	10	▼
Intermediate organisational	Inspection Type (*Random*)			
	A	9	3	▲
	B	7	2	▲
	C	8	4	▲
	D	12	5	▲
	Level of enforcement (*Non-standard*)			
	Standard	3	6	▲
Intermediate vehicle	Length (*Short*)			
	Medium	11	13	▲
	Long	10	9	▲
	Capacity	4	12	▲

▲ = The models return the same effect direction; ▼ The models return an opposite effect direction.

variables—passenger volume and level of occupancy—show such consistency. In the GLRM, both passenger volume ($\beta_k = 0.4159$) and level of occupancy ($\beta_k = 0.6129$) display substantial positive effects, confirming that greater exposure correlates with increased fare evasion, a finding in line with previous studies (e.g., Lee, 2011; Guarda et al., 2016; Cantillo et al., 2022; Barabino et al., 2023). The ANNM similarly identifies these two variables as positively associated with fare evasion. Passenger volume exhibits one of the most consistent and stable effect, with a positive average effect size ($\mu(\partial\phi/\partial\bar{x}) = 0.001$), a notably high probability of positive outcomes ($P(\partial\phi/\partial\bar{x})^{+}$ at 99.91 %), and low variability (CoV $(\partial\phi/(\partial\bar{x}) = 0.453$), indicating a reliable association across observations. Similarly, occupancy shows an even stronger average effect ($\mu(\partial\phi/\partial\bar{x}) = 4.481$) and a 100 % probability of positive influence, further reinforcing its role as a key explanatory factor, in line with the evidence reported by Guarda et al. (2016) and Cantillo et al. (2022), even they used GLRM.

Consistency emerges across the intermediate organisational variables, particularly in relation to inspection strategies. Both the GLRM and ANNM confirm a positive association between targeted inspections—especially types A and B—and fare evasion. In the GLRM, inspection types A and B exhibit positive coefficients ($\gamma_x = 0.476$ and 0.568, respectively). The ANNM supports this finding, with inspection types B and A showing high average effects ($\mu(\partial\phi/\partial\bar{x}) = 6.574$ and $\mu(\partial\phi/\partial\bar{x}) = 4.916$, respectively) and strong probabilities of positive effects ($P(\partial\phi/(\partial\bar{x}))^{+} = 90.60$ % and $P(\partial\phi/(\partial\bar{x}))^{+} = 86.17$ %, respectively). These results align with field evidence suggesting that when inspections follow predictable or targeted patterns, experienced fare evaders may adapt their behaviours, reducing the overall deterrent effect (Dai et al., 2017; Barabino et al., 2023). Assaf and Van den Broeck (2022) demonstrate how virtual communities of fare evaders share real-time inspection information, facilitating the creation of informal maps that help avoid checks. This undermines the intended impact of targeted enforcement and may explain why earlier studies reporting its effectiveness diverge from our results (e.g., Multisystems et al., 2002; Boyd, 2020). In contrast, our findings align with those of Dai et al. (2018), who found that random inspections are more effective at curbing fare evasion. Inspection types C and D also show consistent positive effects across both models, though their relative importance is lower. Overall, the results highlight the potential limitations of targeted inspections when they become predictable, reinforcing the argument for increased randomness and variability in enforcement strategies.

A similar consistency is observed in the direction of the effect of the enforcement level. In the GLRM, the "standard" level of enforcement (i.e., two inspectors per team) is negatively associated with the frequency of fare evasion ($\gamma_x = -0.4301$), suggesting that this configuration is more effective than non-standard ones. The ANNM confirms this, showing a negative average effect ($\mu(\partial\phi/\partial\bar{x}) = -1.561$) and a low probability of positive effects ($(P(\partial\phi/(\partial\bar{x}))^+ = 21.72$ %), indicating that this enforcement setup effectively suppresses evasion. This counterintuitive result—where fewer inspectors yield greater deterrence—has been stated previously in Barabino et al. (2014) and Barabino and Salis (2019), who attribute it to potential inspector crowding when teams are too large.

