



UNIONE EUROPEA
Fondo Sociale Europeo



REACT EU



UNIVERSITÀ
DEGLI STUDI
DI BRESCIA



UNIVERSITÀ
DEGLI STUDI
DI BRESCIA

DEPARTMENT OF ECONOMICS AND MANAGEMENT

Ph.D. Program in

Metodi e Modelli per l'Economia e il Management

Analytics for Economics and Management (AEM)

Scientific sector: SECS-S/06 – METODI MATEMATICI DELL'ECONOMIA

E DELLE SCIENZE ATTUARIALI E FINANZIARIE

XXXVII Cycle

TOWARDS SUSTAINABLE FUTURE:

OPTIMIZATION OF HYDROPOWER MANAGEMENT DECISIONS IN THE NEW STORAGE
MARKET

Author:

Diana Tratta

Supervisor:

Prof. Paolo Falbo – University of Brescia

Co-supervisor:

Prof. Carlo Filippi – University of Brescia

A.Y. 2023/2024

Progetto di ricerca co-finanziato dall'Unione europea nel quadro del Programma PON R&I 2014-2020 risorse FSE
REACT-EU Azione IV.5 "Dottorati su tematiche Green".

Abstract

La transizione verso le Fonti di Energia Rinnovabile (FER) ha generato incertezza e fluttuazioni nel breve periodo, compromettendo la sicurezza dell'approvvigionamento energetico. Per affrontare questa sfida, le tecnologie di accumulo dell'energia sono fondamentali. Queste tecnologie trasformano l'energia da una forma, tipicamente elettrica, in un'altra forma che può essere immagazzinata e successivamente riconvertita in elettricità quando necessario. Questa capacità di accumulo consente di conservare energia nei momenti di bassa domanda e rilasciarla quando la domanda è alta, offrendo così una notevole flessibilità alla rete elettrica e attenuando la variabilità delle FER. L'idea di introdurre un mercato dell'accumulo energetico a lungo termine affronta due sfide principali legate agli investimenti nelle tecnologie di stoccaggio: gli elevati costi e l'alta volatilità dei prezzi dell'energia. L'obiettivo principale di questa tesi è la modellazione e la risoluzione di un problema di massimizzazione del profitto per un produttore di energia avverso al rischio, che possiede un impianto idroelettrico di pompaggio e vuole cogliere l'opportunità di finanziare l'investimento attraverso un mercato dell'accumulo energetico. Le decisioni del proprietario dell'impianto sono state modellate attraverso un problema di ottimizzazione. Sono stati considerati tre potenziali tipi di mercato, dando luogo allo sviluppo di tre modelli: un modello fisico, un modello finanziario e un modello ibrido. Il proprietario trae profitto sia dalla vendita di capacità di accumulo a una controparte, sia partecipando alla speculazione nei mercati spot dell'energia e nei servizi ancillari. Lo studio è stato suddiviso in due parti. Nella prima, i modelli sono stati analizzati in un contesto deterministico; successivamente è stato studiato il caso stocastico, integrando l'analisi dell'incertezza e l'avversione al rischio del cliente. Il principale contributo di questa tesi risiede nell'ottimizzazione congiunta della generazione/pompaggio e della decisione di vendere una parte della capacità di accumulo. Una delle principali opportunità di profitto per il proprietario dell'impianto consiste nell'evasione degli ordini del cliente attraverso contabilizzazione piuttosto che fisicamente, come dimostrano i risultati ottenuti per il modello finanziario e quello ibrido. Questo introduce una complicazione significativa nel modello ibrido: è necessario considerare i movimenti dell'acqua sia da un punto di vista contabile che fisico. L'avversione al rischio del proprietario è stata formulata mediante l'inclusione del Conditional Value-at-Risk come misura di rischio nella funzione obiettivo. Nel caso stocastico, i tempi computazionali per l'ottimizzazione si sono rivelati estremamente elevati, rendendo necessaria l'applicazione di euristiche per identificare soluzioni accettabili entro tempi ragionevoli.

Keywords: impianti di pompaggio idraulico, stoccaggio dell'energia, ottimizzazione, avversione al rischio, mercati energetici

Contents

1	Introduction	11
1.1	Literature Review	13
2	The impact of the storage market in the management of a Hydro Pumping Storage facility	27
2.1	An Hydro Pumping Storage model	29
2.1.1	Decision variables	30
2.1.2	Objective function	31
2.1.3	Profit function	31
2.1.4	Customer's operative decisions	32
2.2	Market models	34
2.2.1	Physical model	34
2.2.2	Financial model	37
2.2.3	Hybrid model	39
2.3	Application	43
2.4	Analysis of results	46
2.4.1	Impact of selling half of the capacity to a counterpart	49
2.5	Conclusion	57
3	Jointly optimal capacity sale decision and management of a Hydro Pumping Storage facility	59
3.1	An Hydro Pumping Storage model under uncertainty	60
3.1.1	Decision variables	60
3.1.2	Objective function	61
3.1.3	Safety measure	62
3.1.4	Profit function	63
3.1.5	Physical model	63
3.1.6	Financial model	66

3.1.7	Hybrid model	66
3.2	Application	68
3.2.1	Smart customer behavior	72
3.2.2	Partially predictable costumer behavior	74
3.2.3	Heuristics	75
3.3	Analysis of Results	82
3.4	Sensitivity analysis	104
3.5	Conclusion	110
4	Conclusions	113

Abstract

The transition to Renewable Energy Sources (RES) has created uncertainty and short-term fluctuations, which can compromise energy supply security. To address this challenge, energy storage technologies are essential. These technologies transform energy from one form, typically electricity, into another form that can be stored and then reconverted into electricity as needed. This storage capability enables energy to be saved when demand is low and released when demand is high, providing significant flexibility to the electrical grid and moderating the variability of RES. The idea of introducing a long-term energy storage market addresses two key challenges in the investment in storage technologies: high costs and high volatility of energy prices. The main goal of this thesis is the modeling and solution of a profit maximization problem for a risk-averse energy producer, who owns a hydroelectric pumping storage facility and aims to seize the opportunity to finance the investment through an energy storage market. The facility owner's decisions were modeled through an optimization problem. Three potential market types were considered, resulting in the development of three models: a physical model, a financial model, and a hybrid model. The owner profits from both selling storage capacity to a counterpart and participating in speculation within the energy spot markets and ancillary services. This study was divided into two parts. First, the models were analyzed in the deterministic case, and subsequently, the stochastic case was studied, integrating the analysis of uncertainty and the customer's risk aversion. The main contribution of this thesis lies in the joint optimization of generation/pumping and the decision to sell part of the storage capacity. One of the key profit opportunities for the plant owner is to fulfill customer orders through accounting rather than physically, as evidenced by the results obtained for the financial model and the hybrid model. This introduces a significant complication in the hybrid model: it is necessary to consider water movements from both an accounting and a physical perspective. The decision maker's risk aversion has been formulated by incorporating the Conditional Value-at-Risk as a risk measure in the objective function. In the stochastic case, optimization computational times proved to be extremely high, necessitating the application of heuristics to identify acceptable solutions within reasonable time frames.

Keywords: hydro-pumping plants, energy storage, optimization, risk aversion, energy market

Contents

Chapter 1

Introduction

The Italian energy market has traditionally been characterized by a predictable and manageable conventional generation mix, mainly composed of thermal and hydroelectric sources. The transition to Renewable Energy Sources (RES) has created uncertainty and short-term fluctuations, which can compromise energy supply security. To address this challenge, energy storage technologies are essential. These technologies transform energy from one form, typically electricity, into another form that can be stored and then reconverted into electricity as needed. This storage capability enables energy to be saved when demand is low and released when demand is high, providing significant flexibility to the electrical grid and moderating the variability of RES. The introduction of a long-term energy storage market aims to mitigate two issues related to storage technologies: the significant expenses associated with investing in storage technologies and the high volatility of energy prices in the spot market. This thesis focuses on the investment decisions faced by the owner of a hydro pumping storage facility, as HPS technology accounts for 96% of the current global energy storage capacity and requires substantial investments. These facilities provide a very large capacity for electricity, with low operating costs and high reliability. Pumped storage facilities offer supplementary services to the power grid, such as regulation of network frequency and creation of reserves . This is attributable to the capacity of hydroelectric power plants and pumped storage systems to rapidly adapt to fluctuations in electricity demand, typically within a few seconds [10]. Depending on the local topographical conditions and water runoff availability, these facilities can be configured as closed- or open-loop. In the latter, a natural inflow feeds either the upper, lower, or both basins, whereas there is no natural inflow in a closed-loop HPS facility.[25]. It is possible to build open-loop HPS facilities from existing conventional hydropower systems by replacing their turbines with reversible ones and constructing a new basin in addition to the existing one [24]. This can lead to substantial investment by facility owners. However, the current framework being discussed by Italian market regulators (legislative decree 210/2021), presents a significant financing opportunity for potential HPS operators, who can obtain a long-term fixed-price payment in exchange for committing a portion of their storage capacity to the energy storage market through. The legislation regulates the possibility of transferring part of one's energy storage capacity through specific auctions within the Mechanism

for the Procurement of Electric Storage Capacity (MACSE). This system is designed to ensure that the required storage capacity is contracted and made available to the national energy system. More specifically participants submit bids specifying the energy capacity (in MWh) and a requested premium (€/MWh-year), ensuring compliance with previously qualified limits set by Terna, which is an Italian Transmission System Operator (TSO) that plays a public service role, essential for ensuring electricity supply to the country and enabling the operation of the entire national electrical system. The selection process prioritizes bids that maximize net system value while adhering to regional and national capacity thresholds. Accepted capacities are contracted with obligations, including ensuring availability, fulfilling implementation deadlines, and complying with payment terms. Participants relinquish any additional public incentives for the contracted portion and must provide pre-auction guarantees to participate.

The main goal of this thesis is to address the challenge of maximizing the profit of a hydroelectric pumped storage (HPS) facility owner, by optimizing the management of the facility jointly with the decision to sell part of the storage capacity to a counterpart. The introduction of a counterpart in the optimization of the management of a Hydro pumping storage facility presented some mathematical challenges. A primary challenge encountered was the development of a practical method for simulating and incorporating the counterpart's orders into the model. To address this issue, two distinct approaches were devised external to the optimization model: one to simulate the behavior of a smart customer, and another to simulate the behavior of a partially predictable customer. The generated orders were subsequently introduced and adjusted through the optimization model. The second main challenge arising from the introduction of a counterpart was that, since the customer's orders were simulated prior to the optimization model, occasionally the customer's orders and the owner's optimal management decision were in opposition. As the majority of HPS facilities have reversible turbines that function as both generators and pumps, it is not feasible to produce energy and pump water simultaneously. To address this problem, three potential market types were considered, resulting in the development of three models: a physical model, a financial model, and a hybrid model. The counterpart is not clearly defined, as the customer could be a Transmission System Operator (TSO), a distributor, or other energy generators. Initially the problem is studied in a deterministic framework and subsequently in a stochastic framework.

The thesis is structured as follows: chapter 1.1 focuses on the literature review, providing an overview of relevant studies about uncertainties in electricity markets and optimization of storage technologies. Chapter 2 introduces the formulation of the optimization models. It integrates the decisions related to energy generation, pumping, and selling storage capacity to a counterpart, presenting three distinct models corresponding to physical, financial, and hybrid approach. The chapter evaluates these models under a deterministic framework using a realistic 24-hour scenario. Chapter 3 reformulates the previous models in a stochastic framework to address market uncertainties, incorporating risk management via CVaR and using historical data for scenario analysis. It also introduces heuristics to manage computational complexity. Finally chapter 3.5 concludes the thesis with a discussion of results and findings of the previous chapters.

1.1 Literature Review

In recent years, a substantial body of literature has emerged on electricity risk management and HPS energy storage. Various studies have been undertaken in the domain of uncertainty in the power market. Kettunen et al. [4] examined the challenges faced by electricity retailers in optimizing the management of an electricity portfolio when dealing with volume risk in meeting load and spot price risk in purchasing from the wholesale market. To address these issues the authors developed a multistage stochastic optimization approach tailored for electricity retailers dealing with load and spot price uncertainties. They proposed a contract portfolio optimization method that integrates the correlation between spot prices and demand, accounts for forward risk premiums, and incorporates risk preferences over time. The optimization process is structured around a scenario tree that models the stochastic processes of prices and loads using mean-reverting Ornstein-Uhlenbeck processes. This tree captured the nonlinear correlation between price and demand, which is crucial for efficient risk management. This study outlines the decision-making framework for an electricity retailer, detailing how demand and spot prices are uncertain and subject to various stochastic influences. The retailer's goal is to minimize costs while adhering to risk constraints, expressed in terms of conditional cash flows at risk. The authors find out that risk-neutral and risk-averse retailers respond differently to changes in market conditions. Risk-neutral retailers focus more on price-related uncertainties, while risk-averse retailers are concerned with load-related uncertainties and the implications of forward risk premiums. They also conclude that including the correlation between load and price in the model is critical, as it influences the risk assessments and overall cost efficiency. John E. Bistline [13] proposed a stochastic research and development (R&D) portfolio management framework that addresses uncertainties associated with research outcomes and market diffusion of energy technologies, and provided numerical results relevant to energy technology strategies amidst fluctuating climate policies and natural gas prices. The paper introduces a stochastic programming capacity planning model that assesses potential benefits from energy R&D expenditures. This model incorporates uncertainties related to market conditions and the likelihood of successful technological outcomes. The author discusses how managing technological change is vital for addressing challenges such as climate change. The framework is designed to help allocate R&D funds effectively across various energy technologies, especially following the Paris Agreement and the rise of initiatives like Mission Innovation. The model suggests that optimal R&D portfolio investments are influenced by technological and market interactions. Investments that might seem suboptimal in isolation may become more valuable when considered collectively, particularly under conditions of market competition. Specifically the author concludes that R&D can be more valuable in environments where decision-makers do not have perfect information about future market conditions. P.V. Hansen[5] examines the impact of uncertainty in water inflow on the performance of hydropower markets under varying market structures. The author highlighted that traditional deterministic models often overlook the complexities introduced by uncertainty, particularly how it enables firms to exercise market power. This analysis, in fact, asserts that uncertainty in inflow influences how firms with market

power adjust their output between different periods. It is established that the shape of the demand function, influenced by consumer preferences, plays a critical role in determining how firms react to this uncertainty. The author developed a two-period model with N identical hydropower producers where water inflow in the first period is deterministic, whereas inflow in the second period is stochastic. The model incorporates constraints on production and reservoir levels, leading to a detailed examination of how producers optimize their output over these periods. In the paper both perfect competition and imperfect competition scenarios are explored. He finds out that under perfect competition, firms respond optimally to uncertainty by saving more output for future periods. Conversely, in scenarios with market power (monopoly or oligopoly), firms may produce less or more than the socially optimal amount depending on the curvature of the demand function: convex demand implies that firms will tend to save more output to accommodate potential low inflow scenarios, while concave demand leads to the opposite behavior. The findings suggest that the interplay between demand elasticity, market structure, and inflow uncertainty significantly affects the strategies employed by hydropower producers. The author's conclusion is that understanding these dynamics is crucial for regulators and market participants to ensure fair pricing and supply security in hydropower markets. Dimoski et al [28] investigated the risk management challenges faced by hydropower producers, particularly regarding operational risk, price variability, and exchange rate fluctuations. The authors addressed the complexities of maximizing cash flow for hydropower producers dealing with unpredictable natural inflows and market risks such as price, exchange rate, and operational risks. The authors proposed a dynamic hedging model that leveraged currency and power futures contracts to mitigate these risks. More specifically they proposed a model structured around two main decision-making problems: production planning and financial hedging. Both problems are formulated as Markov Decision Processes (MDPs): this allows for a sequential approach where production and hedging decisions are made in two stages. The first stage consists of determining the optimal production decisions based on spot prices and inflows. The second stage involves hedging decisions that utilize the outcomes of the first stage. The model incorporates trading in monthly, quarterly, and yearly power futures contracts, which enables more precise control over risk exposure. The risk preferences of the hydropower producer were modeled using the nested Conditional Value-at-Risk (CVaR), since this measure ensures that risk preferences remain time-consistent and coherent across different time steps. To solve the MDPs, the authors employed Approximate Dual Dynamic Programming, which leverages the separation of the state space into resource states and environmental states. This method helps in approximating the value functions associated with the decision processes and allows for efficient computation despite the complexity of the underlying stochastic processes. The analysis reveals that dynamic hedging significantly reduces risk, presenting a 23% increase in the 5%-CVaR of terminal cash flows compared with static strategies. Additionally inclusion of currency forwards in the hedging strategy decreases the standard deviation of terminal cash flows and results in a modest increase in the CVaR, demonstrating the importance of managing currency risk. The authors conclude that dynamic hedging provides a more robust framework for risk management in hydropower production, particularly in the face of fluctuating prices and exchange rates. The model allows

producers to optimize their hedging strategies effectively, thus delivering competitive advantage.

Lu et al. [21] studied the challenges faced by electricity retailers in China's evolving electricity spot market, particularly due to real-time price fluctuations and negative impacts of deviation assessments on costs. The authors introduced a comprehensive electricity cost optimization algorithm that aims to enhance profitability of electricity retailers by optimizing day-ahead (DA) and real-time (RT) electricity costs, as well as deviation assessment costs through effective control of energy storage devices (ESD). They proposed a two-stage comprehensive optimization approach with Day-Ahead (DA) Market Optimization and a Real-Time (RT) Market Optimization. In the first stage electricity retailers declare their expected load based on forecast data. The algorithm then optimizes the declared load to minimize electricity purchase costs in the DA market. In the second stage, following the DA optimization, the load is adjusted based on real-time market conditions. The optimization algorithm takes into account the results from the first stage and the deviation assessment coefficient to manage the difference between declared and actual loads in the RT market. The authors incorporated also the control of energy of energy storage devices (ESD) to facilitate load shifting and optimize costs. More specifically the ESD is used to balance the loads between the Day-Ahead and Real-time markets, thereby reducing deviation assessment costs. The algorithm was tested using trial operation data from the power spot market in Guangdong province, which began the trials between May and June 2019. The electricity price data for several trial days were analyzed to assess the performance of the optimization algorithm. The results revealed that applying the comprehensive optimization algorithm significantly reduced electricity purchase cost. The optimization strategy adjusted the load declaration and controlled energy storage devices (ESD) in real-time to align the actual demand with the declared load, thereby minimizing deviation costs. In particular the comprehensive optimization algorithm demonstrated effectiveness under different scenarios, including varying seasonal loads and deviation assessment policies. Additionally the authors assessed the investment costs of energy storage devices and their recovery period. Various configurations and pricing scenarios for ESD were considered, showing that the investment could be recovered within 3 to 5 years under favorable conditions (e.g., lower unit prices for capacity and power). W.Feng and C.Ruiz [29] explored the management and operation of energy communities that integrate hydrogen production and storage technologies alongside renewable energy sources. Their work primarily focuses on how to manage the risks and uncertainties associated with renewable energy generation and consumption effectively. The authors developed a two-stage stochastic bilevel optimization model that incorporates the interactions between an arbitrageur (acting as the community aggregator) and multiple prosumers (energy consumers who also produce energy). The arbitrageur trades energy in both forward and spot markets while managing the risks posed by uncertain renewable energy generation and demand loads. In particular they emphasized the importance of risk management for energy communities, particularly in the context of integrating hydrogen technologies. To this end, they adopted CVaR as a risk measure. The authors modeled the community's operation as a Stackelberg game, where the arbitrageur (the leader) anticipates the responses of the prosumers (the followers) in their energy trading decisions. A significant focus was placed on the role

of hydrogen production and storage technologies, since those type of technologies, including batteries and hydrogen storage systems, are essential for balancing the supply and demand for energy within an energy community. They help in managing the intermittent nature of renewable energy sources like solar and wind, allowing prosumers to store excess energy generated during peak production times during periods of high or low generations. The authors conducted numerical experiments based on real data to analyze the operation of the energy community under varying technical and economic conditions, emphasizing the role of storage technologies in managing risks and uncertainties associated with energy generation and consumption. In particular the authors concludes that the integration of storage technologies enables prosumers to reduce their reliance on external energy sources and to shift energy consumption to periods of lower prices, thus lowering their operating costs. Also, through hydrogen storage technologies they can diversify revenue streams by participating in hydrogen markets, where they can sell stored hydrogen. They investigated how different levels of risk aversion among community members influence trading strategies in forward and spot markets. They then investigated how different levels of risk aversion among community members influence trading strategies in forward and spot markets. The results indicated that the community's optimal involvement in futures and spot markets is heavily influenced by risk aversion and self-sufficiency. Higher risk aversion tends to lead to more conservative trading decisions, impacting the overall costs for community members. Given the great potential represented by storage technologies in limiting uncertainties in energy markets, many studies have focused on optimizing their management. There are several types of energy storage technologies available, that are broadly classified into mechanical, electro-chemical, chemical, electrical, and thermal energy. Mechanical storage systems includes Hydro Pumping Storage (HPS), Compressed Air Energy Storage (CAES) and flywheel energy storage. The method of hydro pumped storage involves moving water from a lower basin when there is an excess of power generation capacity and releasing it back into the same basin through a turbine when there is a shortage of electricity supply [20]. Compressed air energy storage utilizes underground caverns or aboveground tanks to store compressed air, which is later expanded through turbines to generate electricity while flywheel energy storage systems store kinetic energy in rotating masses, offering high power output and quick response times. Electro-chemical technologies consist of battery energy storage systems, including lithium-ion, lead-acid, and flow batteries, store electrical energy in chemical form and are increasingly popular for their scalability and rapid response times [18]. Emerging technologies such as hydrogen storage and supercapacitors are currently under development to address specific energy storage requirements and overcome limitations of existing methods. Specifically, chemical energy storage technologies represent highly efficient resources, as hydrogen, when used as a fuel, is converted into electricity within a fuel cell. These cells demonstrate high efficiency, with rates of up to 60%, operate without noise, and necessitate minimal maintenance. Nevertheless, hydrogen storage technology is subject to significant costs and infrastructure challenges[30]. Sedghi et al.[11] established an optimal planning framework for battery storage in distribution networks by focusing on determining the ideal location, capacity, and power rating of these storage units. Their objectives included the minimization of the overall costs, which included investment,

operational, and reliability costs, while submitting to technical constraints. Long-term planning is informed by short-term optimal power flow (OPF) analysis that incorporates the uncertainties associated with wind power generation. They demonstrated that optimal planning of battery storage units not only reduced operational costs but also enhanced reliability. The study found that as wind power penetration increases, the required storage capacity and the corresponding costs also increase. Notably, the analysis revealed that the location of storage units is influenced by the level of wind penetration, with certain nodes becoming more favorable for installation depending on the operational conditions. The authors also discussed the impact of reactive power management on the planning of the storage units. They concluded that incorporating reactive power capabilities into the scheduling of batteries significantly improved the overall performance of the distribution network. M-A Hamidan and F.Borouzan[22] provides an in-depth analysis of incorporating distributed generation (DG) and battery energy storage systems (BESS) in electrical distribution networks to enhance reliability and reduce energy losses. The authors utilized a Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) to optimize the placement and sizing of distributed generation and energy storage systems across 30-bus and 69-bus distribution networks. The optimization targets include minimizing the Energy Not Supplied (ENS), reducing load losses, load costs, and voltage drops. The main finding of their work is that the simultaneous use of Distributed Generators and Battery Energy Storage Systems significantly improves reliability and reduces power losses in distribution networks. Specifically, the study demonstrates that the Energy Not Supplied can be reduced by 90% and power losses by 85.20% in a 30-bus distribution network, while in a 69-bus network, Energy Not Supplied is reduced by 85% and power losses by 80%. The optimization was achieved using a Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D), which showed better performance compared to other algorithms like MOPSO and NSGA-II in terms of convergence speed and overall solution quality. Takahashi et al.[17] studied the simultaneous operation scheduling of generators and Battery Energy Storage Systems (BESS) based on actual and forecasted photovoltaic (PV) power outputs. They propose a method utilizing Mixed Integer Linear Programming (MILP) to simultaneously determine and update the generator unit commitment (UC), the scheduled batteries charge/discharge powers, and the actual charge/discharge power at the given time. This method allows for updates based on the actual PV power outputs and forecasted values throughout the day. The authors validate the effectiveness of their approach through numerical simulations in a power system model for the Kanto area in Japan. They proposed a method that involves updating the generator UC and BESS schedules every hour based on real-time PV power outputs, by modifying generator activations six hours after the start of the experiment. They outline the objective function for the MILP, which aims to minimize operating costs for thermal generators while adhering to various constraints including upward reserve capacity, and the upper and lower output limits of generators. The constraints for thermal generators and BESS are then elaborated. These include regulations for reserve capacity, the prohibition of simultaneous charge and discharge for BESS, and conditions regarding initial and termination states of stored energy. The authors also describe how power shortfalls and PV curtailment are calculated, addressing the need for bal-

ancing supply and demand in real-time. Shabani et al. [33] discusses the challenges and solutions related to lithium-ion battery storage in energy systems, particularly focusing on residential grid-connected applications under dynamic electricity pricing conditions. To enhance the practical viability of these batteries for price arbitrage applications, the authors emphasize the need for smart management strategies that optimized battery utilization while effectively addressing these challenges. The study proposes two distinct day-ahead operation scheduling strategies, a short-term profitability optimization and a short-term and long-term profitability optimization, aimed at maximizing both profitability and longevity of battery systems. The first one focuses on immediate revenue gains through fixed charge/discharge rates categorized as high, moderate, and low, and optimizes short-term profitability by identifying the maximum daily electricity price differential while considering battery degradation. The second strategy integrates a smart decision-making process capable of dynamically adjusting various operational parameters (e.g., cycle frequency, timings, durations) to maximize revenue while minimizing degradation costs. The first strategies revealed that higher charge/discharge rates yield better profitability despite shorter lifetimes. The second strategy outperformed the first strategy in terms of long-term profitability and sustainability. The strategies were implemented for two types of Swedish households, one with district heating and one without, using a 5kWh and 10kWh battery systems. The economic performance metrics demonstrated how different charge/discharge rates and operational strategies influence both revenue generation and battery lifespan. The authors concluded that achieving a balance between battery degradation costs and revenue generation is crucial for maximizing profitability to ensure long-term sustainability. R.Wang and R.Zahn [31] explores the integration of wind energy and hydrogen energy storage systems as a sustainable and clean alternative for electricity generation, particularly in off-grid areas. In their work they identifies the HESS as a promising energy storage technology that can enhance the stability and reliability of renewable energy systems. However, the high capital costs associated with HESS are noted as a significant barrier. The authors propose a biological-inspired optimization algorithm, specifically the artificial bee colony optimization method, to optimize the design of an off-grid wind power generator coupled with a hydrogen energy storage system. Their optimization focuses on minimizing two main objectives: the total cost and load loss. The study includes a techno-economic analysis of various configurations using different small horizontal-axis wind turbines. A sensitivity analysis is conducted to evaluate how variations in component costs affect the total cost and load loss. The simulation results indicate that the cost of energy (COE) varies based on load loss, with the lowest COE achieved at $0.5987\$/kWh$ considering the Fortis.-Wind Turbine-hydrogen storage system configuration, which also demonstrated 100% reliability. In addition the performance of the proposed biological inspired optimization algorithm was compared with the Particle Swarm Optimization algorithm. The results show that the proposed algorithm is more effective in achieving lower total costs and quicker convergence. The authors then discuss how fluctuations in the capital costs of wind turbines, fuel cells, and electrolyzers significantly impact the overall the cost of the energy system. The findings suggest that reductions in these costs could facilitate wider adoption of hybrid energy systems.

Xiao et al.[19] studied an innovative wind-electrolytic hydrogen storage system designed to optimize operations within the electricity and hydrogen markets. The proposed system consists of a wind farm (WF), an electrolyzer, a hydrogen storage tank (HST), and connections to both electricity, and hydrogen markets. Their model aims to maximize profits by deciding when to sell electricity directly to the market and when to convert excess electricity into hydrogen for sale to the hydrogen market. The authors formulated the operational problem as a mixed-integer linear programming (MILP) model, incorporating the uncertainties of wind power output and electricity prices. A scenario based stochastic method was employed to analyze these uncertainties, and financial risks were assessed using Conditional Value-at-Risk (CVaR). The results indicate that the expected revenue from the system increases significantly with increasing hydrogen prices and considering an operational strategy that entails selling electricity directly to the electricity market when prices are high and converting excess electricity into hydrogen via an electrolyzer to sell to the hydrogen market when electricity prices are low. D.Yousri et al. [32] presented an integrated model for optimal energy management and demand response of microgrids that utilize hybrid hydrogen-battery (HHB) storage systems. The authors proposed an Integrated Energy Management System (IEMS) that aims to optimize the operation of the HHB storage system while providing demand response capabilities. They focused on minimizing three conflicting objectives: the costs associated with electricity and battery degradation, customer discomfort related to load shifting, and peak-to-average ratio (PAR), which impacts grid stability. They formulated the system as a mixed-integer nonlinear problem solving it using a multi-objective artificial hummingbird algorithm, and evaluated it under four weather scenarios: good, bad, average and forecasted scenarios based on stochastic modeling. The authors then compared the proposed IEMS with conventional energy management strategies and assessed it against a multi-objective particle swarm optimizer. The proposed IEMS demonstrated significant improvements in cost savings (up to 82.95% compared to other conventional energy management strategies) and reductions in greenhouse gas emissions under various weather scenarios. The authors conclude that the implementation of the proposed IEMS significantly enhances the management of hybrid microgrids, leading to substantial economic, and environmental benefits. The use of advanced optimization techniques, such as MOAHA, facilitates improved decision-making processes in the operation of microgrids, particularly in the context of integrating renewable energy and effectively managing demand response. Numerous studies have examined the uncertainty in the electricity market and the optimization of energy generation and storage for HPS facilities both alone and in combination with other renewable energy sources. Abbassi et al. [9] investigated the management of hydropower dams within monopolistic and oligopolistic competition when electricity producers are risk-averse and face demand uncertainty. The authors present a dynamic stochastic model of hydropower management, considering the electricity demand and production dynamics. The first is represented by a linear inverse demand function with a normally distributed price parameter. The production dynamics took into consideration the fact that each dam's water availability is influenced by random precipitation flows, modeled as a random walk. The hydropower operator seeks to maximize expected utility of profit over an infinite horizon. This involves determining the

optimal water release path from the dams to manage electricity production effectively. The authors used a mean-variance utility function to incorporate risk aversion, where the utility depends on the expected profit and variance in profit. The analysis distinguishes between two market structures monopoly and oligopoly. They found that a monopolist manages its hydropower reservoirs through additional pumping or storage based on the relative abundance or scarcity of water in different regions. The monopolist's behavior is characterized by strategic decision-making to smooth out the effects of demand uncertainty on electricity prices. The solution of the optimization model revealed how variations in water inflows affect production levels, leading to strategic decisions on either storing water or generating electricity based on regional water abundance. In the case of oligopoly, the authors explored scenarios with symmetric and asymmetric risk aversion among competing firms. When hydroelectric operators have symmetric attitudes toward risk, the study identifies conditions under which the relative scarcity (or abundance) of water in one operator's dam can lead to additional strategic pumping (or storage) in their competitor's dams. In scenarios where operators exhibit asymmetric risk aversion coefficients, it is shown that a firm's hydroelectric production can increase in parallel with its competitor's risk aversion if the average recharge speed of the competitor's dam exceeds a certain threshold, which is increasingly dependent on the average water inflow. The findings indicate that increases in the risk aversion coefficient of operators tend to reduce overall production levels, as operators become more conservative in their pumping strategies to mitigate risk. In a further study [27], the effects of risk aversion on the output industry were compared. The results showed that in the monopolistic setup, hydroelectric producers allocate water resources intertemporally by adjusting pumping and storage based on water availability across regions. This is aimed at mitigating risks and stabilizing electricity prices amid demand uncertainty. The greater the risk aversion of the producers, the more they tend to store water for future demands, reducing the variability in water pumping. However, in the oligopolistic case, increased risk aversion among oligopolistic producers may either decrease or increase overall hydroelectric output, contingent on specific market conditions and demand fluctuations. The findings demonstrate that the relative scarcity or abundance of water in an operator's dam can influence strategic pumping or storage decisions for competitor dams. This indicates that firms may engage in behavior that prioritizes competitive advantage over efficient resource use. Consequently the combination of uncertainties in water inflows and electricity demand can exacerbate market inefficiencies, leading to strategic behaviors by hydroelectric operators that further distort efficient resource allocation. This study revealed that market power, when coupled with risk aversion, can lead to a misallocation of water resources over time. This necessitates regulatory measures to mitigate adverse effects on intertemporal water allocation. The authors suggest that the lack of markets to share risks or mechanisms such as forward contracts can lead to excessive caution and strategic conservation of water resources by risk-averse generators, further contributing to misallocation. In both of these studies, although they considered the presence of multiple operators, in their work each agent operates its own plant independently, conditioned by the market and the actions of other operators. However, they did not consider the possibility of optimizing the management of a facility while a counterpart has access to the

