Environmental and Resource Economics
https://doi.org/10.1007/s10640-025-00984-0

Brown Price and Green Firms: An ETS Price Floor for a Clean Transition?

Simone Borghesi[1,2] · **Nicola Comincioli**[3,4] · **Peter M. Kort**[5] · **Jacco J. J. Thijssen**[6] · **Sergio Vergalli**[3,4]

Accepted: 6 March 2025

Abstract
We examine the optimal behavior of carbon-emitting companies operating under the European Union Emission Trading System (EU ETS), under which firms are obliged to purchase emission permits on the secondary market if their emissions exceed their allowance. Specifically, we consider the scenario where firms are endowed with the (real) option to undertake a "green" investment to cut their emissions and, thus, permit expenditures. The central challenge is the determination of the optimal time for investment within a stochastic framework characterized by uncertainty in EU ETS permit prices. We address the problem for a heterogeneous group of companies with diverse technological capabilities across industrial sectors. Furthermore, we incorporate a price floor for permit prices to mirror policy efforts aimed at promoting green transition by elevating emission costs. We solve this problem analytically and through numerical simulations calibrated to real market data. In addition to offering insights into individual firm behavior, our findings can support regulators in refining environmental policies, particularly regarding the role of permits price floor and its potential to expedite the green transition.

Keywords EU ETS · Carbon price floor · Real options

✉ Simone Borghesi
simone.borghesi@eui.eu

1 Robert Schuman Centre for Advanced Studies, European University Institute, Fiesole, Italy

2 Università degli Studi di Siena, Siena, Italy

3 Università degli Studi di Brescia, Siena, Italy

4 Fondazione Eni Enrico Mattei, Milan, Italy

5 Tilburg University, Tilburg, Netherlands

6 University of York, York, England

Published online: 02 April 2025

1 Introduction

The accumulation of carbon dioxide and other greenhouse gases (GHGs) in the atmosphere leads to a range of consequences, from rising global temperatures to increased occurances of extreme weather events, causing irreversible damage to ecosystems (Allen et al. 2009; Yi et al. 2015). To tackle the challenge of climate change, it is therefore imperative to reduce carbon emissions, which are the primary drivers of global warming (Yuan et al. 2022). To this end, a range of sustainable practices – "green technologies" – and decisive policy measures must be adopted. A notable example of a broad-based effort to reduce carbon emissions, at the European level, is the "Fit for 55" package. This set of proposals aims to achieve a minimal cut in net emissions of 55% by 2030 through the adoption of a number of tools and regulations, including, for example, Carbon Border Adjustment Mechanisms (CBAMs), Land Use, Land Use Change, and Forestry (LULUCF), the Social Climate Fund (SCF), and more.[1] Among the various policy instruments aimed at reducing carbon emissions (Zhang and Wang 2017), carbon pricing plays a prominent role, with its carbon-reducing effects empirically validated, although these may sometimes appear limited (Green 2021). Carbon pricing has been implemented globally in various forms (Boyce 2018; Anjos et al. 2022), such as (i) Pigouvian taxes, e.g., in Australia (Comincioli et al. 2024), (ii) carbon credits and subsidies, e.g., in Africa (Gujba et al. 2012) and, most notably, (iii) emission trading systems (ETS), particularly in China and in the European Union (EU) (Tang et al. 2020; Verde et al. 2021; Verde and Borghesi 2022).

The EU Emission Trading System (EU ETS) is an essential part of the EU's policy arsenal to combat climate change and the main tool of the European Green Deal for reaching climate neutrality by 2050.[2] As of 2023, the EU ETS operates in 29 countries, i.e. the EU-27 member states, along with Iceland and Norway, and it has been linked to the Swiss ETS since 2020 to enhance market liquidity and stability, as well as to demonstrate political cohesion in the common effort to reduce emissions (Flachsland et al. 2009). The United Kingdom left the EU ETS at the beginning of 2021, as part of its exit from the EU. However, despite the extensive coverage of the EU ETS, the absence of a global carbon market implies that unilateral carbon policies may lead to carbon leakage effects (Antoci et al. 2021). Since its establishment, the EU ETS has served as a regulatory framework for controlling GHG emissions produced by large industry, electricity generation and aviation, which account for roughly 40% of total GHG emissions in the EU. The range of industry covered by this regulation has progressively increased over time and is set to broaden even further in the future. Indeed, a new and separate ETS, the so-called ETS2, will be introduced in 2027 to control emissions from heating buildings, road transport and small industry. The EU ETS is a cap-and-trade mechanism in which permits to emit are primarily auctioned or, to a lesser extent, allocated for free. Allowances can then be traded on a secondary market, where more (less) technologically advanced companies using clean (dirty) technologies can sell (purchase) excess (missing) permits.

Since its implementation in 2005, the EU ETS has helped reduce emissions from power and industrial plants by approximately 47%. The operation of this scheme was divided into four phases. Phase I (2005–2007) was primarily a learning-by-doing period, during which

[1] See: https://www.consilium.europa.eu/en/policies/green-deal/fit-for-55/.

[2] See: https://climate.ec.europa.eu/eu-action/eu-emissions-trading-system-eu-ets_en.

emission allowances were mostly allocated for free, providing little incentive for new competition or green investment. Phase II (2008–2012), which coincided with the first commitment period of the Kyoto Protocol, was largely ineffective due to the financial crisis and its impact on the real economy, leading to an oversupply of permits and subsequent downward pressure on prices. Phase III (2013–2020) introduced a unique EU-wide emission cap, decreasing 1.74% annually, and shifted most of free allowances to auctioning. However, free allowances are still granted to hard-to-abate sectors, that have fewer mitigation options, compared to other industries.[3] Phase IV (2021–2030) progressively accelerated the reduction of the emissions cap, from 2.2% per year over the period 2021–2023, to 4.3% per year over the period 2024–2027 and 4.4% per year from 2028.[4] Reducing the EU-wide cap is a crucial aspect of achieving the EU's policy goals. By decreasing the number of permits available, an upward pressure is applied to carbon price. This increases the cost of maintaining the *status quo* while incentivizing the adoption of more sustainable and less polluting technologies. A firm's decision to go green is obviously influenced by uncertainty regarding the price of carbon. However, since policymakers can affect the price set in the EU ETS and, consequently, the timing of individual companies' investment decisions, this serves as a key lever to promote the green transition and achieve climate neutrality goals. Additionally, by adjusting the amount of free allocations for different sectors, the policymaker can influence the actual emissions costs incurred by firms, thereby shaping their investment decisions.

The carbon price observed in the EU ETS market has shown large fluctuations during the four phases described above. It first decreased down to less than 5 EUR/ton in Phase II due to the financial crisis, and remained very low for many years. The existence of a large oversupply and consequent low prices raised a heated debate and much criticism towards the EU ETS. Indeed, if the price is too low, the polluter pays principle does not fully apply. This led several studies to argue in favor of adopting a price floor in the EU ETS (Pahle et al. 2018; Edenhofer et al. 2019; Flachsland et al. 2020; Hintermayer 2020). A floor prevents the price from collapsing below a minimum level in the system. Some early studies argued in favor of having a price corridor, namely, both a floor and a cap (Burtraw et al. 2010; Borghesi 2011), following the approach adopted in other ETSs worldwide. This idea has been relaunched more recently as a way of facilitating linking among ETSs (Doda et al. 2022). Indeed, adopting a price corridor signals to potential ETS partners the price range that the ETS regulator is willing to accept, and thus may facilitate a linking agreement with other jurisdictions, thus increasing market liquidity.

Rather than adopting a price floor that might be seen as a fiscal policy and thus require unanimity in the EU, the EU regulator preferred a quantity-based price control mechanism that automatically adjusts the supply of allowances to their demand. Indeed, to prevent an oversupply of allowances on the market and the resulting downward pressure on prices, the Market Stability Reserve (MSR) was established in 2018 and became operational in 2019. This reform induced a rapid increase of the price that peaked at around 100 EUR/ton in 2022. Despite the price falls observed in the last few years due to the Ukrainian war and the consequent energy crisis, the EU ETS price remains rather high ranging between 70 and 80 EUR/ton at the moment of writing. While the MSR has been successful in inducing an immediate increase in the carbon price and then in preventing more pronounced price falls

[3] See: https://climate.ec.europa.eu/eu-action/eu-emissions-trading-system-eu-ets/free-allocation_en.

[4] See: https://climate.ec.europa.eu/eu-action/eu-emissions-trading-system-eu-ets/emissions-cap-and-allowances_en.

during recent crises, some adjustments might be needed in the future to address the rapidly changing and increasingly uncertain context and the expected shortage of allowances as the cap moves to zero (Flachsland et al. 2018; Borghesi et al. 2023). Our paper contributes to the growing literature on carbon pricing by modeling the dynamics of the investment decisions of ETS-regulated firms under uncertainty on future carbon prices in a context in which the regulator may intervene by changing the emissions cap, the amount of free allowances and the price floor, when this exists.

Specifically, using a real options model and appropriate numerical simulations, we describe (i) CO_2-emitting firms' optimal choice for green investment under carbon price uncertainty and (ii) how this choice can be influenced, and possibly accelerated, by the policymaker through a price floor. An additional contribution of our work is the use of a Geometric Brownian Motion (GBM) calibrated to real data to account for the uncertainty affecting firms' decisions. A GBM is a stochastic process extensively used in literature to describe the behavior of a range of, e.g., financial, economic and environmental variables over time (Marathe and Ryan 2005). The use of this process is widespread thanks to its ability to capture essential characteristics of real-world variables. Moreover, its simplicity and parsimony, using only two parameters accounting for its trend and volatility, respectively, allows for closed form solutions to a wide range of mathematical problems (Postali and Picchetti 2006). As a consequence, the use of even very complex models is computationally efficient (Lee et al. 2022). A further advantage is the possibility of calibrating process parameters to historical data, enabling a more realistic representation of the dynamics of interest (Comincioli et al. 2021a). The calibration of the GBM describing carbon price to real market data enables a more realistic depiction of the decision-making process. Our goal, then, is to support stakeholders, from policymakers to entrepreneurs, in implementing more robust and realistic plans of action. Moreover, a better understanding of the uncertainty induced by carbon prices can be highly beneficial in macroeconomic and financial modeling, providing valuable insights for all market participants.[5]

The remainder of the paper is structured as follows. Sect 2 describes the theoretical model and the procedure implemented to calibrate the parameters driving uncertainty with real data. The results of numerical simulations are presented in Sect 3. Sect 4 then provides a discussion of the implications of results obtained under different policy options, while Sect 5 finally concludes and provides closing remarks.

2 The Model

The real options model presented in this section describes the decision-making process of a set of representative European carbon-emitting firms, subject to the EU ETS regulation, about undertaking a green investment to eliminate emission. This option is worthy of consideration due to the uncertainty induced by the carbon price. A sufficiently high carbon price might indeed make green investments financially attractive. Two versions of

[5] The use of real options, and thus GBM, has also been extended to environmental economics, for its capacity to examine the optimal timing of the decision process. Indeed, as Pindyck (2002) has argued, "*because of the uncertainties [...] inherent in environmental degradation, its prevention, and its economic consequences, environmental policy design can involve important problems of timing*". In this paper, we present an additional cross-cutting exercise: the calibration of a GBM on the emission quantities of EU ETS member countries, which is detailed in Appendix A.

this model are presented. Sect 2.1 provides the general theoretical framework of the model. Sect 2.2 outlines the decision making process taking place with an unconstrained market price of carbon. Sect 2.3 then introduces a price floor, which can be either constant or a function of firm location Q. Sect 2.4 finally describes the procedure to empirically validate the choice of the stochastic process used to model carbon price-induced uncertainty.