Regarding vehicle-related factors, both models find that medium-length vehicles are associated with increased fare evasion, likely due to their higher capacity and multiple entry points, which complicate inspection. The GLRM reports a positive coefficient for medium vehicles ($\gamma_x = 0.230$) and the ANNM confirms this with a positive average effect ($\mu(\partial\phi/\partial\bar{x}) = 2.017$) and a 78.01 % probability of positive effects. Capacity itself shows a consistent positive relationship in both models ($\gamma_x = 0.0113$ and $\mu(\partial\phi/\partial\bar{x}) = 0.039$), again aligning with the notion that larger, more crowded vehicles provide greater anonymity and opportunities for fare evasion. However, the effect of long vehicles is more complex. In the GLRM, long vehicles are negatively associated with fare evasion ($\gamma_x = -0.502$), a result that diverges from Guarda et al. (2016), but which may be contextually valid for Cagliari, where long vehicles are typically deployed during peak hours when fare evasion is less frequent. The ANNM mirrors this direction, with a negative average effect ($\mu(\partial\phi/\partial\bar{x}) = -0.074$), albeit accompanied by the highest CoV among all variables (31.384), pointing to high context-dependence and strong nonlinearity in this effect.

The only notable divergence between the GLRM and ANNM concerns the "Rush Evening" period (▼ in Table 5). While the GLRM assigns it a negative (although only marginally significant) effect ($\gamma_x = -0.211$), the ANNM identifies a positive one $\mu(\partial\phi/\partial\bar{x}) = 0.551$) with a modest probability of positive outcomes ($(P(\partial\phi/(\partial\bar{x}))^+ = 64.98$ %) and a high CoV (3.709), indicating strong variability in the effect direction. This contrast likely stems from the ANNM's ability to capture nonlinearities and variable interactions—capabilities that the GLRM, by design, lacks. Specifically, the effect of "Rush Evening" may not be uniformly positive or negative across all cases, but rather context-dependent—e.g., varying with factors such as occupancy level, enforcement presence, or service patterns. The GLRM imposes a fixed, average effect across the dataset, whereas the ANNM can dynamically adjust this effect based on surrounding conditions. The direction reversal may thus reflect latent heterogeneity that only the ANNM is capable of modelling.

To summarise, while some discrepancies emerge due to differences in how each model handles nonlinearities and interactions, the overall direction of effects is remarkably consistent.

From a policy perspective, these findings do not imply that different models would lead TAs/PTCs or policymakers to pursue conflicting strategies. Rather, they illustrate that distinct modelling approaches may prioritise certain factors differently, providing a more nuanced basis for decision-making. For instance, both models consistently identify "Passenger volume" as a primary driver of fare evasion, reinforcing its central role in planning interventions. However, while the GLRM ranks "Level of occupancy" highly, the ANNM places greater importance on inspection-related factors, such as "Inspection type." This divergence does not suggest opposing recommendations but rather reflects the value of using complementary modelling approaches. In practice, TAs/PTCs could draw on insights from both models to design a more comprehensive and flexible deterrence strategy—one that addresses both structural demand-side pressures and operational enforcement tactics. Such a strategy could involve adopting dynamic, data-informed inspection policies that focus on routes with high passenger volumes and occupancy levels, while maintaining unpredictable or randomised patterns. This would help concentrate resources on highly exposed services, while preventing experienced fare evaders from adapting their behaviour and undermining the deterrent effect.

4. Conclusions

Fare evasion remains a challenging research domain, continually demanding innovative solutions for Transit Authorities/Public Transport Companies (TAs/PTCs). This study makes a novel contribution to enhance our understanding of fare evasion frequency, which can inform the development of contrasting strategies. Specifically, this study:

- Specifies, calibrates, and validates Generalised Linear Regression Models (GLRMs) and Artificial Neural Network Models (ANNMs) to predict the frequency of fare evasion, considering exposure factors alongside intermediate predictors such as context, organisational, and vehicle-related variables.
- Evaluates and compares the performance of both models by regression plots, error metrics, and the ratio of observed to predicted fare evasion events.
- Assesses and compares the effect of each factor on fare evasion frequency prediction using both models.