same facility. Schill and Kemfert [7] investigated the strategic utilization of electricity storage in imperfect electricity markets, focusing on pumped hydrostorage in Germany. The authors applied a game-theoretic Cournot model to analyze strategic storage utilization in an oligopolistic market environment with imperfectly competitive generators and to assess the impacts of different storage operators on the welfare outcome. Specifically they developed the ElStorM game-theoretic electricity market model, which incorporates hourly time resolution and intertemporal constraints, enabling the examination of strategic interactions between conventional generation and storage operations. The model considers various player types, including merchant storage operators and conventional generation firms, allowing for the exploration of different ownership configurations. In particular, the authors examine how different ownership structures and market power configurations affect the utilization of storage. For instance, it considers cases where storage is operated by strategic players who also have conventional generation capacity, leading to different outcomes in terms of storage utilization and market prices. The analysis performed is based on a winter week in the German electricity market, employing hourly data from the European Energy Exchange (EEX). The study considered six players, including major energy firms like E.ON, RWE, Vattenfall, and EnBW, alongside a competitive fringe and a merchant storage player. This analysis showed how profit-maximizing firms decide on both conventional generation and pumped hydro storage operations, revealing a tendency for strategic operators to under-utilize storage capacity. The introduction of storage tends to decrease total producer surplus while increasing consumer surplus, leading to an overall increase in welfare. Specifically, the extent of these welfare gains is influenced by the ownership structure of storage; higher welfare gains are observed when storage capacity is distributed among multiple operators rather than concentrated in a single provider. In cases where storage is owned by a single oligopolistic generator, in fact, significant underutilization occurs. This particular scenario illustrates the adverse effects of concentrating storage control among strategic players. The author's primary conclusions indicate that Strategic storage is unlikely to represent a significant source of market power in Germany, particularly when storage is distributed among several players. However the potential for welfare losses from strategic storage is considerably lower than those arising from conventional generation. The authors then suggest that economic regulation of existing pumped hydro storage capacity may not be necessary, though future expansions should be monitored to prevent monopolistic control. Furthermore, the study considered various agents, each operating their respective facility. However, the potential for optimizing the behavior of multiple entities using a single facility was not explored in this investigation. Bakanos and Katsifarakis [15] explored the integration of wind energy into a pumped storage hydropower (PSH) system, addressing the challenges posed by the intermittent nature of renewable energy sources. The authors developed a combined simulation and optimization model utilizing genetic algorithms to maximize the economic benefits of this integration. They created a model that integrates wind energy production into a multi-reservoir pumped-storage system, which can operate in either closed-loop or open-loop mode. Next Genetic Algorithms were employed as the optimization tool due to their effectiveness in solving complex, multi-dimensional, nonlinear, and dynamic optimization problems. This method mimics biological evolu-

tion and involves encoding solutions into “chromosomes,” evaluating their fitness based on defined criteria, and applying genetic operators such as selection, crossover, and mutation to evolve better solutions over generations. The primary objective of the model is to maximize the financial benefits from operating the system over a 24-hour period. This was achieved by optimizing the scheduling of energy generation and pumping operations, balancing between hydropower production and wind energy absorption. The case study involved a two-reservoir cascade system with a pumped-storage hydropower system and a conventional hydropower plants along a wind farm. The authors analyzed two operational modes: Closed-Loop Mode and an Open-Loop Mode. In the first case, without considering wind coordination the PSH system generated hydroelectricity based on the market prices, resulting in 9.73% wind energy curtailment. Considering the wind coordination the system absorbed all wind energy, leading to a decrease in hydroelectric generation but an overall increase in economic benefit of 7.02%. With an Open-Loop mode, 18.09% of wind energy is discarded when the wind coordination is not included, while when the wind coordination is considered the system absorbs all wind energy, resulting in a 7.16% increase in total economic benefits, despite a 37.19% reduction in hydroelectric revenue. The study concludes that both closed-loop and open-loop PSH systems, when coordinated with wind farms, achieve greater economic benefits compared to uncoordinated operations. The coordination relies on a combination of market price fluctuations and wind energy absorption, enhancing the flexibility and performance of the system. Sun et al. [23] proposed a hierarchical scheduling control (HSC) method to integrate the variability and uncertainty of a photovoltaic system with a cascade hydro-pumping storage system. They divided their method into two layers: a dynamic optimization layer and a static optimization layer. The first one focus on the safety of the hydro-PV-pumped storage system, emphasizing real-time scheduling, employing Model Predictive Control (MPC) to correct prediction biases and enhances the responsiveness of hydropower generation. The second layer focuses on economic scheduling, optimizing energy allocation for day ahead and hour-ahead periods using heuristic and linear programming algorithms. The authors considered three time horizons for optimization: day-ahead optimization, hour-ahead optimization, and real-time scheduling. They considered a real-world Cascade Hydro-PV-Pumped storage system located in Southwest China as a case study. The authors provides than a comparative analysis of the performance of different scheduling methods under varying weather conditions: sunny, rainy, and cloudy days. For the sunny days the hierarchical scheduling control method demonstrates superior PV follow-up capacities and economic benefits. Considering rainy days, the method maintains system stability even with diminished PV output. Finally, by taking into account cloudy days, the fleXibility and responsiveness of the HSC method prove essential for managing fluctuating energy outputs. The authors concludes that the hierarchical scheduling control method effectively reduces the average relative deviation of PV output, and by effectively addressing the challenges of PV output uncertainty and enhancing operational stability, the HSC method not only maximizes economic benefits but also provides a viable solution for integrating renewable energy sources into the power grids. In addition, some researchers have addressed the investment aspect of hydro-pumping storage facilities. Connolly et al. [6] optimized the maximum feasible profit that a pumped hydroelectric

energy storage (PHES) facility can achieve on an electricity market with perfect pricing foresight for one year. They developed three practical operation strategies: the 24Optimal strategy that optimizes operations for the next day only, requiring good price forecasts, the 24Prognostic strategy that operates based on the average prices predicted for the next 24 hours, and the 24Historical strategy, which Utilizes average prices based on 12 historical and 12 future hours to make operational decisions. The authors then compared these strategies against an optimum profit scenario in which theoretical income was maximized using a full year of hourly electricity price data. In the latter, the hour with the maximum electricity price was identified. Then potential pumping and generation hours based on reservoir capacity are determined, and the operation is executed based on the maximum profit potential. The results have shown that profits from hydro pumping storage operations vary significantly across different electricity markets, influenced by market structures and regulations. The 24Optimal strategy achieved 97% of the profit of the Optimal strategy while the 24Prognostic and 24Historical strategies yield 81% and 83% of the Optimal profits respectively. However Increasing storage capacity from 2 GWh to 8 GWh showed a decrease in the effectiveness of the 24Optimal strategy, highlighting its suitability for a typical 6-8 hour charge/discharge cycle. The authors concludes that there is the necessity for accurate price forecasting and market conditions is critical for the financial viability of PHES investments, and that Without predictable profits or additional revenue from ancillary services, PHES remains a risky investment choice. Toufani et al. [25] investigated various configurations of hydro pumping storage facilities, open-loop and closed-loop systems, under uncertainty in streamflow rates and electricity prices, particularly focusing on their short-term cash flow performances. The study examined five different configurations: a cascading system with an upstream flow, a cascading system with a downstream flow, a non cascading system, also considering both an upstream and a downstream flow, and a closed-loop system. To evaluate those configurations the authors developed a stochastic dynamic programming model to simulate the decision-making process of HPS operators over a finite planning horizon. Their model incorporates the uncertainties in streamflow rates and electricity prices, allowing for a structured analysis of various configurations, since the performance of the HPS configurations is sensitive to the streamflow rates, and frequencies of negative electricity prices (NPF). The analysis, in fact, indicates that higher cash flows are generally observed in months with greater streamflow availability and lower NPF. The authors established theoretical bounds on revenue variations when switching between configurations. The findings indicate that: the open-loop facility with upstream flow can yield cash flows up to four times higher than closed-loop facilities. In fact, operating a large closed-loop facility can be matched by an open-loop facility with 10 times smaller reservoirs. They also find out that if negative electricity prices occur more than 30% of the time, open-loop facilities with downstream flow may outperform those with upstream flow by over 10% in cash flow. The authors suggest future research could integrate upfront installation costs and explore the joint optimization of sizing and operational planning for HPS systems. Additionally, they propose examining the environmental and social implications of various HPS configurations and their integration with other renewable energy sources. They did not consider operational planning for HPS systems, considering that a counterpart can operate in

the same facility. Wesseh et al.[26] studied the critical interplay between storage technologies, particularly Hydro Pumping Storage (HPS), electric vehicles (EVs), and climate policies within the context of imperfect electricity markets using data from China. The authors employed a game-theoretic electricity market model, incorporating assumptions regarding market power and asset ownership. In particular the study assessed both perfect competition and imperfect competition scenarios. The authors also included different ownership structures for pumped hydro storage and electric vehicle operations. They defined scenarios where, storage and electric vehicle operations are monopolized by single firms and storage and electric vehicle operations are operated by a mix of different players, including firms that may or may not own generation capacity. Each player of aims to maximize their profit through hourly decision-making regarding electricity generation, storage loading, and discharging, as well as vehicle loading. The revenue generated by each player is calculated from electricity sales, with specific costs subtracted, including variable generation and storage operational costs. The analysis examined 24 different scenarios that vary based on factors such as, ownership of storage and electric vehicle operations, market power assumptions, presence or absence of climate policy and variable costs associated with pumped hydro storage and electric vehicle operations. The study uses electricity market data for a typical summer week in China, covering 168 hours of model time. The suggested that climate policy may not effectively incentivize the utilization of pumped hydro storage, particularly when firms that have a dominant position in storage operations possess a limited portfolio of generation assets. This implies that the benefits of climate policies are not fully realized in terms of storage usage. The study also highlighted that market power significantly affects storage utilization. In scenarios where market power exists in both power generation and storage operations, the generation from both pumped hydro storage and electric vehicles tends to be zero. This suggests that strategic behavior in markets can adversely affect the expected benefits of these technologies. Finally the study identifies certain strategies as infeasible, particularly those that concentrate storage and electric vehicle operations within a single large generating firm or those that assume zero operational costs in a market with exerted power. This suggests that regulatory and market structure considerations are crucial for the effective operations of these systems. Although the authors have studied the relationships among HPS optimization, electric vehicles, and economic policies, they have not investigated the potential complications of surrendering part of the storage capacity to another agent. Therefore, they did not jointly optimize HPS management with the operations required by a counterpart. Doege et al.[3] modeled the operational flexibility of HPS facilities and explored how they can be utilized to hedge against adverse movements in portfolios. Additionally, they highlighted the importance of an intelligent dispatch policy for managing volume risk. The authors specifically modeled the operational flexibility of hydro pump storage plants to hedge against adverse portfolio movements and manage volume risks that standard power exchange contracts cannot cover. They proposed a linear portfolio optimization model that integrates both financial contracts and physical production assets. This model allows for dynamic dispatch policies based on external drivers such as spot prices, demand, and water inflow. Specifically the hydro plant’s operational decisions involve whether to produce electricity or pump water for storage, where

optimal dispatch strategies depend on real-time market conditions and stochastic nature of water inflow. The optimization problem is formulated to maximize expected profits while controlling risk measured by CVaR. The model incorporates constraints related to production capacity, water levels, and contractual obligations. The authors derive decision variables that determine the dispatch of generation assets. The model was applied to a real-world case study, demonstrating the application of the optimization model using data from the European Energy Exchange (EEX). Key findings highlight the optimal dispatch in response to varying risk levels and market conditions. Additionally, this study emphasizes the intrinsic value of flexibility in power generation. Two main aspects are explored: the hedging value of flexibility and marginal value of flexibility. First the relationship between risk levels and expected profits is assessed, illustrating how flexibility can be leveraged to effectively reduce risk. The authors defined then the marginal value of flexibility in terms of the additional production volume and its impact on risk reduction. The authors indicate plans to extend the model to encompass multiple hydro plants, base-load power plants, and dynamic risk measures, allowing for more comprehensive risk management strategies in electricity production. However, they did not consider the possibility of optimizing the management of the plant jointly with the fact that they ceded part of the storage capacity to a counterpart.

In the existing literature, to the best of my knowledge, no one has ever jointly optimized the operative decisions of the owner with the decision to sell part of the storage capacity to its counterpart. In this study, an optimization model is developed that not only optimizes the owner's management decisions, but also the amount of storage capacity to be transferred to the customer. The main problem raised with the introduction of a counterpart was that, in certain specific hours and scenarios the owner's optimal operative decisions and the customer's pumping and production orders have been in opposition. To face this problem three different types of potential markets are modeled. This results in three different version of the optimization model: a physical model, a financial model and an hybrid model. The models were first evaluated in a deterministic framework using a 24-hour realistic scenario. The owner of this entity is involved in the operation of both day ahead and real-time energy markets. The day ahead market is a forward market, which establishes financial schedules for the consumption and production of energy one day in advance of the operating day. On the other hand, the real-time market is a spot market, where locational marginal prices are determined based on actual grid operating conditions [12]. As a result, the owner has the ability to buy and sell energy through futures contracts and to take advantage of price arbitrage opportunities in the spot market. Specifically, the owner can purchase energy when prices are low during off-peak hours and sell it at higher prices during peak hours. The three models are reformulated within a stochastic framework to address the inherent uncertainties of renewable energy markets. To account for the decision maker's risk aversion, the Conditional Value-at-Risk measure is incorporated into the objective function. The models are tested using 10 generated scenarios. Switching to the stochastic model increases computational complexity and computation time. To efficiently solve the optimization models, two heuristics are applied. In the following chapter, the mathematical formulations of the three models in the deterministic case are presented.

Chapter 2

The impact of the storage market in the management of a Hydro Pumping Storage facility

The Italian energy market, once dominated by a stable mix of thermal and hydroelectric generation, now faces challenges from the variability and uncertainty introduced by renewable energy sources (RES). Energy storage technologies are crucial to address these issues, allowing energy to be stored during low demand and released during high demand, enhancing grid flexibility and stabilizing RES variability. Establishing a long-term energy storage market seeks to address the high investment costs of storage technologies and the price volatility in the spot market. One approach involves upgrading existing hydropower systems to open-loop pumped storage (HPS) facilities by installing reversible turbines and adding new basins, though this requires significant investment from facility owners. The Italian electricity market has been modified by Legislative Decree 210/2021, specifically through Article 18 which specifies that it is possible, for the owner of a storage facility, to finance its investment through selling part of the storage capacity to a counterpart. Here lies the novelty of this work, which is the joint optimization of the energy generating and pumping decisions and the decision to sell part of the storage capacity to a counterpart. In this chapter, a non-linear optimization model is developed to optimize the owner's management decisions considering that a certain amount of storage capacity is transferred to a counterpart. More specifically the original contribution of this study lies in the fact that the owner's production and pumping decisions are optimized by considering that a customer is at the same time managing its storage quota in the same plant. The introduction of a counterpart in the optimization model led to the development of three different potential types of markets, where the interaction between the owner and the customer is addressed from different perspectives. Subsequently, three different versions of the models are developed: a physical model, a financial model, and a hybrid model. Two types of counterparts potentially interested in purchasing storage capacity have been identified. The first is a private individual

or a company that acquires storage capacity through a Power Purchase Agreement (PPA). The second is a Transmission System Operator (TSO), such as Terna in the Italian context. The TSO is responsible for managing the electricity grid to ensure the secure and reliable flow of electricity. This is achieved by issuing commands—real-time instructions to the transmission system—that enable devices, loads, and power plants to maintain the grid within the prescribed frequency and voltage range, while ensuring the transmission of the required amount of power. When the load is not immediately balanced with the power generated, an imbalance occurs. Such imbalances must be corrected, as the power injected into and withdrawn from the grid must always be equal. It is important to distinguish between imbalances and blackouts. A blackout, also referred to as a power outage or system failure, occurs when there is a loss of power in a given area due to an unforeseen or unmanageable event that renders the electricity system so unstable that it collapses. In an electric system, even when loads are curtailed in certain areas, power continues to flow and the unserved load is a result of the need to keep the system operational. In contrast, during a blackout, the current ceases to flow through parts of the system entirely. As such, a blackout represents a pathological condition of the electricity system, which, in principle, should never occur. Blackouts can be highly disruptive, long-lasting, and affect large portions of the network. Furthermore, once a blackout has occurred, it may take considerable time for the system to recover. For this reason, grid management is designed to minimize the risk of blackouts.[16] In this context, the rationale behind a counterpart’s decision to purchase storage capacity is to balance supply and demand, ensure system stability, prevent blackouts, facilitate the integration of renewable energy sources, and maintain grid frequency. Through a Power Purchase Agreement (PPA), an energy producer commits to supplying a specific amount of electricity, while the buyer agrees to purchase it at a predetermined price over a period typically ranging from 5 to 20 years. There are various types of PPAs. In particular, the counterpart under consideration would enter into a virtual (or synthetic) PPA, which is a financial agreement wherein no physical delivery of electricity—or, in this specific case, storage capacity—takes place. As a result, through this type of PPA, the counterpart obtains “virtual” capacity rights, meaning that the capacity remains the property of another entity, while the counterpart merely acquires the associated usage rights. The primary objective of this type of counterpart is the maximization of expected profit, while simultaneously securing a long-term and fixed-price supply. The three models are then evaluated considering a realistic scenario of a facility situated in Italy, employing a deterministic framework. Specifically, the study assessed a 24-hour scenario, initially without considering the presence of a counterpart and subsequently incorporating the hypothesis of allocating half of the storage capacity to the counterpart. By analyzing the models in the deterministic case and considering a limited time range, it is possible to analyze the economic impact, hour by hour, of potential customer behavior on the owner’s ultimate profit. The chapter is structured as follows: Section 2.1 presents the description and mathematical formulation of the developed models. Section 2.3 delineates the characteristics of the models’ application, while Section 2.4 analyzes the obtained results. Finally, Section 2.5 presents the conclusions of the chapter.

2.1 An Hydro Pumping Storage model

The model considers an open-loop facility with upper and lower basins, where energy can be produced by releasing water from the upper basin to the lower basin, and stored by pumping water from the lower basin to the upper basin. There is only one turbine that works both as a generator and pump, so it is not possible to pump water and produce energy simultaneously. Both basins can be fed by natural inflow, such as rainfall, and once they reach their maximum capacity, can be subject to spillovers. The water contained in the basins in each hour t is divided between the owner's and the customer's ownership. In this thesis, I refer to the owner as the decision-maker and to the counterparts as the customers interchangeably.

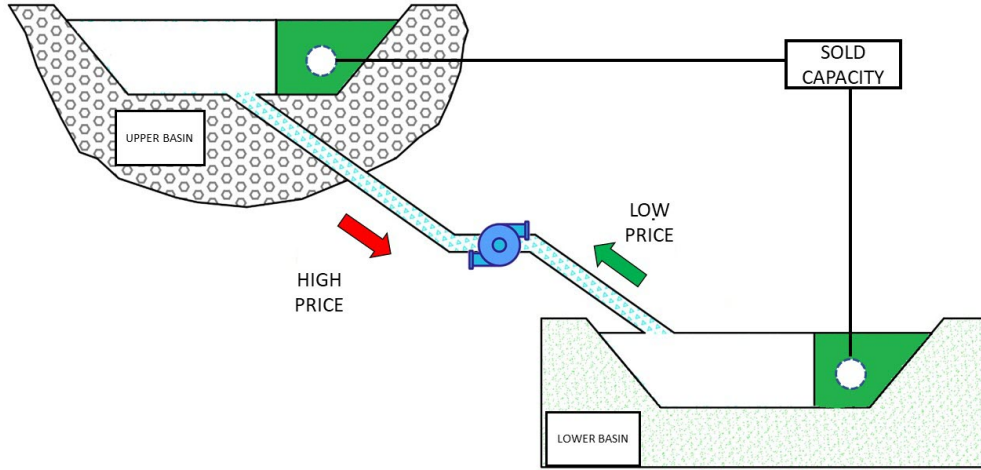


Figure 2.1: Schematic operation of a Hydro Pumping Storage Facility

Figure 2.1 shows the schematic operation of a Hydro Pumping Storage facility considering the possibility to sell part of the storage capacity to a counterpart. At each hour t the total level of the upper and the lower basin are updated as:

$$V_t^{u,tot} = V_{t-1}^{u,tot} - pr_t + pm_t + I_t^u - L_t^u \quad (2.1)$$

The lower basin calculation is, of course, a mirror image for physical production and pumping:

$$V_t^{d,tot} = V_{t-1}^{d,tot} + pr_t - pm_t + I_t^d - L_t^d \quad (2.2)$$

Where:

$V_t^{u,tot}$	Overall upper water level hour t [m^3]
$V_t^{d,tot}$	Overall lower water level in hour t [m^3]
pr_t	Total production in hour t [m^3]
pm_t	Total pumping in hour t [m^3]
L_t^u	Upper basin spillover [m^3]
L_t^d	Lower basin spillover [m^3]
I_t^u	Natural Inflow of the upper basin
I_t^d	Natural Inflow of the lower basin

The calculation of upper and lower spillover required some testing. It is clear that any potential spillover is equal to the positive difference between the water level reached at the end of the hour and the basin's maximum capacity. It is crucial to obtain an exact number; therefore, the best way to determine L_t^u and L_t^d is through the use of maximum functions. The spillovers are calculated as the maximum between the difference between the water level of the basins and the maximum capacity of the upper and lower basins, V_{max}^u and V_{max}^d , and zero. If the difference results in a negative number, the spillovers will be equal to zero, whereas if it results in a positive number, it will be equal to the difference.

The equations for calculating the upper and lower spillover are as follows:

$$L_t^u = \max(V_{t-1}^{u,tot} - pr_t + pm_t + I_t^u - V_{max}^u; 0) \quad (2.3)$$

$$L_t^d = \max(V_{t-1}^{d,tot} + pr_t - pm_t + I_t^d - V_{max}^d; 0) \quad (2.4)$$

In the following sections the remain mathematical specification of the model is defined.

2.1.1 Decision variables

This section introduces the decision variables of the optimization model, representing the key elements to be adjusted in order to achieve the optimal solution. Specifically, the decision variables are as follows:

v_t^+	water released from the upper basin in hour t [m^3]
v_t^-	water pumped from the lower basin in hour t [m^3]
x_t^{spot}	energy sold/bought in the spot market (spot position) in hour t [MWh]
x_i^F	position in future contract i [MWh]

where v_t^+ and v_t^- represent the water moved between the basins in hour t , where t is within the interval $[0, T]$. In particular, v_t^+ quantifies the cubic meters of water released into the upper basin in hour t to produce energy, while v_t^- represents the cubic meters of water pumped from the lower basin to the upper basin, intended to create a storage reserve for future use. Variable x_t^{spot} denotes the MW of energy sold or purchased in the spot market during the specific hour t . Specifically, when x_t^{spot} is negative, it indicates that the owner was required to purchase energy in the spot market to pump water into the upper basin. When x_t^{spot} is positive, it signifies that the owner sold the energy produced in the spot market. Variable x_i^F represents the position adopted by future contract i . A positive value indicates that the owner adopted a short position on that specific future, committing to sell that amount of energy when the future is active. Conversely, if the variable is negative, the owner adopted a long position, indicating that when that specific future is active, the owner is obligated to purchase energy.

2.1.2 Objective function

The objective function aims to maximize the owner's final profits, which are represented by the revenues from selling the energy storage capacity, and the profit obtained through the management of the facility and the operations in the futures and spot markets. The objective function is then define by equation (2.5)

$$\max_{[v_t^+, v_t^-, x_t^{spot}, x_i^F]} [\pi(v_t^+, v_t^-, x_t^{spot}, x_i^F)] \quad (2.5)$$

The profit function $\pi(v_t^+, v_t^-, x_t^{spot}, x_i^F)$ signifies the profit obtained from the management of the facility, and depends on the decision variables previously defined. The subsequent section elucidates the formulation of this function.

2.1.3 Profit function

The owner's profits are calculated as a combination of revenues and costs. The revenue component consist of gains from trading futures and speculating on spot markets. The former is represented by the sum, for each future contract, of the product of the futures position and the future contract price, multiplied by a binary parameter indicating whether the future is active or not. The latter is defined as the product of spot prices and spot position. The cost component is represented by the operative costs associated with the owner's production and pumping decisions.

The following parameters are defined:

A_{it}	Binary parameter = 1 if future contract i is active in time t , = 0 otherwise
r_2	Monthly discount rate
F_i	Unit cost of future contract i [€/MWh]
S_t	Spot price for hour t [€/MWh]
c^+	Marginal production cost of the generator [€/MWh]
c^-	Marginal production cost of the pump [€/MWh]

The profit function is defined as follows:

$$\pi_j = 30 \cdot \sum_{m=1}^{12} e^{-r_2 m} \sum_{k=1}^K \left(\sum_{i=1}^n A_{i,K \cdot m + k} \cdot F_i \cdot x_i^F + S_{K \cdot m + k} \cdot x_{K \cdot m + k}^{spot} - c^+ \cdot v_{K \cdot m + k}^+ \cdot k^+ - c^- \cdot v_{K \cdot m + k}^- \cdot k^- \right) \quad (2.6)$$

In the profit function, an alternative notation is used to indicate hour t . Specifically, t is expressed as $t = K \cdot m + k$, where:

- K represents the total number of hours in a month
- m denotes the index of the month
- k signifies the hour index within a month

For instance, considering a sample of 10h per month, the 3^{rd} hour of the second month can be denoted as the 12^{th} hour of the month (since the count starts from 0, with the second month indicated as month 1, and so on). This type of notation makes it possible to maintain the consecutiveness of the hours while distinguishing the month. The objective function is subject to numerous constraints limiting the counterpart's pumping and production decisions, the balance of the basins, and the accounting offset between the customer's and owner's ownership. The following paragraph will delve into the constraints associated with the counterpart's pumping and production choices.

2.1.4 Customer's operative decisions

I have recognized the necessity to simulate customer orders in a coherent manner with the rest of the problem. Customer's operating decisions cannot be precisely simulated in advance, because they are constrained by the evolution of the system state, which depends on the optimization process. For example, it is not possible to

predict how much the customer will request to pump at time t without knowing the fraction of storage capacity sold to the customer and the average availability at that time, which depends on the actions taken at times $t' < t$. Consequently, production and pumping orders are first generated outside the optimization model. These orders, denoted as v_t^{OC+} for production and v_t^{OC-} for pumping are simulated under the assumption that the customer has been granted full storage capacity, thereby allowing access to the maximum capacity of the turbine X_{max}^+ and pump X_{max}^- . For the deterministic case addressed in this chapter, production and pumping orders were simulated by selecting, among the 24 hours considered, those most likely to correspond to the times when a specific type of customer would request energy production or water pumping (see 2.3). The generated orders are then incorporated into the optimization model to determine the effective customer production orders, represented by v_{jt}^{C+} , and the effective pumping orders, denoted by v_{jt}^{C-} , by applying minimum functions. The following parameters are defined:

v_t^{OC+}	Generated customer's production order [m^3]
v_t^{OC-}	Generated customer's pumping order [m^3]
k^+	Production conversion coefficient [MWh/ m^3]
k^-	Pumping conversion coefficient [MWh/ m^3]
V_{max}^d	Maximum capacity of the lower basin [m^3]

Where the coefficient k^+ allows expressing the energy produced by 1 m^3 in MWh, while k^- convert the energy required to pump 1 m^3 of water into MWh. Also the auxiliary variables are define :

v_t^{C+}	Actual counterpart's production order [m^3]
v_t^{C-}	Actual counterpart's pumping order [m^3]
V_t^{Cu}	Water available for the counterpart in the upper basin hour t [m^3]
V_t^{Cl}	Water available for the counterpart in the lower basin in hour t [m^3]
X_{max}^{C+}	Maximum production capacity of the counterpart [MWh]
X_{max}^{C-}	Maximum pumping capacity of the counterpart [MWh]

The actual customer pumping and production orders are determined as the minimum of three values: the generated orders v_t^{OC-} and v_t^{OC+} and the water available for the counterpart in the lower and upper basin during the previous hour, along with the customer's maximum production capacity expressed in cubic meters (m^3):

$$v_t^{C-} = \min(v_t^{OC-}; V_{t-1}^{Cd}; \frac{X_{max}^{C-}}{k^-}) \quad (2.7)$$

$$v_t^{C+} = \min(v_t^{OC+}; V_{t-1}^{Cu}; \frac{X_{max}^{C+}}{k^+}) \quad (2.8)$$

In constraints (2.7) and (2.8), the terms $\frac{X_{max}^{C-}}{k^-}$ and $\frac{X_{max}^{C+}}{k^+}$ introduces the concept of *storage duration*. In the new legislation regulating the storage market, it is necessary to allocate a minimum storage duration to the counterpart, which is defined as the ratio between the maximum storable energy and the maximum production and pumping capacity. In the case study, a storage duration of 8 h is guaranteed, so

$$8 \leq \frac{\alpha \cdot V_{max}^u \cdot k^+}{X_{max}^{C+}}$$

$$8 \leq \frac{\alpha \cdot V_{max}^u \cdot k^-}{X_{max}^{C-}}$$

where α is a parameter that indicates the percentage of transferred capacity and $\alpha \cdot V_{max}^u \cdot k^+$ is the maximum storable energy reserved for the counterpart and X_{max}^{C+} is the maximum production capacity of the counterpart. Using the above equations, it is possible to express the customer's maximum production and pumping capacity as

$$X_{max}^{C+} = \frac{1}{8} \cdot \alpha V_{max}^u \cdot k^+ \quad (2.9)$$

$$X_{max}^{C-} = \frac{1}{8} \cdot \alpha V_{max}^u \cdot k^- \quad (2.10)$$

To establish resource consistency, a constraint indicating that the maximum energy capacity that can be transferred to the customer must not exceed the maximum capacity of the lower basin is also introduced:

$$\alpha \cdot V_{max}^u \leq V_{max}^d \quad (2.11)$$

2.2 Market models

The introduction of a counterpart presented a challenge in the model, as there was no provision preventing the customer and the owner from producing and pumping simultaneously. To address this, three different types of market are modeled:

- Physical model
- Financial model
- Hybrid model

In the following paragraphs the mathematical development of each model will be described.

2.2.1 Physical model

Under the physical model hypothesis, all the potential conflicts about the use of the turbine (i.e. either pumping or producing at any hour t are settled in favor of the customer. To address this, two binary variables γ_t^- and γ_t^+ were generated together with the generated customer's production and pumping orders (see 2.3), so that:

$$\gamma_t^- = 0 \quad \text{if} \quad v_t^{OC+} > 0 \quad \text{and} \quad \gamma_t^- = 1 \quad \text{if} \quad v_t^{OC+} = 0$$

and

$$\gamma_t^+ = 0 \quad \text{if} \quad v_t^{OC-} > 0 \quad \text{and} \quad \gamma_t^+ = 1 \quad \text{if} \quad v_t^{OC-} = 0$$

For each hour t , the binary variables γ_t^+ and γ_t^- are used to avoid pumping ($v_t^- > 0$) and production whenever the customer orders production ($v_t^{C+} > 0$) or to pump water ($v_t^{C-} > 0$). However, this restriction should be applied only if the decision to sell part of the capacity result positive ($\alpha > 0$).

The following conditions must be enforced:

$$\gamma_t^- = 0 \wedge \alpha > 0 \Rightarrow v_t^- = 0 \quad (2.12)$$

$$\gamma_t^+ = 0 \wedge \alpha > 0 \Rightarrow v_t^+ = 0$$

To do that the following assumption was made: *The fraction of capacity α sold to the customer is either 0 or at least equal to a threshold value δ , with $0 < \delta \leq 1$.*

To enforce this assumption, an additional binary variable λ is introduced, and the following constraints are imposed:

$$\delta \cdot \lambda \leq \alpha \leq \lambda \quad (2.13)$$

Then, condition (2.12) is enforced by the following constraints:

$$v_t^- \leq X_{\max}^-(\gamma_t^- + 1 - \lambda) \quad (2.14)$$

$$v_t^+ \leq X_{\max}^+(\gamma_t^+ + 1 - \lambda) \quad (2.15)$$

Production and pumping decisions

The production capacity of the turbine is limited to $X_{\max}^+$, while the pumping capacity is limited to $X_{\max}^-$. The owner's decisions are influenced by the actions of the customer, such as whether they are also producing energy or physically pumping water. To account for these cases, variables such as X_t^{max+} and X_t^{max-} , which represent the maximum production and pumping capacity of the decision maker in hour t , are introduced and are calculated as the difference between the maximum capacities of the turbine and the operative decision of the counterpart at that time. All quantities are expressed in MWh.

$$X_t^{max+} = X_{\max}^+ - v_t^{C+} \cdot k^+ \quad (2.16)$$

$$X_t^{max-} = X_{\max}^- - v_t^{C-} \cdot k^- \quad (2.17)$$

The production and pumping decisions of the owner, expressed in terms of MWh, are now constrained by X_t^{max+} and X_t^{max-} as follows:

$$0 \leq v_t^+ \cdot k^+ \leq X_t^{max+} \quad (2.18)$$

$$0 \leq v_t^- \cdot k^- \leq X_t^{max-} \quad (2.19)$$

On the top of all the physical decisions, the owner can buy and sell energy taking position on both the spot and the future markets. In any case both physical and financial position must balance in every hour t .

$$v_t^+ \cdot k^+ - v_t^- \cdot k^- - \sum_{i=1}^n A_{it} x_i^F - x_t^{spot} = 0 \quad (2.20)$$

In the following sub-paragraph, the modifications in the basin balance resulting from customers' production and pumping requests as well as the choices made by owners are shown. Both the aggregate level of the basins and the individual level from the perspectives of the counterpart and the decision-maker are considered.

Basin balance

To determine the basin balance, a distinction between the overall basin upper and lower levels and the personal levels of the owner and the customer is made. The overall basin balance is updated through the sum of the owner's and customer's operations, whereas the personal levels are updated by the operations performed by the owner and customer separately. The overall upper and lower water levels should be equal to the sum of the personal levels of the lower and upper basins. The following variables are defined:

V_t^u	Owner's upper water level [m^3]
V_t^{Cu}	Customer's upper water level [m^3]
V_t^d	Owner's lower water level [m^3]
V_t^{Cd}	Customer's lower water level [m^3]

The overall basin levels are determined as shown in equations (2.1) and (2.2) To determine the total production pr_t and total pumping pm_t in hour t , the owner's production (or pumping) decisions were summed with those of the customer:

$$pr_t = v_t^+ + v_t^{C+} \quad (2.21)$$

$$pm_t = v_t^- + v_t^{C-} \quad (2.22)$$

The levels of personal basins are updated simply by the operations performed by the customer and the owner. Thus, customer levels are represented by the following constraints:

$$V_t^{Cu} = V_{t-1}^{Cu} + v_t^{C-} - v_t^{C+} \quad (2.23)$$

$$V_t^{Cd} = V_{t-1}^{Cd} - v_t^{C+} + v_t^{C-} \quad (2.24)$$

The owner's personal levels also consider the natural inflow and any potential spillover, as everything that happens physically to the basin is up to the owner. Owner's personal levels are defined by the following equations:

$$V_t^u = V_{t-1}^u + v_t^- - v_t^+ + I_t^u - L_t^u \quad (2.25)$$

$$V_t^d = V_{t-1}^d - v_t^- + v_t^+ + I_t^d - L_t^d \quad (2.26)$$

For all three models, it was necessary to divide the calculation of the basin update into two steps. First, the basin levels were calculated at hour 0, and then the calculation was performed for the hours to follow. This necessity is due to the fact that the water level prior to hour 0 coincides with the initial water level. It therefore turns out to be necessary to distinguish between the total initial level of the basins $V_0^{u,tot}$ and $V_0^{t,tot}$ and the allocation between owner and customer V_0^u and V_0^{Cu} , for the upper basin and V_0^d and V_0^{Cd} for the lower basin. Specifically in the upper basin, the initial water level is shared between owner and customer as follows:

$$V_0^u = (1 - \alpha)V_0^{u,tot}$$

$$V_0^{Cu} = \alpha V_0^{u,tot}$$

For the lower basin, it is necessary to respect the relation $\alpha V_{max}^u = V_0^{Cu} + V_0^{Cd}$, accordingly:

$$V_0^d = V_0^{d,tot} - (V_{max}^u - V_0^{u,tot})\alpha$$

$$V_0^{Cd} = (V_{max}^u - V_0^{u,tot})\alpha$$

It is now possible to examine the influence of the owner's generation and pumping choices in conjunction with customer orders and financial market speculation on the profits of the facility's owner.