2.1 General Formulation

Let us consider a mass $Q_0 < 0$ of brown firms, uniformly distributed over the interval $[0, Q_0]$. Without any loss of generality, we assume that each firm emits 1 metric ton of CO_2 per period and that at each period their initial endowment of permits is null, so they have to buy one permit at market price.[6] The value of Q_0 therefore measures the amount of CO_2 released into the atmosphere at $t = 0$ by the mass of firms considered.

A specific firm is identified by its "location" $Q \in [0, Q_0]$, which determines the cost associated with making the green investment. Indeed, in contrast to the real option literature where costs are typically assumed to be constant (Comincioli et al. 2021a) or stochastic (Moon and Baran 2018), we introduce a firm-specific cost structure. This choice allows to capture the technological discrepancies among industrial sectors and therefore to account for heterogeneity across firms, that in turn affect their investment cost. In this way, the interval $[0, Q_0]$ can be viewed as a set of firms ordered by their investment cost: those with higher costs are representative of hard-to-abate sectors, while the others reflect industries where carbon abatement is easier and more affordable. The firm-specific investment cost is defined as:

$$C(Q) = \kappa + cQ^{-\gamma}, \tag{1}$$

where κ, c and γ are positive parameters, constants for all firms, allowing to control for the position and the shape of that function. The use of a three-parameters hyperbolic function, allows great flexibility in representing cost structure. Indeed, while κ are c serve as shift and scale parameters, respectively, γ captures the technological discrepancies among industrial sectors, that in turn affect their investment cost, by adjusting the curvature of (1). Indeed, properly calibrating this parameter allows for adjusting the function's shape reflecting the marginal cost, which in turn represents how much easier and cheaper it is for a firm or industry to invest in carbon abatement than another. In light of that, the higher (lower) Q, the more rightward (leftward) the enterprise in the interval $[0, Q_0]$ and then the cheaper (more costly) the investment.

The market for permits is assumed to always be in equilibrium, with a market clearing shadow price that evolves stochastically. This uncertainty is modeled by a probability space $(\Omega, \mathcal{F}, P)$. On this space we define a standard Brownian motion $B = (B_t)_{t \geqslant 0}$ with $B_0 = 0$, P-a.s., and denote its natural filtration (augmented by the P-null sets) by $\mathbf{F} = (\mathcal{F}_t)$. The

[6] Firms subject to the EU ETS typically receive an initial endowment of permits through free allocation and auction purchases. However, we chose to model the scenario where the initial allocation is null, as this is functionally equivalent to studying an investment aimed at reducing emissions exceeding the initial allowance, but offers the advantage of greater simplicity. Moreover, observing the dynamics of Q_t therefore allows to keep easily track of total emission and to compare variations with policy targets.

expectation operator on $(\Omega, \mathcal{F}, P)$ is denoted by E. The market clearing shadow price of permits is the unique strong solution to the geometric Brownian motion (GBM),[7]

$$\mathrm{d}S_t = \alpha S_t \mathrm{d}t + \sigma S_t \mathrm{d}B_t, \tag{2}$$

where $\alpha \in R$ is the drift and $\sigma < 0$ is the volatility of the permit price. For notational convenience, we define a family of probability measures $(P_s)_{s \geqslant 0}$, by, for $s \geqslant 0$,

$$P_s(A) := P(A | S_0 = s), \quad \text{for all S} \in \mathcal{F}.$$

We assume that there exists a risk-free asset in the market with return $r < 0$ as well as a (portfolio of) risky asset(s) that span the uncertainty in the permit price and has a price process that evolves according to the GBM

$$\mathrm{d}X_t = \mu_X X_t \mathrm{d}t + \sigma_X X_t \mathrm{d}B_t,$$

with $\sigma_X < 0$. The Sharpe ratio of the spanning asset is denoted by $h_X := (\mu_X - r)/\sigma_X$. The stochastic discount factor that prices the risk-free and risky asset evolves according to (cf. Thijssen 2010)

$$\mathrm{d}\Lambda_t = -r\Lambda_t \mathrm{d}t - h_X \Lambda_t \mathrm{d}B_t.$$

A straightforward application of the Girsanov theorem now implies that there is a measure $\widetilde{P}$ on the measurable space $(\Omega, \mathcal{F})$, equivalent to P, such that the no-arbitrage value of S, under $\widetilde{P}$ evolves according to the GBM,

$$\mathrm{d}S_t = (r - \delta) S_t \mathrm{d}t + \sigma \mathrm{d}\widetilde{B}_t,$$

Where $\widetilde{B}$ is a $\widetilde{P}$-Brownian motion. We assume that the net conenience yield equals

$$\delta := r + \rho h_X - \alpha > 0.$$

For further reference, the characteristic operator of S (killed at rate r) on $C^2(R_+)$ is given by[8]

$$\varphi(S) \mapsto \mathcal{A}(\varphi)(s) := \frac{1}{2}\sigma^2 S^2 \varphi_{SS}(S) + (r - \delta)\varphi_S(S) - r\varphi(S).$$

In this market, the policymaker can introduce a *price policy*, which is here modeled as a reflecting barrier for the stochastic process S. That is, we define a process $L = (L_t)_{t \geqslant 0}$ on the filtered probability space $(\Omega, \mathcal{F}, \mathbf{F}, P)$, such that L is an $\mathbf{F}$-adapted, non-negative, and non-decreasing stochastic process. The set of policies is denoted by $\mathcal{L}$. Here, L represents the cumulative withdrawal of permits by the regulator to increase the market price of per-

[7] See Sect 2.4 for further discussion about this choice.

[8] We denote partial derivatives by subscripts, e.g., $\varphi_S := \frac{\partial \varphi}{\partial S}$.

mits. The observed permit price is then given by the reflected GBM $P = (P_t)_{t\geqslant 0}$, which, under $\widetilde{P}$, evolves according to

$$\mathrm{d}P_t^L = (r-\delta)P_t\mathrm{d}t + \sigma P_t \mathrm{d}B_t + \mathrm{d}L_t.$$

We assume that the market price, though stochastic, depends in some predictable and smooth way on the total demand for permits, i.e., that there exists some $C^1(R_+)$ function D such that $P_t = D(Q_t)$ with $D' < 0$. Through investment in carbon abatement, the firm can reduce the price it pays for permit by

$$\mathrm{d}U := D'(Q)\mathrm{d}Q.$$

An *investment policy* for the firm is an $\mathbf{F}$-adapted, non-increasing stochastic process Q. The set of price policies is denoted by $\mathcal{Q}$.

Given a price policy L for the regulator and an investment policy Q for the firm, the *regulated price process*, denoted by $P^{L,Q}$ evolves (under $\widetilde{P}$) according to the SDE,

$$\mathrm{d}P_t^{L,Q} = (r-\delta)P_t^{L,Q}\mathrm{d}t + \sigma P_t^{L,Q}\mathrm{d}\widetilde{B}_t + \underbrace{\mathrm{d}L_t}_{\geqslant 0} + \underbrace{D'(Q)\mathrm{d}Q_t}_{\leqslant 0}.$$

Recalling the cost of abating the Q-th unit of carbon (1) and given the regulator's price policy L and an investment policy Q, the firm's carbon (abatement) cost is then[9]

$$F(P,Q) := \widetilde{E}_{P,Q}[\int_0^\infty e^{-rt}\{P_t^{L,Q}Q_t\mathrm{d}t \underbrace{-C(Q_t)\mathrm{d}Q_t}_{\geqslant 0}\}],$$

where the expectation is taken under the risk-neutral measure $\widetilde{P}$. The firm's objective is to solve

$$F^*(P,Q) := \inf_{Q\in\mathcal{Q}} F(P,Q).$$

A *price floor* is a price policy that turns a mapping $Q \mapsto \underline{P}(Q) \geqslant 0$ into an upward reflecting barrier $\underline{P}$. Similarly, a *price ceiling* is an investment policy that turns a mapping $Q \mapsto \overline{P}(Q) \geqslant 0$ into a downward reflecting barrier $\overline{Q}$. With slight abuse of notation we identify price floors and ceilings by the mappings $\underline{P}$ and $\overline{P}$, respectively. We get the following verification theorem.

Theorem 1 *Let $\underline{P}$ be a price floor. Suppose that there exist*

1. a mapping

$$R_+^2 \ni (P,Q) \mapsto \varphi(P,Q),$$

[9] Recall that $\mathrm{d}Q \leqslant 0$, so costs are incurred whenever $\mathrm{d}Q \leqslant 0$.

that is continuously differentiable in Q*, twice continuously differentiable in* P*, and convex in* P*; and*

2. a price ceiling $\overline{P}$*,*

such that, for all $Q \in [0, Q_0]$ *it holds that,*

1. $\mathcal{A}\varphi(P, Q) + PQ = 0$ on $(\underline{P}(Q), \overline{P}(Q))$;
2. $\varphi_P(\overline{P}(Q), Q) = c(Q)/D_Q(Q)$;
3. $\varphi_P(\underline{P}(Q), Q) = 0$; *and*
4. $\varphi_{PQ}(\overline{P}(Q), Q) = 0$

Then $F^* = \varphi$ *and* $\overline{Q}$ *generated by* $\overline{P}$ *is the optimal investment policy.*

Proof. We first show that $F^* \leqslant \varphi$. Fix $T < 0$. Since $\underline{P}$ and $\overline{Q}$ are finite variation processes, it follows from Itō‘s lemma that (under the risk-neutral measure $\widetilde{P}$),

$$\widetilde{E}_{P,Q}\left[e^{-rT}\varphi(P_T^{\underline{P},\overline{Q}}, \overline{Q}_T)\right] = \varphi(P, Q) + \widetilde{E}_{P,Q}[\int_0^T e^{-rt}\mathcal{A}\varphi(P_t^{\underline{P},\overline{Q}}, \overline{Q}_t)\mathrm{d}t$$

$$-\int_0^T e^{-rt}\varphi_P(P_t^{\underline{P},\overline{Q}}, \overline{Q}_t)D_Q(\overline{Q}_t)\mathrm{d}\overline{Q}_t$$

$$+\int_0^T e^{-rt}\varphi_P(P_t^{\underline{P},\overline{Q}}, \overline{Q}_t)\mathrm{d}\underline{P}_t$$

$$-\int_0^T e^{-rt}\varphi_{PQ}(P_t^{\underline{P},\overline{Q}}, \overline{Q}_t)D(\overline{Q}_t)\mathrm{d}\overline{Q}_t]$$

$$\overset{\text{from3,4}}{=} \varphi(P, Q) + \widetilde{E}_{P,Q}[\int_0^T e^{-rt}\mathcal{A}\varphi(P_t^{\underline{P},\overline{Q}}, \overline{Q}_t)\mathrm{d}t$$

$$-\int_0^T e^{-rt}\varphi_P(P_t^{\underline{P},\overline{Q}}, \overline{Q}_t)D_Q(\overline{Q}_t)\mathrm{d}\overline{Q}_t$$

$$\overset{\text{from1}}{=} \varphi(P, Q) - \widetilde{E}_{P,Q}[\int_0^T e^{-rt}P_t^{\underline{P},\overline{Q}}\overline{Q}_t\mathrm{d}t$$

$$-\int_0^T e^{-rt}\varphi_P(P_t^{\underline{P},\overline{Q}}, \overline{Q}_t)D_Q(\overline{Q}_t)\mathrm{d}\overline{Q}_t$$

$$\overset{\text{from2}}{=} \varphi(P, Q) - \widetilde{E}_{P,Q}[\int_0^T e^{-rt}\{P_t^{\underline{P},\overline{Q}}\overline{Q}_t\mathrm{d}t - C(\overline{Q}_t)\mathrm{d}\overline{Q}_t\}].$$

Sending $T \to \infty$ it then follows that

$$\varphi(P,Q)=\widetilde{E}_{P,Q}[\int_0^T e^{-rt}\{P_t^{\underline{P},\overline{Q}}\overline{Q}_t\mathrm{d}t-C(\overline{Q}_t)\mathrm{d}\overline{Q}_t\}].$$

That is, $\varphi(P,Q)$ is the expected discounted cost under the price floor $\underline{P}$ and price cap $\overline{Q}$.

