

## Inventory management for aging products with supply chain finance: the warehouse financing option

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**Abstract:** For businesses specializing in ameliorating goods, such as seasoned cheese, traditional financing models pose unique challenges. Inventory financing relies heavily on past performance, often overlooking the inherent value increase associated with proper aging. This can lead to limited access to capital, hindering growth and operational stability. Warehouse financing emerges as a specialized solution specifically designed for businesses with maturing inventory. Lenders recognize the future value potential of these goods, offering secured loans based on anticipated market appreciation. This approach unlocks immediate cash flow, empowering businesses to cover operational costs, invest in expansion, or manage cash flow fluctuations. This study develops and discusses inventory problems for the specific class of "ameliorating" products, integrated with the warehouse financing technique, to combine the two topics and highlight their main features but above all their scientific and practical importance. The models proposed are focused on a decentralized scenario (single actor perspective) and a centralized scenario (supply chain perspective) to compare the optimal solution in terms of the aging period while maximizing the annual profit. Furthermore, from the supply chain perspective, a multi-supplier single-manufacturer supply chain is proposed with a deteriorating raw material (i.e., fresh milk). While cheese is a prime example, warehouse financing can benefit a diverse range of businesses dealing with ameliorating goods (such as wines, coffee, and aged spirits).

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**Keywords:** supply chain finance, aging products, deteriorating, maturing, warehouse financing, inventory management.

### 1. INTRODUCTION

Recently, companies have been paying more and more attention to issues concerning the production and storage of materials, not only because, if not well implemented and optimized, they generate operational management problems, but also and mainly because they represent a significant part of the costs to be incurred. Specifically, the presented work analyses the inventory models of products that acquire value over time (the so-called "ameliorating items"), principally in the food and beverage sector (e.g., cheese and wine). Ameliorating items present a high sensitiveness to the surrounding environment (e.g., humidity, temperature), hence, they need to be properly stored (i.e., usually in temperature-controlled warehouses) leading to relevant holding costs. However, by deciding to stock an item for longer periods, the product matures, which means higher value and higher selling price (Gonen et al., 2021). This specific class of products constitutes an important slice of products on the market, in contrast to most inventory models in the literature that deals with products that maintain their value over time or, at least, deteriorate progressively.

In the current economic environment, on one hand, the risk of insolvency increases, and on the other hand the necessity of financing by companies rises, mainly due to the payment

difficulties of customers and the high volatility of prices, caused by the rapid expansion of emerging markets. In this regard, supply chain finance (SCF) emerged as a mean for efficiency which consists of the inter-company optimization and integration of the financial flows (Pfohl and Gomm, 2009). Recently, many inventory models have been developed for maturing/aging items, while analyzing the interesting role of financial aspects (Marchi et al., 2023). However, previous works are mainly focused on trade credit solutions among the partners and do not address the potential of the warehouse financing technique that is gaining more and more interest among ameliorating items producers. Thus, the proposed inventory model is a new design for supply chains of aging products, implementing this new financing strategy. The present study was developed to introduce and discuss inventory problems for the specific class of "ameliorating" products, incorporated with the warehouse financing technique, to combine the two topics and highlight their main features but above all their scientific and practical importance.

### 2. MODEL FORMULATION

The idea of the following model is to associate the maturing and aging process of products in the warehouse with the strategy of warehouse financing. Initially, a decentralized scenario with a single-actor perspective (e.g., a single cheese producer) is considered. The focus then shifts to a centralized

scenario in which various actors involved in the supply chain are introduced in the optimization process. The scenarios that will be studied in the development of the model are the following:

- Single manufacturer (decentralized scenario): the producer produces ameliorating items to cover the final demand. In this context, raw materials storage is not considered because it is assumed that when the orders with fresh items from the farmers arrive, processing starts immediately, with raw materials directly entering the production process. Moreover, the aging process only takes place in the manufacturer's warehouse.
- Multi-supplier single-manufacturer supply chain (centralized scenario): in addition to the amelioration aspect due to the maturing and aging process of the finished products in the warehouse, the deterioration of the raw material is added. For instance, milk, for cheese production, is among the most rapidly perishable fresh products and must therefore be stored in a controlled environment, at appropriate temperatures, and for a limited period.