Profit function

In the physical model there are no added sources of profit and cost, therefor profit part is still calculated as described by section 2.1.3. Given the specific characteristics of this type of market, the counterparty most compatible with a physical market model is a Transmission System Operator (TSO). In the case of a TSO counterparty, the objective of pumping water may not be to subsequently generate electricity for resale, but rather to absorb excess energy from the system. Therefore, even in situations where the counterparty's operational decisions run counter to the profit-maximization goals of the asset owner, a TSO customer is guaranteed priority access to the use of the turbine.

2.2.2 Financial model

In the financial model, as the name suggests, all of the customer's operational decisions are compensated solely through financial transactions. This model can be described as two distinct facilities: the physical facility,

to which the owner has unrestricted access, and a virtual entirely managed by the customer. Customer production and pumping orders can now be summarized for each hour t by the variable x_t^{stockC} , expressed in MWh. This variable can be positive when the customer requests energy production, or negative when the customer decides to pump water. Accordingly x_t^{stockC} is calculated as follow:

$$v_t^{C+} \cdot k^+ - v_t^{C-} \cdot k^- - x_t^{stockC} = 0 \quad (2.27)$$

Subsequently, this positions are compensated through financial transactions in the profit function.

Production and pumping decisions

As said in the previous paragraph, customer orders are the result of a hypothetical facility, and are never physically executed. This explains why, in contrast to the physical contract, in the model generated by the financial contract, the owner's optimal production and pumping decisions are limited only by the maximum capacity of the turbines and not by customer orders. The owner's decisions to pump or produce are then only bounded by physical capacity of the turbine:

$$0 \leq v_t^+ \leq X_{max}^+ \quad (2.28)$$

$$0 \leq v_t^- \leq X_{max}^- \quad (2.29)$$

As for the previous model, constraint [2.20] was applied.

Basin balance

In the case of the financial model, the capacity, and consequently the total physical level of the reservoirs, is no longer physically divided between the owner and the customer. The customer draws on a fictitious plant, theoretically separate from the real one under the owner's sphere. As a result, the basins of the owner and customer, respectively, are updated as separate entities, depending on the operations carried out by the two agents. It is no longer necessary to calculate the total volume of the basins, since they correspond to the volumes resulting from the operations performed by the owner. The owner and customer basin levels are simply calculated as shown by constraints from (2.3) to (2.26).

Profit function

For the financial model, the profit function defined by [2.6] is modified slightly. The profit (cost) obtained by financial compensation is considered through the variable x_t^{stockC} multiplied by the spot price in each hour t . The profit function is modified as follows:

$$\begin{aligned} \pi = 30 \cdot \sum_{m=1}^{12} e^{-r_2 m} \sum_{k=1}^K \left(\sum_{i=1}^n (A_{i,K \cdot m+k} \cdot F_i \cdot x_i^F) + S_{K \cdot m+k} \cdot x_{K \cdot m+k}^{spot} \right. \\ \left. - S_{K \cdot m+k} \cdot x_{K \cdot m+k}^{stockC} \right. \\ \left. - c^+ \cdot v_{K \cdot m+k}^+ \cdot k^+ - c^- \cdot v_{K \cdot m+k}^- \cdot k^- \right) \end{aligned} \quad (2.30)$$

Thus, when the customer asks to produce energy ($x_t^{stockC} > 0$), the owner faces a cost because it has to compensate financially for the energy demand. When the customer is required to pump water ($x_t^{stockC} < 0$), the owner obtains revenue because, in this case, the customer pays the owner for the stored potential energy, even if fictitious. The counterpart targeted by this type of model is a producer operating under a Power Purchase Agreement (PPA), through which it acquires virtual rather than physical storage capacity. With such a customer, it is indeed possible to financially settle both pumping and generation orders, allowing for flexible participation in the market without the need for physical ownership of the storage asset. In the next section the last type of potential contract is analyzed.

2.2.3 Hybrid model

For the hybrid model, the problem becomes in a sense asymmetrical, since two different priorities to the producing and the pumping orders of the customer are distinguished. The first case is simple to address. If the customer wants to produce energy, they have priority. When the customer requests energy production, it means they need that specific amount of energy at that moment. Therefore, the owner cannot pump the water back and must allow the customer to use the turbine. To address this, the method presented for the physical contract was used. In this case only the binary variable γ^- was generated as seen for the physical contract. Constraint (2.14) was then applied.

The second case is more involved: when the counterpart demands to pump water, the underlying assumption made here is that he simply wants to have a certain amount of water available for future. To achieve this result, pumping water from the lower to the upper basin is not the only necessary way. On the contrary there are several other ways that the owner can adopt. The following alternative solutions, in order of priority, are available to the owner to satisfy the pumping order of the customer:

1. Excess water within the decision-maker's ownership
2. Physical upper inflow (i.e., I_t^u)
3. Owner production decision (i.e., v_t^+)

The first resource that appears logically targetable is the potential excess water available to the owner in specific scenarios and hours. This quantity is calculated as the difference between the water allocated to the owner in the upper basin during the previous hour (V_{t-1}^u) and the water that remains the owner's property after part of the storage capacity has been sold ($(1 - \alpha)V_{max}^u$). If this difference is positive, it indicates that the owner has an excess of water, represented by $c_t^{u,max}$:

$$c_t^{u,max} = \max(V_{t-1}^u - (1 - \alpha)V_{max}^u; 0) \quad (2.31)$$

This excess water originates from the inflow accumulated in previous hours, as all physical changes to the reservoir fall under the owner's responsibility. The amount of the customer's pumping order that can be

covered by this excess is represented by c_t^u :

$$c_t^u = \min(c_t^{u,max}; v_t^{C-}) \quad (2.32)$$

If c_t^u is equal to the customer's order v_t^{C-} it means that the excess water $c_t^{u,max}$ was sufficient to meet the counterpart request. If this is not the case, it is possible to draw from the inflow of the upper basin, arrived in hour t . To calculate the amount of natural inflow required to compensate for any remaining water, the remaining pumping order $v_t^{C-} - c_t^u$ that needs to be fulfilled, is compared with the natural upper inflow I_t^u , and the smaller value as the required amount of natural inflow, I_t^{c1} is selected.

$$I_t^{c1} = \min(v_t^{C-} - c_t^u; I_t^u) \quad (2.33)$$

The amount of natural inflow that remains under the owner's ownership after this compensation is defined as I_t^c :

$$I_t^c = I_t^u - I_t^{c1} \quad (2.34)$$

If (2.34) results equal to zero the upper inflow does not offset the counterpart's order. In this situation, it is necessary to determine the remaining amount of water required to fulfill the customer's pumping order, v_t^{C1-} which is given by the following equation:

$$v_t^{C1-} = \max(v_t^{C-} - c_t^u - I_t^c; 0) \quad (2.35)$$

If the result is positive, it indicates that part of the order still needs to be fulfilled. In this case, the owner can satisfy the remaining portion of the order by transferring water to the upper basin. To achieve this, the owner can adjust part of their production decision v_t^+ . The water now physically used for energy production is denoted as $\hat{v}_t^+$, and it is calculated as follows:

$$\hat{v}_t^+ = \max(v_t^+ - v_t^{C1-}; 0). \quad (2.36)$$

After the order of pumping has been compensated using all three possible resources, it is possible to calculate the amount of water remaining in the order that is physically pumped, $\hat{v}_t^{C-}$:

$$\hat{v}_t^{C-} = \max(v_t^{C1-} - \hat{v}_t^+; 0), \quad (2.37)$$

Table 2.1 summarizes an example of a customer pumping order compensation.

	Variable	Description	Value
Starting pumping Value	V_t^c	Initial customer pumping order	40
Excess of water	$c_t^{u,max} = \max(V_{t-1}^u - (1 - \alpha)V_{max}^u; 0)$	Excess water within the decision-maker's ownership	30
1st compensation	$c_t^u = \min(c_t^{u,max}; v_t^{C-})$	Amount of pumping order covered by the excess of water	30
Inflow	I_t^u	Natural upper Inflow	15
2nd compensation	$I_t^{c1} = \min(v_t^{C-} - c_t^u; I_t^u) = \min(70; 15)$	Amount of pumping order covered by the natural Inflow	15
Remaining order	$v_t^{C1-} = \max(v_t^{C-} - c_t^u - I_t^u; 0)$	Remaining order after the second compensation	55
Production decision	V_t^+	Optimal production decision	30
3rd compensation	$\hat{v}_t^+ = \max(v_t^+ - v_t^{C1-}; 0)$	Final pumping value	40

Table 2.1: Pumping orders compensation process

The introduction of constraints outlined in this paragraph has enabled us to delineate the counterpart's operative decisions and their impact on the owner's own production and pumping decisions. In the subsequent paragraphs, it will be demonstrate how the owner's optimal operative decisions are not only influenced by the presence of a counterpart, but also by the technical attributes of the facility.

Production and pumping decisions

As for the Physical model, the owner's optimal production and pumping decisions are constrained by both the turbine and pump capacity and what the customer is physically doing in that specific hour t . The constraints that delineate these limitations are analogous to those presented for the physical contract (2.16–2.20), with the exception that, with the hybrid contract, there is a distinction between the optimal owner's production decision (v_t^+) and the amount of water that is physically able to use to produce energy ($\hat{v}_t^+$). The distinction between the customer's pumping order (v_t^{C-}) and what is actually pumped for him ($\hat{v}_t^{C-}$) is also considered. The constraints previously presented are then modified as follow:

$$X_t^{max+} = X_{max}^+ - v_t^{C+} \cdot k^+ \quad (2.38)$$

$$X_t^{max-} = X_{max}^- - \hat{v}_t^{C-} \cdot k^- \quad (2.39)$$

$$0 \leq \hat{v}_t^+ \cdot k^+ \leq X_t^{max+} \quad (2.40)$$

$$0 \leq v_t^- \cdot k^- \leq X_t^{max-} \quad (2.41)$$

The owner's physical production is constrained by the turbine's maximum capacity and the customer's production order. Instead, the pumping decision is constrained by the amount physically pumped for the

customer and the pump's maximum capacity.

Constraint (2.20) is subsequently modified by compensating in this way:

$$\hat{v}_t^+ \cdot k^+ - v_t^- \cdot k^- - \sum_{i=1}^n A_{it} x_i^F - x_t^{spot} = 0 \quad (2.42)$$

The energy sold in the spot market is now $\hat{v}_t^+ \cdot k^+$, so the energy physically produced, and not the optimal production decision v_t^+ . In the following paragraph, the modifications in the basin balance resulting from customers' production and pumping requests, as well as the choices made by owners, are illustrated. The analysis considered both the total water level of the basin and the individual levels of the owner and the customer, whose combined levels equal the total.

Basin balance

To determine the basin balance, a distinction was made between the overall basin upper and lower levels, distinguishing between the levels pertaining to the owner and the customer. The overall basin balance is updated through physical transactions, such as $\hat{v}_t^+$ and $\hat{v}_t^{C-}$, whereas the personal levels are updated through accounting transactions, such as v_t^+ and v_t^{C-} . The overall upper and lower water levels should be equal to the sum of the personal levels of the lower and upper basins. The overall basin balance is updated through physical movements. To determine the total physical production, the owner's physical production and production order are summed. Similarly, the total physical pumping was determined by summing the pumping decision and the quantity physically pumped by the customer:

$$pr_t = \hat{v}_t^+ + v_t^{C+} \quad (2.43)$$

$$pm_t = v_t^- + \hat{v}_t^{C-} \quad (2.44)$$

.

The overall basin balance was then determined in the same way as presented for the physical contract. Thus, the constraints from (2.1) to (2.4) also apply here.

Personal levels are updated through considering accounting transactions, next to physical movements. The customer's personal level is determined as follows:

$$V_t^{Cu} = V_{t-1}^{Cu} + v_t^{C-} - v_t^{C+} \quad (2.45)$$

$$V_t^{Cd} = V_{t-1}^{Cd} - v_t^{C+} + v_t^{C-} \quad (2.46)$$

The owner's personal levels are defined by the following equations:

$$V_t^u = V_{t-1}^u + v_t^- - v_t^+ - c_t^u + I_t^c - L_t^u \quad (2.47)$$

$$V_t^d = V_{t-1}^d - v_t^- + v_t^+ + c_t^u + I_t^d + I_t^{c1} - L_t^d \quad (2.48)$$

The owner's personal levels account for natural inflow and potential spillovers, as the owner controls all basin activities. If the owner compensates for the counterpart's pumping order using excess water and natural inflow I_t^u , this is reflected in the personal level update. In the upper level, the owner transfers c_t^u to the lower basin. The owner only accesses a portion of the natural inflow, I_t^c , while the rest, needed to offset I_t^{c1} , is allocated to the lower basin.

It is now possible to examine the influence of the owner's generation and pumping choices in conjunction with customer orders and financial market speculation on the profits of the facility's owner.

Profit function

The profit function defined previously has been modified as follows:

$$\begin{aligned} \pi = 30 \cdot \sum_{m=1}^{12} e^{-r_2 m} \sum_{k=1}^K \left(\sum_{i=1}^n A_{i,K \cdot m+k} \cdot F_i \cdot x_i^F + S_{K \cdot m+k} \cdot x_{K \cdot m+k}^{spot} \right. \\ \left. + S_{K \cdot m+k} \cdot (v_{K \cdot m+k}^{C-} - \hat{v}_{K \cdot m+k}^{C-}) \cdot k^- \right. \\ \left. - c^+ \cdot \hat{v}_{K \cdot m+k}^+ \cdot k^+ \right. \\ \left. - c^- \cdot v_{K \cdot m+k}^- \cdot k^- \right) \end{aligned} \quad (2.49)$$

It emerges that a significant profit opportunity arises from fulfilling customer orders through accounting, rather than physically. This profit is defined as the difference between the actual pumping order and the physical pumping for the customer multiplied by the spot prices for each hour t . Thus, this additional profit is quantified in the profit function as:

$$S_{K \cdot m+k} \cdot (v_{K \cdot m+k}^{C-} - \hat{v}_{K \cdot m+k}^{C-}) \cdot k^-$$

This opportunity, as will be seen by analyzing the results, provides a real option to the owner to exercise each time he receives a pumping order from the customer, and to reflect it in the profit distribution of the owner. As in the case of the financial market, the most suitable counterpart for the hybrid model is a producer operating under a Power Purchase Agreement (PPA).

2.3 Application

The study evaluated a 24-hour scenario, with data for this period selected from a database of previously collected historical data. Specifically, the randomly selected date was May 22, 2023. To test the models, a

realistic case of a plant located in Italy was considered. More specifically, the facility is located in Southern Italy, in the Campania region. The collected data comprises the spot prices and the natural inflow observed in the designated area during that particular day. The spot prices¹ were collected from the GME (Gestore del Mercato Elettrico) dataset, while the natural inflow data² were collected from Copernicus data store, more specifically from the ERA 5 dataset. Two different futures contract were considered: Future 1 with a price equal 90 €/MWh, and Future 2 with a price equal to 120€/MWh. Figure 2.2 illustrates the values of futures contracts 1 and 2 during their active periods within the 24-hour time frame.

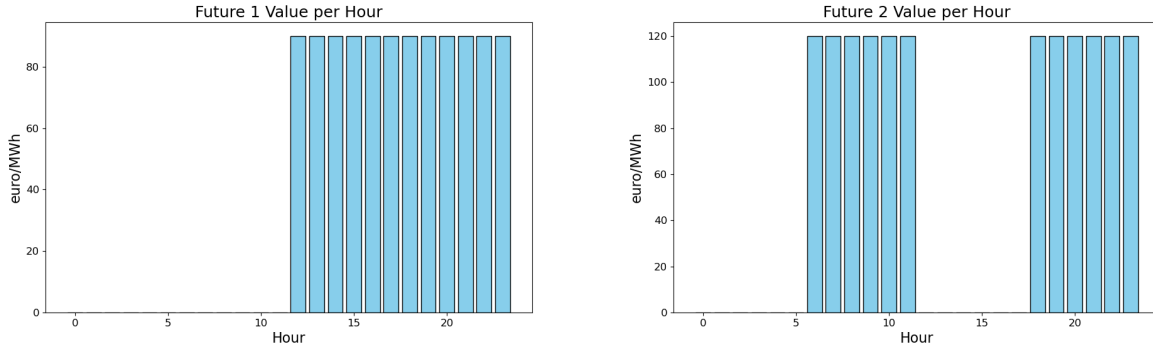


Figure 2.2: Value of the future contracts within 24 hours

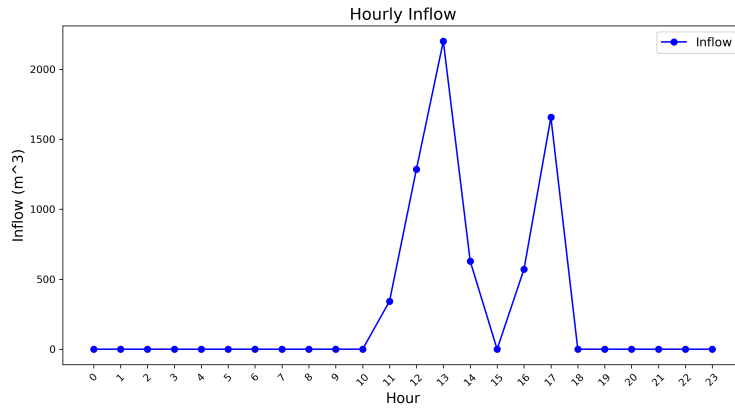


Figure 2.3: Natural inflow per hour

Figure 2.3 shows the rainfall that occurred during the 24 hours, in the area where the hydro pumping storage facility is located. It can be seen that precipitation of average flow rate occurred in the middle hours of the day. The technical parameters considered are listed in Table 2.2.

¹Source of data: Gestore del Mercato Elettrico (GME), <https://www.mercatoelettrico.org/It/download/DatiStorici.aspx>

²Source of data: Copernicus Data Store ERA5 dataset, <https://cds.climate.copernicus.eu/datasets/reanalysis-era5-single-levels?tab=download>

	Definition	Value
d (h)	Facility Duration	15
d_s (h)	Sold Duration	8
X_{max}^u (MWh)	Maximum storage capacity	9,000
V_{max}^u (m^3)	Maximum capacity of the upper basin	7,000,000
V_{max}^d (m^3)	Maximum capacity of the lower basin	125,000,000
V_0^u (m^3)	Initial water level in the upper basin	3,500,000
V_0^d (m^3)	Initial water level in the lower basin	100,000,000
X_{max}^+ (MW)	Maximum capacity of the turbine	572
X_{max}^- (MW)	Maximum capacity of the pump	628
η_T	efficiency of the turbine	92.6%
η_P	efficiency of the pump	79.5%
k^+	Production conversion coefficient	0.001261023
k^-	Pumping conversion coefficient	0.00171024
c^+ (€)	Production operative cost	1
c^- (€)	Pumping operative cost	1

Table 2.2: Technical parameters

The three models were two different type of potential customer:

- A smart customer
- A Partially predictable customer

The first is a type of counterpart that operates strategically, requesting energy production when prices are high to maximize profitability and water pumping when prices are low to minimize costs. The latter represents a category of consumer who alternates between making optimal and suboptimal decisions. Figure 2.4 shows the potential behavior of a smart customer, which generally decides to produce energy at relatively high prices and pump water at relatively low prices. Figure 2.5 shows partially predictable customer production and pumping orders confronted with spot prices. As it can be seen, the customer sometimes expresses pumping and production orders that are more intelligent, while at other times they are, admittedly, more naive. For example, in hour 18 the customer expresses a production order when the spot price is quite high (about 120 €/MWh), and asks to pump water in hour 1, with a price that, for the period under consideration, is low (about 90 €/MWh). In hour 19, however, it expresses a pumping order, with a price of about 130 €/MWh, and requests to produce power in hour 22, for a price of about 110 €/MWh.

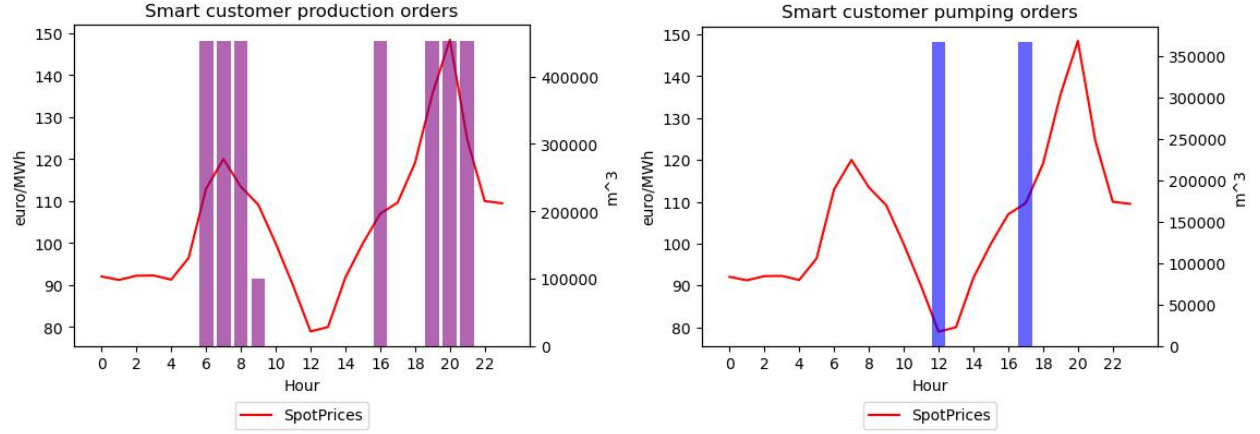


Figure 2.4: Smart customer production and pumping orders confronted with the spot prices

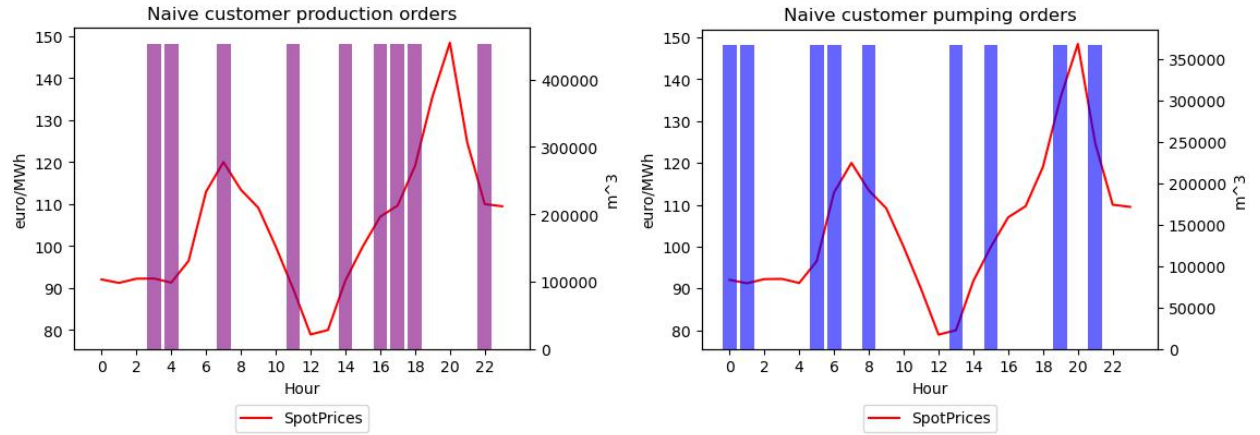


Figure 2.5: Partially predictable customer production and pumping orders confronted with the spot prices

The models were initially evaluated in a deterministic scenario spanning a limited number of hours, first without considering the customer's influence by setting α to zero, and then incorporating it by allocating half of the capacity ($\alpha = 0.5$) and the full capacity ($\alpha = 1$). This approach allows for an hour-by-hour analysis of the customer's impact, or lack thereof, on the owner's economic strategy. The models were solved using Gurobi as the optimizer and Python as the programming language. The tests were conducted on a computer equipped with 16GB of RAM. The following paragraph will present the obtained results.

2.4 Analysis of results

Initially, the models were evaluated by setting the value of α to zero, in order to analyze the optimal management strategy of the owner, without the influence of the customer. Figure 2.6 presents the owner's

optimal management decisions. The positive section of the graph illustrates the pumping decision, while the negative part depicts the production decision. The orange line represents the spot prices over the 24-hour period, while the dashed lines indicate the maximum capacity of the pump and the turbine expressed in m^3 . The optimal solution indicates that it was advantageous to generate energy during periods of positive peak prices, specifically in the morning and evening. With the exception of hour 17, which exhibits the lowest price during peak hours, the owner consistently produced energy at maximum turbine capacity. Despite the presence of several low-price hours throughout the 24-hour period, it is optimal to pump at maximum capacity only during hour 12. Given that the upper basin begins at half capacity (containing 3,500,000 cubic meters of water), it is not necessary to pump water from the lower basin on multiple occasions to ensure sufficient water availability for power generation during high-price hours.

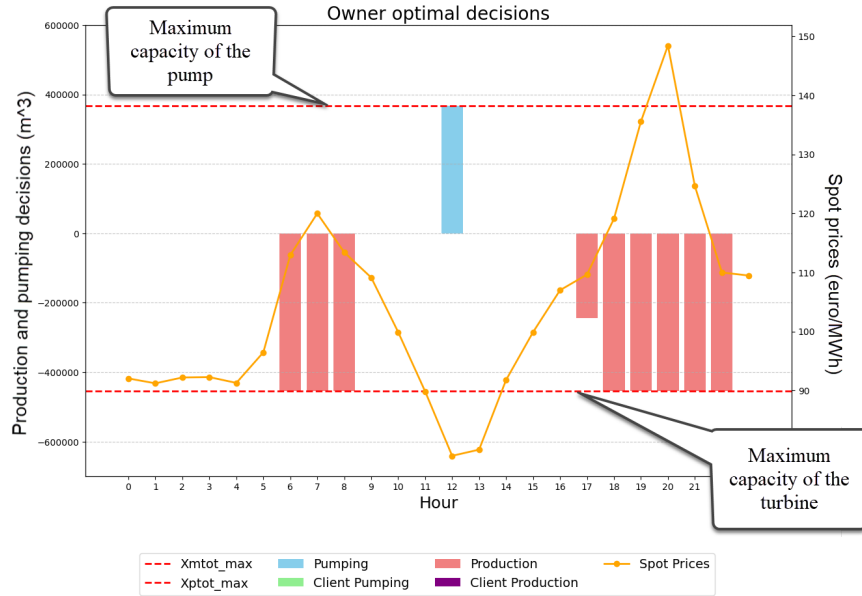


Figure 2.6: Owner's optimal decisions without considering the customer

Figure 2.7 shows the 24-hour basin update, for both the upper and the lower basin, in relation with the spot prices. In the upper basin it can be seen that when the price increases, the level of the basin decreases: in fact with high prices it is optimal to produce energy, consequently water is released from the upper basin to the lower basin. It can also be seen that it is optimal to empty the upper basin by the end of the time horizon. In the lower basin, the opposite happens. When prices rise the level of the basin rises, because water comes in from the upper basin, while when prices fall the level falls, since it is convenient to pump water from the lower basin.

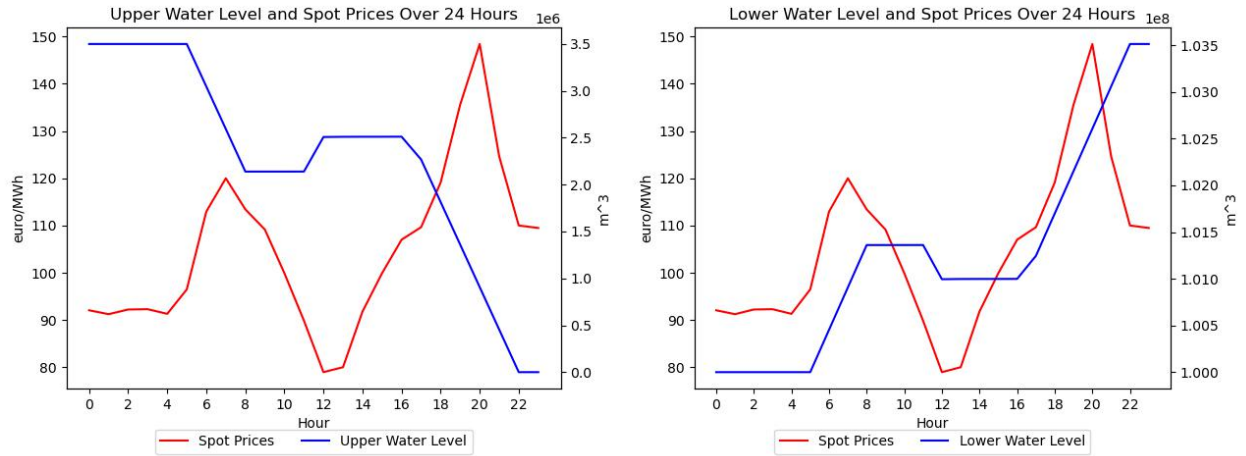


Figure 2.7: 24-hour basin update

As shown in Figure 2.8, the position adopted in the spot market is negative in most case. This means that, even though energy was produced and sold on the market during many hours, it was still necessary to purchase additional energy from the spot market to meet demand.

This can be better understood by examining the relationship between the futures position and the spot position. When a futures contract is active, it is optimal to adopt a positive position, which corresponds to selling energy during that hour. However, the facility's maximum production capacity ($X_{max}^+ = 572$ MW) is often insufficient to fully meet the contract's requirements. As a result, additional energy must be purchased from the spot market to fulfill the contractual obligations.



Figure 2.8: Position adopted in spot and futures market

For example, during hours 6, 7, and 8, it was necessary to purchase less energy from the spot market compared to hours 9, 10, and 11. This is because, during those earlier hours, the facility operated at maximum turbine capacity, producing as much energy as possible to meet demand. The optimal future position x_i^F was determined to be 0 for Future 1 and 8,333 MWh for Future 2. When Future 2 is active, the owner is required to purchase energy from the market. However, given that the Future 2 price is 120€, it remains economically advantageous to sell energy through Future 2 and acquire it in the spot market. This analysis demonstrates that the optimization of the facility is relatively marginal when compared to the overall result, as the majority of costs and revenues are derived from financial operations.

The optimized profit of the owner results to be 937,689 € as detailed in Table 2.4.

Profit π	937,689
Loss incurred in the spot market	-11,056,797
Profit obtained through future contracts	12,000,000
Operative production costs	4,885
Operative pumping costs	628

Table 2.3: Profits obtained by the owner

In the next section, the impact of selling half of the storage capacity to a counterpart, by fixing a value of α at 0.5, will be analyzed.

2.4.1 Impact of selling half of the capacity to a counterpart

Smart customer results

The impact of selling half of the storage capacity to a counterpart was examined for all three models, considering first the impact of a smart customer and subsequently that of a less informed partially predictable customer. An alpha value of 0.5 was selected to study the impact of capacity sharing with the customer, without allocating an excessively small or excessively large portion to them. Figure 2.9 illustrates the impact of a smart customer in a physical, financial and hybrid framework. Considering a smart customer, the most relevant hour to analyze is hour 17, as it features a spot price that is neither excessively high nor excessively low. During this hour, the customer requested water pumping, whereas the owner finds it more advantageous to produce energy.

Physical model The first graph in Figure 2.9 presents the results obtained for the Physical model. When the owner's and customer's decisions align, as in hours 12 and 8, the two agents equally share the capacity of the pump and the turbine. At hour 17, the price is neither excessively high nor excessively low. During this particular hour, the customer requested water pumping, while the owner identified energy production as the optimal choice. In this case, the owner deferred to the customer's request and allocated the turbine to them.

Financial model Considering the financial model, when the decisions of the customer and the owner coincide,

both orders can be executed. In particular, the owner can take full advantage of the maximum capacity of the pump and turbine, as shown by Figure 2.9. In hour 17, when the two orders are in opposition these can both be executed, since in the financial model the orders are only financially compensated, and do not have to be physically executed.

Hybrid model When analyzing the hybrid model, several noteworthy aspects come to light. As shown in Figure 2.9, at hour 12, both the owner and the customer engage in water pumping, sharing the pump's capacity. Although the owner could have offset the customer's pumping request by transferring water from the upper basin, this option was not economically viable due to the extremely low price. The low price rendered it uneconomical to fully utilize the turbine's capacity through such an offset, as the potential gain would have been outweighed by the operational costs of maximizing pump usage. Instead, the optimal strategy for the owner was to delay pumping until the following hour, ensuring sufficient reserves to generate energy during peak demand hours. Figure 2.9 highlights the facility's physical operations. At hour 17, no physical operations were performed. However, as illustrated in Figure 2.10, a change in the ownership of water in the upper basin occurred during this hour. This exemplifies the concept of a real option: the owner can choose to fulfill a customer's pumping request without initiating physical operations. This is further clarified in Figure 2.11, which compares the customer's accounting pumping orders with the owner's physical offsets. At hour 17, a pumping order was placed by the customer but was offset by a water release in the upper basin, as determined by the owner's optimal production strategy.