To prove the converse inequality, again fix $T<0$. Choose any feasible investment policy Q. Since φ is convex in P it follows that

$$0\leqslant\varphi_P(P,Q)\leqslant C(Q)/D_Q(Q),$$

for all Q. Therefore, Itō's lemma gives,

$$\widetilde{E}_{P,Q}[\int_0^T e^{-rt}\{P_t^{\underline{P},Q}Q_t\mathrm{d}t-C(Q_t)\mathrm{d}Q_t\}]\geqslant\widetilde{E}_{P,Q}[\int_0^T e^{-rt}\varphi_P(P_t^{\underline{P},Q},Q_t)\mathrm{d}Q_t$$

$$-\int_0^T e^{-rt}\mathcal{A}\varphi(P_t^{\underline{P},Q},Q_t)\mathrm{d}t]$$

$$=\varphi(P,Q)-\widetilde{E}_{P,Q}[e^{-rT}\varphi(P_T^{\underline{P},Q},Q_T)].$$

Letting $T\to\infty$, we see that

$$\varphi(P,Q)\leqslant\widetilde{E}_{P,Q}[\int_0^T e^{-rt}\{P_t^{\underline{P},Q}Q_t\mathrm{d}t-C(Q_t)\mathrm{d}Q_t\}].$$

Since Q was an arbitrary investment policy it, thus, follows that,

$$\varphi(P,Q)\leqslant\inf_Q\widetilde{E}_{P,Q}[\int_0^T e^{-rt}\{P_t^{\underline{P},Q}Q_t\mathrm{d}t-C(Q_t)\mathrm{d}Q_t\}]=F^*(P,Q).$$

Therefore, $\varphi=F^*$ and the investment policy $\overline{Q}$ induced by the price ceiling $\overline{P}$ achieves this minimal cost.■

2.2 Unconstrained Carbon Price Solution

To get a feeling for this result, we explicitly solve for the price ceiling when $\underline{P}\equiv 0$, i.e. when the regulator does not intervene in the market. If there is no price floor, the observed price equals the unconstrained shadow price (2), i.e. $P_t=S_t$, as long as the representative firm does not exert control. Given an initial permit price $P<0$, the firm's uncontrolled (i.e. $\mathrm{d}Q\equiv 0$) discounted expected costs are,

$$R(P):=\widetilde{E}_P\left[\int_0^\infty e^{-rt}P_tQ_t\mathrm{d}t\right]=\frac{PQ}{\delta}, \tag{3}$$

where, recall,

$$\delta = r + \sigma h_X - \alpha < 0,$$

is the asset's net convenience yield and $r < 0$ is the risk-free rate. Under the risk-neutral measure, the evolution of P_t is then given by

$$\frac{\mathrm{d}P_t}{P_t} = (r-\delta)\mathrm{d}t + \sigma \mathrm{d}\widetilde{B}_t, \tag{4}$$

where $\widetilde{B}$ is a $\widetilde{P}$-Brownian motion.

At any time t, each firm has the possibility to undertake an emission-cutting green investment. In other words, it faces the choice between (i) paying the sunk cost that allows to avoid permits purchase at an uncertain price and (ii) continuing the business as usual, being subject to price-related uncertainty. For the Q-th unit this suggests that there exists a barrier $\overline{P}(Q)$ at which the firm located at Q is indifferent between investing in green technology and waiting.[10] If undertaken, its investment will keep the market price just below the barrier. While $\mathrm{d}Q = 0$ the firm's expected discounted costs follow from Hamilton–Jacobi–Bellman (HJB) equation:[11]

$$rF(P,Q) = \lim_{\mathrm{d}t \downarrow 0} \frac{1}{\mathrm{d}t} \widetilde{E}_{P,Q}\left[PQ\mathrm{d}t + \mathrm{d}F(P_{\mathrm{d}t}, Q_{\mathrm{d}t})\right].$$

A standard application of Itō's lemma shows that the HJB equation reduces to the second-order PDE (Dixit and Pindyck 1994),

$$\frac{1}{2}\sigma^2 P^2 F_{PP}(P,Q) + (r-\delta)PF_P(P,Q) + PQ = rF(P,Q).$$

The general solution to this PDE is

$$F(P,Q) = \frac{PQ}{\delta} + A(Q)P^{\beta_1} + B(Q)P^{\beta_2}, \tag{5}$$

where $A(Q)$ and $B(Q)$ are firm-specific constants, i.e. given Q, to be determined, and $\beta_1 < 1$ and $\beta_2 < 0$ are the roots of the fundamental quadratic equation (Dixit and Pindyck 1994):

$$\mathcal{Q}(\beta) \equiv \frac{1}{2}\sigma^2\beta(\beta-1) + (r-\delta)\beta - r = 0. \tag{6}$$

At the barrier, we have to apply appropriate boundary conditions. Firstly, when $P = 0$, which is an absorbing state, the firm's expected discounted costs are null, i.e. $F(0,Q) = 0$. It, therefore, must hold that $B(Q) = 0$ for all $Q \in (0, Q_0]$. Secondly, once $Q = 0$ there are no more firms who need to buy permits, i.e. $F(P,0) = 0$ for all P. This requires that

[10] Note that, in assessing its own investment problem, the firm also needs to take into account the investment decisions of the other firm. This is why we use the notion of a representative firm to analyze the problem.

[11] Time dependency is omitted to lighten the notation.

$A(0) = 0$. Then, at the barrier $\overline{P}(Q)$ the marginal benefit of investment should equal its marginal cost, i.e. $F_Q(\overline{P}(Q), q) = cq^{-\gamma}$, for all $Q \in (0, Q_0]$. Finally, the barrier should be chosen optimally, i.e. $F_{QP}(\overline{P}(Q), Q) = 0$, for all $Q \in (0, Q_0]$.

From the last condition, it follows that:

$$A'(Q)P^{\beta_1} = -\frac{P}{\delta\beta_1}, \tag{7}$$

which, once plugged into the third one, leads to the optimal trigger:

$$\overline{P}(Q) = \frac{\beta_1\delta}{\beta_1 - 1}\left(\kappa + cQ^{-\gamma}\right). \tag{8}$$

It is worth noting that $\overline{P}(Q)$ is decreasing in Q, so that a firm investing later in the sequence has a higher price trigger.[12]

Broadening the perspective from firm-specific decisions to the aggregate outcome of the non-contemporaneous decision of the plurality of companies subject to the EU ETS, it appears crucial for the authority in charge to keep track of this process. Indeed, in the framework of this study, green transition *is* the process that sees brown companies undertake carbon abatement investment, at their optimal time, thereby reducing total emissions generated by the mass of firms considered. We, therefore, introduce the following Cumulative Investment Function (CIF):

$$CIF(t) = Q_0 - \overline{P}^{-1}\left(\max_{i \leqslant t} P_i\right), \tag{9}$$

where Q_0 represents the total emissions at $t = 0$ for the mass of firms considered, and $\overline{P}^{-1}$ is the inverse function of the investment trigger (8), that returns the share of firms that have not yet met the investment until t. Being the CIF the complement to Q_0 of the companies that still have to undertake green investment, it effectively tracks the share of firms that have implemented carbon abatement and cut their emissions. For this reason, the ratio between $CIF(t)$ and Q_0 represents the total percentage reduction in emission achieved, providing a valuable metric for the policymakers to assess progress towards long-term climate goals. By construction, the CIF is monotonically increasing due to irreversibility of investment decisions. Moreover, because when the investment trigger is hit following an upward price fluctuation, this is surpassed, not being a reflecting barrier. As a consequence, multiple firms undertake green investment simultaneously, leading to a stepwise behavior of the function.

2.3 Price Floor-Constrained Solution

To model the policy scenario in which the regulator intervenes in the market, the model detailed in Sect 2.1 needs to be solved for the case when $\underline{P} \neq 0$. While an explicit price

[12] The investment trigger $\overline{P}(Q)$ can be interpreted also in terms of expected investment timing $E[T]$. Specifically, given a price at some $t = 0$, the following relation holds (under the historic measure P, Wong 2007): $E[T] = \ln \frac{\overline{P}(Q)}{P} \left|\alpha - \frac{\sigma^2}{2}\right|^{-1}$.

floor is not currently in place, this scenario could offer valuable insights for discussions surrounding the potential introduction of this policy option (Hintermayer 2020). Let the social planner introduce a price floor $\underline{P}$, that can be either constant or adjusted with respect to a firm's location Q, acting as a lower reflecting barrier. The observed price P_t must therefore be distinguished from the "true" shadow price following the unconstrained GBM S_t. It can be defined as:

$$P_t = \begin{cases} S_t & \text{if} \quad \mathrm{S_t} < \underline{\mathrm{P}} \\ \underline{P} & \text{if} \quad \mathrm{S_t} \leqslant \underline{\mathrm{S}} \end{cases}, \tag{10}$$

The expected discounted costs for the representative firm then equal:[13]

$$F(P, Q) = \begin{cases} \frac{PQ}{\delta} + A(Q)P^{\beta_1} + B(Q)P^{\beta_2} & \text{if} \quad \mathrm{S_t} < \underline{\mathrm{P}} \\ \frac{\underline{P}Q}{r} + D(Q)s^{\beta_1} + E(Q)S^{\beta_2} & \text{if} \quad \mathrm{S_t} \leqslant \underline{\mathrm{P}} \end{cases}, \tag{11}$$

where firm-specific constants $A(Q)$, $B(Q)$, $D(Q)$ and $E(Q)$ are yet to be determined. For this purpose, we exploit the following conditions. In correspondence of the absorbing state $S = 0$, a firm's expected discounted costs must be nil, i.e. $E(Q) = 0$ for all Q. Then, at the floor, i.e. when $S = \underline{P}$, the following conditions must hold to guarantee continuity and differentiability:

$$\frac{\underline{P}Q}{\delta} + A\left(Q\right)\underline{P}^{\beta_1} + B\left(Q\right)\underline{P}^{\beta_2} = \frac{\underline{P}Q}{r} + D\left(Q\right)\underline{P}^{\beta_1} \tag{12}$$

and:

$$\frac{\underline{P}Q}{\delta} + \beta_1 A\left(Q\right)\underline{P}^{\beta_1} + \beta_2 B\left(Q\right)\underline{P}^{\beta_2} = \frac{\underline{P}Q}{r} + \beta_1 D\left(Q\right)\underline{P}^{\beta_1} \tag{13}$$

Moreover, the firm's investment response is to invest as soon as the price hits a trigger $E(Q)$. This happens when the firm is indifferent between undertaking the investment and continuing with "business as usual," i.e.:

$$\frac{\overline{P}(Q)}{\delta} + A'(Q)\overline{P}(Q)^{\beta_1} + B'(Q)\overline{P}(Q)^{\beta_2} = cQ^{-\gamma}, \tag{14}$$

for all $Q \in (0, Q_0]$. Finally, the investment trigger is chosen optimally, i.e.:

$$\frac{1}{\delta} + \beta_1 A'(Q)\overline{P}(Q)^{\beta_1 - 1} + \beta_2 B'(q)\overline{P}(Q)^{\beta_2 - 1} = 0, \tag{15}$$

for all $q \in (0, Q_0]$.