An experiment was conducted using real-world data sourced from a medium-sized PTC in Cagliari, Italy. The findings showed the pronounced influence of passenger volume, level of occupancy, and inspection methodologies on the frequency estimation. Moreover, in this specific context, the ANNM demonstrates slightly superior performance compared to the GLRM, as evidenced by higher correlation coefficients, lower error metrics, and a closer alignment between observed and predicted fare evasion events. This finding suggests that advanced machine learning techniques might be more effective in capturing the complexities inherent in fare evasion dynamics, at least within the context of the analysed dataset. Furthermore, although both models often yield similar effects, each may emphasise separate aspects of the factors influencing fare evasion due to their different capacities to capture nonlinear relationships. Notably, these divergences are not limitations per se but rather reflect distinct modelling philosophies: interpretability and transparency in the GLRM versus precise forecasting in the ANNM.

Furthermore, the models were validated using internal cross-validation —i.e., testing and validation subsets from the same data distribution— but no external validation —i.e., testing on data from other cities, transit systems, or time periods— was performed. Therefore, while such findings may be transferable to contexts sharing similar features, e.g. ticketing systems, inspection strategies, passenger volumes, and inspection policy employed by the TAs/PTC, they cannot be automatically generalised to all transit systems without further validation. Thus, applying the proposed modelling framework to datasets from different urban transit systems represents a crucial next step. Such efforts would help better evaluate the transferability of the approach and potentially reinforce or refine the conclusions presented here.

These findings offer actionable insights for TAs/PTCs to enhance fare enforcement:

- Passenger volume is the main predictor of fare evasion in both GLRM and ANNM, indicating that high-ridership routes should be prioritised-ensuring fairness through randomised inspections. For instance, a peak-hour route with heavy ridership may require more inspections, but at varied times to avoid profiling.
- Model selection should align with operational goals. GLRMs may be ideal for transparent policy evaluation, while ANNMs might better support real-time applications and adaptive planning. For instance, GLRMs offer interpretable estimates of factor effects (e.g., the expected increase in fare evasion frequency with rising passenger volume). In contrast, ANNMs can identify high-priority segments (e.g., those with elevated evasion frequency) with strong predictive performance.
- Both models can support planning for new routes by enabling the anticipation of fare evasion measurement before service deployment. For instance, a future suburban route projected to have high volume

and vehicle occupancy could be flagged in advance. This would allow TAs/PTCs to pre-emptively assign, e.g., randomised patrols to this route, even before it becomes operational, thereby enhancing enforcement while optimising resource use.

The study lays the groundwork for future research. First, fare inspectors may concentrate on specific routes, time periods, or service types, which could introduce a sampling bias. Although our models attempt to control for potential selection bias by including detailed predictors (e.g., inspection type, time period, vehicle characteristics), selection bias cannot be fully eliminated due to the observational nature of the data. Therefore, future work should incorporate more stratified inspection strategies during data collection to ensure broader coverage and minimise bias. Second, novel intermediate factors could enrich the understanding of this multifaceted phenomenon. Third, sensitivity analyses or scenario simulations may help evaluate the effectiveness of specific policy interventions, for instance, by simulating changes in fare evasion frequency under varying conditions using the trained models. Fourth, exploring diverse machine learning models, such as boosting techniques and Support Vector Machine methods, holds promise for enhancing predictive accuracy, particularly in data-scarce settings. Fifth, future research could reframe fare evasion frequency prediction as a classification task by applying thresholds (e.g., top 10 %) to identify high-risk zones. This would enable the use of classification metrics (e.g., confusion matrices, precision, recall, AUC-ROC) and support TAs/PTCs in prioritising areas for targeted interventions more effectively. Finally, in this study, the frequency of fare evasion for both models was compared; however, no information was reported regarding the severity of fare evasion. Thus, novel research could evaluate the severity of fare evasion, providing a comparison between both models.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the authors utilised ChatGPT (OpenAI) to enhance the English language and readability. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