Figure 2.9: Owner's optimal decisions in physical, financial, and hybrid settings considering a smart customer.

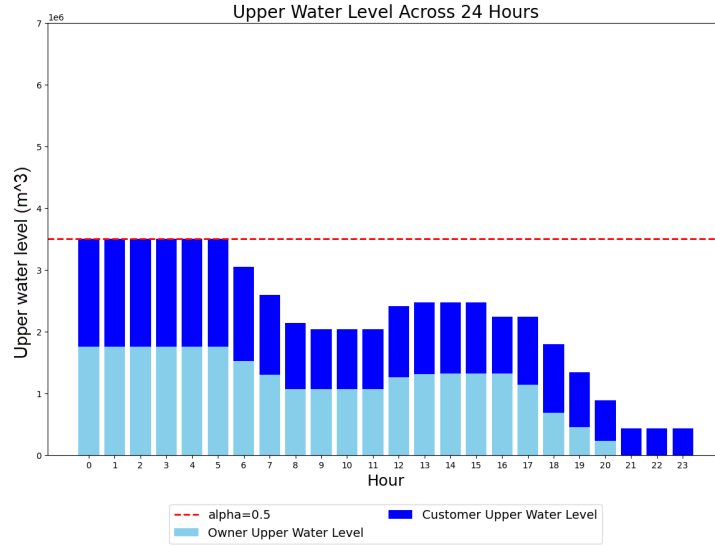


Figure 2.10: 24-hour smart customer and owner upper basin update in a hybrid settings

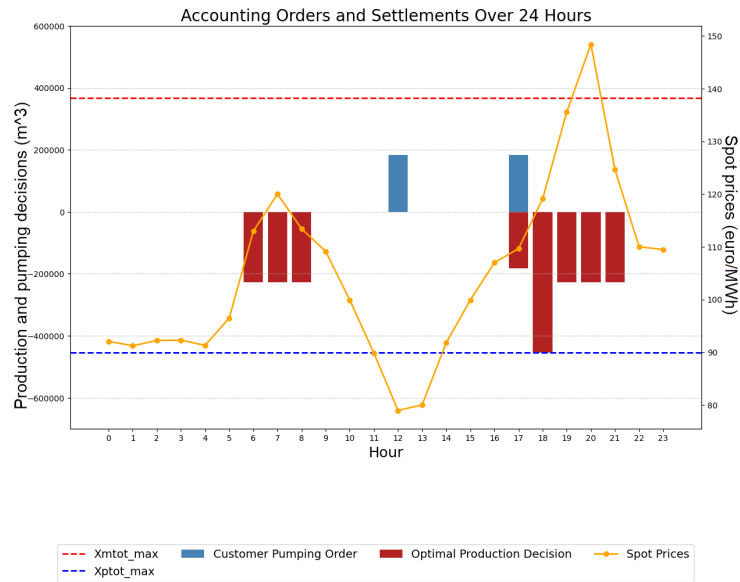


Figure 2.11: Accounting pumping orders and physical settings over 24 hours

The allocation of half of the storage capacity to a smart costumer had a significant impact on the owner's ultimate profit. Table 2.4 summarizes the profits and costs incurred by the owner, taking into account the presence of a smart costumer.

	No customer	Physical model	Financial model	Hybrid model
Obj (€)	937,689	669,956	738,116	679,080
Profit/loss obtained in the spot market (€)	-11,056,797	-11,327,282	-11,056,797	-11,352,815
Profit obtained through future contracts (€)	12,000,000	12,000,000	12,000,000	12,000,000
Operative production costs (€)	4,885	2,446	4,885	2,288
Operative pumping costs (€)	628	314	628	414
Pumping profits (€)	0	0	58,594	34,604

Table 2.4: Owner’s profits and costs considering a smart customer

Considering selling half of the storage capacity to a smart customer, the owner’s ultimate profit decreases significantly. While this scenario introduces new complexities, the impact on profit varies depending on the management model employed.

Financial model

In the financial model, the owner acts solely as a financial counterpart to the customer. This means that while the operating costs for production and pumping remain unchanged from the optimized model without a customer, the financial relationship with the customer alters the final profit. The key factor here is the financial compensation tied to the customer’s production and pumping orders. By offsetting two specific pumping orders executed by the customer, the owner manages to mitigate losses and achieve a profit of 58,594€. This highlights how the financial model relies on offset strategies to recover some of the profit lost due to transferring storage capacity

Hybrid model

The hybrid model demonstrates the second-highest profit among the scenarios. In this case, the management strategy changes significantly to adapt to the shared use of the infrastructure. Operating costs for both production and pumping decrease, indicating improved efficiency in certain areas. However, this efficiency comes at a cost—losses in the spot market increase. A notable factor is the reduced energy production, as the owner now has to share the turbine capacity with the customer. This reduction in output leads to less energy being sold on the market, forcing the owner to purchase additional energy to meet futures contract obligations. Despite these challenges, the owner still secures a profit of 36,604€, largely thanks to the strategic offsetting of a pumping order at hour 17.

Physical model

The physical model results in the largest decline in profit among the three scenarios. Unlike the financial model, where the customer’s presence is primarily a financial factor, the physical model places direct constraints on the owner’s operations. The customer’s orders determine what the owner can and cannot physically produce, which creates additional complexities in managing the plant. When the customer’s or-

ders align with the owner’s optimal decisions, the two parties must share the pump and turbine capacity. However, when their strategies conflict—as seen at hour 17—the owner is forced to relinquish the ability to produce, prioritizing the customer’s access to the turbine. This operational constraint leads to reduced production and increased reliance on the spot market, compounding the overall loss.

Partially predictable customer results

With a smart customer, only hour 17 represents a potential source of profit for the owner. Upon testing the three models considering a partially predictable customer, the results exhibited slight differences.

Physical model

As illustrated in Figure 2.12, within a physical setting, the naive pumping decisions of the customer influenced the owner’s production strategy during several hours, such as hour 19. Notably, the owner’s optimal pumping decision in hour 12 was significantly reduced compared to the optimal strategy without the customer’s involvement. This reduction stems from the physical constraints on the owner’s production decisions: pumping additional water is neither optimal nor necessary, as the excess water cannot be effectively utilized for energy production.

Financial model

With the financial model, as shown in the second graph of Figure 2.12, several opportunities for profit are identified. During hours 6, 8, 19, and 21, the customer requests water pumping at a higher price. In a financial context, the owner not only continues to produce energy at the turbine’s maximum capacity but also receives compensation from the customer to maintain a water reserve for future use.

Hybrid model

In a hybrid context, the more naive pumping decisions not only create potential profit opportunities but also impact the owner’s production strategy. As shown in Figure 2.12, the owner produces minimal energy while pumping more water compared to the optimal strategy without the customer. This approach reflects the owner’s effort to balance a consistent number of pumping requests by reallocating water in the upper basin, as depicted in Figure 2.14. As a result, the owner must increase water pumping during periods of lower prices, ensuring maximum turbine capacity utilization during peak spot prices in hours 19 and 20. This compensation is evident in the customer’s share of the upper basin. For example, during hours 5 and 6, the owner’s water level decreases while the customer’s level increases, even though no physical water transfer occurs.

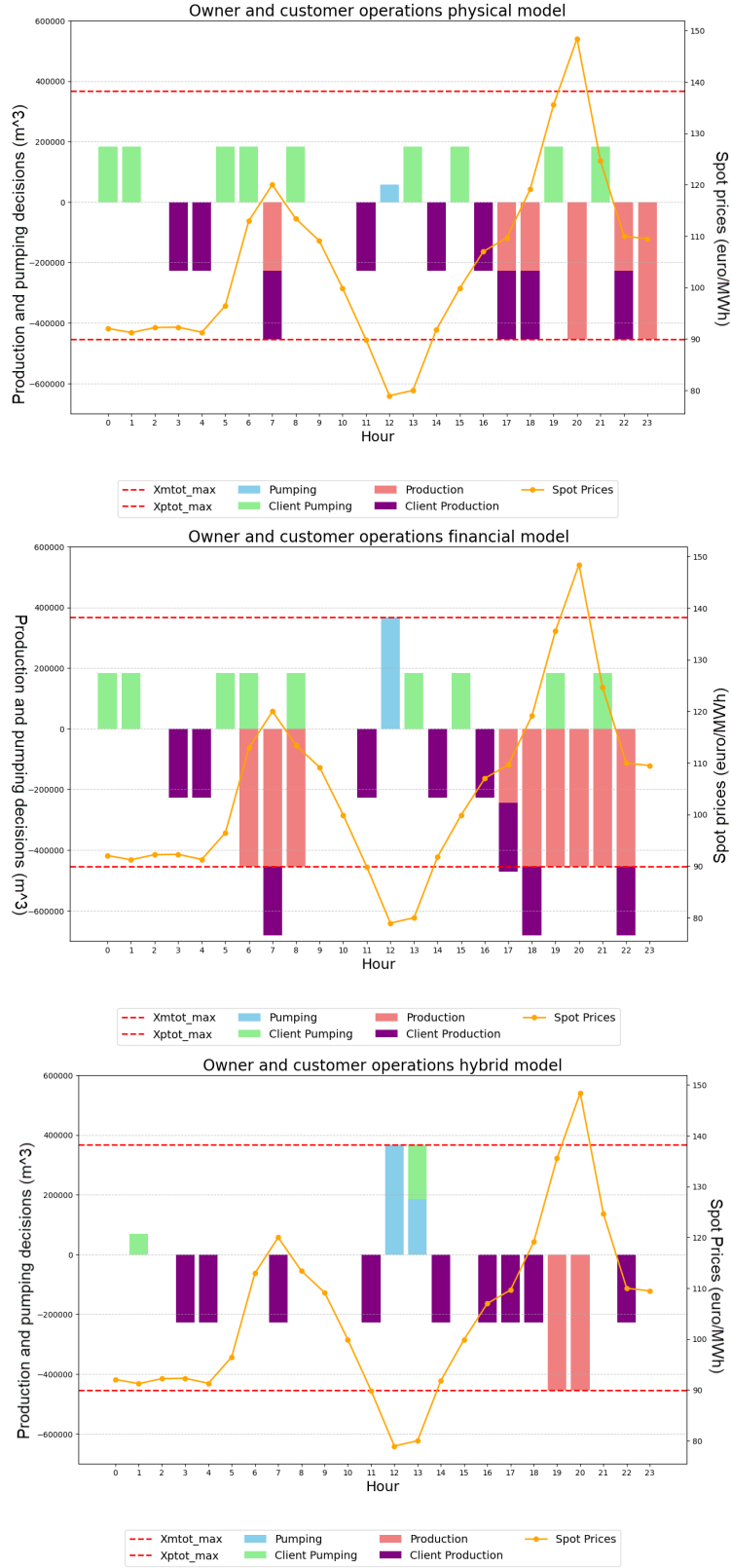


Figure 2.12: Owner's optimal decisions in physical, financial, and hybrid settings considering a Partially predictable customer.

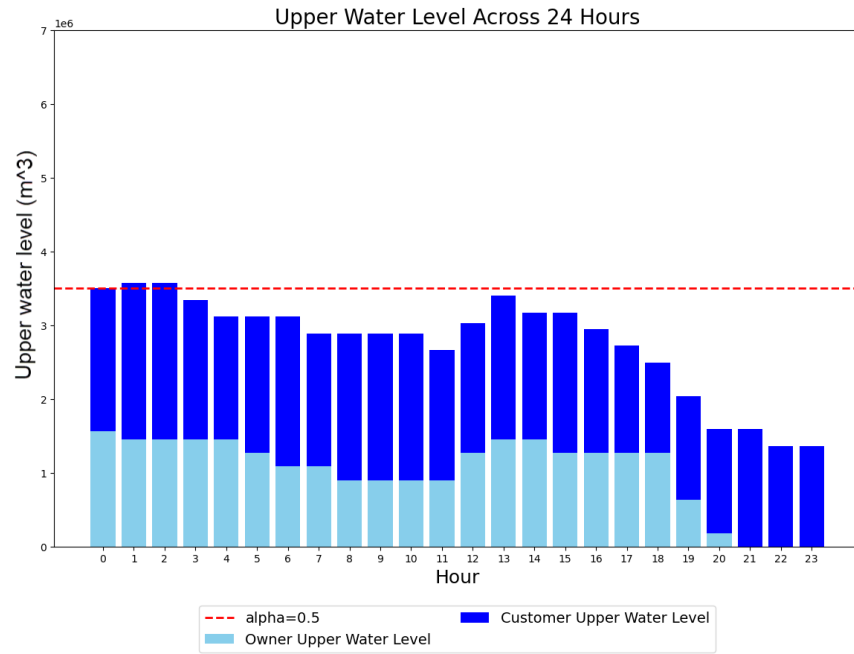


Figure 2.13: 24-hour Partially predictable customer and owner upper basin update in a hybrid setting

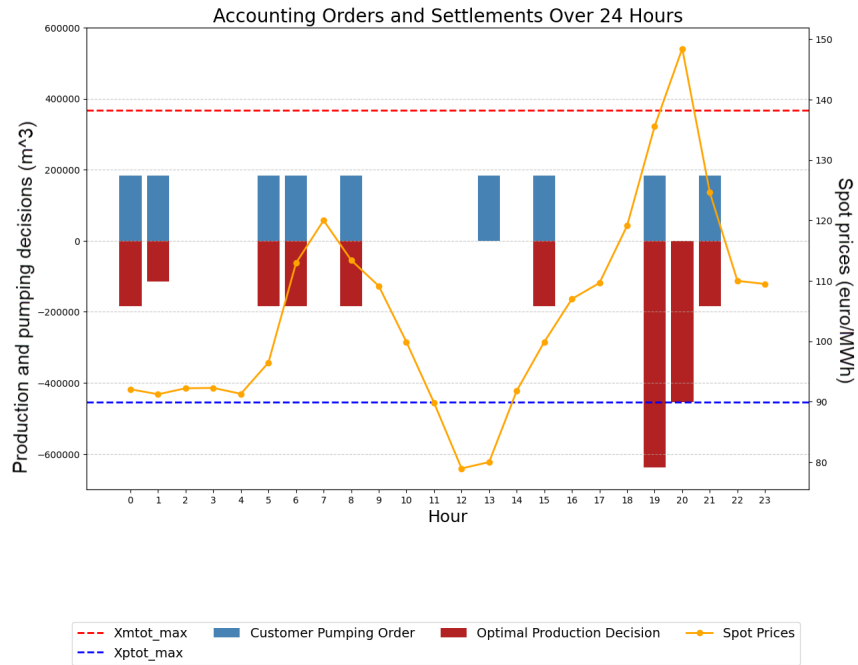


Figure 2.14: Accounting pumping orders and physical settings over 24 hours

The introduction of a partially predictable customer, rather than a smart one, had a substantially more

significant impact on the owner’s final profit, as demonstrated in Table 2.5.

	No customer	Physical model	Financial model	Hybrid model
Obj (€)	937,689	664,535	968,288	742,956
Profit/loss obtained in the spot market (€)	-11,056,797	-11,333,078	-11,056,797	-11,516,560
Profit obtained through future contracts (€)	12,000,000	12,000,000	12,000,000	12,000,000
Operative production costs (€)	4,885	2,288	4,885	1144
Operative pumping costs (€)	628	99	628	946
Pumping profits (€)	0	0	263,689	261,605

Table 2.5: Owner’s profits and cost considering a Partially predictable customer

The financial model turns out, again, to have a higher final profit, exceeding even the profit obtained without the customer. It is particularly noticeable that, compared to the previous case with a smart customer, the profit from offsetting pump orders increased, reaching a value of 263,689€. This is due to the larger amount of pumping orders compared to the smart customer. The hybrid model also has higher profits from offsetting pumping orders than the previous case. This leads to a higher profit than selling half the capacity to a smart customer. It can be seen that the production operating costs turn out to be half that of the previous case, indicating that the actual energy production has halved. It then turned out to be necessary to pump more water in order to have sufficient water in the upper basin to transfer to the customer, and to produce energy during peak hours. This is reflected in a higher loss in the spot market. Again, the physical model turns out to have a lower ultimate profit. This is a predictable result since a partially predictable customer has a greater impact, compared to a smart customer, on what the owner can physically do. For example, in hours 6, 18, 19, and 21, the customer’s pumping orders prevented the owner from producing energy, negatively affecting the final profit.

2.5 Conclusion

A hydro-pumping storage facility’s operational decisions were optimized to maximize profit by jointly optimizing pumping water and producing energy, and considering that a customer is at the same time managing its storage quota in the same plant. The model was applied to an open-loop facility consisting of an upper and lower basin, both fed by natural inflow. After introducing the customer’s potential production and pumping orders, an important problem emerged: at certain times the customer’s orders went in opposition to the owner’s optimal decisions. This led to the identification of three different types of potential market setup, distinguishing, as a result, three models: physical, financial and hybrid. The three models were implemented considering a scenario of 24 hours and two types of potential customers: a smart customer and a partially predictable one. The technical specifications of the facility were obtained from an installation located in Italy. The models were initially evaluated without considering the influence of the customer by setting α to

zero, and subsequently by incorporating the customer's influence, allocating half of the capacity ($\alpha = 0.5$) and the full capacity ($\alpha = 1$). The selling price P was set equal to 0, in order to examine solely the impact of customer behavior on the three models. The results indicated that for both smart and partially predictable customers, the financial model proved most profitable, followed by the hybrid model. However, only in the case of a less informed customer did the financial model exceed the profit generated without considering customer influence. The physical model proved to be the most restrictive for the owner. A further significant finding was the substantial profit potential for the plant owner from compensating for the customer's pumping orders in both the financial and hybrid models. In the financial model, this was achieved through financial compensation, while in the hybrid model, it was accomplished by fulfilling the counterpart's pumping order through accounting rather than physical movement. This can be achieved by selling water, such as excess amounts or the physical inflow that feeds the upper basin, and the owner's own production. In certain hours, it was more profitable for the owner to cede their production capacity to the customer rather than use it to produce energy and sell it in the spot market.

Chapter 3

Jointly optimal capacity sale decision and management of a Hydro Pumping Storage facility

In the sector of renewable energy production, uncertainty is a structural component. Renewable energy sources such as solar and wind are inherently variable and challenging to predict accurately. The prices and costs in the energy markets exhibit extreme volatility, as they are influenced by external factors such as demand, energy policies, and climatic conditions. Energy storage systems are crucial for managing this variability; however, optimal management of these technologies necessitates models capable of considering both expected profits and associated risks. In Chapter 2, the models were developed considering a deterministic framework. This type of framework is useful in contexts where uncertainties are limited or for preliminary analyses. However, a deterministic optimization may lead to suboptimal too risky decisions when real-world conditions differ from the assumptions. The models described and implemented in the previous chapter have been reformulated stochastically. A stochastic model allows, in fact, to face the complexity of energy markets in a realistic way, providing more realistic solutions than a deterministic model. To do that, several scenarios of spot energy prices and inflow of water into the basins were considered. These scenarios were generated from data collected over 10 years, and are denoted in the models as j . The calculation of Conditional Value at Risk ($CVaR$) is introduced in the objective function. In this way, the models not only maximize profits but also take into account the risks associated with worst-case scenarios. In the stochastic model, optimization balances profitability and risk, with a more conservative and realistic approach. The models are now formulated as two-stage nonlinear optimization problem. The amount of storage capacity sold, α , is now considered a decision variable, determined at in the first stage of the optimization process. The models were tested considering 10 scenarios of 8760 hours each. The switch to the stochastic model involves an increase in computational complexity and consequently in computation time. In order to be able to solve the optimiza-

tion models, it was therefore necessary to apply two heuristics. A sensitivity analysis was then performed on all three models, considering different values of turbine capacity. The chapter is structured as follows: Section 3.1 describes the stochastically reformulated models. Section 3.2 delineates the characteristics of the models' application, while Section 3.3 analyzes the obtained results. The results of the sensitivity analysis are presented in Section 3.4. Finally, Section 3.5 presents the conclusions of the chapter.

3.1 An Hydro Pumping Storage model under uncertainty

The model developed in a deterministic framework in the previous chapter has been adapted to a stochastic framework. The generic characteristics of the model remained consistent with the deterministic case. The model incorporates two basins where energy is generated through water release and stored via pumping. A single turbine facilitates both energy production and water pumping, precluding simultaneous operation. Natural inflows can augment the basins, which overflow when at capacity. Water is allocated between the owner and the customer. The model is now optimized considering various scenarios j . The subsequent subsections delineate the updated model.

3.1.1 Decision variables

This section introduces the decision variables that characterize the optimization model implemented in the stochastic framework. The decision variables previously presented, such as v^+ , v^- , and x^{spot} , are updated to account for the impact of various scenarios j . The position taken on the futures contracts x_i^F does not depend on the scenario considered, so the variable is not changed from the deterministic case. Compared with the model applied to the deterministic case, two more decision variables are considered in the stochastic case: α and Var_β . As in the deterministic framework α represents the percentage of storage capacity sold to the customer, and is a number between 0 and 1. While in the deterministic case, however, it was regarded as a known parameter of the model, in the stochastic framework it becomes a decision variable of the model. Var_β identifies the minimum acceptable profit in the $(1 - \beta)\%$ of worst-case scenarios. In practice Var_β denotes the threshold value below which the worst-case $(1 - \beta)\%$ of the profits stays. The determination of Var_β is necessary for the safety measure $CVar_\beta$, discussed in the following subsection.

The decision variables are now defined as follows:

α	percentage of transferred storage capacity
v_{jt}^+	water released from the upper basin in scenario j and hour t [m^3]
v_{jt}^-	water pumped from the lower basin in hour t , under scenario j [m^3]
x_{jt}^{spot}	energy sold/bought in the spot market (spot position) [MWh]
x_i^F	position in future contract i [MWh]
VaR_β^1	β -quantile of the loss and profit distribution

The model is formulated as a two-stage nonlinear optimization problem. Two-Stage Stochastic Programming is a widely used optimization framework for decision-making under uncertainty, particularly in situations where decisions need to be made sequentially. It is based on the concept of splitting decisions into two stages to account for uncertain future outcomes:

- First stage: decisions are made before the uncertainty is revealed. These are often strategic or long-term decisions, such as determining investment levels, production capacities, or resource allocations.
- Second stage: after the uncertainty is revealed (e.g., demand fluctuations, market prices, or weather conditions), additional decisions are made to adapt to the realized scenario.

Variables such as α , x_i^F , and VaR_β are determined at time 0 during the optimization process, corresponding to the first stage. In contrast, variables like v_{jt}^+ , v_{jt}^- , and x_{jt}^{spot} are subsequently decided for each scenario j and hour t , during the second stage. With the decision variables defined, the formulation of the objective function will be explained in the following paragraph.

3.1.2 Objective function

In a stochastic context, it is necessary to redefine the objective function in order to include uncertainty management within the optimization model. The objective function to maximize is a combination of expected profits and risk: ν multiplies the profit part, while its complement multiplies the safety-measure. The profit part is represented by the mean of the profits calculated in each scenario j , where J represents the numbers of scenarios.

$$\max_{[\alpha, v_{jt}^+, v_{jt}^-, x_{jt}^{spot}, x_i^F, VaR_\beta]} \underbrace{\nu \left[\left(\frac{1}{J} \sum_{j=1}^J \pi_j \right) \right]}_{\text{Expected Profit}} + (1 - \nu) \underbrace{\left[VaR_\beta - \frac{1}{J \cdot \beta} \sum_{j=1}^J z_j \right]}_{\text{Safety-measure}} \quad (3.1)$$

¹For convenience, the variable is referred to as VaR_β , although its value represents the VaR only when the optimum is reached. In a feasible solution, VaR_β does not correspond to the VaR associated with that solution.

The safety measure is defined as the profit's Conditional Value at Risk (CVaR) and its implementation will be discussed in the following paragraph.

3.1.3 Safety measure

The profit Conditional Value at Risk was applied as a safety measure. The Conditional Value at Risk ($CVaR_\beta$) is a risk measure that evaluates the average expected loss in worst-case scenarios, that is, those below the threshold value defined by VaR_β . While VaR_β identifies the smallest profit (or largest loss) π_β such that the probability of getting a profit not larger than π_β is $1 - \beta$, the $CVaR_\beta$ extends this notion by considering the conditional average profit or loss in those scenarios. The $CVaR_\beta$ is commonly defined as:

$$CVaR_\beta = \mathbb{E}[\pi \mid \pi \leq VaR_\beta],$$

which represents the expected profit (or expected loss) conditional on the profit π being less than or equal to VaR_β [8], where:

$$VaR_\beta = \inf\{\pi : P(\Pi \leq \pi) \geq 1 - \beta\}$$

The profit CVaR for the β -quantile was chosen over other risk measures because of its characteristics. First, it can be introduced into an optimization model using a convex optimization formulation [1]. Second, it is a coherent risk measure because it fulfills all four requirements on coherent risk measure. Considering the risk measure $\rho \in \mathbb{R}$, where x and y are random returns, the requirements of a coherent risk measure are as follows [2]:

- Translation invariance $\rho(x + a) = \rho(x) - a \quad \forall a \in \mathbb{R}$.
- Subadditivity $\rho(x + y) \leq \rho(x) + \rho(y) \quad \forall x, y$.
- Positive homogeneity $\rho(\lambda x) = \lambda \rho(x) \quad \forall \lambda \geq 0$.
- Positivity $\rho(x) \geq 0 \quad \forall x \geq 0$.

The main objective is, to maximize the profits. The $CVaR_\beta$ then provides an estimate of the average profit in worst-case scenarios. The higher the $CVaR$, the higher the average profit in the adverse scenarios. The profit's $CVaR$ for the β -quantile can be incorporated in the objective function as follows:

$$CVaR_\beta = \left[VaR_\beta - \frac{1}{J \cdot \beta} \sum_{j=1}^J z_j \right]$$

The z_j are auxiliary variables that represent the excess loss over the VaR_β in worst-case scenarios. Those variable must be non negative and at least equal to the difference between VaR_β and the profit determined in each scenario j :

$$z_j \geq VaR_\beta - \pi_j \tag{3.2}$$

$$z_j \geq 0 \tag{3.3}$$

In the subsequent section, the profit function is modified to account for various scenarios.

3.1.4 Profit function

In a stochastic framework, the profits generated by the management of the facility are calculated as a combination of revenues and costs, discounted at a monthly rate, and are computed for each scenario j . The revenue component is updated solely in the profit portion obtained in the spot market. The spot position x_{jt}^{spot} is dependent on the energy trade in the spot market in each hour t and scenario j ($v_{jt}^+ \cdot k^+$ and $v_{jt}^- \cdot k^-$). Consequently, the profit or loss incurred in the spot market is calculated as the spot position multiplied by the spot price observed in each hour t of each scenario j , $S_{j,t}$. The cost component is additionally revised to account for the operational hourly expense of energy production and pumping in each scenario j .

The profit function is updated as follows:

$$\pi_j = 30 \cdot \sum_{m=1}^{12} e^{-r_2 m} \sum_{k=1}^K \left(\sum_{i=1}^n A_{i,K \cdot m+k} \cdot F_i \cdot x_i^F + S_{j,K \cdot m+k} \cdot x_{j,K \cdot m+k}^{spot} - c^+ \cdot v_{j,K \cdot m+k}^+ \cdot k^+ - c^- \cdot v_{j,K \cdot m+k}^- \cdot k^- \right) \quad (3.4)$$

The notation introduced in the previous chapter, in subsection 2.1.3, for defining hours while maintaining the distinction of months, has been retained. This type of notation, in fact, facilitates the examination of hourly dependencies between consecutive months, while preserving the monthly distinctions and independence between scenarios. In the subsequent subsections, the physical, financial, and hybrid models, updated to account for various scenarios, are presented.

3.1.5 Physical model

The physical model presented in section 2.2.1 has been revised to incorporate the presence of diverse scenarios.

$$\max_{[\alpha, v_{tj}^+, v_{tj}^-, x_{tj}^{spot}, x_i^F]} \nu \left[\left(\frac{1}{J} \sum_{j=1}^J \pi_j \right) \right] + (1 - \nu) \left[VaR_\beta - \frac{1}{J \cdot \beta} \sum_{j=1}^J z_j \right] \quad (3.5)$$

$$\text{s.t.} \quad z_j \geq VaR_\beta - \pi_j \quad (3.6)$$

$$z_j \geq 0 \quad (j \in J) \quad (3.7)$$

$$\pi_j = 30 \cdot \sum_{m=1}^{12} e^{-r_2 m} \sum_{k=1}^K \left(\sum_{i=1}^n A_{i,K \cdot m+k} \cdot F_i \cdot x_i^F + S_{j,K \cdot m+k} \cdot x_{j,K \cdot m+k}^{spot} - (c^+ \cdot v_{j,K \cdot m+k}^+ \cdot k^+ + c^- \cdot v_{j,K \cdot m+k}^- \cdot k^-) \right) \quad (3.8)$$

$$\text{Customer's priority} \begin{cases} \delta \cdot \lambda \leq \alpha \leq \lambda \\ v_{jt}^- \leq X_{max}^-(\gamma_{jt}^- + 1 - \lambda) \\ v_{jt}^+ \leq X_{max}^+(\gamma_{jt}^+ + 1 - \lambda) \end{cases} \quad (3.9)$$

$$\text{Customer's orders} \begin{cases} v_{jt}^{C+} = \min(v_{jt}^{OC+}; V_{j,t-1}^{Cu}; X_{max}^{C+}) \\ v_{jt}^{C-} = \min(v_{jt}^{OC-}; V_{j,t-1}^{Cd}; X_{max}^{C-}) \\ X_{max}^{C+} = \frac{1}{8} \cdot \alpha V_{max}^u \cdot k^+ \\ X_{max}^{C-} = \frac{1}{8} \cdot \alpha V_{max}^u \cdot k^- \end{cases} \quad (3.10)$$

$$\text{Turbine capacity} \begin{cases} X_{jt}^{max+} = X_{max}^+ - v_{jt}^{C+} \cdot k^+ \\ 0 \leq v_{jt}^+ \cdot k^+ \leq X_{jt}^{max+} \\ X_{jt}^{max-} = X_{max}^- - v_{jt}^{C-} \cdot k^- \\ 0 \leq v_{jt}^- \cdot k^- \leq X_{jt}^{max-} \end{cases} \quad (3.11)$$

$$\text{Energy balance} \quad v_{jt}^+ \cdot k^+ - v_{jt}^- \cdot k^- - \sum_{i=1}^n A_{it} x_i^F - x_{jt}^{spot} = 0 \quad (3.12)$$

$$\text{Customer's Basins} \begin{cases} V_{jt}^{Cu} = V_{j,t-1}^{Cu} + v_{jt}^{C-} - v_{jt}^{C+} \\ V_{jt}^{Cd} = V_{j,t-1}^{Cd} - v_{jt}^{C-} + v_{jt}^{C+} \end{cases} \quad (3.13)$$

$$\text{Owner's basins} \begin{cases} V_{jt}^u = V_{j,t-1}^u + v_{jt}^- - v_{jt}^+ + I_{jt}^u - L_{jt}^u \\ V_{jt}^d = V_{j,t-1}^d - v_{jt}^- + v_{jt}^+ + I_{jt}^d - L_{jt}^d \end{cases} \quad (3.14)$$

$$\text{Basins' balance} \begin{cases} pr_{jt} = v_{jt}^+ + v_{jt}^{C+} \\ pm_{jt} = v_{jt}^- + v_{jt}^{C-} \\ V_{jt}^{u,tot} = V_{j,t-1}^{u,tot} - pr_{jt} + pm_{jt} + I_{jt}^u - L_{jt}^u \\ V_{jt}^{d,tot} = V_{j,t-1}^{d,tot} + pr_{jt} - pm_{jt} + I_{jt}^d - L_{jt}^d \\ L_{jt}^u = \max(V_{j,t-1}^{u,tot} - pr_{jt} + pm_{jt} + I_{jt}^u - V_{max}^u; 0) \\ L_{jt}^d = \max(V_{j,t-1}^{d,tot} + pr_{jt} - pm_{jt} + I_{jt}^d - V_{max}^d; 0) \end{cases} \quad (3.15)$$

Compared with the model presented in the previous chapter, the calculation of $CVaR$ for the β -quantile in the objective function is introduced. Consequently, the constraints (3.6) and (3.7), related to the tail of worst-case profits, are also introduced. The constraints described in detail in 2.2.1 were updated by considering the presence of the j scenarios in order to reevaluate the variables computed and determined for each t hour of each scenario j . The following sections show the financial model and the hybrid model described in 2.2.2 and 2.2.3, updated in a similar manner.