The coefficients $A(Q)$, $B(Q)$ and $D(Q)$, as well as investment trigger $\overline{P}$, are uniquely determined by numerically solving the system made of (12) to (15). The following observations on the coefficients' sign are worth to highlight. $A(Q)$ is negative because if P increases

[13] Time dependency is omitted to lighten the notation.

and hits the trigger, investment takes place. Consequently, a downward pressure is exerted on P, making business-as-usual cheaper for those firms that have not yet invested. $B(Q)$, on the other hand, is positive, because when P decreases and hits the floor $\underline{P}$, the pressure on the price is upwards. Finally, $D(Q)$ is also positive, because when S is below the trigger and the price floor, thus, bites, there is a positive chance that $\underline{P}$ is hit after which the permit costs increase.

Since, unlike the case without floor, the investment trigger is now obtained numerically, it is not possible to apply the definition (9) to obtain the CIF and keep track of emissions. The evolution of Q_t is therefore also studied numerically.

2.4 GBM Validation and Estimation

To assess whether carbon price can be described by (2), the GBM validation procedure must be carried out (Biondi and Moretto 2015). More specifically, the following features must be tested: log-returns $r_t = \ln S_{t+1} / \ln S_t$ should be (i) not autocorrelated, (ii) normally distributed and (iii) stationary. For this purpose, the execution of a Ljung-Box test, a Chi-square goodness-of-fit or a Jarque-Bera test, and an Augmented Dickey-Fuller test is needed, respectively. If these tests confirm that r_t has all the desired characteristics, not being able to reject the null hypothesis of absence of autocorrelation and normality, as well as rejecting the null hypothesis of unit root, it can be said that S_t can be drawn by a GBM and, therefore, its parameters α and σ can obtained by the following procedure (Ladde and Wu 2009). Firstly, applying Itô's lemma to the function $\ln S_t$ leads to:

$$d \ln S_t = \left(\alpha - \frac{\sigma^2}{2} \right) dt + \sigma dW_t, \tag{16}$$

which, using an Euler-type discretization over the time interval $[t-1, t]$, can be rewritten as:

$$\ln \frac{S_t}{S_{t-1}} = r_t = \left(\alpha - \frac{\sigma^2}{2} \right) \Delta t + \sigma \left(W_t - W_{t-1} \right), \tag{17}$$

with $\Delta t = 1$. Finally, $\alpha - \frac{\sigma^2}{2}$ can be obtained as the average value of r_t and σ as its standard deviation. The same result can be obtained using the OLS estimating $\alpha - \frac{\sigma^2}{2}$ as the unique coefficient of (17), and then solving for α, given σ, although this estimate has no statistical significance due to the relatively small sample size.

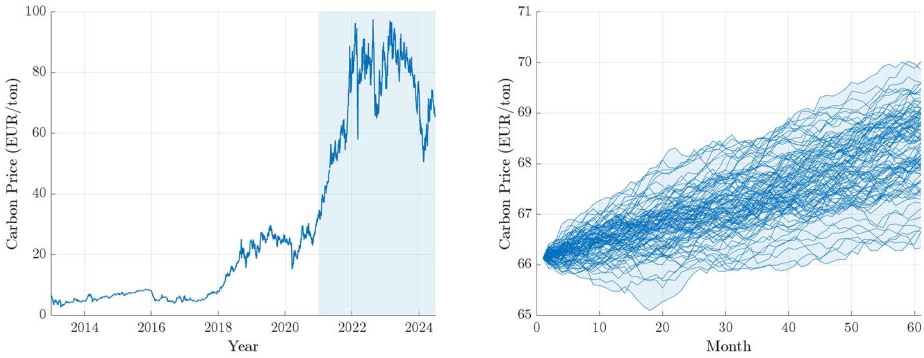
3 Numerical Analysis

3.1 Simulation Setup

The numerical analysis presented in this section is based on the parameters depicted in Table 1, which provides baseline values as well as additional ones used for robustness

Table 1 Parameter values for the baseline case and for sensitivity analysis

Parameter	Value(s)		Description
α	0.0280		GBM drift
σ	0.1051	0.0946, 0.1156	GMB diffusion
P_0	66.1300		GBM initial value
κ	1125		Cost function shift parameter
c	1000		Cost function scale parameter
γ	1000	0.9000, 1.1000	Cost function shape parameter
r	0.0500		Risk-free interest rate



(a) Observed prices.

(b) Simulated prices.

Fig. 1 Carbon price observed in the EU ETS secondary market from the beginning of Phase III to date. Parameter estimation is based on prices observed from the start of Phase IV to the present, represented by the shaded area (left panel). Outcome of 100 GBM trajectories simulated based on estimated parameters (right panel)

check.[14] The dynamics of investment decisions and the clean transition are analyzed over an arbitrary five-year time frame. As GBM drift and diffusion coefficients, namely α and σ, we use the outcome of the two-stage methodology described in Sect 2.4. We applied this procedure to EU ETS carbon price, depicted in the left panel of Fig. 1, retrieved from the International Carbon Action Partnership (ICAP).[15] We focus on monthly data from the beginning of EU ETS phase IV to date, i.e. between January 2021 and June 2024. The GBM hypothesis is validated as the statistical tests executed confirm all the desired features, with a significance level of 1%, and the following estimates are $\alpha = 0.0280$ and $\sigma = 0.1051$, respectively. The presence of a positive drift parameter is consistent with the recent upward shift observed on the EU ETS, primarily attributable to the significant decrease in the cumulative supply of emission allowances (Gerlagh et al. 2022). The deterministic part of (2) is indeed the standard differential equation for exponential growth, or decay, with rate parameter α. In light of the importance of volatility in investment decision (Comincioli et al.

[14] Results presented in this section have been subjected to extensive sensitivity analysis. Most relevant ones, i.e. those with respect to γ, are presented in the main body of the manuscript, while other, i.e. those focusing on σ, are in Appendix B. Robustness check regarding other parameters, which we omit here for the sake of brevity, confirm the robustness of the results and are available upon request.

[15] See: https://icapcarbonaction.com/en/ets-prices

2021b), in addition to estimated value, two more values for σ obtained varying the estimate by $\pm 10\%$, have been tested for robustness check.

The estimates of α and σ have been used to generate 100 trajectories of the GBM (2), shown in the right panel of Fig. 1, to obtain a Monte Carlo driven simulation on the corresponding CIFs (9). This low number of simulations was chosen to enhance graph readability and to highlight the stepwise dynamics of the CIF. To ensure result reliability, we repeated the exercise with 10,000 simulations, finding only $\pm 1\%$ differences on variables on which we later draw policy implications. As GBM initial value, we used the carbon price observed in EU ETS secondary market on June 30th 2024, i.e. 66.13 EUR per metric ton. This date was chosen because it aligns with the end the time series used for the previous estimation. In addition, given P_o, the parameters of the cost function (1), i.e. κ, c and γ, have been calibrated in order to generate an investment trigger comparable to market price. Results of numerical simulations remain therefore robust to changes in the GBM's initial value, provided the cost function is recalibrated. Finally, risk-free interest rate is set to the standard level of 5% (Dixit and Pindyck 1994).

We execute three sets of numerical analysis, to account for different policy scenarios. The first one reflects the current formal state of the EU ETS, where no explicit floor is imposed on the permit market price. This business-as-usual (BAU) case corresponds to the solution detailed in Sect 2.2, and is referred to as the "no price floor" case. Then, to assess how the green transition process can be influenced by alternative policy options, we introduce a price floor as outlined in Sect 2.3. The price floor is declined twofold. On the one hand, we examine a constant one, which impacts all firms equally, regardless of their industry or green investment cost. This scenario is referred to as the "constant price floor" case. On the other hand, we introduce a hypothetical, asymmetric price floor designed to impact firms differently based on their sector and, therefore, abatement ability. While not feasible for real-world implementation, as market price is unique, this scenario serves as an intellectual exercise, mimicking the effect of free allowance allocations granted to hard-to-abate sectors.[16] Specifically, since these industries receive substantial free allowances due to their relevance, also measured in terms of trade intensity (Verde et al. 2019; Abrell et al. 2022), their marginal cost of brown production is effectively reduced. This, in turn, makes carbon abatement less attractive, further delaying the green investment decision, an outcome already expected given their higher abatement costs compared to other industries. Hereafter, we refer to this scenario as the "refined price floor" case. Additionally, in order to facilitate comparison between the constant and the refined price floors, both are calibrated to generate the same tax revenue for the authority.

Later in this section, results for the three policy scenarios are shown and commented, while Table 2 provides a summary of numerical results. Tables 4 and 5 in Appendix B provide the outcomes of the sensitivity analysis performed with respect to σ.

3.2 Scenario 1: No Price Floor

Figure 2 focuses on the basic framework, detailed in Sect 2.2, where the carbon price is not constrained by any price floor. The left panel shows the investment trigger as a function of firm location, Q, for different levels of the elasticity (in absolute value) of the firm's marginal investment cost, γ. The right panel depicts, for the base case value of γ, the CIF (9)

[16] See: https://climate.ec.europa.eu/eu-action/eu-emissions-trading-system-eu-ets/free-allocation_en

Table 2 CIF's 10th, 50th (median) and 90th percentiles over a 5 years horizon, sampled with a semiannual frequency, with baseline $\sigma = 0.1051$

Years	No floor			Constant floor			Refined floor		
	10th	50th	90th	10th	50th	90th	10th	50th	90th
$\gamma = 0.9$									
0.0	–	–	–	–	–	–	–	–	–
0.5	–	–	–	–	–	–	–	7.0	20.0
1.0	–	–	–	–	–	25.0	0.5	17.0	32.5
1.5	–	–	23.0	–	13.5	43.0	6.5	26.5	44.0
2.0	–	–	39.5	–	25.5	52.0	14.0	33.0	52.0
2.5	–	8.0	50.0	–	35.0	59.0	17.5	38.5	58.0
3.0	–	29.0	62.5	15.0	46.0	68.0	27.5	47.0	67.0
3.5	–	42.5	64.5	26.5	54.0	69.5	33.0	53.5	68.5
4.0	9.0	51.5	66.0	35.5	60.0	71.0	39.0	59.0	69.5
4.5	28.5	60.0	70.0	45.5	66.0	74.0	46.5	64.5	73.0
5.0	32.5	65.0	74.0	48.0	69.5	76.5	48.0	68.5	75.5
$\gamma = 1.0$									
0.0	–	–	–	–	–	–	13.0	13.0	13.0
0.5	–	–	–	–	5.0	34.5	14.5	26.0	39.0
1.0	–	–	29.5	–	30.0	51.0	18.5	37.0	51.0
1.5	–	13.5	50.0	4.0	44.0	62.0	25.5	45.5	60.0
2.0	–	30.5	59.5	23.5	51.5	67.5	33.5	51.0	65.5
2.5	–	41.5	66.0	30.5	57.0	71.5	37.0	55.5	70.5
3.0	15.0	53.0	74.0	45.5	64.0	77.0	46.5	62.0	76.0
3.5	31.0	61.0	75.5	52.0	68.5	78.0	51.5	67.0	77.0
4.0	41.5	67.0	76.0	57.0	72.0	79.0	56.0	71.0	78.0
4.5	53.0	72.0	79.0	63.5	76.0	81.0	62.0	74.5	80.0
5.0	55.5	75.5	81.5	65.0	78.0	82.5	63.0	77.0	82.5
$\gamma = 1.1$									
0.0	–	–	–	–	–	–	28.0	28.0	28.0
0.5	–	–	25.5	5.0	37.0	55.0	29.5	42.0	54.0
1.0	–	16.5	52.0	18.5	52.0	65.5	33.5	52.0	63.0
1.5	–	42.0	65.0	35.5	61.0	72.0	41.0	59.0	71.0
2.0	2.0	52.5	70.5	48.0	66.0	76.0	48.5	63.5	74.5
2.5	18.0	59.0	75.5	52.5	69.5	79.0	52.0	67.5	77.5
3.0	43.0	67.0	80.0	61.5	74.0	83.0	59.5	72.0	82.0
3.5	53.0	72.0	81.5	66.0	77.0	83.5	63.5	75.0	82.5
4.0	60.0	76.0	82.0	69.5	79.0	84.0	67.5	78.0	83.0
4.5	67.0	79.0	84.0	73.5	82.0	85.0	72.0	81.0	85.0
5.0	68.0	81.5	85.5	74.0	83.5	86.5	73.0	82.5	86.0

for each simulated path, as well as their median, and 10th and 90th percentiles. A sensitivity analysis of the CIFs with respect to γ is shown later.