CRediT authorship contribution statement

Benedetto Barabino: Writing – review & editing, Writing – original draft, Supervision, Resources, Methodology, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Roberto Ventura:** Writing – review & editing, Writing – original draft, Visualization, Software, Methodology, Formal analysis, Data curation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

The initial phase of this paper, including the completion of the first draft, occurred while the first author was employed at CTM SpA. We extend our gratitude to the General Management of CTM SpA for their support of this research and for allowing access to their real data for experimentation. This research received partial funding from the Department of Civil, Environment, Land and Architecture Engineering and Mathematics (DICATAM) at the University of Brescia through the research grant titled "Valuation of the Risk of Fare Evasion in an Urban Public Transport Network," CUP: D73C22000770002. Moreover, the publication has been partially funded by Fondazione Unibs (https://www.fondazioneunibs.eu/). The opinions, findings, conclusions, or recommendations expressed in this material are solely those of the authors and do not necessarily represent the views of CTM. Any remaining errors are the sole responsibility of the authors.

Appendix

List of Acronyms

Acronym	Full Form
ANNM	Artificial Neural Network Model
AUC	Area Under the Curve
DM	Descriptive Model
EM	Econometric Model
GIS	Geographic Information System
GLRM	Generalised Linear Regression Model
IoT	Internet of Things
MLM	Machine Learning Model
MOEs	Measures of Errors
NBR	Negative Binomial Regression
NLP	Natural Language Processing
PFI	Permutation Feature Importance
POP-TS	Proof-of-Payment Transit System
PTC	Public Transport Company
ROC	Receiver Operating Characteristic
RP	Regression Plot
TA	Transit Agency
USA	United States of America

The acronyms are ordered alphabetically.

List of Variables

Variable	Definition
R	Set of transit routes
$S(r)$	Set of homogeneous segments of route $r \in R$
T	Set of time windows during the monitoring period
X	Set of fare evasion frequency predictors
$x(s,r,t)$	Value of predictor $x \in X$ on segment $s \in S(r)$ of route $r \in R$ during time window $t \in T$
$K \subset X$	Set of exposure factors
$k(s,r,t)$	Value of exposure factor $k \in K$ on segment $s \in S(r)$ of route $r \in R$ during time window $t \in T$
$freq(s,r,t)$	Observed fare evasion frequency on segment $s \in S(r)$ of route $r \in R$ during time window $t \in T$
TR	Training subset
TE	Testing subset
$\widetilde{freq}(s,r,t)$	Predicted fare evasion frequency on segment $s \in S(r)$ of route $r \in R$ during time window $t \in T$
α	Constant term in GLRM
β_k	Coefficient for exposure factor $k \in K$ in GLRM
γ_x	Coefficient for predictor $x \in X$ in GLRM
p_{gl}	Global p-value of the GLRM
p_x	Individual p-value for predictor $x \in X$
dr	Deviance ratio of the GLRM
VA	Validation subset
ϕ	Function representing the ANNM
N	Set of neurons in the hidden layer of the ANNM
$\overrightarrow{\lambda}$	Parameter vector of the ANNM
$\overrightarrow{\lambda_0}$	Optimal parameter vector after training
ρ_x	Permutation feature importance of predictor $x \in X$
$\varepsilon(r,s,t)$	Residual error for prediction on segment $s \in S(r)$ of route $r \in R$ during time window $t \in T$
MSE	Mean Squared Error
$\overline{x}$	Generic predictor in partial derivative analysis
$\frac{\partial \phi}{\partial \overline{x}}(s,r,t)$	Partial derivative of ANNM output with respect to $\overline{x}$
$\mu\left(\frac{\partial \phi}{\partial \overline{x}}\right)$	Mean of partial derivatives for $\overline{x} \in X$
$\sigma\left(\frac{\partial \phi}{\partial \overline{x}}\right)$	Standard deviation of partial derivatives for $\overline{x} \in X$
$CoV\left(\frac{\partial \phi}{\partial \overline{x}}\right)$	Coefficient of variation of partial derivatives for $\overline{x} \in X$
$P\left(\frac{\partial \phi}{\partial \overline{x}}\right)^+$	Probability that the partial derivative is positive
$\Delta \overline{x}$	Marginal increase in $\overline{x} \in X$
R	Correlation coefficient in Regression Plots
MAE	Mean Absolute Error
$RMSE$	Root Mean Squared Error
CoV	Coefficient of variation
π	Ratio of predicted to observed fare evasion events