3.1.6 Financial model

The financial model separates the operation into a physical facility controlled by the owner and a virtual one managed by the customer, with all operational decisions compensated solely through financial transactions.

$$\max_{[\alpha, v_{tj}^+, v_{tj}^-, x_{tj}^{spot}, x_i^F]} \nu \left[\left(\frac{1}{J} \sum_{j=1}^J \pi_j \right) \right] + (1 - \nu) \left[VaR_\beta - \frac{1}{J \cdot \beta} \sum_{j=1}^J z_j \right] \quad (3.16)$$

$$\text{s.t.} \quad z_j \geq VaR_\beta - (\pi_j + P \cdot V_{max}^u \cdot \alpha) \quad (3.17)$$

$$z_j \geq 0 \quad (3.18)$$

$$\begin{aligned} \pi_j = 30 \cdot \sum_{m=1}^{12} e^{-r_2 m} \sum_{k=1}^K \left(\sum_{i=1}^n A_{i,K \cdot m+k} \cdot F_i \cdot x_i^F + S_{j,K \cdot m+k} \cdot x_{j,K \cdot m+k}^{spot} \right. \\ \left. - S_{j,K \cdot m+k} \cdot x_{j,K \cdot m+k}^{stockC} \right. \\ \left. - (c^+ \cdot \hat{v}_{j,K \cdot m+k}^+ \cdot k^+ + c^- \cdot v_{j,K \cdot m+k}^- \cdot k^-) \right) \end{aligned} \quad (3.19)$$

$$\text{Customer's orders} \begin{cases} v_{jt}^{C+} = \min(v_{jt}^{OC+}; V_{j,t-1}^{Cu}; X_{max}^{C+}) \\ v_{jt}^{C-} = \min(v_{jt}^{OC-}; V_{j,t-1}^{Cd}; X_{max}^{C-}) \\ X_{max}^{C+} = \frac{1}{8} \cdot \alpha V_{max}^u \cdot k^+ \\ X_{max}^{C-} = \frac{1}{8} \cdot \alpha V_{max}^u \cdot k^- \end{cases} \quad (3.20)$$

$$\text{Owner's decisions} \begin{cases} 0 \leq v_{jt}^+ \cdot k^+ \leq X_{max}^+ \\ 0 \leq v_{jt}^- \cdot k^- \leq X_{max}^- \\ v_{jt}^+ \cdot k^+ - v_{jt}^- \cdot k^- - \sum_{i=1}^n A_{it} x_i^F - x_{jt}^{spot} = 0 \end{cases} \quad (3.21)$$

$$\text{Owner's basins balance} \begin{cases} V_{jt}^u = V_{j,t-1}^u - v_{jt}^+ + v_{jt}^- + I_{jt}^u - L_{jt}^u \\ V_{jt}^d = V_{j,t-1}^d + v_{jt}^+ - v_{jt}^- + I_{jt}^d - L_{jt}^d \\ L_{jt}^u = \max(V_{j,t-1}^u - v_{jt}^+ + v_{jt}^- + I_{jt}^u - V_{max}^u; 0) \\ L_{jt}^d = \max(V_{j,t-1}^d + v_{jt}^+ - v_{jt}^- + I_{jt}^d - V_{max}^d; 0) \end{cases} \quad (3.22)$$

$$\text{Customer's basins balance} \begin{cases} V_{jt}^{Cu} = V_{j,t-1}^{Cu} + v_{jt}^{C-} - v_{jt}^{C+} \\ V_{jt}^{Cd} = V_{j,t-1}^{Cd} - v_{jt}^{C-} + v_{jt}^{C+} \end{cases} \quad (3.23)$$

$$\text{Customer's compensation} \begin{cases} v_{jt}^{C+} \cdot k^+ - v_{jt}^{C-} \cdot k^- - x_{jt}^{stockC} = 0 \end{cases} \quad (3.24)$$

3.1.7 Hybrid model

In the hybrid model, two cases are distinguished: energy production takes precedence over pumping. When producing energy, the customer's needs must be met immediately, so the owner cannot pump water. However, when the customer requests pumping, it is only to store water for the future. The owner can compensate

the customer's pumping order by using resources in the upper basin, employing various methods beyond just pumping water to the upper basin.

$$\max_{[\alpha, \lambda, v_{tj}^+, v_{tj}^-, x_{tj}^{spot}, x_i^F]} \nu \left[\left(\frac{1}{J} \sum_{j=1}^J \pi_j \right) \right] + (1 - \nu) \left[VaR_\beta - \frac{1}{J \cdot \beta} \sum_{j=1}^J z_j \right] \quad (3.25)$$

$$\text{s.t.} \quad z_j \geq VaR_\beta - (\pi_j + P \cdot V_{max}^u \cdot \alpha) \quad (j \in J) \quad (3.26)$$

$$z_j \geq 0 \quad (3.27)$$

$$\begin{aligned} \pi_j = 30 \cdot \sum_{m=1}^{12} e^{-r_2 m} \sum_{k=1}^K \left(\sum_{i=1}^n A_{i,K \cdot m+k} \cdot F_i \cdot x_i^F + S_{j,K \cdot m+k} \cdot x_{j,K \cdot m+k}^{spot} \right. \\ \left. + S_{j,K \cdot m+k} \cdot (v_{j,K \cdot m+k}^{C-} - \hat{v}_{j,K \cdot m+k}^{C-}) \cdot k^- \right. \\ \left. - (c^+ \cdot \hat{v}_{j,K \cdot m+k}^+ \cdot k^+ + c^- \cdot v_{j,K \cdot m+k}^- \cdot k^-) \right) \end{aligned} \quad (3.28)$$

$$\text{Customer's priority} \begin{cases} \delta \cdot \lambda \leq \alpha \leq \lambda \\ v_{jt}^- \leq X_{max}^-(\gamma_{jt} + 1 - \lambda) \end{cases} \quad (3.29)$$

$$\text{Customer's orders} \begin{cases} v_{jt}^{C+} = \min(v_{jt}^{OC+}; V_{j,t-1}^{Cu}; X_{max}^{C+}) \\ v_{jt}^{C-} = \min(v_{jt}^{OC-}; V_{j,t-1}^{Cd}; X_{max}^{C-}) \\ X_{max}^{C+} = \frac{1}{8} \cdot \alpha V_{max}^u \cdot k^+ \\ X_{max}^{C-} = \frac{1}{8} \cdot \alpha V_{max}^u \cdot k^- \end{cases} \quad (3.30)$$

$$\text{Turbine capacity} \begin{cases} X_{jt}^{max+} = X_{max}^+ - v_{jt}^{C+} \cdot k^+ \\ 0 \leq \hat{v}_{jt}^+ \cdot k^+ \leq X_{jt}^{max+} \\ X_{jt}^{max-} = X_{max}^- - \hat{v}_{jt}^{C-} \cdot k^- \\ 0 \leq v_{jt}^- \cdot k^- \leq X_{jt}^{max-} \end{cases} \quad (3.31)$$

$$\text{Energy balance} \begin{cases} \hat{v}_{jt}^+ \cdot k^+ - v_{jt}^- \cdot k^- - \sum_{i=1}^n A_{it} x_i^F - x_{jt}^{spot} = 0 \end{cases}$$

$$\text{Excess water compensation} \begin{cases} c_{jt}^u = \min(v_{jt}^{C-}; c_{jt}^{max}) \\ c_{jt}^{u,max} = \max(V_{t-1}^u - (1 - \alpha) V_{max}^u; 0) \end{cases} \quad (3.32)$$

$$\text{Inflow compensation} \begin{cases} I_{jt}^{c1} = \min(v_{jt}^{C-} - c_{jt}^u; I_{jt}^u) \\ I_{jt}^c = I_{jt}^u - I_{jt}^{c1} \end{cases} \quad (3.33)$$

$$\text{Physical production and pumping} \begin{cases} \hat{v}_{jt}^+ = \max(v_{jt}^+ - v_{jt}^{C1-}; 0) \\ v_{jt}^{C1-} = \max(v_{jt}^{C-} - c_{jt}^u - I_{jt}^u; 0) \\ \hat{v}_{jt}^{C-} = \max(v_{jt}^{C1-} - v_{jt}^+; 0) \end{cases} \quad (3.34)$$

$$\text{Basin's balance} \begin{cases} pr_{jt} = v_{jt}^+ + v_{jt}^{C+} \\ pm_{jt} = v_{jt}^- + v_{jt}^{C-} \\ V_{jt}^{u,tot} = V_{j,t-1}^{u,tot} - pr_{jt} + pm_{jt} + I_{jt}^u - L_{jt}^u \\ V_{jt}^{d,tot} = V_{j,t-1}^{d,tot} + pr_{jt} - pm_{jt} + I_{jt}^d - L_{jt}^d \\ L_{jt}^u = \max(V_{j,t-1}^{u,tot} - pr_{jt} + pm_{jt} + I_{jt}^u - V_{max}^u; 0) \\ L_{jt}^d = \max(V_{j,t-1}^{d,tot} + pr_{jt} - pm_{jt} + I_{jt}^d - V_{max}^d; 0) \end{cases} \quad (3.35)$$

$$\text{Owner's basins} \begin{cases} V_{jt}^u = V_{j,t-1}^u + v_{jt}^- - v_{jt}^+ - c_{jt}^u + I_{jt}^c - L_{jt}^u \\ V_{jt}^d = V_{j,t-1}^d - v_{jt}^- + v_{jt}^+ + c_{jt}^u + I_{jt}^d + I_{jt}^{c1} - L_{jt}^d \end{cases} \quad (3.36)$$

$$\text{Customer's basins} \begin{cases} V_{jt}^{Cu} = V_{j,t-1}^{Cu} + v_{jt}^{C-} - v_{jt}^{C+} \\ V_{jt}^{Cd} = V_{j,t-1}^{Cd} - v_{jt}^{C-} + v_{jt}^{C+} \end{cases} \quad (3.37)$$

3.2 Application

The three models were tested considering 10 different scenarios of 8,760 hours each. The stochastic scenarios were generated from data collected over a decade, spanning the period from 2014 to 2023. To generate realistic scenarios, data were collected regarding spot energy prices and energy demand in the C-SUD energy zone in Italy. Data regarding precipitation were then collected in order to determine the potential inflow into basins in the zone of interest. The spot prices² and the load data³ were collected from the GME (Gestore del Mercato Elettrico) dataset. Precipitation data⁴ were collected from Copernicus data store, more specifically from the ERA 5 dataset. The simulation method employed is an enhancement of the approach proposed by Cerqueti et al.[14], which is grounded in Markov chain theory. This method allows series to be simulated from historical data, maintaining the correlation that has occurred historically between the series. For example, it is known that there is a correlation between energy prices and energy demand. Figure 3.1 illustrates the average daily energy load and spot prices for the specified region in Italy during 2020. Figure 3.2 depicts the daily average energy load and spot prices simulated from this historical data. The graphical representation

²Source of data: Gestore del Mercato Elettrico (GME), <https://www.mercatoelettrico.org/It/download/DatiStorici.aspx>, day-ahead market single national price (C-SUD energy zone)

³Source of data: Gestore del Mercato Elettrico (GME), <https://www.mercatoelettrico.org/It/download/DatiStorici.aspx>, day-ahead market total purchases, C-SUD energy zone

⁴Source of data: Copernicus Data Store ERA5 dataset, <https://cds.climate.copernicus.eu/datasets/reanalysis-era5-single-levels?tab=download>

provided displays the arithmetic mean of spot prices and energy load, as the hourly representation of these data proved to be excessively confusing. It should therefore be noted that the graphs may appear slightly distorted, as the load varies across different hours of the day. It is observable that the positive correlation between energy demand and energy prices has been maintained in the simulated series: when a peak occurs in the energy demand series, it is also present in the price series, as illustrated in Figure 3.2.

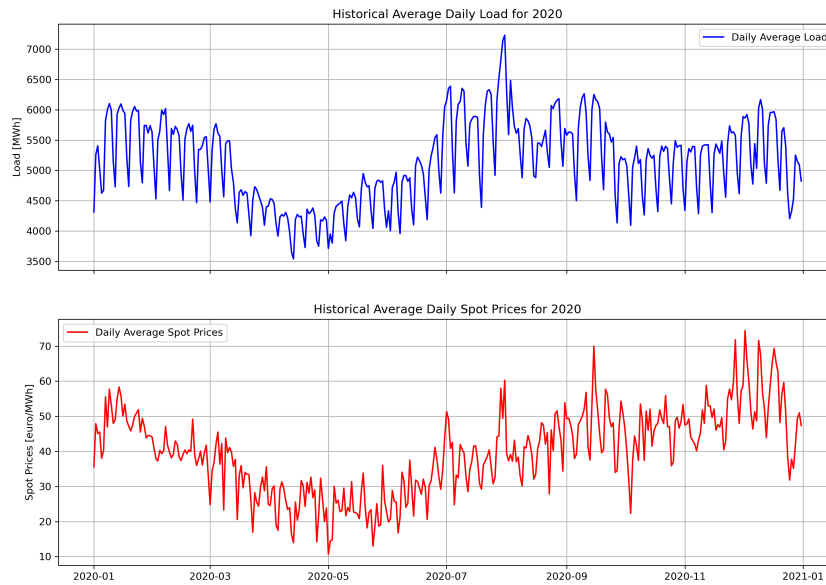


Figure 3.1: Average daily Load and daily Spot Prices in 2020

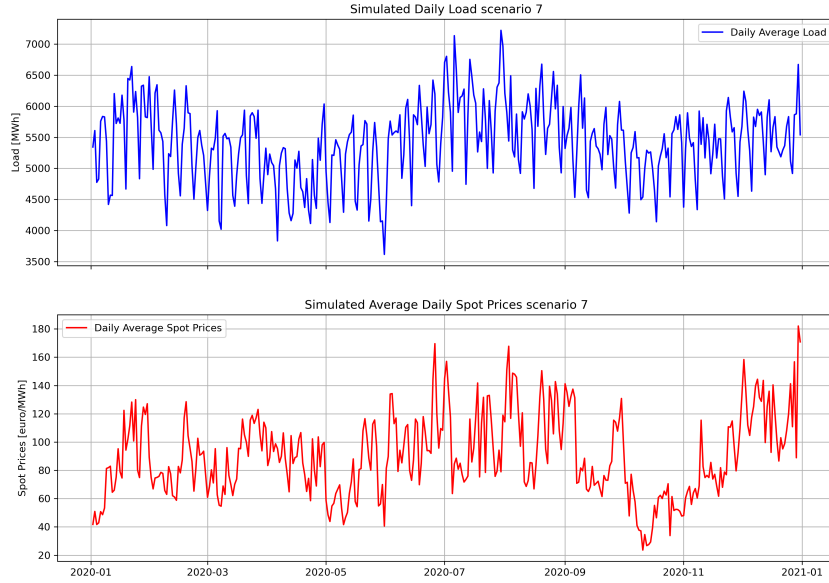


Figure 3.2: Average daily simulated Load and daily Spot Prices in 2020

The ERA 5 datasets provide the total precipitation that occurred in a specific area, expressed in m . Specifically, they provide the accumulated liquid and frozen water, comprising rain and snow, that falls to the Earth's surface. This represents the sum of large-scale precipitation and convective precipitation. To introduce these parameters into the models, the source data were used to determine the inflows into the basins based on the total precipitation. To do that it was necessary to convert the precipitation from m to m^3 by multiplying the precipitation expressed in m by the area of the basins, which is approximately $6,000,000m^2$. Figures 3.3 and 3.4 present the simulated average daily inflow for scenario 7 and scenario 10, respectively, based on precipitation data collected for 2020 and 2023. While the simulated series do not perfectly replicate the historical data, they maintain consistent statistical characteristics.

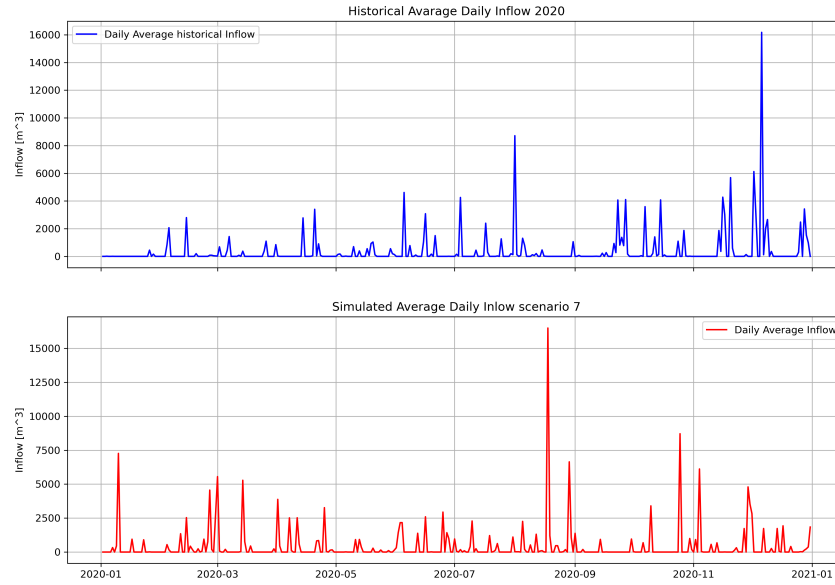


Figure 3.3: Average daily simulated Inflow scenario 7

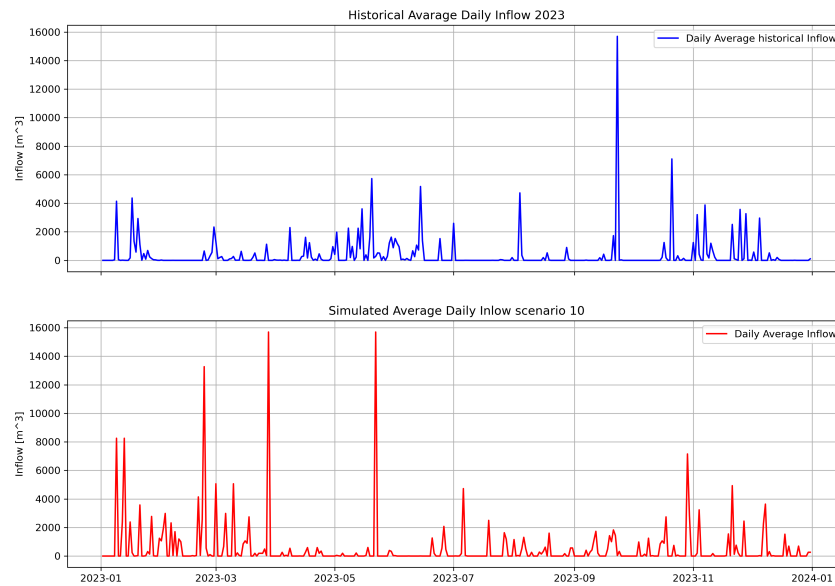


Figure 3.4: Average daily simulated Inflow in scenario 10

The technical parameters that characterize the facility are the same presented in Table2.2 in 2.3. In

the stochastic case, the models were also tested considering a smart customer and a partially predictable customer. Each production and pumping order from the customer was simulated for every hour (8,760) and for each scenario (10), totaling 87,360 production orders and 8,7600 pumping orders, for both the smart customer and the partially predictable customer. Given the high number of customer decisions to simulate in both cases, it was necessary to automate the simulation of customer behavior. In the following subsections, the methods for simulating the behavior of the smart customer and the partially predictable customer are presented.

3.2.1 Smart customer behavior

Because the smart customer behaves consistently with prices evolution in the spot market, the counterpart's preliminary orders are generated through an optimization model, where v_{jt}^{OC+} and v_{jt}^{OC-} are decision variables. The optimization model is a simplification of the descriptive models of the three types of contracts: this type of simplified model determines the optimal positions taken by the customer, assuming that the entire capacity of the plant is given to him. The objective function to maximize, in this case, considers just the expected customer profit.

$$\max_{[v_{jt}^{OC+}, v_{jt}^{OC-}, x_{j,t}^{spotC}]} \left[\left(\frac{1}{J} \sum_{j=1}^J \pi_j^C \right) \right] \quad (3.38)$$

where π_j^C is the customer's profit for each scenario j and is determined as the discounted revenue obtained by the energy trading in the spot market:

$$\pi_j^C = 30 \cdot \sum_{m=1}^{12} e^{-r_2 m} \sum_{k=1}^K \left(S_{j,K \cdot m+k} \cdot \hat{x}_{j,K \cdot m+k}^{spotC} \right). \quad (3.39)$$

The position taken by the customer in the spot market is denoted by $\hat{x}_{j,K \cdot m+k}^{spotC}$, and is determined by the following relation:

$$v_{jt}^{OC+} \cdot k^+ - v_{jt}^{OC-} \cdot k^- - \hat{x}_{jt}^{spotC} = 0 \quad (3.40)$$

As in constraint (2.20), when the value of $\hat{x}_{jt}^{spotC}$ is positive, it indicates that the customer has issued an order to sell energy in the spot market. Conversely, if it is negative, the customer has purchased energy from the spot market to pump water into the upper basin. The optimal operative decision of the customer are limited by the maximum capacity of the turbine and the pump:

$$0 \leq v_{jt}^{OC+} \cdot k^+ \leq X_{max}^+ \quad (3.41)$$

$$0 \leq v_{jt}^{OC-} \cdot k^- \leq X_{max}^- \quad (3.42)$$

The orders calculated in this way update the lower and upper basin levels as described by the following constraints:

$$V_{jt}^{Cu} = V_{j,t-1}^{Cu} - v_{jt}^{OC+} + v_{jt}^{OC-} - L_{j,t}^u \quad (3.43)$$

$$V_{jt}^{Cd} = V_{j,t-1}^{Cd} + v_{jt}^{OC+} - v_{jt}^{OC-} - L_{j,t}^d \quad (3.44)$$

The introduction of constraints from (3.41) to (3.44) ensure that the simulated orders are also consistent with the facility equilibrium.

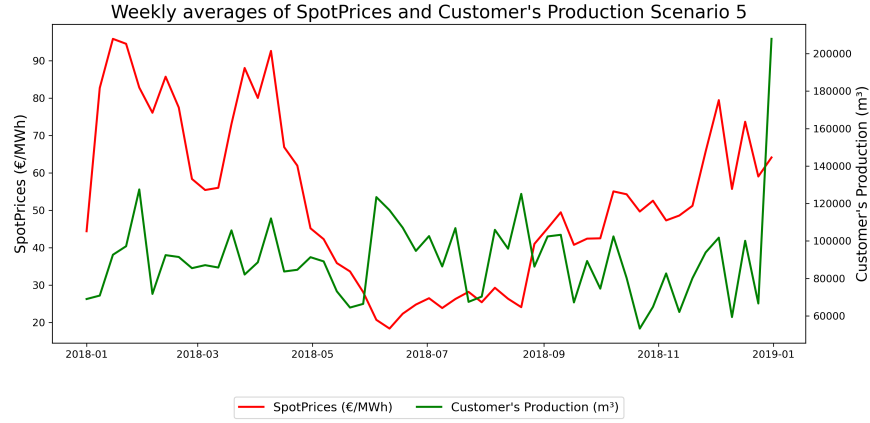


Figure 3.5: Average weekly smart customer's production order's compared with spot prices

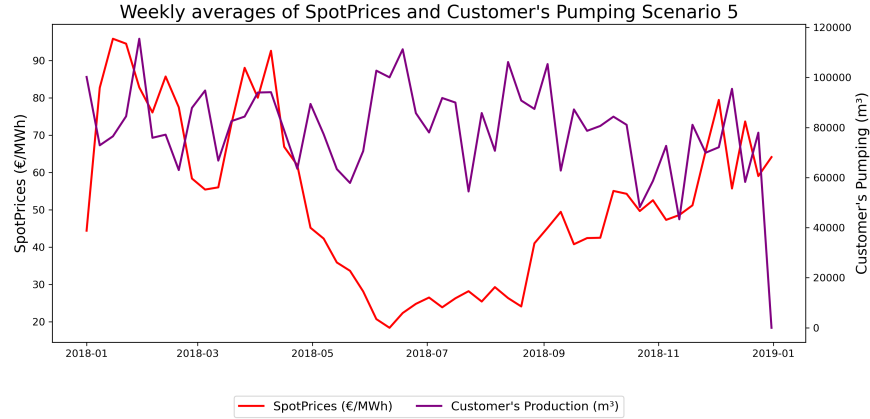


Figure 3.6: Average weekly smart customer's production order's compared with spot prices

Figures 3.5 and 3.6 illustrate the average weekly customer production and pumping orders generated for scenario 5, compared with the spot prices generated for that scenario. The data demonstrate that, even on a weekly average, customers tend to produce energy when prices increase and pump water when prices decrease.

3.2.2 Partially predictable customer behavior

A realistic behavior for a less sophisticated customer is a more random one: occasionally capable of making more profitable choices, while at other times falling into errors.

To define a partially predictable customer orders, random sequences $\tilde{\alpha}_{jt}^{f+}$ and $\tilde{\alpha}_{jt}^{f-}$ are first generated where:

- $\tilde{\alpha}_{jt}^{f+} \sim U(0, 1)$ is associated with customer production
- $\tilde{\alpha}_{jt}^{f-} \sim U(0, 1)$ is associated with customer pumping

These sequences are generated such that they are orthogonal, i.e. $\tilde{\alpha}_{jt}^{f+} \cdot \tilde{\alpha}_{jt}^{f-}$ for all j , is equal to zero, thereby ensuring that the customer cannot simultaneously engage in production and pumping activities.

The generated orders are then compared with the water availability in the basin.

$$v_{jt}^{OC+} = \min(\tilde{\alpha}_{jt}^{f+} \cdot v_{max}^+; V_{j,t-1}^u)$$

$$v_{jt}^{OC-} = \min(\tilde{\alpha}_{jt}^{f-} \cdot v_{max}^-; V_{j,t-1}^d)$$

$$V_{jt}^u = V_{j,t-1}^u - v_{jt}^{OC+} + v_{jt}^{OC-}$$

$$V_{jt}^d = V_{j,t-1}^d + v_{jt}^{OC+} - v_{jt}^{OC-}$$

Where:

$$v_{max}^+ = \frac{X_{max}^+}{k^+}$$

$$v_{max}^- = \frac{X_{max}^-}{k^-}$$

In this way, it is possible to generate orders that are coherent within the facility equilibrium but follow a more random path than the previous method.

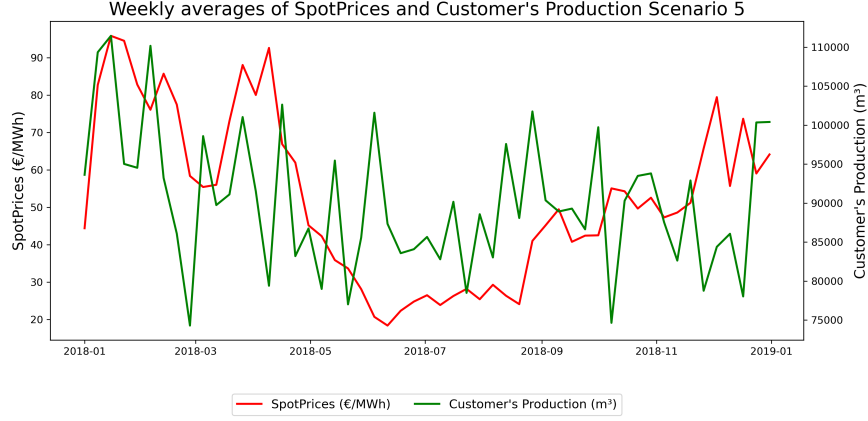


Figure 3.7: Average weekly Partially predictable costumer's production order's compared with spot prices

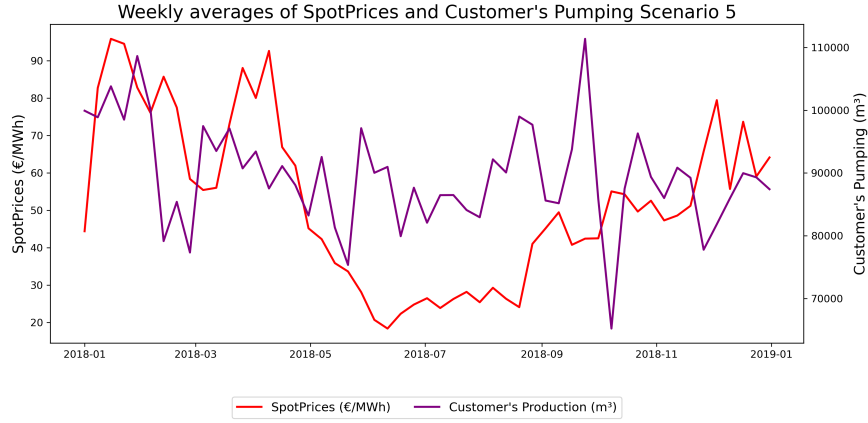


Figure 3.8: Average weekly Partially predictable costumer's production order's compared with spot prices

Figures 3.7 and 3.8 reports the weekly average of partially predictable customer's production and pumping orders, confronted with the spot prices. When compared with the smart customer's orders in Figures 3.5 and 3.6, it can be seen that the partially predictable customer has a more unpredictable behavior on average, producing and pumping water, sometimes when it is most convenient, sometimes when prices are not favorable for production and pumping operations.

3.2.3 Heuristics

The transition from deterministic to stochastic models involves an increase in computational complexity and, consequently, in computational time. The models must compute different solutions for each scenario, and then integrate these with the calculation of the *CVaR* and the decision to sell capacity to a counterpart. Variables defined across both scenarios and hours generate 87,360 groups, for a total of approximately 2,620,800 variables resulting in a substantial increase in computational time. The hybrid model and the physical model

incorporate even a greater number of maximum and minimum constraints. As these constraints are nonlinear, the solver must linearize them to determine the optimal solution, which further increases the computational time. The physical model was evaluated considering 10 scenarios with 8,760 hours each, implemented in Python language using Gurobi 11.0 as optimization solver and run on a Workstation HP Intel(R)-Xeon(R) at 3.5 GHZ with 64GB RAM. More than 10 days were required to obtain an optimal solution. Consequently, the necessity to implement heuristics arose, with the objective of identifying a satisfactory solution.

Heuristics play a crucial role in solving optimization problems characterized by a large number of variables, where the exact approach is often impractical because of the combinatorial explosion of possible solutions. In such contexts, model optimization may not provide solutions in acceptable time due to the associated computational complexity. Heuristics, on the other hand, provide satisfactory solutions in significantly reduced time by exploiting strategies that do not guarantee the optimal solution but reduce the computational complexity. Heuristic techniques, such as local search, simulated annealing or genetic algorithms, are able to efficiently explore the space of solutions, avoiding exhaustive examination of all possible combinations. This approach yields useful results in situations where the size of the problem makes the application of exact methods infeasible. The importance of heuristics thus lies in their ability to balance computational efficiency with the quality of solutions, enabling them to successfully address complex problems that would otherwise be difficult to solve.

Two simple heuristics, which can be classified as matheuristics, were applied. These heuristics are based on a simplification of the underlying mathematical model, making it easier to solve. An optimal solution to the simplified model will be feasible for the original model but potentially suboptimal.

The first heuristic is based on the following observations. Compared to most of the decision variables, which pertain to operational actions, the single variable α is of a strategic nature, as it determines the fraction of storage that cannot be managed by the owner. From a mathematical perspective, the variable α constrains all other variables. Empirical observations show that fixing the value of α significantly simplifies the model. Moreover, α can vary within the closed interval $[0, 1]$. A dynamic grid search refinement approach was applied. This technique creates a grid of values for the variable α :

$$0, \frac{1}{n}, \frac{2}{n}, \dots, \frac{n-1}{n}, 1$$

and solves the restricted model with α fixed at $\frac{\tau}{n}$, for all $\tau = 0, 1, \dots, n$. In these tests, $n = 10$ was chosen. Subsequently, the most promising α regions are identified. Within these intervals, the models were further tested using a denser α grid, with α increasing in increments of 0.01. The solution corresponding to the highest optimal objective value is selected.

The second heuristic is based on the observation that, although there are 365 days in a year, most of them can be grouped into a small number of patterns. To simulate sampling from such patterns, the model is restricted to consider only the first week of each month. This results in a total of 2,016 hours per scenario instead of 8,760, thus reducing the model size to approximately one-fourth. The financial model turned out as the faster in terms of computational time. This could be expected since it contains a smaller number of

nonlinear constraints compared with the other two. For this model, it was sufficient to apply only the first type of heuristics. In contrast, to obtain acceptable solution for the physical model and the hybrid model, it was necessary to apply both heuristics. Hypothesis tests were then performed to verify that the difference in the distributional characteristics of the random variables, considering 8,760 hours and considering 2,016 and 288 hours, was not statistically significant. The random variables analyzed were Spot Prices and Inflow. For each random variable, the difference between the following was tested, considering a significance level of 0.05:

- Mean
- Variance
- Standard Deviation
- Skewness
- Kurtosis
- Median
- Mean

The results of the hypothesis tests are reported below.