The investment trigger, being a linear function of marginal investment costs (cf., (8)), is also decreasing with respect to a firm's location Q. Firms located farther to the left (right) have a higher (lower) investment cost, so that a higher (lower) carbon price is needed to make green action worthwhile. As the shape parameter γ increases we observe *ceteris paribus* a reduction in the investment trigger. This happens because an increase (decrease) of

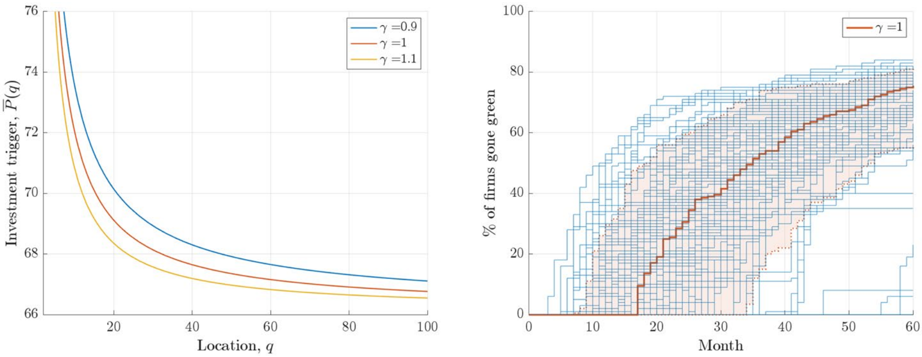


Fig. 2 Investment trigger as a function of Q for different values of γ (left panel) and CIF, with $\gamma = 1.0$, for each simulated price trajectory (right panel), without any price floor. Thick line represents median values and shaded area shows the difference between 10th and 90th percentile

γ reduces (increases) the investment cost, again because of the linear relationship with the investment trigger.

The trajectory of the CIFs is always a monotonically increasing piecewise constant function, where discontinuities appear when the simulated outcome of the GBM for the specific Q at that time hits the corresponding trigger. In addition, when the trigger is hit, a mass of firms undertakes green investment, i.e. those whose specific trigger is lower than P_t, leaving the function constant until the next overrun of the (new) lowest investment trigger. The thick line represents the median of all CIFs, while the shaded area comprises those between the 10th and 90th percentiles, offering several key insights as well as a confidence interval. Firstly, the policymaker is given an estimate of when, on average, the green transition process begins, i.e. after 18 months in this simulation. Additionally, the median CIF also show what is the expected emission cut by 5 years, i.e. 75.5%. Note that the steepness of the CIF also provides valuable information. For any given pair of transition staring times and final emission cut, a faster investment process, i.e. a steeper CIF, is preferable as it minimizes the total amount of emissions. Finally, it is worth noting that the log-normal behavior of the GBM is evident, as we observe a significantly higher density of trajectories above the median compared to those below.

Figure 3 shows the CIFs, based on the same GBM simulations of the previous one, for two additional levels of γ, selected to model a scenario with a higher (lower) elasticity of marginal investment costs, depicted in the left (right) panel. As expected, if the cost of green investment increases, a higher carbon price is required to make continuing with the BAU approach less attractive, i.e. to incentivize emissions cuts. This resulting effect has two components. On the one hand, a delay of the begin of transition process, which begins on average after 27 months, compared with the 18 months of the baseline case, is observed. In addition, the expected reduction in emissions after 5 years is lower, that is 65.0% instead of 75.5%. Conversely, a lower investment cost yields the opposite effects and is, thus, more favorable from an environmental perspective, as it accelerates emission reductions. In this case, the first firm's investment is expected after 11 months, while the final total emission cut is expected to be 81.5%.

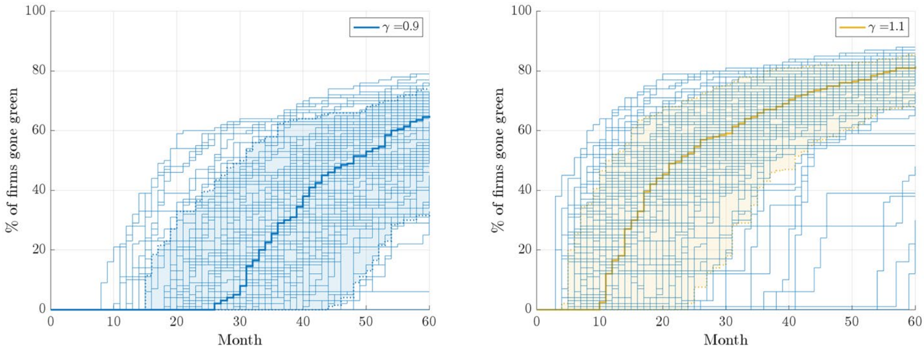


Fig. 3 CIFs with $\gamma = 0.9$ (left panel) and $\gamma = 1.1$ (right panel), for each simulated price trajectory, without any price floor. Thick lines represent median values and shaded areas show the difference between 10th and 90th percentiles

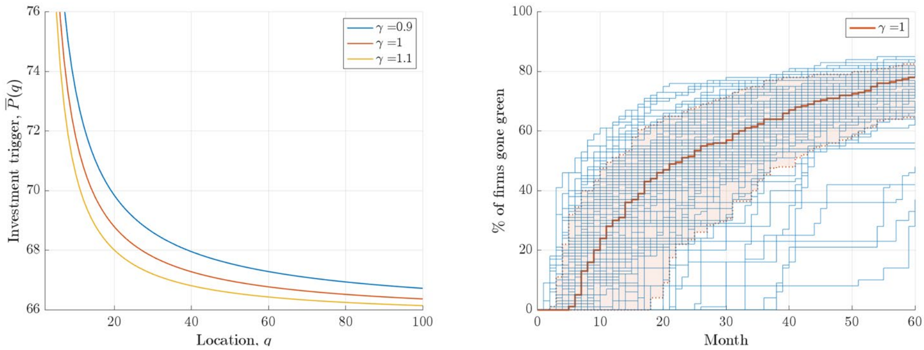


Fig. 4 Investment trigger as a function of Q for different values of γ (left panel) and CIF, with $\gamma = 1.0$, for each simulated price trajectory (right panel), with a constant price floor. Thick line represents median values and shaded area shows the difference between 10th and 90th percentile

3.3 Scenario 2: Constant Price Floor

This section presents the results of the scenario where the carbon price is subject to a policy-imposed floor, as modeled in Sect 2.3. The aim of these numerical simulations, based on the same GBM trajectories as above, is then to determine numerically whether the imposition of a price floor influences investment decisions and, if so, to quantify the extent of this impact. Although it cannot be highlighted analytically, since the system of (12) to (15) must be solved numerically, an impact of the floor on the investment decision is expected, as it appears from (11). For this purpose, we test the effects of both the constant and the refined price floors.

Figure 4 focuses on a constant price floor of 40 EUR/ton and shows the investment trigger for different levels of γ, as well as the CIFs for its baseline value.[17] Investment triggers depicted in the left panel show a downward shift, −0.63% on average, compared to the no floor scenario. With a lower investment trigger, the investment decision will be

[17] Simulation results are robust with respect to different levels of the price floor.

made sooner, as the dynamics in the right panel clearly show. More specifically, the first green investment is expected to take place after 6 months, i.e. 12 months earlier than in the BAU scenario. In addition, after 5-years, the expected emission cut is 78%, i.e. 2.5% points higher than in the no floor case. As one might expect, the magnitude of this effect depends on the level of the floor. However, it is also noteworthy that the impact on the investment trigger occurs even with a price floor set significantly lower than recent market prices. This is crucial from the policymaker's perspective, because incentivizing green investments does not require a price floor high enough to actually constrain the market price. It is sufficient that there is a *possibility* of the floor being reached, i.e., the "threat is enough". A sensitivity analysis on these results with respect to γ is shown in the Appendix by Fig. 7.

3.4 Scenario 3: Refined Price Floor

Figure 5 illustrates the results obtained in the case of the refined price floor, i.e. a function $\underline{P}(Q)$ that allows to mimic the effect on decision making process of hard-to-abate firms that benefit from a permit cost reduction due to a relevant supply of free permits. More specifically, we set $\underline{P}(Q) = 35 + 0.1Q$.

As in the previous case, investment triggers shows an overall downward shift compared to the BAU scenario, regardless of the level of γ, with an accelerating effect on green transition. However, the introduction of a refined price floor based on firms' location has a non-linear impact on the investment trigger. Hard-to-abate companies, thanks to the additional free allowances and to the consequent decrease in cost for permits, indeed undertake green investment at a higher market price. For this reason, their trigger is higher with respect to the constant floor case. In our framework, this distortion is observed at $Q = 50$ to the left and the gap increases as Q decreases. The opposite holds for firms for which green investment is cheaper, as their investment decision is accelerated. This dynamics can be seen also in Fig. 6, which is later object of discussion in Sect. 4. The effect of a refined floor can be quantified observing the CIFs depicted in the right panel. More specifically, first investment takes places immediately for roughly the 13% of firms and after 5 years the expected emission cut is 77%, 1% lower than in the constant floor scenario. Moreover, compared to the previous cases, the CIF exhibits a more linear behavior, which reduces the area under the

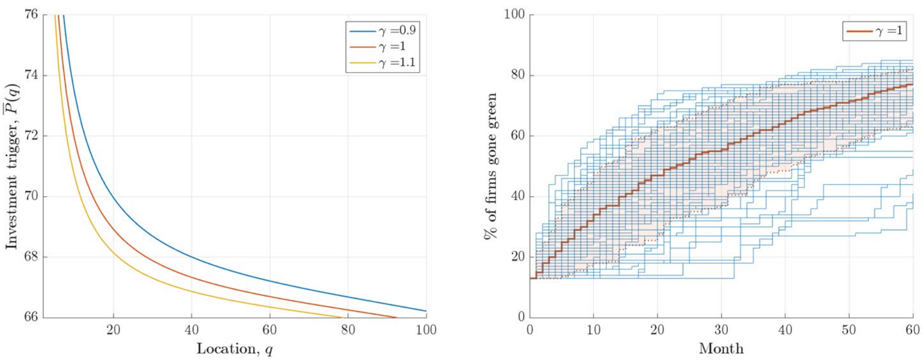


Fig. 5 Investment trigger as a function of Q for different values of γ (left panel) and CIF, with $\gamma = 1.0$, for each simulated price trajectory (right panel), with a refined price floor. Thick line represents median values and shaded area shows the difference between 10th and 90th percentile

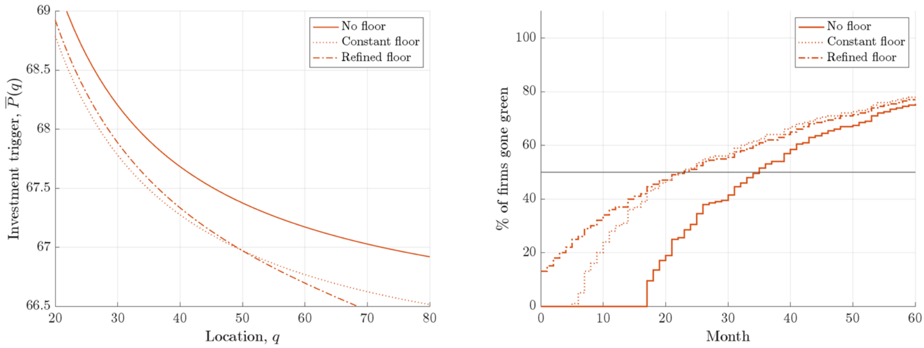


Fig. 6 Investment trigger as a function of Q (left panel) and median CIF (right panel) for different policy options and $\gamma = 1$. Horizontal line corresponds to a cut of emissions by 50%

curve, *ceteris paribus*. A sensitivity analysis on these results with respect to γ is shown in the Appendix by Fig. 8.