The variables are listed in the order of their first appearance.

Data availability

The data that has been used is confidential.

References

Adnan, N., Nordin, S.M., Rahman, I., Noor, A., 2018. The effects of knowledge transfer on farmers decision making toward sustainable agriculture practices: In view of green fertilizer technology. World J. Sci. Technol. Sustain. Dev. 15 (1), 98–115.

Alhassan, I., Matthews, B., Toner, J., Susilo, Y., 2022. Seamless public transport ticket inspection: exploring users' reaction to next-generation ticket inspection. J. Public Transp. 24, 100004.

Assaf, C., Van den Broeck, P., 2022. The quest for a fairer formula. how re-institutionalisation begins with neo-illegal transport communities at the fringe of social innovation in Brussels. European Journal of Spatial. Development 19 (4), 72–98.

Barabino, B., Bonera, M., Maternini, G., Porcu, F., Ventura, R., 2024a. Refining a crash risk framework for urban bus safety assessment: evidence from Sardinia (Italy). Reliab. Eng. Syst. Saf. 245, 110003.

Barabino, B., Carra, M., Currie, G., 2024b. Fare inspection in proof-of-payment transit networks: a review. J. Public Transp. 26, 100101.

Barabino, B., Di Francesco, M., Ventura, R., Zanda, S., 2024c. Toward real-time deterrence against fare evasion risk in public transport. Transp. Res. Interdiscip. Perspect. 27, 101238.

Barabino, B., Di Francesco, M., Ventura, R., 2023. Evaluating fare evasion risk in bus transit networks. Transp. Res. Interdiscip. Perspect. 20, 100854.

Barabino, B., Lai, C., Olivo, A., 2020. Fare evasion in public transport systems: a review of the literature. Public Transp. 12, 27–88.

Barabino, B., Salis, S., 2023. Segmenting fare-evaders by tandem clustering and logistic regression models. Public Transp. 15 (1), 61–96.

Barabino, B., Salis, S., 2020. Do students, workers, and unemployed passengers respond differently to the intention to evade fares? an empirical research. Transport. Res. Interdiscipl. Perspect. 7, 100215.

Barabino, B., Salis, S., 2019. Moving towards a more accurate level of inspection against fare evasion in proof-of-payment transit systems. Netw. Spat. Econ. 19 (4), 1319–1346.

Barabino, B., Salis, S., Useli, B., 2015. What are the determinants in making people free riders in proof-of-payment transit systems? evidence from Italy. Transp. Res. A 80, 184–196.

Barabino, B., Salis, S., Useli, B., 2014. Fare evasion in proof-of-payment transit systems: Deriving the optimum inspection level. Transp. Res. B Methodol. 70, 1–17.

Barabino, B., Salis, S., Useli, B., 2022. Assessing the Intention to Evade Fares for Demographic Segments of Passengers: Empirical Research in Italy for Building Smart (er) Cities. J. Urban Plann. Dev. 148 (1).

Botchkarev, A. (2018). Performance metrics (error measures) in machine learning regression, forecasting, and prognostics: Properties and typology. arXiv. https://arxiv.org/abs/1809.03006.

Boukerche, A., Wang, J., 2020. Machine learning-based traffic prediction models for intelligent transportation systems. Comput. Netw. 181.