Hypothesis tests on the difference in the distributional characteristics of Spot Prices

	<i>Mean (8760)</i>	<i>Mean (2016)</i>		<i>Mean (8760)</i>	<i>Mean (288)</i>
Mean	82.9254	86.8433	Mean	82.9254	88.5197
Variance	1122.4691	1514.7188	Variance	1122.4691	1791.8773
Observations	10	10	Observations	10	10
Pearson correlation	0.8804		Pearson correlation	0.9371	
Hypothesized mean difference	0		Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9		<i>df</i> (degrees of freedom)	9	
t statistic	-0.6708		t statistic	-1.1049	
$P(T \leq t)$ one-tail	0.2595		$P(T \leq t)$ one-tail	0.148922074	
Critical t one-tail	1.8331		Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.5191		$P(T \leq t)$ two-tail	0.2978	
Critical t two-tail	2.2621		Critical t two-tail	2.2621	

Table 3.1: Hypothesis Test on Spot Prices Means

	<i>Variance (8760)</i>	<i>Variance (2016)</i>
Mean	2677.0314	3242.8727
Variance	28,876,364.82	29,113,163.94
Observations	10	10
Pearson correlation	0.9324	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	-0.9038	
$P(T \leq t)$ one-tail	0.1948	
Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.3896	
Critical t two-tail	2.2621	

	<i>Variance (8760)</i>	<i>Variance (288)</i>
Mean	2677.0314	3701.5225
Variance	28,876,364.82	52,077,369.61
Observations	10	10
Pearson correlation	0.9839	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	-1.5035	
$P(T \leq t)$ one-tail	0.0835	
Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.1669	
Critical t two-tail	2.2621	

Table 3.2: Hypothesis Test on Spot Prices Variance

	<i>Std_dev (8760)</i>	<i>Std_dev (2016)</i>
Mean	40.9642	45.3478
Variance	1109.9540	1318.2768
Observations	10	10
Pearson correlation	0.9006	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	-0.8777	
$P(T \leq t)$ one-tail	0.2014	
Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.4028	
Critical t two-tail	2.2621	

	<i>Std_dev (8760)</i>	<i>Std_dev (288)</i>
Mean	40.964	46.550
Variance	1109.954	1705.159
Observations	10	10
Pearson correlation	0.9597	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	-1.3374	
$P(T \leq t)$ one-tail	0.1069	
Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.2139	
Critical t two-tail	2.2621	

Table 3.3: Hypothesis Test on Spot Price Standard deviation

	<i>Skewness (8760)</i>	<i>Skewness (2016)</i>
Mean	1.0314	1.0495
Variance	0.0456	0.1059
Observations	10	10
Pearson correlation	0.7993	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	-0.2848	
$P(T \leq t)$ one-tail	0.3911	
Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.7823	
Critical t two-tail	2.2621	

	<i>Skewness (8760)</i>	<i>Skewness (288)</i>
Mean	1.0314	0.9114
Variance	0.0456	0.1249
Observations	10	10
Pearson correlation	0.1724	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	0.9983	
$P(T \leq t)$ one-tail	0.1721	
Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.3442	
Critical t two-tail	2.2621	

Table 3.4: Hypothesis Test on Spot Price Skewness

	<i>Kurtosis (8760)</i>	<i>Kurtosis (2016)</i>
Mean	1.5537	1.4693
Variance	0.8872	2.4055
Observations	10	10
Pearson correlation	0.6296	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	0.2215	
$P(T \leq t)$ one-tail	0.4148	
Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.8296	
Critical t two-tail	2.2621	

	<i>Kurtosis (8760)</i>	<i>Kurtosis (288)</i>
Mean	1.5537	0.9620
Variance	0.8873	3.1071
Observations	10	10
Pearson correlation	0.0374	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	0.9511	
$P(T \leq t)$ one-tail	0.1832	
Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.3664	
Critical t two-tail	2.2622	

Table 3.5: Hypothesis Test on Spot Price Kurtosis

	<i>Median (8760)</i>	<i>Median (2016)</i>
Mean	71.2372	72.8586
Variance	324.5679	533.0676
Observations	10	10
Pearson correlation	0.9009	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	-0.4930	
$P(T \leq t)$ one-tail	0.3169	
Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.6338	
Critical t two-tail	2.2622	

	<i>Median (8760)</i>	<i>Median (288)</i>
Mean	71.2372	74.2723
Variance	324.5679	555.6155
Observations	10	10
Pearson correlation	0.9268	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	-0.9953	
$P(T \leq t)$ one-tail	0.1728	
Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.3456	
Critical t two-tail	2.2621	

Table 3.6: Hypothesis Test on Spot Price Median

	<i>Minimum (8760)</i>	<i>Minimum (2016)</i>
Mean	2.7327	7.6677
Variance	37.2227	36.7379
Observations	10	10
Pearson correlation	0.6409	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	-3.0285	
$P(T \leq t)$ one-tail	0.0071	
Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.6338	
Critical t two-tail	2.2622	

	<i>Minimum (8760)</i>	<i>Minimum (288)</i>
Mean	2.7327	16.0518
Variance	37.2226	183.5851
Observations	10	10
Pearson correlation	0.7068	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	-4.1317	
$P(T \leq t)$ one-tail	0.0013	
Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.0025	
Critical t two-tail	2.2621	

Table 3.7: Hypothesis Test on Spot Price Minimum

	<i>Maximum (8760)</i>	<i>Maximum (2016)</i>		<i>Maximum (8760)</i>	<i>Maximum (288)</i>
Mean	286.5221	272.9746	Mean	286.5220	234.2349
Variance	14179.05	17838.70	Variance	14179.0495	15943.1564
Observations	10	10	Observations	10	10
Pearson correlation	0.8754		Pearson correlation	0.9411	
Hypothesized mean difference	0		Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9		<i>df</i> (degrees of freedom)	9	
t statistic	0.6632		t statistic	3.8728	
$P(T \leq t)$ one-tail	0.2619		$P(T \leq t)$ one-tail	0.0019	
Critical t one-tail	1.8331		Critical t one-tail	1.8331	
$P(T \leq t)$ two-tail	0.5238		$P(T \leq t)$ two-tail	0.0038	
Critical t two-tail	2.2622		Critical t two-tail	2.2621	

Table 3.8: Hypothesis Test on Spot Price Maximum

Analyzing the results for the distributions of spot prices, it is observed that p-values exceed 0.05 in the majority of tests. This indicates that the null hypothesis—stating that the difference between the distributions of the 8760-hour series and, first, the 2016-hour series and then the 288-hour series—is equal to zero—is accepted. The only exceptions are the tests performed on the maximum and minimum values when comparing with the 288-hour series. In these cases, the p-value is extremely small, leading to the rejection of the null hypothesis. Consequently, the difference between the means of the distributions is significantly different from zero.

Hypothesis tests on the difference in the distributional characteristics of Inflow

	<i>Mean (8760)</i>	<i>Mean (2016)</i>		<i>Mean (8760)</i>	<i>Mean (288)</i>
Mean	454.6583	466.4972202	Mean	454.6582944	490.2666
Variance	7118.83565	12222.89545	Variance	7118.83565	141,792.2
Observations	10	10	Observations	10	10
Pearson correlation	0.587809798		Pearson correlation	0.358511	
Hypothesized mean difference	0		Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9		<i>df</i> (degrees of freedom)	9	
t statistic	-0.409079347		t statistic	-0.31706	
$P(T \leq t)$ one-tail	0.346022507		$P(T \leq t)$ one-tail	0.379215	
Critical t one-tail	1.833112933		Critical t one-tail	1.833112933	
$P(T \leq t)$ two-tail	0.692045014		$P(T \leq t)$ two-tail	0.75843	
Critical t two-tail	2.262157163		Critical t two-tail	2.262157163	

Table 3.9: Hypothesis Test on Inflow Means

	Variance (8760)	Variance (2016)
Mean	8,995,690	8,682,005
Variance	7.35e+12	2.32e+13
Observations	10	10
Pearson correlation	0.521346	
Hypothesized mean difference	0	
df (degrees of freedom)	9	
t statistic	0.241053	
P($T \leq t$) one-tail	0.407458	
Critical t one-tail	1.833112933	
P($T \leq t$) two-tail	0.814916	
Critical t two-tail	2.262157163	

	Variance (8760)	Variance (288)
Mean	8,995,689.787	7,211,718.39
Variance	7.35354e+12	9.4028e+13
Observations	10	10
Pearson correlation	0.59632603	
Hypothesized mean difference	0	
df (degrees of freedom)	9	
t statistic	0.674178971	
P($T \leq t$) one-tail	0.258568814	
Critical t one-tail	1.833112933	
P($T \leq t$) two-tail	0.517137628	
Critical t two-tail	2.262157163	

Table 3.10: Hypothesis Test on Inflow Variance

	Std_dev (8760)	Std_dev (2016)
Mean	2966.637422	2854.353313
Variance	216,391.3248	594,079.873
Observations	10	10
Pearson correlation	0.514439683	
Hypothesized mean difference	0	
df (degrees of freedom)	9	
t statistic	0.534339345	
P($T \leq t$) one-tail	0.303026171	
Critical t one-tail	1.833112933	
P($T \leq t$) two-tail	0.606052342	
Critical t two-tail	2.262157163	

	Std_dev (8760)	Std_dev (288)
Mean	2966.637	2272.705
Variance	216,391.3	2,273,925
Observations	10	10
Pearson correlation	0.534455	
Hypothesized mean difference	0	
df (degrees of freedom)	9	
t statistic	1.663333	
P($T \leq t$) one-tail	0.065303	
Critical t one-tail	1.833112933	
P($T \leq t$) two-tail	0.130607	
Critical t two-tail	2.262157163	

Table 3.11: Hypothesis Test on Inflow Standard deviation

	Skewness (8760)	Skewness (2016)
Mean	13.76822	10.67016
Variance	4.865846	4.779153
Observations	10	10
Pearson correlation	-0.05394	
Hypothesized mean difference	0	
df (degrees of freedom)	9	
t statistic	3.07278	
P($T \leq t$) one-tail	0.006647	
Critical t one-tail	1.833112933	
P($T \leq t$) two-tail	0.013294	
Critical t two-tail	2.262157163	

	Skewness (8760)	Skewness (288)
Mean	13.76822	6.870808
Variance	4.865846	4.233476
Observations	10	10
Pearson correlation	-0.34838	
Hypothesized mean difference	0	
df (degrees of freedom)	9	
t statistic	6.228898	
P($T \leq t$) one-tail	7.67e-05	
Critical t one-tail	1.833112933	
P($T \leq t$) two-tail	0.000153	
Critical t two-tail	2.262157163	

Table 3.12: Hypothesis Test on Inflow Skewness

	<i>Kurtosis (8760)</i>	<i>Kurtosis (2016)</i>
Mean	279.7716569	156.7543
Variance	8783.084981	6270.694
Observations	10	10
Pearson correlation	-0.097719638	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	3.028092414	
$P(T \leq t)$ one-tail	0.007145382	
Critical t one-tail	1.833112933	
$P(T \leq t)$ two-tail	0.014290765	
Critical t two-tail	2.262157163	

	<i>Kurtosis (8760)</i>	<i>Kurtosis (288)</i>
Mean	279.7717	57.33682
Variance	8783.085	1202.697
Observations	10	10
Pearson correlation	-0.25551	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	6.517818	
$P(T \leq t)$ one-tail	5.46e-05	
Critical t one-tail	1.833112933	
$P(T \leq t)$ two-tail	0.000109	
Critical t two-tail	2.262157163	

Table 3.13: Hypothesis Test on Inflow Kurtosis

	<i>Maximum (8760)</i>	<i>Maximum (2016)</i>
Mean	87398.01	56083.5
Variance	3.12e+08	3.71e+08
Observations	10	10
Pearson correlation	-0.01872	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	3.753612	
$P(T \leq t)$ one-tail	0.002265	
Critical t one-tail	1.833112933	
$P(T \leq t)$ two-tail	0.004529	
Critical t two-tail	2.262157163	

	<i>Maximum (8760)</i>	<i>Maximum (288)</i>
Mean	87,398.00566	23,704.5
Variance	312,230,369.6	4.35e+08
Observations	10	10
Pearson correlation	0.358581089	
Hypothesized mean difference	0	
<i>df</i> (degrees of freedom)	9	
t statistic	9.165142416	
$P(T \leq t)$ one-tail	3.6781e-06	
Critical t one-tail	1.833112933	
$P(T \leq t)$ two-tail	7.35621e-06	
Critical t two-tail	2.262157163	

Table 3.14: Hypothesis Test on Inflow Maximum

The results for the inflow distributions reveal some additional differences. Skewness, kurtosis, and the maximum yield p-values that lead to the rejection of the null hypothesis of equality among the distributions. Nonetheless, the equality of the means, variances, and standard deviations was considered sufficient to justify the use of this sampling technique. Moreover, since the median and the minimum are exactly the same across the three series, no further hypothesis testing was necessary. The following section show the results obtained. I analyze them first by commenting on the case without a customer, which I take as the benchmark case, and then I compared it with the other models.

3.3 Analysis of Results

In the stochastic case, similar to the deterministic case discussed in the previous chapter, the models were evaluated without considering the customer's presence. As a result, an α value of 0 was set. Figure 3.9 shows the results obtained, presenting the mean spot prices and the total number of hours during which energy was produced or water was pumped for each scenario. Scenario 2 exhibits the highest profit among the 10

scenarios, despite having fewer production hours. This outcome can be attributed to the occurrence of higher average spot prices in scenario 2. In Table 3.15, the average daily spot price spread and the average margin per generation hour are reported. The former represents the mean difference between the highest and lowest prices observed in a day for each scenario⁵. The latter represents the average profit obtained for each hour of production per MW generated. Consequently, in scenario 2, the profit obtained by the owner for each MW produced was, on average, 215.74, significantly higher than the profit obtained in the other scenarios.

	Daily average Spot prices spread (€/MWh)	Average Profit per generation hour (€/MWh)
Scenario 1	51.68	91.43
Scenario 2	99.56	215.74
Scenario 3	60.25	110.29
Scenario 4	37.25	68.64
Scenario 5	37.78	72.31
Scenario 6	49.51	101.23
Scenario 7	66.05	129.63
Scenario 8	52.26	100.02
Scenario 9	69.20	127.31
Scenario 10	70.93	132.74

Table 3.15: Average spot prices daily spread and profit per generation hour

⁵ $\frac{\text{Highest } S_{jt} - \text{Lowest } S_{jt}}{365}$

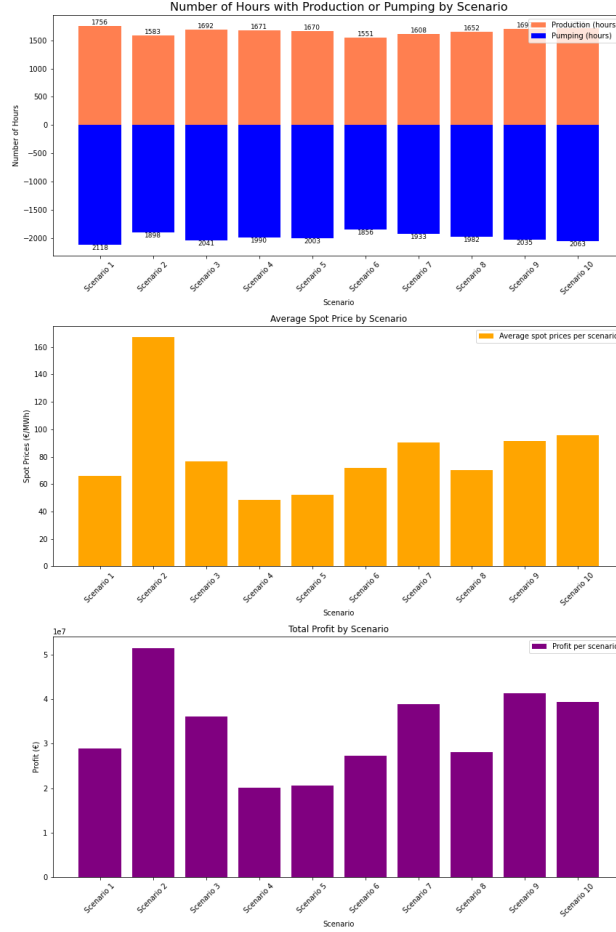


Figure 3.9: Profit obtained for each scenario without customer

In the first plot of Figure 3.9, it is observable that, across the 10 scenarios, the owner utilizes the facility infrequently. Taking scenario 1 as an example, the owner employs the facility to generate energy for 1,756 hours and to pump water for 2,118 hours. In total, the facility is utilized for 3,874 hours out of 8,760. Scenario 6 represents the least utilized configuration of the plant. The facility operates for energy production for 1,551 hours and water pumping for 1,856 hours, totaling 3,407 hours of operation. Figures 3.10 and 3.11 illustrate the average daily spot position assumed by the owner in the first and second months of scenario 6, respectively. It is evident that the owner did not engage in energy sales on the spot market (thus not producing energy) and did not purchase energy (thus not pumping water) for the majority of days.

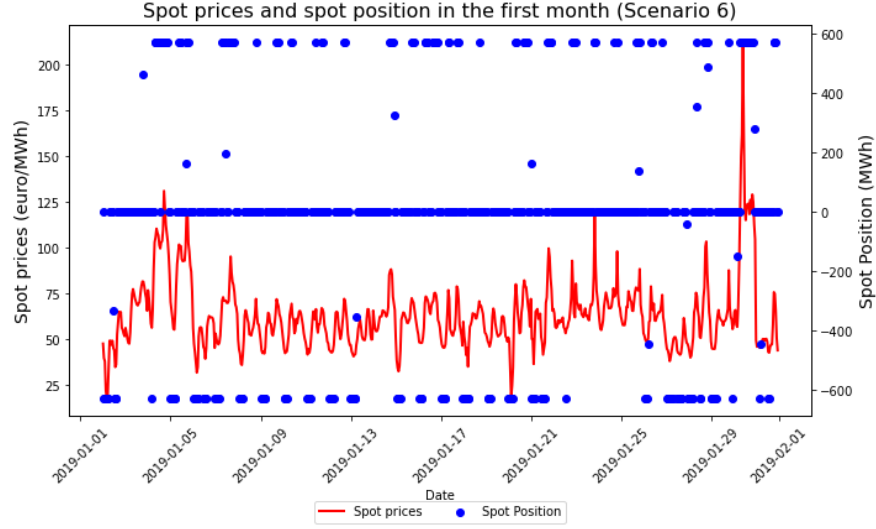


Figure 3.10: Average daily spot position in the first month of Scenario 6

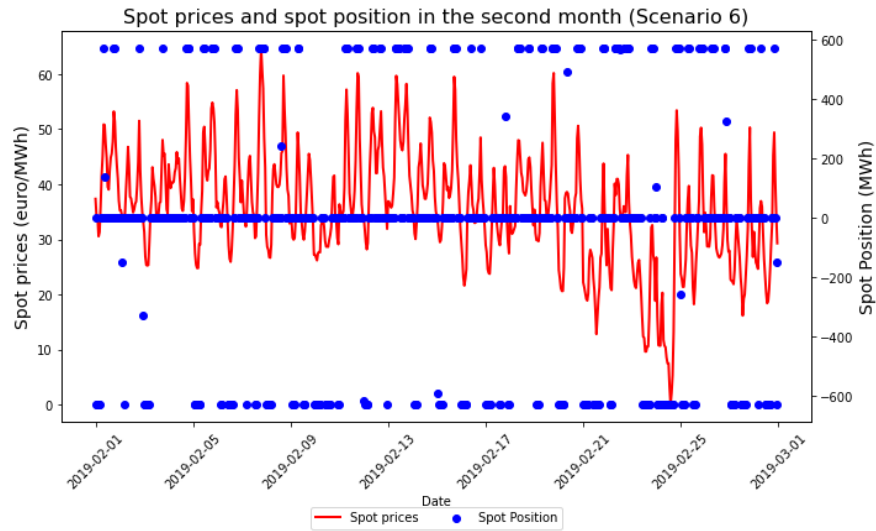


Figure 3.11: Average daily spot position in the second month of Scenario 6

The origin of such a consistent quantity of idle hours is simply due to the efficiency of the turbine. More precisely considering typical efficiency coefficients for generation (η_T) and pumping (η_P) :

- $\eta_T = 92.6\%$
- $\eta_P = 79.5\%$

These efficiency values indicate that utilizing the equivalent of 1 MW of water in the upper basin for energy production will yield only 0.926 MW of water. Furthermore, pumping the equivalent of 1 MW of water from the lower basin to the upper basin will result in only 0.795 MW of water being obtained. Figure 3.12

illustrates the impact of these efficiencies on production and pumping. At $t = 1$, the equivalent of $\frac{1}{\eta_T} \approx 1.26$ MW of water is pumped from the lower basin to the upper basin. Due to the efficiency of the pump, the potential energy obtained in the upper basin is about 1 MW. When this 1 MW is generated, the potential energy obtained in the lower basin, accounting for the efficiency of the turbine, is about $\eta_P = 0.93$ MW.

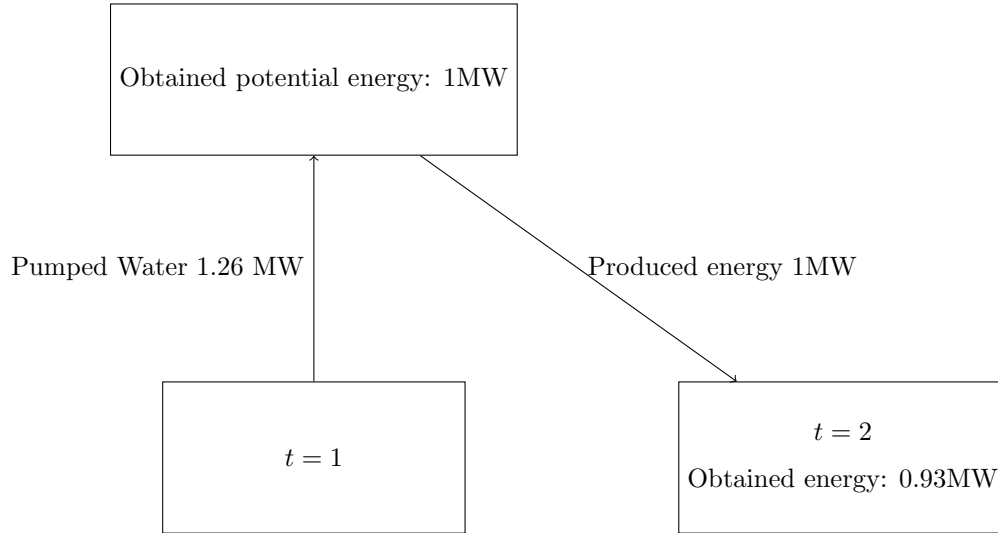


Figure 3.12: Loss of energy due to the efficiency of the turbine

The ratio between the initial potential energy and the obtained energy at the end is:

$$\frac{1}{\eta_T \eta_P} \approx 1.36$$

The concept of price power ratio is subsequently introduced. When the ratio between the highest spot price and the lowest spot price, identified within a specific time frame, is less than 1.36, it is advantageous for the plant to remain inactive. That is to say that the ratio suggests that, after pumping water, it is preferable to convert the water reserve into sold energy if the production price is at least 36% higher than the cost of pumping. Figure 27 illustrates the price power ratio observed in the first month of scenario 6. Each graph represents a week of the month, with four price ratios calculated daily, one for each quarter of the day. It is evident that the price power ratio frequently remains below the red dotted line, which denotes the target ratio of 1.36. This observation explains why, in the first month, the spot position was often equal to 0, as depicted in Figure3.10.

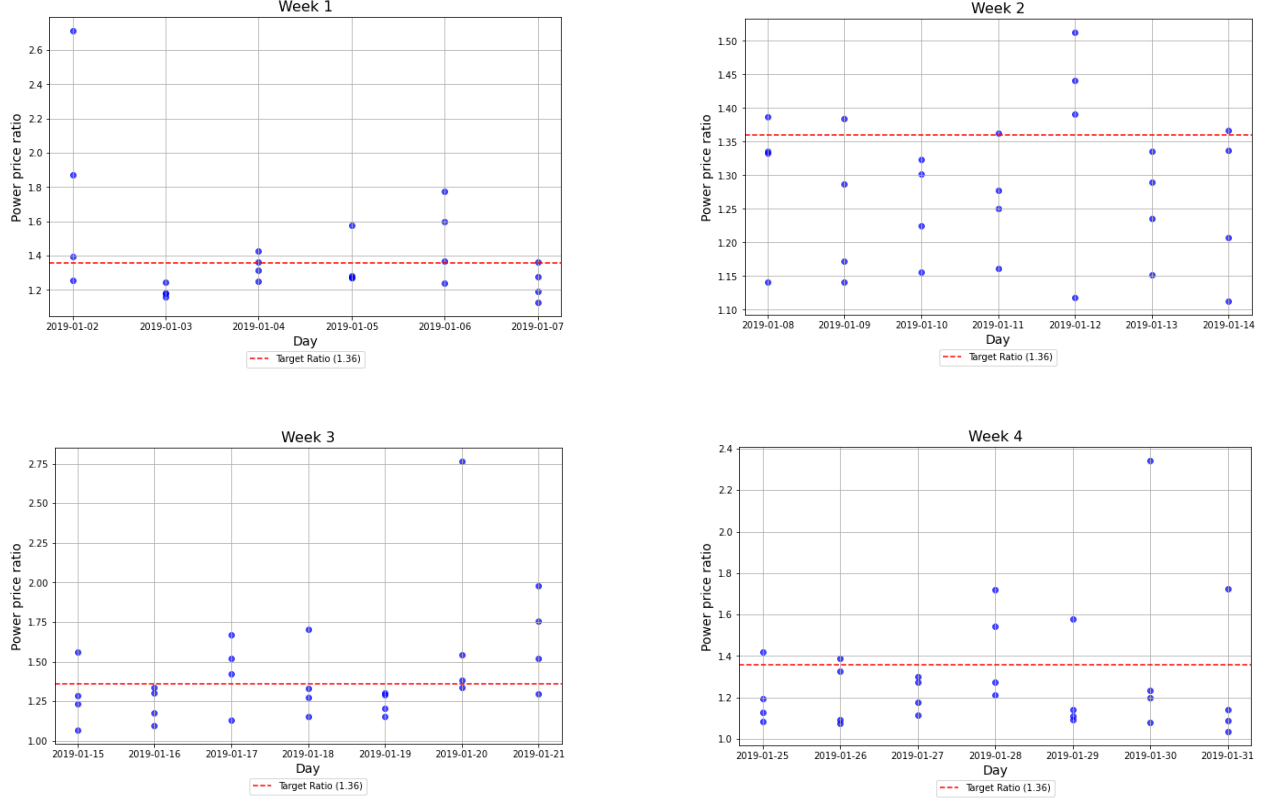


Figure 3.13: Power price ratio in the first month of Scenario 6

The concept of price power ratio was used to simulate a more conservative smart customer. The smart costumer was simulated as a customer that pumps or produce when the ratio between the maximum price and the minimum is equal to 1.5. To do that, the efficiency of the turbine available for the customer was modified such that:

$$1.5 = \frac{1}{\eta_P \cdot \tilde{\eta}_T}$$

The obtained $\tilde{\eta}_T$ (83.9%) is then used to calculate the maximum capacity of the turbine utilize in the simulation of the smart customer:

$$\tilde{X}_{max}^+ = \frac{\tau_T \cdot d_w \cdot g \cdot \tilde{\eta}_T \cdot h}{10^6}$$

Where the value of each parameter is given in Table 3.16.

	Definition	Value
$\tau_T (m^3/s)$	Turbine flow	126
$d_w (Kg/m^3)$	Water density	1000
$g(m/s^2)$	Gravity	9.81
$\tilde{\eta}_T$	Efficiency of the turbine	83.9%
$h (m)$	height of the falls	500

Table 3.16: Turbine’s capacity parameters

By constraining $\tilde{X}_{max}^+$ as the maximum capacity of the turbine in the smart customer simulation, it was possible to simulate a customer that makes intelligent decisions, but is also more conservative than the owner. Following the analysis of the model’s properties without considering the customer presence, the optimal solution of the three models is examined, reintroducing the possibility of selling a portion of the storage capacity. Due to the high computational time, several heuristics were implemented. The financial model was the easiest to solve, as it involved fewer non-linear constraints, so only the first heuristic was required. For the physical and hybrid models, however, multiple heuristics had to be applied. Initially, a single heuristic was used: a grid of α values was created, ranging from 0 to 1 with increments of 0.1. This approach was sufficient to find an acceptable solution for the financial model. However, the computational time remained excessively high for the physical and hybrid models, as shown in Table 3.17.

	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$
Financial model	1,170s $\approx 19min$	1,213s $\approx 20min$	1,221s $\approx 20min$
Physical model	1,217s $\approx 20min$	10,980s $\approx 3h$	11,450s $\approx 3h$
Hybrid model	10,076s $\approx 3h$	169,730s $\approx 2days$	171,932s $\approx 2days$

Table 3.17: Computational time applying the first heuristic

The second heuristic consists of selecting only the first week of each month for each scenario j , resulting in a total of 10 scenarios of 2016 hours each. This approach allows for the consideration of a smaller data frame while still encompassing each day of the week.

Upon application of the second type of heuristic, the computational time for the physical model reduced significantly. However, for the hybrid model, the computational time remained excessively high. A noteworthy observation regarding the hybrid model is that the computational time increases with the fixed alpha value, as demonstrated in Table 3.18.

	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$
Physical model	$938s \approx 15min$	$989s \approx 16min$	$984s \approx 16.4min$
Hybrid model	$1,078s \approx 17min$	$33,731s \approx 9h$	$37,224s \approx 10h$

Table 3.18: Computational time applying the first and the second heuristic

It therefore became necessary to apply a more stringent heuristic to the hybrid model. In this case, the first day of each month was selected, resulting in a total of 10 scenarios of 288 hours each. Unfortunately, the necessity of applying different heuristics for each model led to models that are not fully comparable in their results, as they are optimized based on different set of scenarios. The results obtained for each model are therefore only comparable within the same model. In the subsequent paragraphs, the results of the three models are analyzed.

Financial model results

Smart customer Table 3.19 reports the results obtained considering a smart customer, applying a grid of α values ranging from 0 to 1, with a step size of 0.1. Table 3.19 presents the optimal objective, which represents the expected profits of the owner, while also taking into account its risk aversion, for each α considered. With a smart customer, the greater the storage capacity transferred, the greater the loss for the owner. This is a logical conclusion, as, by construction a smart customer makes profitable decisions. Since the owner is the financial counterpart of the customer he symmetrically loses money as the customer makes profits. This phenomenon is also reflected in the customer's average profit. As the α increases, the profit of the smart customer also increases. The average customer's profits are calculated as the mean of the profit resulting to him over the 10 scenarios. The *transfer price* P was calculated ex-post, as the price per MWh at which the storage capacity must be sold in order to recover the loss incurred from the transfer of storage capacity. More specifically was calculated as:

$$P = \frac{\Pi_{\alpha>0} - \Pi_{\alpha=0}}{\alpha X_{max}^u}$$

where $\Pi_{\alpha>0}$ is the expected profit obtained by selling part of the storage capacity and αX_{max}^u is the relinquished storage capacity. It is observed that the increase in loss, associated with the rise in α , is also reflected in the transfer price, which likewise increases as α grows. The Var_{β} decreases with the reduction of the optimal objective. Figure 3.14 presents the graphical representation of the results obtained.

α	Objective (€)	Average customer's profit (€)	Transfer price (€/MWh)	Var_{β} (€)
0	31,910,101	0	0	20,121,318
0.1	28,940,046	3,092,074	3006	18,255,115
0.2	25,980,041	6,175,321	3215	16,398,690
0.3	23,022,776	9,255,078	2143	14,543,778
0.4	20,067,994	12,333,323	3318	12,690,157
0.5	17,106,733	15,416,012	3340	10,831,755
0.6	14,136,320	18,504,594	3355	8,971,856
0.7	11,157,716	21,596,810	3366	6,968,762
0.8	8,173,975	24,695,011	3376	5,015,170
0.9	5,182,463	27,802,117	3384	3,063,998
1	2,185,491	30,914,195	3391	1,108,381

Table 3.19: Results of the financial model considering a smart customer

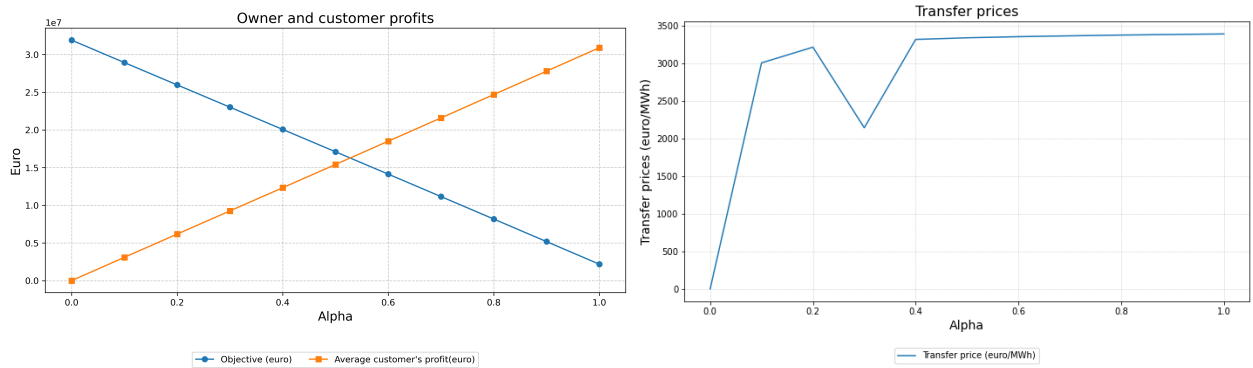


Figure 3.14: Results of the financial model considering a smart customer

The most promising α region was identified between 0 and 0.1. Within this interval, the model were further tested using a denser α grid, with α increasing in increments of 0.01. As shown in figure 3.15, considering a smart counterpart and a financial model, the optimal region was identified for an α between 0 and 0.1. Testing the model within this region, it is observed that the final expected profit decreases linearly as the transferred storage capacity increases. Figure 3.16 illustrates the relationship between the expected profit and the profit generated on average across 10 scenarios by the counterpart. As can be seen, even within the optimal region, the greater the transferred storage capacity, the higher the customer's profit, and conversely, the lower the owner's final profit.

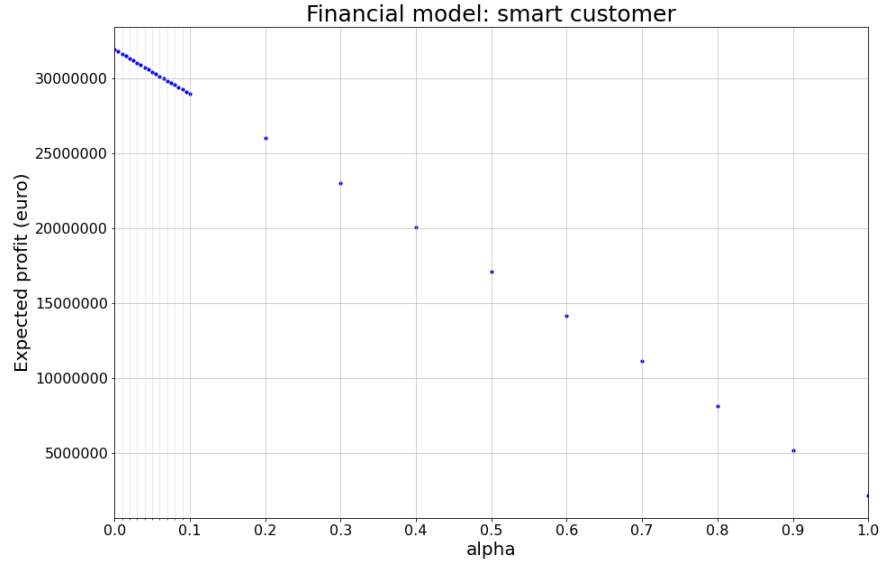


Figure 3.15: Trend of optimal expected profit as a function of α for a smart customer in a financial model

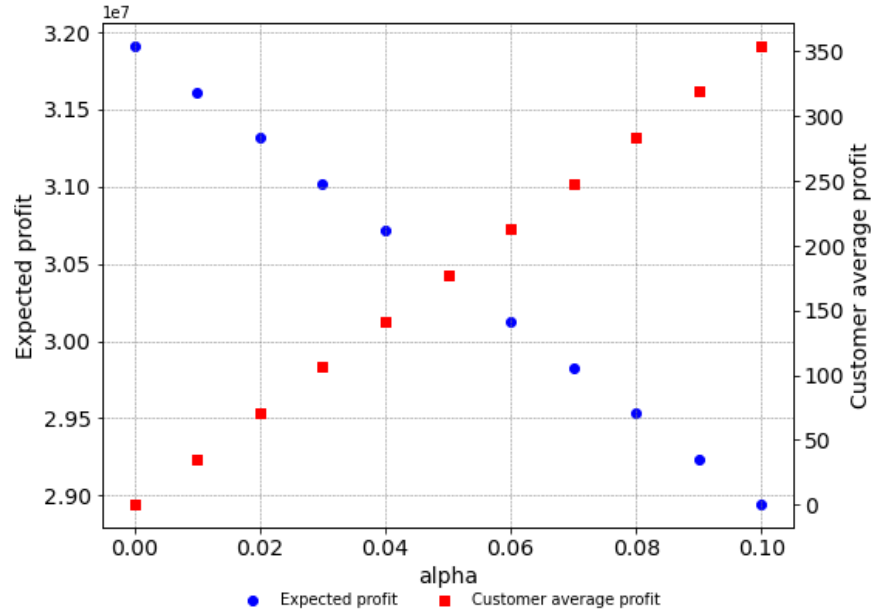


Figure 3.16: Expected profits and customer's average profit depending on α considering a smart customer

Partially predictable customer Table 3.20 presents the results obtained for a partially predictable customer. It should be noted that the optimal objective increases proportionally with the percentage of transferred capacity. Under the financial market model, the model is required to compensate the customer for production orders. This dynamic is also reflected in the average customer's profit, which decreases as the sold storage capacity increases. Figure 3.17 presents the graphical representation of the results obtained.