4 Discussion

The numerical simulations presented in Sect 3 illustrate the dynamics of the green transition process, providing insights in the timing of green investment and the emissions reduction achieved within a given time frame under the (i) "no price floor", (ii) "constant price floor" and (iii) "refined price floor" scenarios. The effectiveness of one policy option over another can be assessed by comparing their respective median CIFs, which indicate, on average, the expected timing of the investments and the projected emissions reduction. While these results stem from a model that substantially simplifies the complexity of the decision-making process, they nonetheless confirm the effectiveness of a price floor in reducing emissions (Clò et al. 2013). However, the comparison between the constant and refined price floors reveals a trade-off, discussed later in this section, between an earlier start to the transition and a greater reduction in emissions over the medium to long term. This information is valuable for regulators seeking to design optimal environmental policies, as it helps determine which price floor design best aligns with their objectives.

Figure 6 provides a comparison among the three scenarios considered. The left panel presents investment triggers. Since a constant price floor affects the investment decision of all firms uniformly, the investment trigger is simply shifted downward from its BAU level. This happens because a constant price floor does not introduce any distortion based on firm's location or, equivalently, industrial sector. In contrast, a refined price floor which mimics the effect of sector-specific free allowances, alters both the slope and position of the trigger. Indeed, if hard-to-abate sectors receive larger free allocations, then the investment trigger becomes steeper, while for industries with lower investment costs, it flattens. The right panel shows how this distortion is reflected in the percentage of firms undertaking green investment. As expected, the lowest CIF corresponds to the scenario without a price floor. Then, the comparison between the CIFs of the two other cases reveals the following key dynamic. While the refined floor accelerates investment for firms already inclined to transition early, it further delays the decision for hard-to-abate industries. Consequently, the CIF under the refined floor initially exceeds that of the constant floor, but over time, the trend reverses, with the constant floor leading to greater emission reductions in the long run.

However, the analysis of first investment timing and of the emissions reduction achieved at the end of the time frame do not provide a complete picture. A policy decision based solely on these metrics would overlook a crucial aspect captured by the CIF, i.e., the subtended area, which depends on the CIF's concavity. Indeed, this area represents the cumulative stock of avoided emissions over a given period, and it is therefore essential to consider this factor when selecting the optimal policy option. From Fig. 6 it is clear that a refined floor generates a larger area compared to the constant one, despite its CIF being higher only in the short term. However, these results are presented for an arbitrary five-year time frame, while the effects of the policy extend beyond this period. Therefore, the choice between the two policies is not straightforward, as it involves a trade-off between achieving a faster emissions reduction in the short run and a higher overall reduction in the long run. In light of this, an authority with a short-term (long-term) perspective might favor a refined (constant) price floor. However, this decision critically depends on the policymaker's objective function and the rate used to discount future costs and benefits arising from their choices. Using information provided by the area subtended by the CIF to complement previous findings could therefore support regulators in shaping their environmental policies to balance the long-term target of achieving a specific emissions reduction within a specific deadline with the goal of minimizing cumulative emissions within that period. This comparison is based on the outcomes obtained with the baseline value $\gamma = 1$. However, a different level of investment cost does not affect the quality of results, as shown in the Appendix B by Figs. 9 and 10.

The CIF path represents the green transition process, tracking the percentage of firms that have invested in emissions reduction at each time t. This pathway can be shaped primarily by policy decisions regarding the implementation of a price floor for permit prices, but it is also influenced by other parameters mentioned above and analyzed in the sensitivity analysis, e.g. investment cost, price volatility or interest rate. All these variables, together with the policy option chosen, represent the leverages available to the policymaker to shape green transition in order to meet specific goals.

5 Conclusion

This paper whishes to contribute to both the scientific literature and the policy debate by presenting a rigorous model describing the green investment decision-making process at the firm level and the consequent broader dynamics of the green transition. Our model demonstrates how this transition can be effectively incentivized through targeted policy instruments. Additionally, through calibrated numerical simulations based on real data, we quantify the effects of different policy approaches, offering insights into their impact on both the timing of green investment and emission reduction over a specified time frame. These findings provide regulators with a valuable tool for selecting and fine-tuning the most effective policies to accelerate the green transition and achieve emission reduction targets.

A key policy instrument for fostering the green transition is the price floor, which is the central focus of this study. The introduction of a price floor in the EU ETS market is an ongoing topic of discussion (Hintermayer 2020), though the reduction in total allowances may effectively act as an implicit floor. Implementing a price floor supports the green transition by both accelerating initial investments and achieving a greater reduction in emissions over a given time horizon (Ohlendorf et al. 2022). Through a set of numerical simulations calibrated on real data, we assess the impact of two alternative price floor designs. First, we examine the effects of a constant floor, which affects all firms uniformly. Second, we analyze a refined price floor, designed to simulate the distortion induced by the substantial amount of free allowances granted to hard-to-abate sectors. Our results provide useful insights for regulators in selecting the most appropriate type of price floor and optimizing its design to achieve specific objectives.

Despite its importance, the price floor is not the only lever available to regulators to accelerate the green transition. As shown in the sensitivity analyses, other variables at least partially within the control of the policymaker also play a significant role in investment decisions. Among these, the cost of green investment emerges as a key driver, as even small changes have a relevant impact on investment timing, potentially advancing or delaying the transition process by several months. Reducing the cost of green investment could therefore serve as a complementary instrument. Various policy options could achieve this goal, including direct subsidies or tax credit, which reduce upfront costs and enhance financial viability of green investment (Aldy and Stavins 2012; Bigerna et al. 2023). Additionally, green bonds and low-interest financing could lower long-term financial burdens, making green investment more attractive (Ehlers and Packer 2017). Further measures include facilitating Public-Private Partnerships (PPPs) and supporting economies of scale, to drive down unit costs (Vona and Consoli 2015). Finally, funding Research & Development (R&D) initiatives could push technological innovation, leading to more cost-effective green investment options (Costantini et al. 2015).

The volatility of permit market price also influences investment decisions, although its effect might be ambiguous (Feng et al. 2011). On the one hand, higher volatility raises the investment trigger, making firms less inclined to invest. On the other hand, it increases the probability of hitting the trigger, with an opposite effect on investment decision (Sarkar 2000). Within our framework, the first effect dominates, leading to delayed investments as volatility rises. Although policymakers cannot directly control market price volatility, they can still mitigate its impact through mechanisms such as the Market Stability Reserve (MSR), which absorbs excess supply during periods of low demand, thereby stabilizing prices.[18] Conversely, during periods of excess demand, adjustments to market liquidity could effectively reduce volatility (Narassimhan et al. 2018). The interest rate also plays a role in shaping investment decisions, as it affects the present value of future costs. Indeed, an increase (decrease) in the interest rate shifts the investment trigger upward (downward), delaying (accelerating) the transition process (Wong 2007). While primarily under the control of monetary authorities rather than environmental regulators, interest rate policies can still interact with climate policies, influencing the pace of green investment.

While our findings provide valuable insights into the effectiveness of different price floor designs, some limitations should be acknowledged. Firstly, our model abstracts from real-world complexities, relying on a limited set of variables to describe the decision-making process. Additionally, firm heterogeneity extends beyond differences in cost functions and is more complex than how it is represented in our framework. These limitations are inher-

[18] See: https://climate.ec.europa.eu/eu-action/eu-emissions-trading-system-eu-ets/market-stability-reserve_en

ent to the real option approach, even when calibrated on real data. Regarding the policy application, the instruments available to regulators are not limited to price floors, and price floors themselves can be designed in ways beyond the two configurations analyzed in this study. For instance, a time-dependent price floor that increases over time could exert greater upward pressure on prices as the green transition progresses. While the limitations of the real option model are intrinsic to this methodological choice, exploring alternative price floor designs and other regulatory approaches beyond price floors remains an avenue for future research.

Moreover, it is worth noting that the carbon price does not operate in isolation within the EU ETS, being affected by other factors beyond the mere demand and supply of allowances. Indeed, the market price is also influenced by broader energy market dynamics. Previous research has found asymmetric dependencies between EU ETS prices and fossil fuel prices, particularly for oil and natural gas, with stronger correlations in downturns (Soliman and Nasir 2019). While these interactions are beyond the scope of our model, they could represent an additional channel through which policy interventions, such as price floors, might indirectly affect firms' investment decisions, within a carbon-constrained economy.

In conclusion, our findings on the two policy options, i.e. constant floor and refined floor, highlight distinct effects on the green transition. The constant floor delays the beginning of the transition but achieves greater emissions reductions over the medium to long term, i.e. within five years in our exercise. Conversely, the refined floor encourages earlier investment but results in a lower percentage of firms undertaking the green investment over time. Thus, the optimal choice between these policy instruments depends on the policymaker's priorities. A regulator with a short-term perspective might favor the refined price floor, which accelerates early investment. On the contrary, a policymaker focused on long-term emissions reductions may prefer the constant floor. Ultimately, the policymaker's decision between the two alternatives will depend on their utility function and the discount rate applied to future costs and benefits, as these factors influence the perceived trade-off between immediate reductions and long-term outcomes.

Appendix

A GBM Validation and Estimation for Emission Quantities

Greenhouse gas emission data used for this exercise are provided by the Emissions Database for Global Atmospheric Research (EDGAR) (Crippa et al. 2022).[19] Carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) and fluorinated gases emissions data are available for 210 countries from 1970 (2000) with yearly (monthly) frequency. In this exercise, we focus CO_2 yearly data until 2021, to take advantage of the greater historical depth,[20] limited to EU ETS participating countries, either individually or aggregated.

[19] See: https://edgar.jrc.ec.europa.eu/dataset_ghg70

[20] Another rationale of this choice stems from the significant autocorrelation observed in monthly data, due to the seasonal pattern proper of many energy-related series (Comincioli et al. 2024), which makes the procedure explained in this section not applicable.

The results of GBM validation procedure are collected in Table 3 where, for sake of brevity,[21] a check mark indicates that log-returns r_t of a specific series (i) show no autocorrelation, (ii) are confirmed normal by at least one test and (iii) are stationary, all at least with a 5% significance level. These results can be analyzed both on a country basis and at an aggregate level.