Boyd, C., 2020. Revisiting the foundations of fare evasion research. Transp. Res. A 137, 313–324.

Breiman, L., 2001. Random forests. Machine Learn. 45, 5–32.

Bucciol, A., Landini, F., Piovesan, M., 2013. Unethical behavior in the field: Demographic characteristics and beliefs of the cheater. J. Econ. Behav. Organ. 93, 248–257.

Busco, C., González, F., Jaqueih, Y., Jiménez, F., Alonso, B., 2021. Understanding transantiago users' motivations for paying or evading payment of bus fares. J. Public Transp. 23 (2), 1–15.
Cantillo, A., Raveau, S., Muñoz, J.C., 2022. Fare evasion on public transport: who, when, where and how? Transp. Res. A Policy Pract. 156, 285–295.
Cao, L., 2017. Data science: a comprehensive overview. ACM Comput. Surveys (CSUR) 50 (3), 1–42.
Cools, M., Fabbro, Y., Bellemans, T., 2018. Identification of the determinants of fare evasion. Case Study Transport Policy 6 (3), 348–352.
Chang, L.Y., 2005. Analysis of freeway accident frequencies: negative binomial regression versus artificial neural network. Saf. Sci. 43 (8), 541–557.
CTM, 2023. Carta mobilità 2023-2024. Available at: https://ctmcagliari.portaletrasparenza.net/it/trasparenza/servizi-erogati/carta-dei-servizi-e-standarddi-qualita.html. Accessed on 16.07.2024.
Currie, G., Delbosc, A., 2017. An empirical model for the psychology of deliberate and unintentional fare evasion. Transp. Policy 54, 21–29.
Dai, Z., Galeotti, F., Villeval, M.C., 2017. The efficiency of crackdowns: a lab-in-the-field experiment in public transportations. Theor. Decis. 82, 249–271.
Dai, Z., Galeotti, F., Villeval, M.C., 2018. Cheating in the lab predicts fraud in the field: an experiment in public transportation. Manag. Sci. 64 (3), 1081–1100.
Delbosc, A., Currie, G., 2016. Cluster analysis of fare evasion behaviours in Melbourne, Australia. Transp. Policy 50, 29–36.
Escalona, P., Brotcorne, L., Fortz, B., Ramirez, M., 2024. Fare inspection patrolling under in-station selective inspection policy. Ann. Oper. Res. 332 (1), 191–212.
Fatima, M., Pasha, M., et al., 2017. Survey of machine learning algorithms for disease diagnostic. J. Intell. Learn. Syst. Appl. 9 (01), 1.
Ferlitsch, A., 2021. Deep Learning patterns and Practices. Simon and Schuster.
Flach, P., 2012. Machine learning: the art and science of algorithms that make sense of data. Cambridge University Press.
Fujiyoshi H, Hirakawa T, Yamashita T. (2019). Deep learning-based image recognition for autonomous driving. IATSS Res. 2019; 43(4):244–52.
Gonzalez, F., Busco, C., Codocedo, K., 2019. Fare evasion in public transport: grouping Transantiago users'behavior. Sustainability 11 (23), 6543.
Greenland, S., Senn, S.J., Rothman, K.J., et al., 2016. Statistical tests, P values, confidence intervals, and power: a guide to misinterpretations. Eur. J. Epidemiol. 31 (4), 337–350.
Guzman, L.A., Arellana, J., Camargo, J.P., 2021. A hybrid discrete choice model to understand the effect of public policy on fare evasion discouragement in Bogota's Bus Rapid Transit. Transp. Res. A 151, 140–153.
Guarda, P., Galilea, P., Paget-Seekins, L., Ortúzar, J.d.D., 2016. What is behind fare evasion in urban bus systems? an econometric str. Transp. Res. A Policy Pract. 84, 55–71.
Hegde, J., Rokseth, B., 2020. Applications of machine learning methods for engineering risk assessment–a review. Saf. Sci. 122, 104492.