### 2.1 Problem description

As a traditional supply chain with ameliorating and deteriorating items, the case of cheese production is shown (Figure 1).

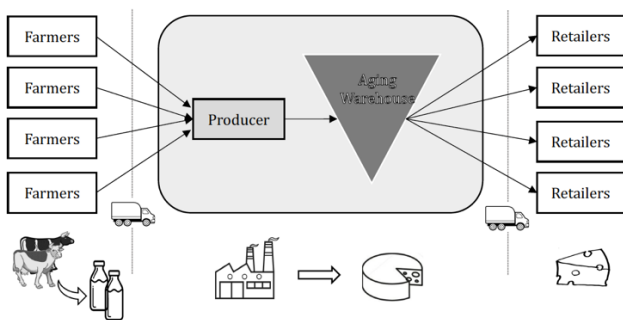


Figure 1. The supply chain of cheese aging warehouse

The characteristic steps of the production process are milk collection, delivery from the farmers to the cheese producer, cheese manufacturing process, cheese maturing/aging period in the warehouse, delivery to retailers, and the final customers. The producer places an order to the suppliers (i.e., the farmers) and then incurs a certain setup cost due to receiving the raw material and preparation for the production process. The raw material to produce cheese is mainly milk, but also salt and rennet (the ferment) are ingredients in cheese: their quantities are so small, and above all, they are not fresh products, which is why their procurement is omitted from the supply chain considered in this model. Furthermore, as mentioned above, it has been assumed that the cheese production process starts immediately when the producer receives the fresh items, so no deterioration/degradation of the products is considered before the start of the processing into cheese. Cheese manufacturing puts a certain amount of stock in the finished product warehouse, the 'aging warehouse'. The finished product, i.e.,

the matured and aged cheese, will remain in stock in the producer's warehouse for a certain period: the older it gets, the more its value (and therefore the price) increases, but at the same time the demand decreases. After an appropriate maturing/aging period, the products are ready and thus sold to customers (that may be a combination of retailers or independent final customers). The warehouse financing technique, which allows a lower interest rate for the manufacturer from the financial institution, is applied only to the finished products warehouse (i.e., the aging warehouse where the goods mature over time) and not to the raw material stocks or even to the retailer's warehouse of finished products: this is because being perishable products (as raw materials and aged products are) would increase the risk and therefore they cannot be used as a guarantee. The structure described so far specifically concerns the case of the cheese supply chain, but the model could be extended to other types of items that ameliorate and therefore increase their value over time if kept in stock. Thus, other types of products characterized by a "maturing/aging" process (such as wine, spirits, etc.) could be further identified. In the case of cheese stock, it is not a question of continuous maturing, but rather of intervals (cheeses are sold matured 12, 18, 24, etc. months): however, in the following model as well as in the one studied by Zanoni et al. (2019), continuous maturation is assumed and finished products will remain in stock and mature for a specific period. In this way, the model will identify the optimal maturing/aging period of the stock cheese to maximize the total annual profit of the producer. The continuous maturation in question is certainly a simplification but allows the model to be more generalizable and thus to include different types of ameliorating products with continuous maturation or aging process. The implementation of the WF technique introduces benefits mainly in the evaluation of the costs for the producer: this practice allows banks to lower the interest rates charged to the debtor requesting the loan (in our case the owner of the aging warehouse, i.e., the cheese producer himself), which will consequently lead to a change in capital costs, and thus to a reduction in the holding costs of the warehouse. Since in our study we are dealing with ameliorating items, it has on the other hand an additional impact on the price, and since demand is considered price-dependent in the model formulated below, it also affects the demand. The impact of warehouse financing on the batch and on the optimal maturing time of the products will then be compared.