On the one hand, it is noteworthy that countries whose emissions can be described by a GBM model, that is meeting all the criteria outlined in Sect 2.4, account for 66.6% of the total. Those that instead fail at most one normality test correspond to 23.6% while finally those for whom the GBM validation procedure fails account for 9.8% of total emissions. In addition, focusing on the ten largest emitters, it is worth noting that (i) eight of the ten, the only exceptions being Spain and Poland, passes all the tests required by the GBM validation procedure and that (ii) half of them present a negative deterministic drift. The latter result

Table 3 Share of total emissions as of December 2021, result of GBM validation procedure and, provided a positive outcome, estimate of GBM parameters for countries currently in the EU ETS, either individually and aggregated. Countries in italics are those for which one of the two normality tests fails

Country	Share 2021	Test GBM	μ	σ
AUT	0.0260	√	0.0093	0.0392
BEL	0.0315	√	−0.0038	0.0485
BGR	0.0153	√	−0.0040	0.0645
CYP	0.0020	-	-	-
CZE	0.0345	√	−0.0084	0.0353
DEU	0.2217	√	−0.0063	0.0329
DNK	0.0125	√	−0.0049	0.0739
ESP	0.0787	√	0.0136	0.0557
EST	0.0057	-	-	-
FIN	0.0297	√	0.0094	0.0588
FRA	0.1072	√	−0.0053	0.0419
GRC	0.0169	-	-	-
HRV	0.0070	√	0.0002	0.0569
HUN	0.0183	-	-	-
IRL	0.0108	√	0.0125	0.0485
ISL	0.0009	√	0.0102	0.0621
ITA	0.1086	√	0.0038	0.0370
LTU	0.0062	-	-	-
LUX	0.0027	√	-0.0106	0.0746
LVA	0.0041	-	-	-
MLT	0.0005	√	0.0272	0.1347
NDL	0.0487	√	0.0052	0.0390
NOR	0.0142	√	0.0060	0.0460
POL	0.1033	√	0.0033	0.0433
PRT	0.0169	-	-	-
ROU	0.0280	-	-	-
SVK	0.0129	√	−0.0005	0.0417
SVN	0.0048	√	0.0119	0.0630
SWE	0.0303	√	−0.0015	0.0399
EU ETS	**1.0000**	√	**−0.0010**	**0.0286**

[21] Detailed results are available upon request.

is in line with the global effort of carbon dioxide emissions and supports empirical literature assessing that greenhouse gases emission can be effectively reduced by environmental taxes and promotion of cleaner energy sources (Ghazouani et al. 2021). On the other hand, focusing on EU ETS participating countries combined, emissions series is proved to be possibly drawn by a GBM, with parameters $\mu = -0.0010$ and $\sigma = 0.0286$, respectively. This negative drift parameter indicates that the deterministic linear trend of combined emission is decreasing. In addition, its modest magnitude assess that combined emissions are yes decreasing, but very slowly. This result is in line with the majority of empirical studies, suggesting that the aggregate reduction of carbon dioxide emissions are generally limited between 0% and 1.5% per year, especially in the case of ETS (Green 2021).

B Sensitivity Analysis

Figures 7 and 8 display the results of a sensitivity analysis on the CIF with respect to γ, for both the "constant price floor" and "refined price floor" scenarios. The left (right) panel of each figure corresponds to the case of a 10% reduction (increase) of γ from the baseline level, which makes investment more (less) expensive, in line with (1). These results, net of the effect of higher or lower investment costs, confirm the key findings presented in Sect 3. Specifically, (i) the introduction of a price floor accelerates the green transition process, also increasing the total emission reduction by the end of the period, and (ii) a refined price floor accelerates the transition in the earlier part of the period, but slows it down thereafter, making the CIF behave in a more linear manner.

These dynamics are also confirmed by Figs. 9 and 10, showing the comparison between investment trigger (left panel) and CIF (right panel), for the lower and higher level of γ, respectively.

Finally, Tables 4 and 5 present the results of the sensitivity analysis with respect to σ, whose baseline value is reduced and increased by 10% from the baseline case, respectively. These tables summarize the CIF values over the timeframe of interest, sampled at a semi-annual frequency, for the three policy scenarios under investigation, using the three different γ levels tested. Analysis of these results shows that a reduction (increase) in vola-

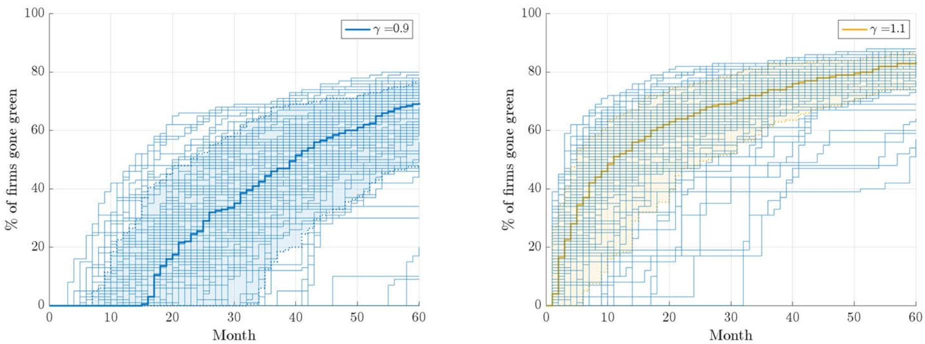


Fig. 7 CIFs with $\gamma = 0.9$ (left panel) and $\gamma = 1.1$ (right panel), for each simulated price trajectory, with a constant price floor. Thick lines represent median values and shaded areas show the difference between 10th and 90th percentiles

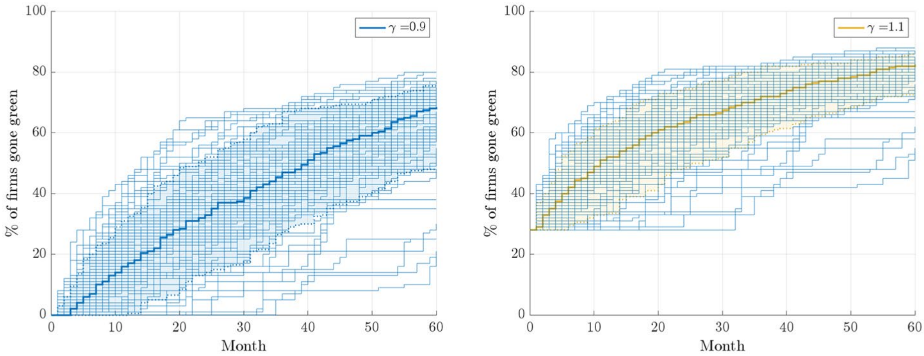


Fig. 8 CIFs with $\gamma = 0.9$ (left panel) and $\gamma = 1.1$ (right panel), for each simulated price trajectory, with a refined price floor. Thick lines represent median values and shaded areas show the difference between 10th and 90th percentiles

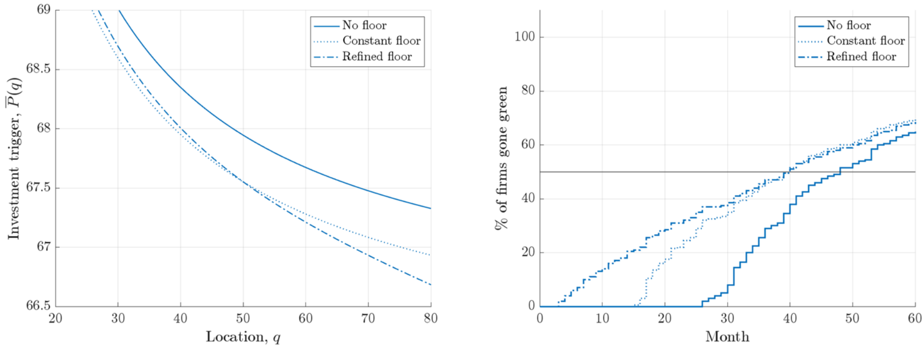


Fig. 9 Investment trigger as a function of Q (left panel) and median CIF (right panel) for different policy options and $\gamma = 0.9$. Horizontal line corresponds to a cut of emissions by 50%

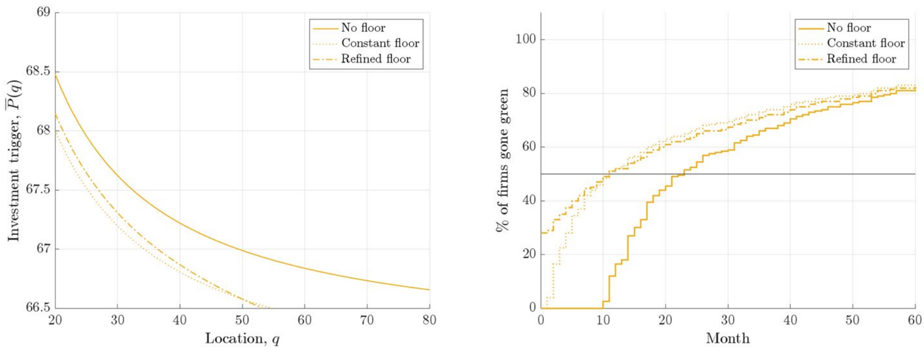


Fig. 10 Investment trigger as a function of Q (left panel) and median CIF (right panel) for different policy options and $\gamma = 1.1$. Horizontal line corresponds to a cut of emissions by 50%

Table 4 CIF's 10th, 50th (median) and 90th percentiles over a 5 years horizon, sampled with a semiannual frequency, with baseline $\sigma = 0.0946$

Years	No floor			Constant floor			Refined floor		
	10th	50th	90th	10th	50th	90th	10th	50th	90th
$\gamma = 0.9$									
0.0	50.0	50.0	50.0	55.0	55.0	55.0	55.0	55.0	55.0
0.5	52.0	57.0	63.0	57.0	61.0	66.0	56.5	60.0	65.0
1.0	54.5	63.0	68.5	59.0	65.5	70.5	58.0	65.0	70.0
1.5	58.0	66.0	73.0	61.5	69.0	74.0	61.0	68.0	74.0
2.0	60.5	70.0	75.0	64.0	72.0	76.5	63.0	71.0	75.5
2.5	63.0	73.0	78.0	66.0	74.5	79.0	65.0	74.0	78.0
3.0	66.0	74.0	79.0	68.5	76.0	80.0	68.0	75.0	80.0
3.5	68.0	76.0	81.0	70.0	77.0	82.0	69.5	77.0	82.0
4.0	71.0	78.0	82.0	73.0	79.0	83.0	72.0	78.0	83.0
4.5	73.0	79.0	83.0	74.0	80.0	83.0	74.0	80.0	83.0
5.0	74.0	80.5	84.0	76.0	81.0	84.0	75.0	81.0	84.0
$\gamma = 1.0$									
0.0	66.0	66.0	66.0	69.0	69.0	69.0	68.0	68.0	68.0
0.5	67.0	70.0	74.0	70.0	73.0	76.0	69.5	72.0	75.0
1.0	69.0	74.0	77.5	71.5	76.0	79.0	71.0	75.0	78.0
1.5	71.0	76.0	80.0	73.0	78.0	81.0	72.0	77.0	81.0
2.0	72.5	78.0	81.5	74.5	79.5	82.5	74.0	79.0	82.0
2.5	74.0	80.0	84.0	76.0	81.0	84.0	75.0	81.0	84.0
3.0	76.0	81.0	85.0	77.5	82.0	85.0	77.0	82.0	85.0
3.5	77.0	82.0	86.0	78.5	83.0	86.0	78.0	83.0	86.0
4.0	79.0	84.0	87.0	80.0	84.0	87.0	80.0	84.0	87.0
4.5	80.0	84.0	87.0	81.0	85.0	87.0	81.0	85.0	87.0
5.0	81.0	85.0	88.0	82.0	86.0	88.0	82.0	86.0	88.0
$\gamma = 1.1$									
0.0	75.0	75.0	75.0	77.0	77.0	77.0	76.0	76.0	76.0
0.5	76.0	78.0	80.0	78.0	80.0	82.0	77.0	79.0	81.0
1.0	77.0	80.0	83.0	79.0	82.0	84.0	78.0	81.0	83.0
1.5	78.0	82.0	85.0	80.0	83.0	85.0	79.0	82.5	85.0
2.0	79.5	83.0	86.0	81.0	84.0	86.0	80.0	84.0	86.0
2.5	80.5	85.0	87.0	82.0	86.0	88.0	81.0	85.0	87.0
3.0	82.0	86.0	88.0	83.0	86.0	88.0	82.5	86.0	88.0
3.5	83.0	86.0	89.0	84.0	87.0	89.0	83.0	87.0	89.0
4.0	84.0	87.0	89.0	85.0	88.0	89.5	84.0	87.0	89.0
4.5	85.0	88.0	89.0	86.0	88.0	90.0	85.0	88.0	90.0
5.0	85.5	88.0	90.0	86.0	89.0	90.0	86.0	88.5	90.0

tility ultimately leads to an anticipation (delay) in investment. This suggests that, within the simplified framework of this study, the delaying effect of increased uncertainty on the investment decision offsets the anticipating one due to the increased probability of reaching the investment trigger. Consequently, the ambiguity of the net effect discussed in Sect 5 is not observed.