Ian, G., Yoshua, B., Aaron, C., Yoshua, B.D., 2016. Deep Learning 1.
Jhaveri, R.H., Revathi, A., Ramana, K., Raut, R., Dhanaraj, R.K., 2022. A review on machine learning strategies for real-world engineering applications. Mob. Inf. Syst. 2022 (1), 1833507.
Lee, J., 2011. Uncovering San Francisco, California, Muni's Proof-of-payment patterns to help Reduce Fare Evasion. Transportat. Res. Rec. J. Transport. Res. Board 2216, 75–84.
Marchand, A., Marx, P., 2020. Automated product recommendations with preference-based explanations. J. Retail. 96 (3), 328–343.
Marquardt, D.W., 1963. An algorithm for least-squares estimation of nonlinear parameters. J. Soc. Ind. Appl. Math. 11 (2), 431–441. https://doi.org/10.1137/0111030.
MathWorks. (2025). TrainingOptionsLM: Levenberg–Marquardt algorithm options. MATLAB & Simulink. Retrieved June 23, 2025, from https://it.mathworks.com/help/deeplearning/ref/nnet.cnn.trainingoptionslm.html.
Multisystems, Inc., Mundle, Associates, Inc., Parsons Transportation Group, Inc., 2002. A Toolkit for Self-Service, Barrier-Free Fare Collection. Transit Cooperative Research Program, Report 80.
Otter, D.W., Medina, J.R., Kalita, J.K., 2020. A survey of the usages of deep learning for natural language processing. IEEE Trans Neural Netw Learn Syst.
Reddy, A.V., Kuhls, J., Lu, A., 2011. Measuring and controlling subway fare evasion. Transp. Res. Rec. 2216, 85–99.
Romero Reyes, I.V., Fedyushkina, I.V., Skvortsov, V.S., Filimonov, D.A., 2013. Prediction of progesterone receptor inhibition by high-performance neural network algorithm. Internat. J. Math. Models Methods Appl. Sci. 7, 303–310.
Sarker, I.H., Hoque, M.M., Uddin, M.K., Alsanoosy, T., 2021. Mobile data science and intelligent apps: concepts, AI-based modeling and research directions. Mobile Networks Appl. 26 (1), 285–303.
Sarker, I.H., 2021. Machine learning: Algorithms, real-world applications and research directions. SN Comput. Sci. 2 (3), 160.
Sasaki, Y., 2014. Optimal choices of fare collection systems for public transportations: Barrier versus barrier-free. Transp. Res. B Methodol. 60, 107–114.
Schmidhuber, J., 2015. Deep Learning in neural networks: an overview. Neural Netw. 61, 85–117.
Ślusarczyk, B., 2018. Industry 4.0: are we ready? Polish J. Manag. Stud. 17.
Sramka, M., Slovak, M., Tuckova, J., Stodulka, P., 2019. Improving clinical refractive results of cataract surgery by machine learning. PeerJ 7, e7202.
Strobl, C., Boulesteix, A.L., Zeileis, A., Hothorn, T., 2007. Bias in random forest variable importance measures: Illustrations, sources and a solution. BMC Bioinf. 8, 1–21.
Troncoso, R., de Grange, L., 2017. Fare evasion in public transport: a time series approach. Transp. Res. A 100, 311–318.
Ventura, R., Barabino, B., Maternini, G., 2024a. Estimating the frequency of traffic overloading on road bridges. Journal of Traffic and Transportation Engineering (English Edition) 11 (4), 776–796.
Ventura, R., Maternini, G., Barabino, B., 2024b. Traffic hazards on main road's bridges: real-time estimating and managing the overload risk. IEEE Trans. Intell. Transp. Syst.
Wolfgram, L., Pollan, C., Hostetter, K., Martin, A., Spencer, T., Rodda, S., Amey, A., 2022. Measuring and managing Fare Evasion. Transportation Research Board, Washington, D.C.
Zhang, P., 2010. Industrial control system simulation routines. Adv. Indust. Control Technol. 784.