α	Objective (€)	Average customer's profit (€)	VaR_β (€)
0	31,910,101	0	20,121,318
0.1	37,794,848	-6,113,385	23,948,322
0.2	43,060,444	-11,583,625	27,372,120
0.3	47,683,647	-16,382,312	30,415,967
0.4	51,664,450	-20,510,068	34,100,964
0.5	55,019,019	-23,985,995	35,336,542
0.6	57,636,431	-26,830,256	37,184,434
0.7	59,892,470	-29,041,488	38,571,624
0.8	61,383,712	-30,592,913	40,889,917
0.9	62,266,638	-31,511,621	41,503,926
1	62,531,147	-31,787,086	41,672,848

Table 3.20: Results of the financial model considering a Partially predictable customer

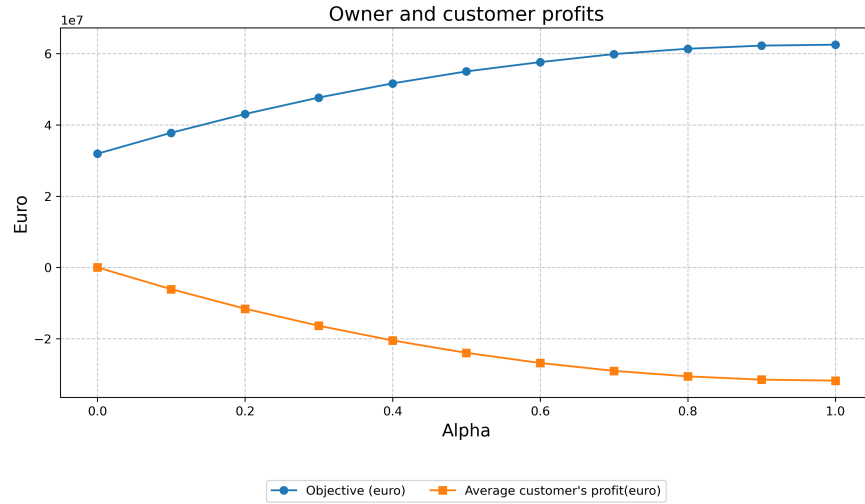


Figure 3.17: Results of the financial model considering a Partially predictable customer

The most promising region is identified for an α between 0.9 and 1. As shown in Figure 3, the growth in expected profit slows within the optimal region until it levels off. As demonstrated in figure 3.19, the customer's average profit decreases in a manner that mirrors the increase in expected profit. This increase is due to a rise in the customer's pumping decisions, as illustrated in figure 3.20. Additionally, the growth curve of the customer's pumping decisions also decelerates as α increases. This outcome is attributable to the pump's capacity, which limits the number of cubic meters the customer can utilize each hour to produce energy.

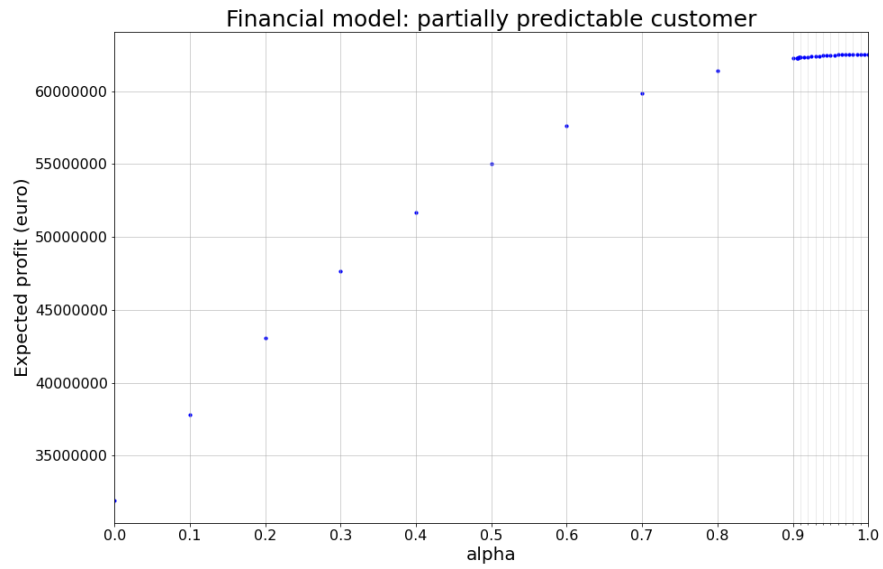


Figure 3.18: Trend of optimal expected profit as a function of α for a partially predictable customer in a financial model

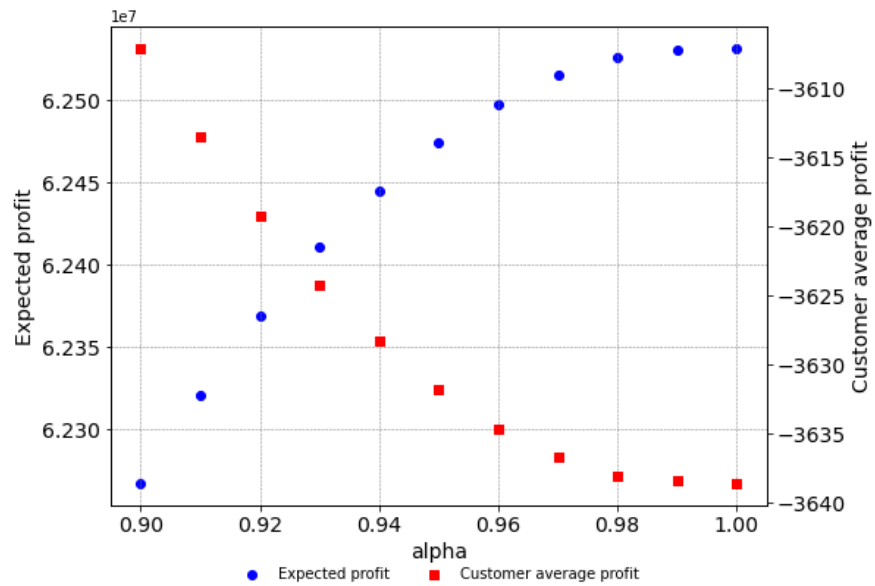


Figure 3.19: Expected profits and customer's average profit depending on α considering a partially predictable customer

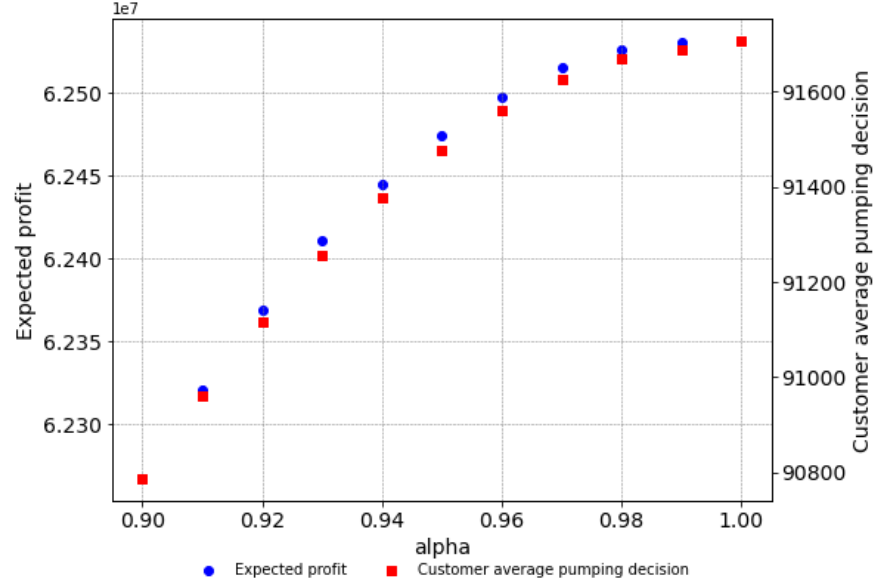


Figure 3.20: Expected profits and customer's average pumping decisions depending on α considering a partially predictable customer

Physical model results

The analysis of the results obtained for the physical model yields several noteworthy observations. As evident from a comparison of the results reported in Tables 3.21 and 3.22, considering first a smart customer and then a partially predictable customer, in both cases the owner incurs losses when selling storage capacity to the counterpart. It is observed, however, that as long as the ceded storage capacity is between 10% and 60%, the owner's final profit decreases in both cases, albeit more rapidly when considering a partially predictable customer. This outcome can be explained considering that a partially predictable customer often prevents the owner from producing energy or pumping water when optimal, if these choices conflict with those of the customer. Consequently, the owner is significantly constrained in his opportunities to generate profit. However the owner is also limited by a smart customer. Despite the owner and the smart customer often agree about the "operating direction" of the turbine the two are forced to share the turbine. However, when the owner begins to cede from 70% to 100% of the storage capacity to the counterpart, the situation reverses. In this instance, it is less onerous to interact with a partially predictable customer rather than a smart customer. By selling storage capacity, the owner also relinquishes the capacity of the turbine and pump. Consequently, with a smart customer, it becomes nearly impossible to access the pump and turbine during the most profitable hours as the ceded capacity increases. With a partially predictable customer, there remains the possibility of accessing the turbine if the customer is not utilizing it, even when selling the entirety of the storage capacity. Indeed, what is sold is the capacity in MW of the upper reservoir. The owner, therefore, still has access to the water in the lower reservoir and has the opportunity to utilize it during hours when the customer is not using the facility. By interfacing with a partially predictable customer, it is

thus possible to exploit the facility during certain more profitable hours.

α	Objective (€)	Transfer price (€/MWh)	Var_{β} (€)
0	8,670,354	0	4,260,797
0.1	7,863,703	946	5,046,414
0.2	7,058,415	945	3,457,785
0.3	6,253,466	945	3,057,849
0.4	5,448,159	945	2,659,183
0.5	4,642,058	945	2,260,591
0.6	3,833,950	946	1,859,541
0.7	3,022,742	946	1,454,610
0.8	2,204,802	948	1,049,085
0.9	1,373,411	951	642,373
1	270,924	985	120,532

Table 3.21: Results of the physical model considering a smart customer

The calculated transfer price and the Var_{β} exhibit a similar pattern. In the case of a partially predictable customer, the transfer is higher when considering a lower alpha, but consistently decreases. Conversely, with a smart customer, the transfer price demonstrates less variability but tends to increase. Figures 3.21 and 3.22 provide a graphical representations of the obtained results.

α	Objective (€)	Transfer price (€/MWh)	Var_{β} (€)
0	8,670,354	0	4,260,797
0.1	5,289,534	3959	2,526,030
0.2	4,930,312	2189	2,330,422
0.3	4,599,793	1608	2,149,995
0.4	4,302,980	1277	1,990,249
0.5	4,033,048	1085	1,841,958
0.6	3,798,601	950	1,716,852
0.7	3,600,377	847	1,615,713
0.8	3,442,669	764	2,076,395
0.9	3,325,076	694	1,450,945
1	3,246,148	634	1,934,088

Table 3.22: Results of the physical model considering a partially predictable customer

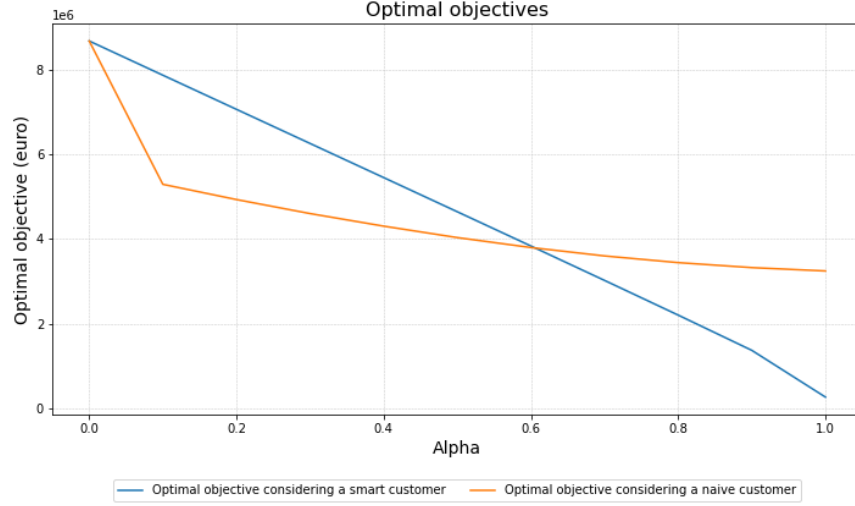


Figure 3.21: Optimal objective considering a smart and a partially predictable customer

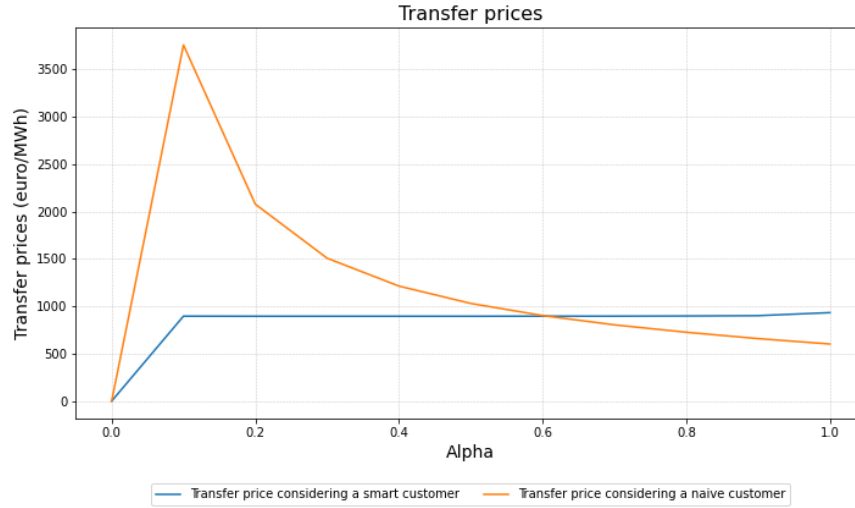


Figure 3.22: Transfer prices considering a smart and a Partially predictable customer

In both cases the most promising region was identified for an α between 0 and 0.1. Considering a smart customer, in the optimal region identified can be seen that as the transferred storage capacity increases, the final expected profit decreases (see figure 3.23). This outcome is explained by the fact that as storage capacity is transferred, turbine capacity is also transferred. Consequently, the higher the α , the greater the customer's production and the lower the owner's production, which is forced to share the turbine capacity with the counterpart during the hours when both make production decisions. This phenomenon is depicted in figure 3.24.

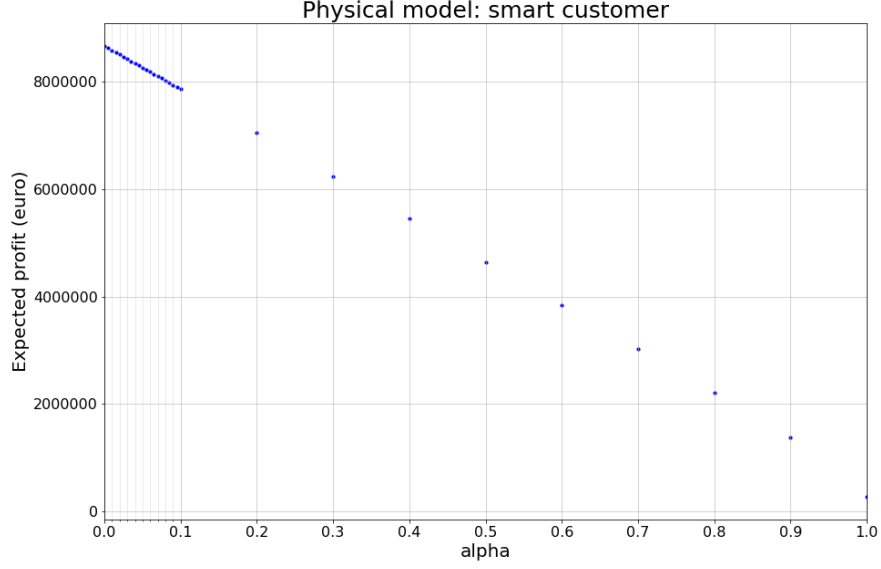


Figure 3.23: Trend of optimal expected profit as a function of α for a smart customer in a physical model

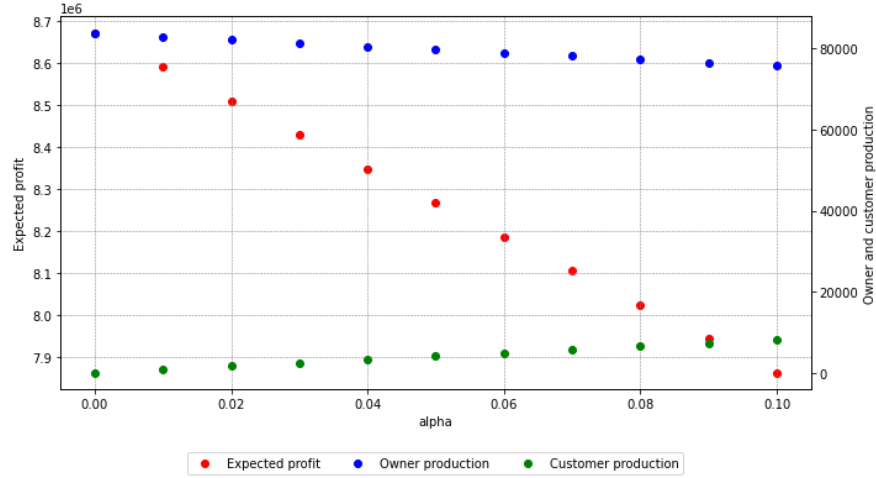


Figure 3.24: Expected profits and production decisions depending on α considering a smart customer

Taking into account a partially predictable counterpart, it is observed that transferring a minimal portion of storage capacity, specifically 1%, is sufficient to significantly reduce the expected profit, from approximately 9,000 approximately to approximately 6,000,000. When more than 1% storage capacity is transferred, the expected profit continues to decrease, although at a slower rate. This phenomenon is explained by the frequency of optimal production choices. With a partially predictable counterpart, even a small transfer of storage capacity is enough for the owner to be penalized by the counterpart's opposing production choices. If the counterpart decides to pump water when it is optimal for the owner to generate energy, the owner consequently cedes the turbine to the counterpart, thereby losing profit opportunities. Figure 3.26 illustrates the comparison between the frequency of production orders when the counterpart is not considered and when

part of the storage capacity has been transferred to the counterpart. As can be seen, after transferring 1% of the storage capacity to the counterpart, the frequency of production choices executed by the owner drops sharply, then remains constant as the transferred storage capacity increases.

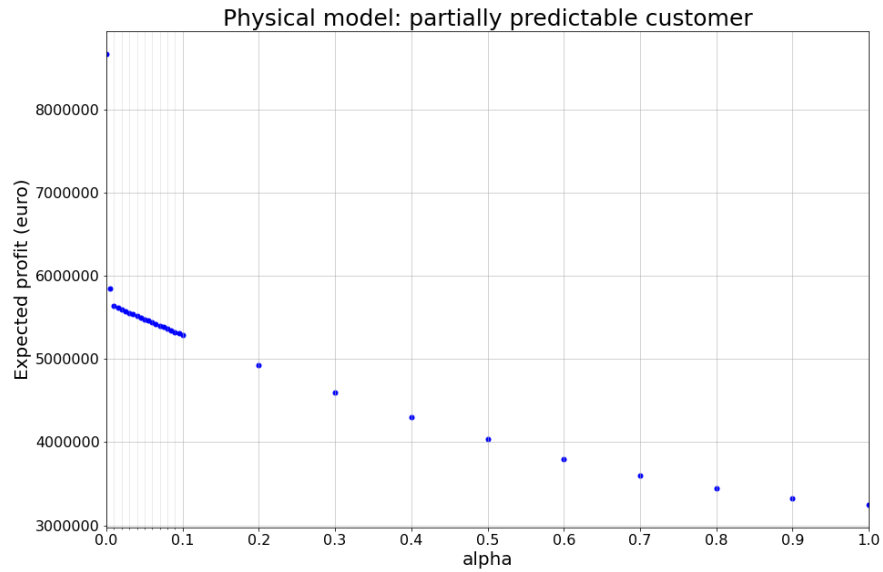


Figure 3.25: Trend of optimal expected profit as a function of α for a partially predictable customer in a physical model

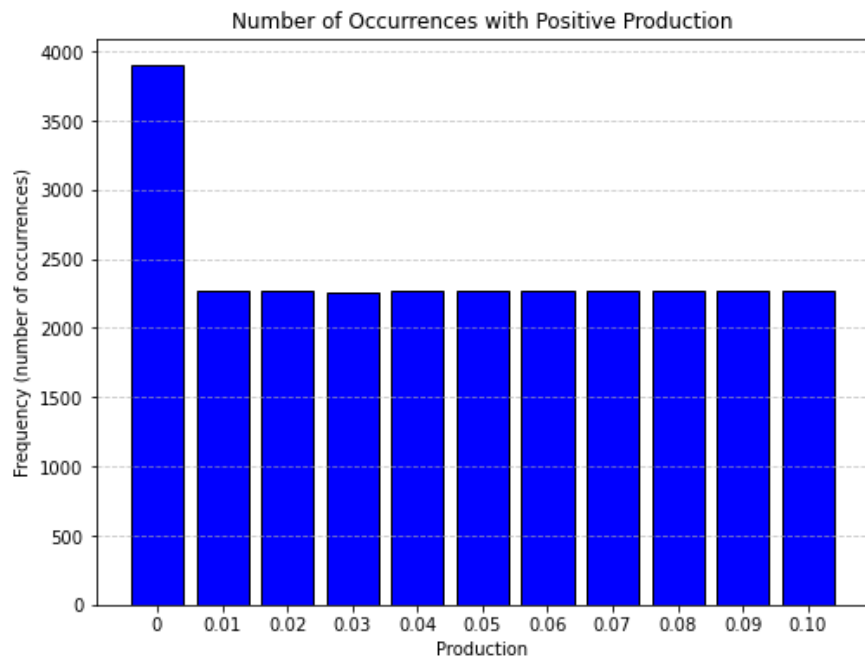


Figure 3.26: Number of occurrences with positive production considering a partially predictable customer in a physical setting

Hybrid model results

For the hybrid model, it was necessary to apply a stricter heuristic. Instead of selecting the first week for each month of each scenario, only the first day was selected, for a total of 288 hours for each scenario j . This allowed for a significant reduction in computation time, as observed in Tables 3.23 and 3.24. The hybrid model is the most computationally intensive, as it contains the highest number of non-linear constraints. The majority of these constraints are associated with customer orders and their potential compensation. This explains why higher α values result in increased computational time. It is also noteworthy that the optimization of the model considering a partially predictable customer required more computation time than the optimization with a smart customer. This can be attributed to the fact that the smart customer was simulated to behave in an optimal manner and to be more conservative than the owner⁶. Consequently, there were fewer orders to process and compensate compared to the partially predictable customer, which was simulated in a more random manner.

Tables 3.23 and 3.24 present the obtained results for the hybrid model, considering first a smart customer and then a partially predictable customer. In both cases, the optimal objective decreases with the increase in transferred storage capacity. It is noteworthy that by transferring only 10% or 20% of the storage capacity to a smart counterpart rather than a partially predictable one, the optimal objective experiences a less severe reduction. This can be attributed to the fact that in these two cases, the average profit obtained by the owner for each scenario j , considering a partially predictable customer, was significantly lower than the average profit obtained considering a smart customer. As shown in Table 3.25, considering an α of 0.2, this is due to the fact that with a partially predictable customer, the owner was unable to produce the same amount of energy as in the case with a smart customer, since more water in the upper basin was utilized to compensate for the customer's pumping orders. As the transferred capacity remains relatively small, the pumping compensation profits were insufficient to offset the lost production.

As the transferred storage capacity increases, the situation changes, with the results obtained from the partially predictable customer demonstrating a more favorable outcome compared to those from a smart customer. This phenomenon is attributed to the fact that with higher transferred capacity, the profits obtained from the compensation of pumping orders also increase. In the case of a partially predictable customer, there are more opportunities to compensate pumping orders, thereby mitigating the lost profits resulting from the introduction of the customer.

⁶The smart customer considers a price power ratio threshold of 1.5 instead of 1.36

α	Objective (€)	Transfer price (€/MWh)	Average pumping profits (€)	Var_{β} (€)	Computational time
0	2,343,233	0	0	1,112,607	137s
0.1	2,192,886	174	39,330	1,024,929	140s
0.2	2,042,520	174	80,213	935,456	183s
0.3	1,889,124	116	125,297	966,034	190s
0.4	1,735,589	222	237,135	756,322	303s
0.5	1,579,204	177	237,135	665,795	334s
0.6	1,419,496	178	308,321	572,714	416s
0.7	1,255,327	180	383,092	476,550	745s
0.8	1,092,299	182	622,798	380,103	1668s
0.9	963,594	178	833,789	320,470	5640s
1	808,992	179	1,159,921	243,634	5920s

Table 3.23: Results of the hybrid model considering a smart customer

α	Objective (€)	Transfer price (€/MWh)	Average pumping profits (€)	Var_{β} (€)	Computational time
0	2,343,233	0	0	1,112,607	137s
0.1	2,012,476	383	553,666	890,325	498s
0.2	1,972,246	214	1,028,340	869,969	705s
0.3	1,919,156	163	1,435,532	840,564	1005s
0.4	1,843,048	144	1,778,380	802,700	1572s
0.5	1,743,798	139	2,103,297	746,143	1174s
0.6	1,655,311	133	2,405,923	727,378	1555s
0.7	1,561,929	129	2,658,350	686,515	27346s
0.8	1,455,262	129	2,890,417	601,888	16065s
0.9	1,343,661	129	3,167,396	505,444	18008s
1	1,227,367	128	3,492,624	410,624	23,130s

Table 3.24: Results of the hybrid model considering a Partially predictable customer

	Smart Customer	Partially predictable Customer
Average profit (€)	2,165,526	2,094,720
Profit/loss obtained in the spot market on average (€)	2,155,211	1,121,749
Energy produced and sold in the spot market on average (MWh)	31,848	27,128

Table 3.25: Owner's profits with an α of 0.2

Whether considering a smart customer or a partially predictable customer, the expected profit decreases

within the identified most promising area, between 0 and 0.1. With a smart customer, a linear decrease is observed. As shown in Figure 10, this decline is due to a drop in physical production and, consequently, in the energy that can be sold on the spot market. However, this loss is mitigated by the profits obtained from compensating the customer's pumping orders.

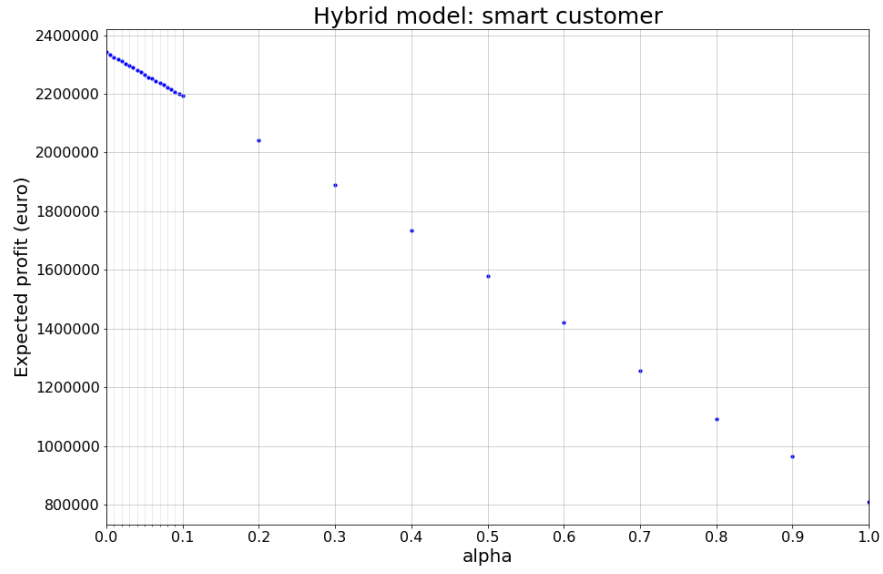


Figure 3.27: Trend of optimal expected profit as a function of α for a smart customer in a hybrid model

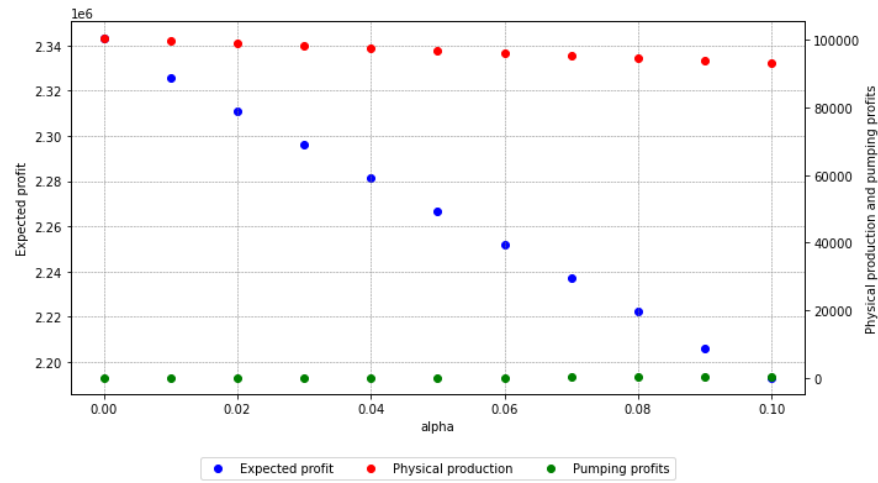


Figure 3.28: Expected profits, physical production and pumping profits considering a smart customer

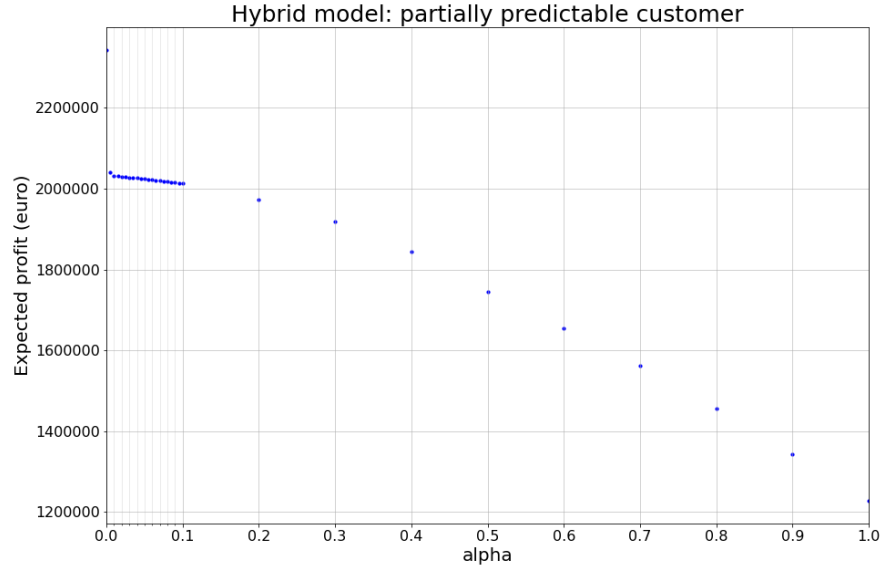


Figure 3.29: Trend of optimal expected profit as a function of α for a partially predictable customer in a hybrid model

With a partially predictable customer, after an initial rapid decline, the expected profit tends to decrease more slowly, thanks to the higher profit achieved through compensating the customer's orders. As shown in Figure 3.31, the profits derived from the compensations are higher than those obtainable with a smart customer.