Table 5 CIF's 10th, 50th (median) and 90th percentiles over a 5 years horizon, sampled with a semiannual frequency, with baseline $\sigma = 0.1156$

Years	No floor			Constant floor			Refined floor		
	10th	50th	90th	10th	50th	90th	10th	50th	90th
$\gamma = 0.9$									
0.0	–	–	–	–	–	–	–	–	–
0.5	–	–	–	–	–	–	–	–	–
1.0	–	–	–	–	–	–	–	–	–
1.5	–	–	–	–	–	–	–	–	–
2.0	–	–	–	–	–	–	–	–	2.5
2.5	–	–	–	–	–	–	–	–	13.0
3.0	–	–	–	–	–	6.5	–	–	26.0
3.5	–	–	–	–	–	20.0	–	4.5	32.0
4.0	–	–	–	–	–	34.5	–	11.5	39.0
4.5	–	–	13.0	–	–	47.0	–	21.0	47.5
5.0	–	–	33.5	–	3.0	55.5	–	25.0	54.5
$\gamma = 1.0$									
0.0	–	–	–	–	–	–	–	–	–
0.5	–	–	–	–	–	–	–	–	–
1.0	–	–	–	–	–	–	–	–	–
1.5	–	–	–	–	–	–	–	–	5.5
2.0	–	–	–	–	–	–	–	–	17.5
2.5	–	–	–	–	–	7.0	–	2.0	30.0
3.0	–	–	–	–	–	40.0	–	10.5	43.5
3.5	–	–	–	–	–	48.5	–	20.0	49.0
4.0	–	–	22.0	–	1.5	56.5	–	28.0	55.0
4.5	–	–	44.5	–	30.5	64.0	–	38.5	62.0
5.0	–	–	56.0	–	38.5	69.5	1.0	42.0	67.0
$\gamma = 1.1$									
0.0	–	–	–	–	–	–	–	–	–
0.5	–	–	–	–	–	–	–	–	–
1.0	–	–	–	–	–	–	–	–	4.5
1.5	–	–	–	–	–	–	–	–	17.5
2.0	–	–	–	–	–	–	–	2.0	30.5
2.5	–	–	–	–	–	38.0	–	13.0	43.0
3.0	–	–	–	–	–	58.5	–	22.0	56.0
3.5	–	–	26.0	–	9.0	64.0	–	33.0	61.0
4.0	–	–	47.5	–	35.0	69.0	0.5	42.0	66.0
4.5	–	–	61.0	–	52.5	74.0	5.5	52.0	71.5
5.0	–	–	68.5	–	57.0	77.5	11.0	55.0	75.0

Funding Open access funding provided by European University Institute - Fiesole within the CRUI-CARE Agreement.

References

Abrell J, Cludius J, Lehmann S, Schleich J, Betz R (2022) Corporate emissions-trading behaviour during the first decade of the eu ets. Environ Resour Econ 83(1):47–83

Aldy JE, Stavins RN (2012) The promise and problems of pricing carbon: theory and experience. J Environ Dev 21(2):152–180

Allen MR, Frame DJ, Huntingford C, Jones CD, Lowe JA, Meinshausen M, Meinshausen N (2009) Warming caused by cumulative carbon emissions towards the trillionth tonne. Nature 458(7242):1163–1166

Anjos MF, Feijoo F, Sankaranarayanan S (2022) A multinational carbon-credit market integrating distinct national carbon allowance strategies. Appl Energy 319:119181

Antoci A, Borghesi S, Iannucci G, Sodini M (2021) Should I stay or should I go? Carbon leakage and ETS in an evolutionary model. Energy Econ 103:105561

Bigerna S, Hagspiel V, Kort PM, Wen X (2023) How damaging are environmental policy targets in terms of welfare? Eur J Oper Res 311(1):354–372

Biondi T, Moretto M (2015) Solar grid parity dynamics in Italy: a real option approach. Energy 80:293–302

Borghesi S (2011) The European emission trading scheme and renewable energy policies: credible targets for incredible results? Int J Sustain Econ 3(3):312–327

Borghesi S, Pahle M, Perino G, Quemin S, Willner M (2023) The market stability reserve in the EU emissions trading system: a critical review. Annu Rev Resour Econ 15(1):131–152

Boyce JK (2018) Carbon pricing: effectiveness and equity. Ecol Econ 150:52–61

Burtraw D, Palmer K, Kahn D (2010) A symmetric safety valve. Energy Policy 38(9):4921–4932

Clò S, Battles S, Zoppoli P (2013) Policy options to improve the effectiveness of the eu emissions trading system: a multi-criteria analysis. Energy Policy 57:477–490

Comincioli N, Guerini M, Vergalli S (2024) Carbon Taxation and Electricity Price Dynamics: empirical Evidence from the Australian Market. Environ Resour Econ 1–31

Comincioli N, Hagspiel V, Kort PM, Menoncin F, Miniaci R, Vergalli S (2021a) Mothballing in a duopoly: evidence from a (shale) oil market. Energy Econ 104:105583

Comincioli N, Panteghini PM, Vergalli S (2021b) Welfare effects of business taxation under default risk. Int Tax Public Finan 28(6):1412–1429

Costantini V, Crespi F, Martini C, Pennacchio L (2015) Demand-pull and technology-push public support for eco-innovation: the case of the biofuels sector. Res Policy 44(3):577–595

Crippa M, Guizzardi D, Banja M, Solazzo E, Muntean M, Schaaf E, Pagani F, Monforti-Ferrario F, Olivier J, Quadrelli R, et al. (2022) CO2 emissions of all world countries. *JRC Science for Policy Report*, *European Commission, EUR*, 31182.

Dixit A, Pindyck R (1994) Investment under Uncertainty. Princeton University Press, Princeton

Doda B, Verde SF, Borghesi S (2022). ETS Alignment: a price collar proposal for carbon market integration. Technical report, European University Institute.

Edenhofer O, Flachsland C, Kalkuhl M, Knopf B, Pahle M (2019) Options for a carbon pricing reform. Mercator Research Institute on Global Commons and Climate Change (MCC). *Berlin*

Ehlers T, Packer F (2017) Green bond finance and certification. BIS Quarterly Review September

Feng Z-H, Zou -L-L, Wei Y-M (2011) Carbon price volatility: evidence from eu ets. Appl Energy 88(3):590–598

Flachsland C, Marschinski R, Edenhofer O (2009) To link or not to link: benefits and disadvantages of linking cap-and-trade systems. Clim Policy 9(4):358–372

Flachsland C, Pahle M, Burtraw D, Edenhofer O, Elkerbout M, Fischer C, Tietjen O, Zetterberg L (2018) Five myths about an EU ETS carbon price floor. CEPS Policy Insights 17

Flachsland C, Pahle M, Burtraw D, Edenhofer O, Elkerbout M, Fischer C, Tietjen O, Zetterberg L (2020) How to avoid history repeating itself: the case for an EU Emissions Trading System (EU ETS) price floor revisited. Clim Policy 20(1):133–142

Gerlagh R, Heijmans RJ, Rosendahl KE (2022) Shifting concerns for the EU ETS: are carbon prices becoming too high? Environ Res Lett 17(5):054018

Ghazouani A, Jebli MB, Shahzad U (2021) Impacts of environmental taxes and technologies on greenhouse gas emissions: contextual evidence from leading emitter European countries. Environ Sci Pollut Res 28:22758–22767

Green JF (2021) Does carbon pricing reduce emissions? A review of ex-post analyses. Environ Res Lett 16(4):043004

Gujba H, Thorne S, Mulugetta Y, Rai K, Sokona Y (2012) Financing low carbon energy access in Africa. Energy Policy 47:71–78

Hintermayer M (2020) A carbon price floor in the reformed EU ETS: design matters! Energy Policy 147:111905

Ladde G, Wu L (2009) Development of modified geometric brownian motion models by using stock price data and basic statistics. Nonlinear Anal Theory Methods Appl 71(12):e1203–e1208
Lee SL, Liew CY, Chen CK, Voon LL (2022) Geometric Brownian motion-based time series modeling methodology for statistical autocorrelated process control: logarithmic return model. Int J Math Math Sci
Marathe RR, Ryan SM (2005) On the validity of the geometric Brownian motion assumption. Eng Econ 50(2):159–192
Moon Y, Baran M (2018) Economic analysis of a residential pv system from the timing perspective: a real option model. Renew Energy 125:783–795
Narassimhan E, Gallagher KS, Koester S, Alejo JR (2018) Carbon pricing in practice: a review of existing emissions trading systems. Clim Policy 18(8):967–991
Ohlendorf N, Flachsland C, Nemet GF, Steckel JC (2022) Carbon price floors and low-carbon investment: a survey of german firms. Energy Policy 169:113187
Pahle M, Burtraw D, Flachsland C, Kelsey N, Biber E, Meckling J, Edenhofer O, Zysman J (2018) Sequencing to ratchet up climate policy stringency. Nat Clim Change 8(10):861–867
Pindyck RS (2002) Optimal timing problems in environmental economics. J Econ Dyn Control 26(9-10)
Postali FA, Picchetti P (2006) Geometric Brownian motion and structural breaks in oil prices: a quantitative analysis. Energy Econ 28(4):506–522
Sarkar S (2000) On the investment–uncertainty relationship in a real options model. J Econ Dyn Control 24(2):219–225
Soliman AM, Nasir MA (2019) Association between the energy and emission prices: an analysis of eu emission trading system. Resour Policy 61:369–374
Tang L, Wang H, Li L, Yang K, Mi Z (2020) Quantitative models in emission trading system research: a literature review. Renew Sustain Energy Rev 132:110052
Thijssen J (2010) Irreversible investment and discounting: an arbitrage pricing approach. Ann Finance 6(295)
Verde SF, Borghesi S (2022) The international dimension of the EU emissions trading system: bringing the pieces together. Environ Resour Econ 83(1):23–46
Verde SF, Galdi G, Alloisio I, Borghesi S (2021) The EU ETS and its companion policies: any insight for China's ETS? Environ Develop Econ 26(3):302–320
Verde SF, Teixidó J, Marcantonini C, Labandeira X (2019) Free allocation rules in the eu emissions trading system: what does the empirical literature show? Clim Policy 19(4):439–452
Vona F, Consoli D (2015) Innovation and skill dynamics: a life-cycle approach. Ind Corp Change 24(6):1393–1415
Wong KP (2007) The effect of uncertainty on investment timing in a real options model. J Econ Dyn Control 31(7):2152–2167
Yi C, Pendall E, Ciais P (2015) Focus on extreme events and the carbon cycle. Environ Res Lett 10(7)
Yuan X, Su C-W, Umar M, Shao X, Lobon O-R (2022) The race to zero emissions: can renewable energy be the path to carbon neutrality? J Environ Manage 308:114648
Zhang X, Wang Y (2017) How to reduce household carbon emissions: a review of experience and policy design considerations. Energy Policy 102:116–124