### 2.2 Notation and Assumptions

The main assumptions that are used to construct the proposed model are given as follows: i) the inventory system involves only one type of item; ii) the time horizon is infinite; iii) the value of the price of the final product  $p(t)$  [€/unit] increases over time due to the aging/maturing effect; this increase in value is assumed to be homogeneous, so no product deterioration (only amelioration) in the aging warehouse is considered. Weight losses occurring in the aging period are due to physical processes, hence, can be predicted and are included in the market price; iv) the annual demand rate  $D$  [unit/year] is a decreasing linear function of the selling price, i.e., price-dependent, and therefore it follows:

$$D(t) = a - b \cdot p(t) \quad (1)$$

where  $p(t) < a/b$  represents the unit selling price of the item at the time when it is sold,  $a$  is the maximum demand when the price is zero, and  $b > 0$  is the price elasticity of demand; v) replenishment for the manufacturer is considered instantaneous, so lead time is zero; vi) the production rate of the manufacturer,  $P$ , is assumed to be higher than the demand rate to ensure the delivery of all products required by the final costumers, i.e.,  $P > D$ ; vii) fixed period policy, i.e., the constant reorder period policy, is adopted to manage the warehouse of the “maturing/aging” products: orders are placed at a fixed rate (the constant aging period  $T$  [year]) and the items are sold in batches of size  $Q$  [unit] of aged products to retailers to meet the final demand,  $D$ ; viii) to produce a final batch of aged products, it is assumed that the manufacturer orders a quantity of raw materials, i.e., fresh items, from the suppliers equal to  $Q\gamma$  [unit], where  $\gamma$  is the coefficient of conversion between the raw material and the finished products involved in the production process of the manufacturer. A lot-for-lot policy is therefore assumed, i.e., the producer produces exactly the quantities ordered; ix) the capacity of the warehouse is considered large enough to contain the entire quantity of the batch size sold to retailers every time interval equal to the aging period; x) the physical component of the holding cost  $h_{stock}$  [€/unit year] also includes all the costs of auxiliary services, including energy, water and everything else that is needed to keep the inventory stocked; xi) the capital-constrained holding costs of the producer  $h_{fin}$  [€/unit year] can be calculated as follows:

$$h_{fin} = \rho' \cdot P_{avg} \quad (2)$$

where  $\rho' = \rho_0 + \frac{\rho}{\delta \cdot \theta \cdot \omega \cdot I_m \cdot P}$  represents the discount rate applied to the cost of capital (Marchi et al., 2020),  $\rho_0$  and  $\rho$  represents the components of the cost of capital respectively not affected and affected by the discount [€/€·year],  $\delta$  is the coefficient defining the inventory level elasticity of the cost of capital,  $\theta$  the share of the stock level used as a guarantee,  $\omega$  is the liquidity of the goods which defines their ability to be converted into cash (the faster and cheaper this happens, the more liquid the goods can be considered),  $I_m \cdot P$  is the average value of the goods, given by the product of the average inventory level and the price of the goods inside the warehouse, and  $P_{avg}$  [€/unit] is the average value of the aged product over the aging period  $T$ , i.e. the value acquired by the cheese seasoned  $T$  years according to Zanoni et al. (2019).

$$P_{avg} = \frac{\int_1^T p(t) dt}{T-1} = \frac{\int_1^T \alpha \cdot t^{\beta-1} dt}{T-1} = \frac{\alpha}{T-1} \cdot \frac{T^{\beta+1}-1}{\beta+1} \quad (3)$$

where  $\alpha$  is the scale parameter used to model the aging effect on the price of the cheese items, and  $\beta$  is the shape parameter of the price function; xii) degradation has been assumed at the suppliers (fresh products) and the retailers if not stored properly, i.e., at inappropriate temperatures or for an excessive time; xiii) suppliers make the same raw material price for the manufacturer  $P_s$  [€/unit] (equivalent to the purchase cost of

fresh items for the producer), being the price determined by the market (e.g., quoted product for the milk). The same applies to the selling price of the aged products for the retailers, which was assumed to be equal to  $P_r$  [€/unit]; xiv) all the end customers see the same utility in maturing, and therefore an equal price sensitivity of the buyers of the different retailers has been assumed. Thus, the parameters for the formulation of seasoning will be unchanged; xv) no stockout is allowed at the customer end; xvi) all the units produced in the same batch enter the warehouse at the same time. After a proper fixed aging period  $T$  [years] in the aging warehouse of finished goods, the manufacturer sells the batch of aged products to the retailers (Figure 2).