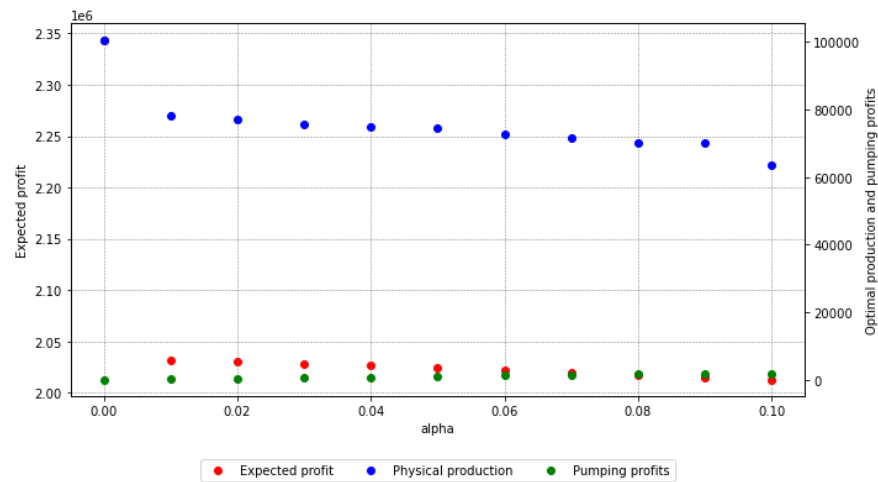


Figure 3.30: Expected profits, physical production and pumping profits considering a partially predictable customer

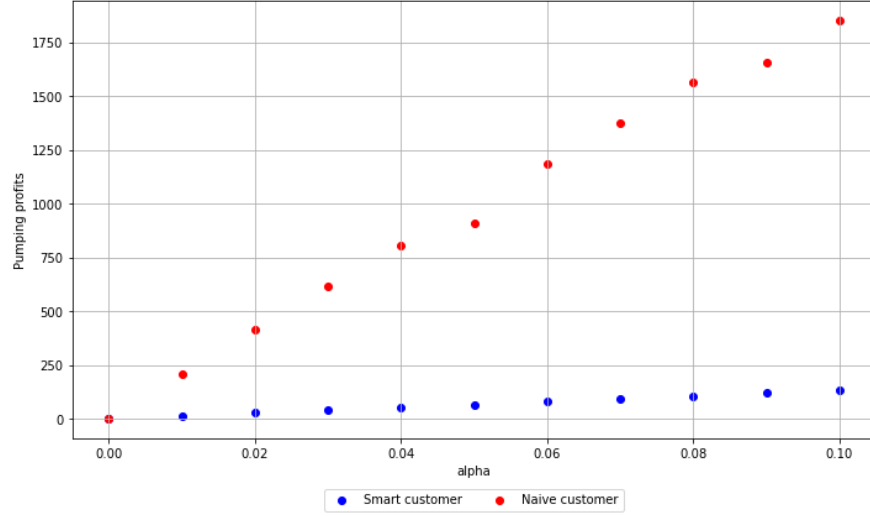


Figure 3.31: Pumping profit obtained with a smart and a naive customer

When considering a partially predictable customer, an interesting phenomenon is observed: when 50% of the capacity is transferred, the profits derived from the compensation of the customer's pumping orders exceed the optimal objective. Comparing the profits obtained in the spot market by the owner, considering an α of 0.4 and 0.5, it is evident that transferring half of the capacity to the counterpart results in negative profits. To compensate for more partially predictable pumping orders, it is necessary to pump more water from the lower basin and produce less energy, ensuring sufficient water in the upper basin. As shown in Table 3.26, a higher α necessitates more pumping, consequently resulting in significantly lower average profits for the owner in the spot market. However, the increased profit obtained from compensating the customer's pumping orders helps mitigate this loss.

	$\alpha = 0.4$	$\alpha = 0.5$
Profit/loss obtained in the spot market on average (€)	230,819	-201,542
Energy produced and sold in the spot market on average (MWh)	24,119	22,017
Energy required to pump water on average (MWh)	34,637	35,474

Table 3.26: Owner's profits considering a Partially predictable customer and an α of 0.4 and 0.5

It is observed that, when considering a partially predictable counterpart, the transfer price also decreases as α increases. This indicates that the higher the storage capacity relinquished, the lower the price at which this capacity must be sold to compensate for the loss. Conversely, if the counterpart is a smart customer, it is consistently more profitable to refrain from selling storage capacity, as there are insufficient opportunities for profiting from the customer's pumping orders in this scenario.

3.4 Sensitivity analysis

A sensitivity analysis was conducted by testing the model with different turbine capacities. The maximum pumping capacity and the plant duration were updated accordingly. The initial values of the turbine and pump capacities were modified by increasing both by 15% in three successive steps and decreasing them by 15% in two additional steps, resulting in a total of six capacity values. The full set of values considered is presented below, where the values highlighted in yellow represent the original technical specifications considered for the plant.

Turbine Production Capacity (MW)	Turbine Pumping Capacity (MW)	Facility duration (h)
413	453	21
486	533	18
572	628	15
657	722	13
756	830	12
869	955	10

The different turbine capacity values were assigned exclusively to the owner. The customer, at most, was granted a fixed storage duration of 8 hours, a maximum turbine capacity of 572 MWh, and a maximum pumping capacity of 628 MWh. In future analyses, the models will be tested by assigning the owner and the customer the same maximum generation and pumping capacities.

The results obtained so far are presented below.

Financial model

The results are presented through 3D plots, where the y-axis represents the turbine capacity, the x-axis the values of α , and the z-axis the expected final profit in millions of euros. Starting from the financial model and considering a smart-type customer, it can be observed that, for each turbine capacity considered, the expected profit decreases as α increases, as previously analyzed by the dynamic grid search refinement. However, an interesting detail emerges: when the turbine capacity increases relative to the initial technical specifications, and for values of α greater than 0.7, the expected profit still decreases, but at a slower rate compared to other cases, resulting in a noticeable bend in the surface, as shown in figure 3.32. This phenomenon can be explained by the fact that even with an α equal to 1, the customer has access to a maximum capacity of 572 MW, whereas the owner benefits from higher capacities. As a result, the owner can partially offset the customer's actions by producing a greater amount of energy and reselling it on the spot market, thus mitigating the financial impact of the counterpart compensation.

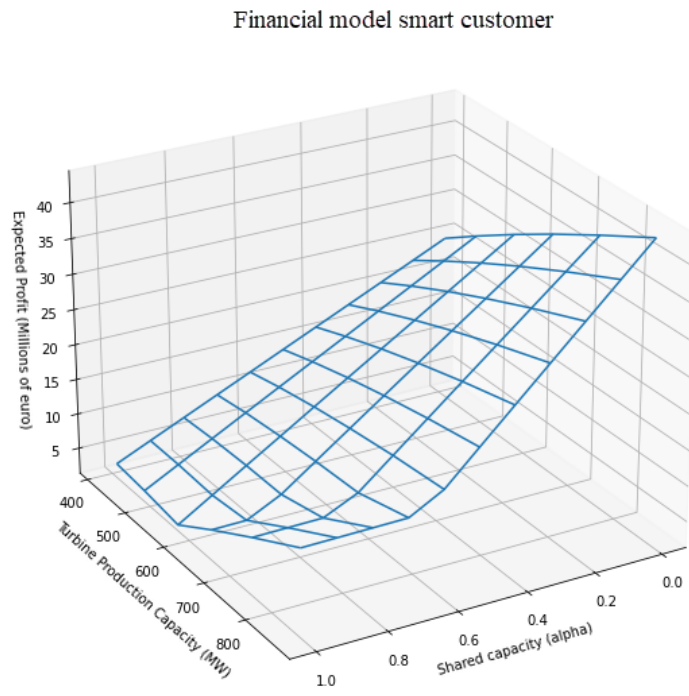


Figure 3.32: Expected profit considering different turbine capacity with a smart customer and a financial model

Moving to a partially predictable counterpart, the results once again reflect the patterns observed in previous analyses. As shown in figure 3.33, the greater the capacity allocated to the customer, the higher the expected final profit obtained by the owner, at all levels of turbine capacity considered. Similarly, it can be observed that, as the turbine capacity increases and for values of α greater than 0.7, the expected profit increases more sharply compared to previous cases. This behavior can again be attributed to the greater generation capacity available to the owner in comparison to the counterpart. The higher profit obtained by the owner is attributable to the greater amount of energy that can be sold on the market, relative to the energy requested by the customer.

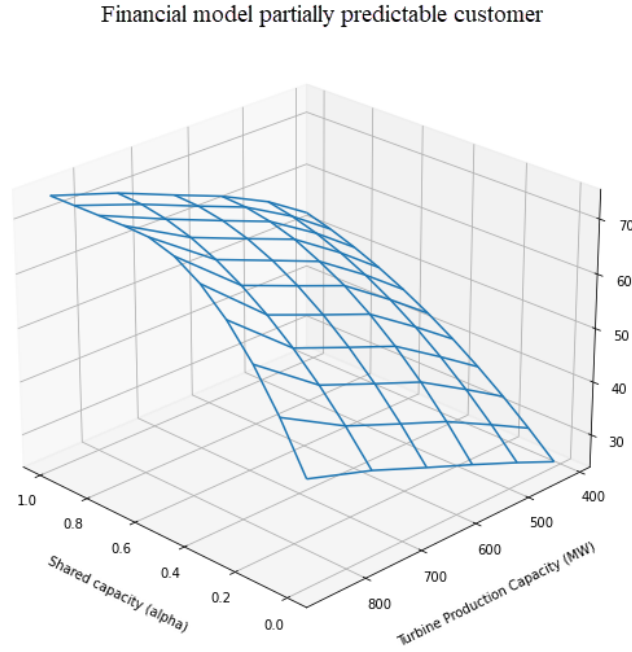


Figure 3.33: Expected profit considering different turbine capacity with a partially predictable customer and a financial model

Physical model

When considering a physical market and a smart counterpart, the same phenomenon observed in the financial model can be identified. As shown in figure 3.34, and by examining the results for each turbine capacity individually, it becomes evident that the final profit decreases as α increases. However, when looking at the overall 3D surface, a noticeable fold emerges for higher turbine capacities (relative to the original value) and α values greater than 0.7, indicating that, in these cases, the expected profit decreases less sharply. Compared to the results obtained for the same smart customer in the financial model (figure 3.32), this discrepancy in the rate of decline appears to be less pronounced. In fact, within a physical model, the owner interacting with a smart customer must share the ability to generate electricity. Nevertheless, since the customer is granted a maximum of 572 MW, and the owner retains access to a higher generation capacity, the owner can

partially mitigate the losses resulting from the production constraints imposed by the presence of the smart counterpart.

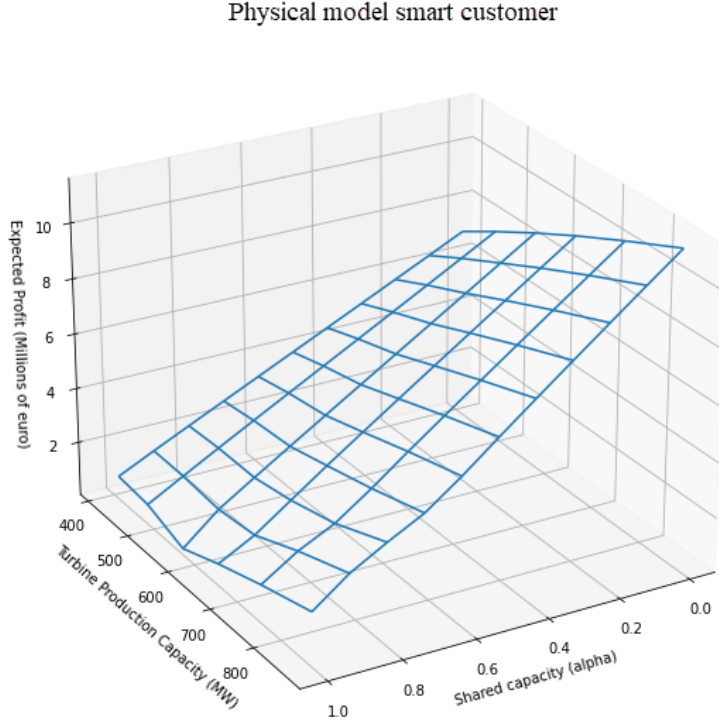


Figure 3.34: Expected profit considering different turbine capacity with a smart customer and a physical model

When moving to a partially predictable customer, the expected final value again exhibits patterns similar to those previously observed. As shown in figure 3.35, for each turbine capacity considered, there is a significant drop in the expected profit when moving from $\alpha = 0$ to $\alpha = 0.1$. As demonstrated in Section 3.3, in the presence of a partially predictable counterpart, even the transfer of a small share of storage capacity can drastically reduce the owner's production opportunities and, consequently, the expected value. For α values greater than 0.7, and when the turbine capacity exceeds 572 MW, a slowdown in the decline of the expected value can be observed—although this effect is less pronounced than in previous cases. This is because, when dealing with a partially predictable customer in a physical market, the main factor affecting expected profit is the lost opportunity to generate energy.

Physical model partially predictable customer

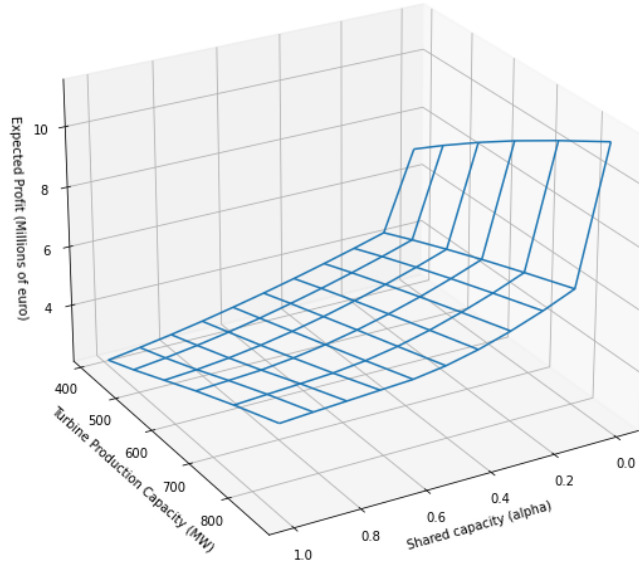


Figure 3.35: Expected profit considering different turbine capacity with a partially predictable customer and a physical model

As a result, even though the owner has access to a larger turbine capacity than the customer, this does not significantly impact the expected profit. The customer, in fact, prevents the owner from generating energy whenever such actions conflict with their own dispatch orders.

Hybrid model

In the case of a smart customer and a hybrid model, it can be observed that as the turbine capacity increases, the final profit increases, as shown in Figure 10. Conversely, the evolution of the expected value with respect to α is decreasing. For α values greater than 0.7 and turbine capacities greater than 572 MW, the expected profit decrease slows slightly. With a smart-type customer, the expected profit decreases because the owner is required to share the turbine capacity with the counterpart. However, when the owner has access to a capacity greater than 572 MW, this loss is mitigated. Even when 100% of the capacity is assigned to the customer, the latter is entitled to a maximum of 572 MW, allowing the owner to continue producing energy and selling it on the market.

3D Wireframe Plot of Expected Profit

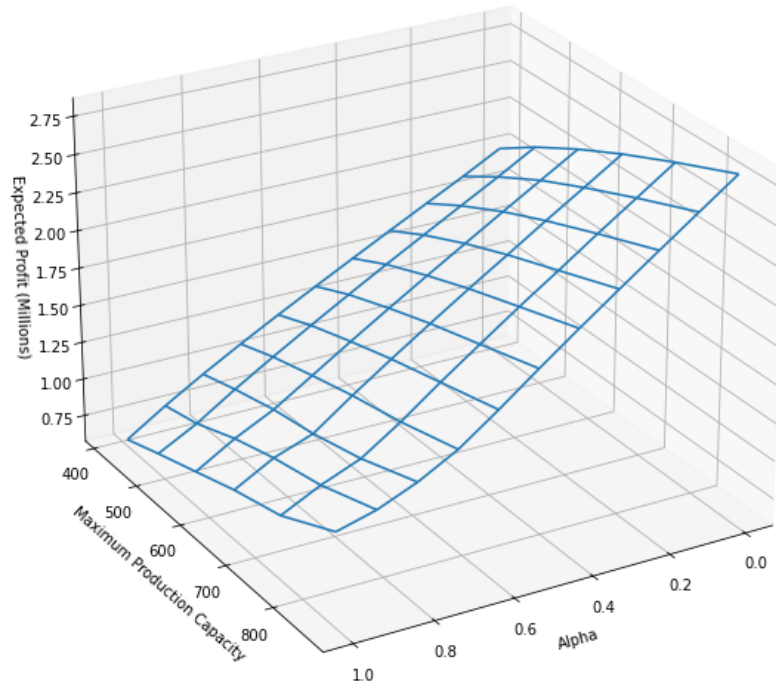


Figure 3.36: Expected profit considering different turbine capacity with a smart customer and a hybrid model

Hybrid model partially predictable customer

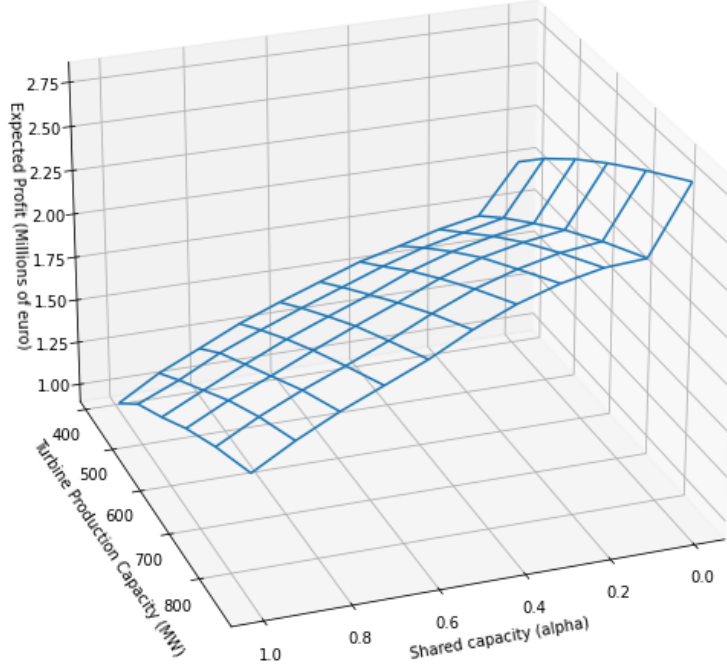


Figure 3.37: Expected profit considering different turbine capacity with a partially predictable customer and a hybrid model

When moving to a partially predictable customer, the optimal expected final profit exhibits a pattern comparable to that analyzed in Section 3.3, as already observed in previous cases. As α increases from 0 to 0.1, a sharp decline in expected profit can be observed across all the turbine capacity levels considered. This decline slows as α continues to increase. As previously explained, this deceleration in the profit decrease is due to the higher revenues resulting from the compensation of the customer's orders, compared to the interaction with a smart-type customer.

Further sensitivity analyses will be conducted. First, additional turbine capacity values will be considered, and the same analyses will be performed by assigning the same turbine capacity to both the customer and the owner. Moreover, a sensitivity analysis will be carried out by considering spot price scenarios characterized by different means and standard deviations.

3.5 Conclusion

In this chapter, the models described and implemented previously have been reformulated stochastically. To accomplish this, several scenarios of spot energy prices and inflow of water into the basins were considered.

These scenarios were generated from data collected over 10 years. The calculation of Conditional Value at Risk ($CVaR$) is introduced in the objective function to account for the owner's risk aversion. The models were tested considering 10 different scenarios. However, the transition to the stochastic model resulted in increased computational complexity and, consequently, computation time. To solve the optimization models, it was therefore necessary to apply three heuristics. Specifically, the three models presented different computational time challenges due to their varying levels of difficulty. The financial model was the least complex to solve, as it contains the lowest number of non-linear constraints. In this case, it was necessary to apply only the first heuristic: a grid of α values was set from 0 to 1 with a step of 0.1. The physical and hybrid models were more computationally demanding due to the higher number of non-linear constraints. An additional heuristic was applied: only the first week of each month for each scenario was selected, resulting in a total of 10 scenarios with 2016 hours each. The higher complexity characterizing the hybrid model, however, necessitated a more restrictive version of this heuristic: only the first day of each month was selected, resulting in a total of 10 scenarios with 288 hours each. The application of these heuristics enabled the determination of an acceptable solution for the models. Nevertheless, since the models were evaluated using different set of scenarios, it was not possible to compare them with one another. The models were first evaluated without considering the impact of a customer, by fixing the α to zero. In the analysis of the 10 scenarios considered, it emerges that the facility is generally used sparingly by the owner, with the lowest utilization recorded in scenario 6, where the facility operates for a total of 3407 hours out of 8,760. This behavior is influenced by the limited efficiency of the turbine, with the generator efficiency at 92.6% and the pump efficiency at 79.5%, resulting in significant cumulative energy losses. When the ratio between the maximum and minimum energy prices in the spot market is less than 1.36, operating the facility is not economically advantageous. In scenario 6, this ratio is often below 1.36, leading to frequent inactivity, as evidenced by the spot market position data for the first two months. To simulate a more conservative and intelligent customer, the concept of a "price power ratio" of 1.5 was introduced. This was achieved by reducing the turbine efficiency to 83.9% and limiting its maximum capacity. This approach allows for the optimization of operational decisions while ensuring greater caution in balancing costs and benefits. Then the three models were tested for different value of α . The optimizations were conducted with a transfer price P set to zero, isolating the impact of customer behavior on the owner's final profit. With the financial model, when a smart customer is considered, relinquishing a larger portion of storage capacity results in greater losses for the owner. This is logical, as a smart customer makes decisions aligned with the spot market, such as requesting energy generation when prices are high and water pumping when prices are low. The owner is obligated to fulfill energy generation requests at higher prices and receives compensation for pumping at lower prices. This dynamic is also reflected in the customer's average profit: as α increases, the customer's profit also rises. However, with a partially predictable customer the owner's optimal objective increases proportionally with the percentage of transferred capacity. A partially predictable customer makes less rational decisions compared to the owner, whose strategies are optimized. Under the financial contract, the owner compensates customers for production orders even when

prices are suboptimal and is compensated for pumping orders made at less profitable times. Consequently, the average profit for a partially predictable customer decreases as more storage capacity is sold. In summary, the results for the physical model reveal significant trends when considering both smart and partially predictable customers. In both cases, the owner’s profit diminishes with increasing transferred storage capacity. However, the profit reduction is deeper for a partially predictable customer, as their decisions often prevent the owner from optimizing energy generation or water pumping schedules. When the owner transfers 70% or more of the storage capacity, the scenario reverses: it becomes less burdensome to interact with a partially predictable customer than a smart one. A smart customer strategically utilizes the turbine and pump during the most profitable hours, significantly constraining the owner’s opportunities for profit generation. Conversely, a partially predictable customer occasionally leaves the turbine accessible during optimal times, enabling the owner to exploit profitable opportunities despite limited capacity. Considering the hybrid model, for smaller α values, transferring capacity to a smart customer results in less severe profit reductions compared to a partially predictable customer. This occurs because a partially predictable customer’s suboptimal pumping orders often reduce the owner’s opportunities to generate profits. As α increases, the pattern shifts: the owner derives more compensation from partially predictable customer pumping orders, mitigating profit losses from reduced energy production. When transferring 50% of the storage capacity to a partially predictable customer, the profits from compensating pumping orders exceed the optimal objective. This is due to the fact that higher α values necessitate more pumping, leading to reduced spot market profits for the owner. However, increased pumping compensation helps offset these losses.

A sensitivity analysis was performed to assess the impact of varying turbine capacities on the model outcomes. By progressively increasing and decreasing both turbine and pumping capacities relative to the initial technical specifications, it was found that, across all models considered (financial, physical, and hybrid), a higher turbine capacity allocated to the owner enables a partial mitigation of the negative effects associated with resource sharing with the customer. In particular, for values of $\alpha > 0.7$, the decline in expected profit becomes significantly less steep, especially when interacting with a smart-type customer. This behavior can be attributed to the upper limit imposed on the customer’s capacity (572 MW), which allows the owner to continue generating and selling energy on the market. Further analyses will explore scenarios in which the owner and customer are assigned equal capacities, as well as spot price conditions characterized by different means and standard deviations.

The three developed models – financial, physical, and hybrid – provide complementary analyses that enable a comprehensive understanding of the effects of capacity transfer on profitability, risk, and operational dynamics. Each offers a unique perspective: the financial model highlights the economic impact of decisions, the physical model emphasizes operational implications, while the hybrid model integrates both dimensions, proving essential for optimizing strategic choices in complex and realistic scenarios.

Chapter 4

Conclusions

This thesis explored the optimization of operational and strategic decisions for a hydro-pumping storage (HPS) facility owner, focusing on the interplay between energy generation, water pumping, and the sale of storage capacity to external customers. Three distinct models—financial, physical, and hybrid—were developed to analyze these decisions under both deterministic and stochastic frameworks, offering complementary insights into profitability, risk management, and operational trade-offs.

The deterministic framework revealed the influence of customer behavior on the profitability of HPS operations. It was observed that the financial model generally outperformed the others in terms of profitability, particularly when interacting with customers following a randomic strategy. Conversely, smart customers often constrained the owner’s ability to optimize operations, reducing profit potential. The hybrid model demonstrated a balance between financial compensation and operational flexibility, while the physical model proved to be the most restrictive, offering limited room for maneuverability.

Transitioning to the stochastic framework introduced additional complexities due to uncertainties in energy prices and natural inflows. These uncertainties were addressed using Conditional Value at Risk (CVaR) to incorporate the owner’s risk aversion. The results underscored the challenges posed by the non-linear constraints of the physical and hybrid models, which required computational heuristics to ensure acceptable solutions. Despite these challenges, the models provided valuable insights into the conditions under which HPS facilities operate most effectively, highlighting the critical role of energy price dynamics and facility efficiencies.

The findings of this thesis emphasize that the introduction of a customer has a very strong impact on the facility owner’s expected profit and risk levels. This impact is strongly influenced by the type of customer, with smart customers introducing more complexity and potentially reducing profitability due to their rational strategy. partially predictable customers, on the other hand, generally resulted in increased profits for the owner, particularly in the financial and hybrid models, as their suboptimal decisions created opportunities for the owner to optimize operations. Furthermore, the type of market setup considered, whether financial, physical, or hybrid, plays a key role in shaping the overall expected profit. The financial model demonstrated

superior profitability, especially in interactions with partially predictable customers, while the hybrid model offered a balanced approach by combining financial compensation with operational flexibility. The physical model, although restrictive, highlighted critical operational constraints and risks associated with HPS management. Furthermore, the hybrid model emerged as a promising approach, balancing financial incentives and operational efficiency to optimize overall performance. Overall, the findings of this thesis emphasize that customer agreements and market structures must be carefully designed to align with the operational and financial objectives of HPS facility owners. The type of customer and market setup profoundly influence profitability and risk, making it essential to tailor strategies based on these factors. Looking ahead, the methodologies and insights developed in this thesis can inform regulatory frameworks and market designs aimed at fostering the deployment of energy storage solutions. By enabling a deeper understanding of the trade-offs between risk, profit, and operational flexibility, this work contributes to advancing the integration of renewable energy into the power grid while supporting the financial viability of large-scale energy storage systems.

Bibliography

- [1] R Tyrrell Rockafellar, Stanislav Uryasev, et al. “Optimization of conditional value-at-risk”. In: *Journal of risk* 2 (2000), pp. 21–42.
- [2] Freddy Delbaen. “Coherent risk measures on general probability spaces”. In: *Advances in finance and stochastics: essays in honour of Dieter Sondermann* (2002), pp. 1–37.
- [3] Jorg Doege, Philippe Schiltknecht, and Hans-Jakob Lüthi. “Risk management of power portfolios and valuation of flexibility”. In: *Or Spectrum* 28.2 (2006), pp. 267–287.
- [4] Janne Kettunen, Ahti Salo, and Derek W Bunn. “Optimization of electricity retailer’s contract portfolio subject to risk preferences”. In: *IEEE Transactions on Power Systems* 25.1 (2009), pp. 117–128.
- [5] Petter Vegard Hansen. “Inflow uncertainty in hydropower markets”. In: *Scandinavian Journal of Economics* 111.1 (2009), pp. 189–207.
- [6] David Connolly et al. “Practical operation strategies for pumped hydroelectric energy storage (PHES) utilising electricity price arbitrage”. In: *Energy Policy* 39.7 (2011), pp. 4189–4196.
- [7] Wolf-Peter Schill and Claudia Kemfert. “Modeling strategic electricity storage: the case of pumped hydro storage in Germany”. In: *The Energy Journal* (2011), pp. 59–87.
- [8] John Hull. *Risk management and financial institutions, + Web Site*. Vol. 733. John Wiley & Sons, 2012.
- [9] Abdesslem Abbassi, Ahlem Dakhlaoui, and Lota D Tamini. “Risk Aversion and Dynamic Games Between Hydroelectric Operators under Uncertainty”. In: (2014).
- [10] Juan I Pérez-Díaz et al. “Trends and challenges in the operation of pumped-storage hydropower plants”. In: *Renewable and Sustainable Energy Reviews* 44 (2015), pp. 767–784.
- [11] Mahdi Sedghi, Ali Ahmadian, and Masoud Aliakbar-Golkar. “Optimal storage planning in active distribution network considering uncertainty of wind power distributed generation”. In: *IEEE Transactions on Power Systems* 31.1 (2015), pp. 304–316.
- [12] Asmae Berrada, Khalid Loudiyi, and Izeddine Zorkani. “Valuation of energy storage in energy and regulation markets”. In: *Energy* 115 (2016), pp. 1109–1118.
- [13] John E Bistline. “Energy technology R&D portfolio management: Modeling uncertain returns and market diffusion”. In: *Applied Energy* 183 (2016), pp. 1181–1196.

- [14] Roy Cerquetti, Paolo Falbo, and Cristian Pelizzari. “Relevant states and memory in Markov chain bootstrapping and simulation”. In: *European Journal of Operational Research* 256.1 (2017), pp. 163–177.
- [15] PI Bakanos and KL Katsifarakis. “Optimizing operation of a large-scale pumped storage hydropower system coordinated with wind farm by means of genetic algorithms”. In: *Global NEST Journal* 21.4 (2019), pp. 495–504.
- [16] Anna Creti` and Fulvio Fontini. *Economics of electricity: Markets, competition and rules*. Cambridge University Press, 2019.
- [17] Kohei Takahashi et al. “Simultaneous operation scheduling of generators and battery energy storage system based on actual and forecasted photovoltaic power outputs”. In: *2019 International Conference on Smart Energy Systems and Technologies (SEST)*. IEEE. 2019, pp. 1–6.
- [18] Md Mustafizur Rahman et al. “Assessment of energy storage technologies: A review”. In: *Energy Conversion and Management* 223 (2020), p. 113295.
- [19] Pengfei Xiao et al. “Optimal operation of a wind-electrolytic hydrogen storage system in the electricity/hydrogen markets”. In: *International Journal of Hydrogen Energy* 45.46 (2020), pp. 24412–24423.
- [20] Andrew Blakers et al. “A review of pumped hydro energy storage”. In: *Progress in Energy* 3.2 (2021), p. 022003.
- [21] Ting Lu, Weige Zhang, and Xiaowei Ding. “Operation strategy of electricity retailers based on energy storage system to improve comprehensive profitability in China’s electricity spot market”. In: *Energies* 14.19 (2021), p. 6424.
- [22] Mohammad-Ali Hamidan and Farzaneh Borousan. “Optimal planning of distributed generation and battery energy storage systems simultaneously in distribution networks for loss reduction and reliability improvement”. In: *Journal of Energy Storage* 46 (2022), p. 103844.
- [23] Zhang Sun et al. “Hierarchical Scheduling Control Method for Cascade Hydro-PV-Pumped Storage Generation System”. In: *International Transactions on Electrical Energy Systems* 2022 (2022).
- [24] Parinaz Toufani, Emre Nadar, and Ayse Selin Kocaman. “Operational benefit of transforming cascade hydropower stations into pumped hydro energy storage systems”. In: *Journal of Energy Storage* 51 (2022), p. 104444.
- [25] Parinaz Toufani, Emre Nadar, and Ayse Selin Kocaman. “Short-term assessment of pumped hydro energy storage configurations: Up, down, or closed?” In: *Renewable Energy* 201 (2022), pp. 1086–1095.
- [26] Presley K Wesseh, Nelson I Benjamin, and Boqiang Lin. “The coordination of pumped hydro storage, electric vehicles, and climate policy in imperfect electricity markets: Insights from China”. In: *Renewable and Sustainable Energy Reviews* 160 (2022), p. 112275.

- [27] Abdessalem Abbassi, Ahlem Dakhlaoui, and Nihed Ben Khalifa. “Risk Aversion and Hydro-energy Reservoirs Management under Monopolistic and Oligopolistic Industrial Structures”. In: *International Journal of Energy Economics and Policy* 13.6 (2023), p. 581.
- [28] Joakim Dimoski et al. “Dynamic hedging for the real option management of hydropower production with exchange rate risks”. In: *OR Spectrum* 45.2 (2023), pp. 525–554.
- [29] Wenxiu Feng and Carlos Ruiz. “Risk management of energy communities with hydrogen production and storage technologies”. In: *Applied Energy* 348 (2023), p. 121494.
- [30] Babangida Modu et al. “A systematic review of hybrid renewable energy systems with hydrogen storage: Sizing, optimization, and energy management strategy”. In: *International Journal of Hydrogen Energy* (2023).
- [31] Ruilian Wang and Rongxin Zhang. “Techno-economic analysis and optimization of hybrid energy systems based on hydrogen storage for sustainable energy utilization by a biological-inspired optimization algorithm”. In: *Journal of Energy Storage* 66 (2023), p. 107469.
- [32] Dalia Yousri et al. “Integrated model for optimal energy management and demand response of micro-grids considering hybrid hydrogen-battery storage systems”. In: *Energy Conversion and Management* 280 (2023), p. 116809.
- [33] Masoume Shabani et al. “Smart and optimization-based operation scheduling strategies for maximizing battery profitability and longevity in grid-connected application”. In: *Energy Conversion and Management: X* 21 (2024), p. 100519.

Ringraziamenti

Vorrei prima di tutto ringraziare i miei relatori, il Professor Paolo Falbo e il Professor Carlo Filippi. Senza la loro guida e pazienza, questo lavoro non sarebbe stato possibile e io non sarei cresciuta così tanto.

Ringrazio poi Roberto Bracco e Francesco Mattion, che mi hanno aiutata a perfezionare e rendere concreto questo progetto.

Una menzione d'onore va alla dottoressa Valentina Morandi, che ha avuto la pazienza di condividere con me un server remoto durante la stesura della tesi.

Ringrazio mia madre, Agata, e mio padre, Ivan, grazie ai quali ho deciso di intraprendere questo percorso e che mi hanno supportata fino alla fine.

Ringrazio mia sorella Elena, che non ha mai smesso di credere in me e che mi è stata d'ispirazione lungo tutto il cammino. Un grazie anche a mio fratello Bruno, che mi ha sempre tenuta con i piedi per terra.

Ringrazio il mio compagno Oladele, che insieme a me ha rinunciato a tante cose per potermi stare vicino e supportarmi durante la stesura della tesi. Un ringraziamento va ai miei colleghi di dottorato: Matteo, Annagiulia, Mateusz e Tommaso. Non avrei potuto desiderare compagni di avventura migliori. Infine, grazie alle mie amiche: Elisa, Maria, Gemma, Michela, Chiara e Ilaria, che riescono sempre a regalarmi un sorriso e un momento di leggerezza.