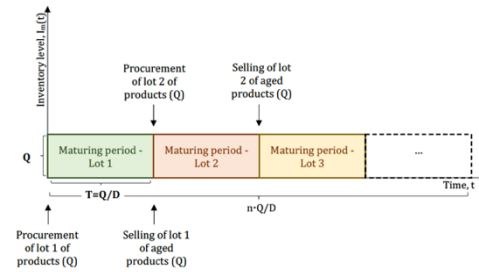


Figure 2. Inventory level of the aged warehouse: stock of finished products at the producer's warehouse

The objective function is based on the maximization of the annual average profit [€/year]: the number of aging months in the warehouse ( $T$ ) will be assumed as the decision variable.

## 2.3 Model development

### 2.3.1 Single-manufacturer model

Considered as a single actor, the total annual cost faced by the manufacturer is the sum of three contributions:

- Annual purchase costs, i.e., the costs of acquiring raw materials (i.e., fresh items and in our specific case milk):

$$\text{Annual purchasing cost} = P_s \cdot D \cdot \gamma \quad (4)$$

- Annual setup costs, i.e., the cost of starting the production of a lot:

$$\text{Annual setup cost} = A/T \quad (5)$$

where  $A$  represents the single setup cost [€/setup] for the manufacturer.

- Annual costs of maintaining stocks, i.e., holding costs of the finished products, according to Figure 2:

$$\text{Annual holding cost} = (h_{stock} + h_{fin})Q \quad (6)$$

The average annual profit of the manufacturer  $AP(T)_{sm}$ , which should be maximized, will be the difference between the profit obtained from the sale of the aged items and the sum of all these costs previously introduced.

$$AP(T)_{sm} = (a - b \cdot \alpha \cdot T^\beta) \left[ \alpha \cdot T^\beta - P_s \cdot \gamma - T \cdot \left[ h_{stock} + \left( \frac{\alpha}{T-1} \cdot \frac{T^{\beta+1}-1}{\beta+1} \right) \cdot \left( \rho_0 + \frac{\rho}{\delta \cdot \theta \cdot \omega \cdot T \cdot (a-b \cdot \alpha \cdot T^\beta) \cdot \alpha \cdot T^\beta} \right) \right] \right] - \frac{A}{T} \quad (7)$$

### 2.3.2 Multi-supplier single-manufacturer model

In this scenario, the supply chain of the manufacturer of aged products integrated with raw materials suppliers is analyzed. The aging process and the warehouse financing technique, both of which affect only the manufacturer's finished goods warehouse (i.e., the aging warehouse), have no impact on suppliers. On the other hand, the deteriorating issue will be introduced concerning the raw material produced by the farmer. The farmers must provide every period  $T$  the total quantity of raw materials needed for production. To meet the final demand of consumers  $D$ , the manufacturer produces a batch of finished products equal to  $Q$  [unit] which corresponds to a quantity of raw materials equal to  $Q\gamma$  [unit]. Furthermore, to identify the demand that each supplier must cover with his raw materials production and supply to the manufacturer, a variable has been inserted:  $\varphi_{s,j}$  represents the fraction of the manufacturer's final demand that the  $j$ -th supplier satisfies. The sum of the fractions of demand that the  $m$  suppliers cover must surely amount to the total demand, so the following equation will certainly be verified:  $\sum_{j=1}^m \varphi_{s,j} = 1$ . The quantity produced and delivered by the  $j$ -th farmer is assumed to be  $q_{s,j}$ , with  $j = 1, \dots, m$ . So, the whole production can be summarized with the following expression:

$$Q \cdot \gamma = \sum_1^m q_{s,j} \quad (8)$$

Raw materials, once delivered to the manufacturer, directly enter the production process. The producer will wait for the arrival of the raw material from all suppliers and therefore the time that determines the start of the producer's production is given by:  $\max\left\{\frac{q_{s,j}}{p_{s,j}}\right\}$ , where  $p_{s,j}$  is the production rate of the  $j$ -th supplier. The raw materials remain in stock at the raw material warehouses at the suppliers' site, equal to:  $\max_j\left\{\frac{q_{s,j}}{p_{s,j}}\right\} - \frac{q_{s,j}}{p_{s,j}}$ . Figure 3 shows the evolution of the supplier's stock level over time compared to that of the producer.

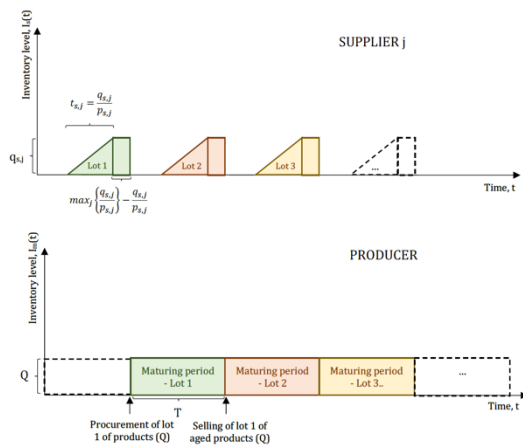


Figure 3. Inventory levels of the warehouses: stock of raw materials at the  $j$ -th supplier and stock of finished products at the producer warehouse

As far as suppliers of fresh products are concerned, the deterioration of the stocked goods is also introduced, i.e., the

loss of quality that raw materials suffer if they are not properly stocked (i.e., temperature and timing). Considering the above aspects and analyzing the supplier individually, the total annual cost faced is the sum of four contributions:

- Annual production costs for the  $j$ -th supplier ( $PC_j$ ), i.e., the costs of acquiring raw materials (i.e., fresh items and in our specific case milk)

$$PC_j = r_s \cdot d_{s,j} \cdot \gamma \quad (9)$$

where  $d_{s,j} = D\varphi_{s,j}$  represents the portion of the manufacturer's demand for raw material that the  $j$ -th supplier covers, while  $r_s$  is the production cost for the supplier [€/unit].

- Annual setup costs for the single supplier ( $SC_j$ )

$$SC_j = \frac{A_{s,j}}{T} \quad (10)$$

where  $A_{s,j}$  is the single setup cost for supplier  $j$ .

- Annual holding costs of the raw material for the  $j$ -th supplier ( $HC_j$ )

$$HC_j = (h_{s,j} \text{ stock} + \rho_{s,j} \cdot r_s) \cdot \left( \frac{t_{s,j}}{2} + \left( \max\left\{\frac{q_{s,j}}{p_{s,j}}\right\} - \frac{q_{s,j}}{p_{s,j}} \right) \right) \cdot d_{s,j} \quad (11)$$

where  $t_{s,j}$  is the period of production of raw material for the  $j$ -th farmer, and  $\rho_{s,j}$  the Interest rate of the  $j$ -th supplier.

- Annual deterioration cost for the  $j$ -th supplier ( $DC_j$ ), i.e., the cost due to the loss of quality of raw material over time (Zanoni and Zavarella, 2012):

$$DC_j = \frac{d_{s,j}}{q_{s,j}} \cdot P_s \cdot \left( q_{s,j} - \frac{d_{s,j}}{b(T_{s,j})} \left( 1 - e^{-\frac{b(T_{s,j})q_{s,j}}{d_{s,j}}} \right) \right) \quad (12)$$

where  $T_{s,j}$  is the temperature at which the goods are stocked, which might vary from supplier to supplier.

The average annual profit  $AP(T, t_{s,j})_{ms-sm}$  of the two echelons considered in the integrated supply chain described so far is defined in equation (13).

$$AP(T, t_{s,j})_{ms-sm} = (\alpha \cdot T^\beta - r_s \cdot \gamma) \cdot (a - b \cdot \alpha \cdot T^\beta) - T \cdot (a - b \cdot \alpha \cdot T^\beta) \cdot \left[ h_{stock} + \left( \frac{\alpha}{T-1} \cdot \frac{T^{\beta+1}-1}{\beta+1} \right) \cdot \left( \rho_0 + \frac{\rho}{\delta \theta \cdot \omega \cdot T \cdot (a-b \cdot \alpha \cdot T^\beta) \cdot \alpha \cdot T^\beta} \right) \right] - \frac{A}{T} - \sum_1^m \left[ \frac{A_{s,j}}{T} + (h_{s,j} \text{ stock} + \rho_{s,j} \cdot r_s) \cdot \left( \frac{t_{s,j}}{2} + \left( \max\left\{\frac{q_{s,j}}{p_{s,j}}\right\} - \frac{q_{s,j}}{p_{s,j}} \right) \right) \cdot \varphi_{s,j} \cdot \gamma \cdot (a - b \cdot \alpha \cdot T^\beta) + \frac{\varphi_{s,j} \cdot \gamma \cdot (a-b \cdot \alpha \cdot T^\beta)}{t_{s,j}} \cdot P_s \cdot \left( t_{s,j} - \frac{1}{b(T_{s,j})} \left( 1 - e^{-b(T_{s,j}) \cdot t_{s,j}} \right) \right) \right] \quad (13)$$

### 3. THE SEASONED CHEESE CASE STUDY

This section presents a numerical analysis of an Italian seasoned cheese case study with two suppliers, to show the applicability of the model to real cases. The data used are shown in Table 1 and come from national regulations, market prices, and data collected on the field.

Table 1. Data used for the numerical analysis.

Parameter	Value	Description
$\alpha$	3.6	Scale parameter of price function for aged products
$\beta$	0.35	Shape parameter of price function for aged products
$a$	10000 <sup>12</sup>	Scale parameter for demand formulation
$b$	500	Price elasticity of demand
$m$	1	Constant empirically determined for food degradation
$T_c$	$T_{c, \text{milk}} = 5$ $T_{c, \text{cheese}} = 14$	Constant empirically determined for food degradation
$T_s$	$T_{s, \text{milk}} = 1 \text{ } ^\circ\text{C}^{13}$ $T_{s, \text{r cheese}} = 5 \text{ } ^\circ\text{C}^{14}$	Temperature at which the food is stocked

MANUFACTURER		
$P_s$	0.40 €/unit <sup>15</sup>	Purchase cost of raw materials (i.e., fresh items) for the manufacturer
$A$	500 €/setup	Setup cost of the manufacturer production process
$\gamma$	15.15 <sup>16</sup> unit/unit	Conversion coefficient of raw materials into finished products for the manufacturer
$h_{\text{stock}}$	0.025 €/unit-year	Physical component of holding costs for finished products in the aging warehouse for the manufacturer
$\rho_0$	0.05 €/€·year	Component of the cost of capital not affected by discount
$\rho$	0.15 €/€·year	Component of the cost of capital affected by discount
$\delta$	0.0005	Coefficient defining the inventory level elasticity of the cost of capital
$\theta$	0.6	Coefficient defining the share of the stock level used as a guarantee
$\omega$	0.2	Liquidity of the goods (i.e., their ability to be converted into cash)

SUPPLIERS		
$m$	2	Number of suppliers
$r_s$	0.2 €/unit	Purchase cost for the supplier
$A_{s,j}$	$A_{s,1} = 550 \text{ €/process}$ $A_{s,2} = 500 \text{ €/process}$	Setup cost of the j-th supplier related to the extraction/processing of raw material
$p_{s,j}$	$p_{s,1} = 1000000 \text{ unit/year}^{17}$ $p_{s,2} = 800000 \text{ unit/year}$	Production rate of the j-th supplier
$h_{s,j \text{ stock}}$	$h_{s,1 \text{ stock}} = 0.02 \text{ €/unit-year}$ $h_{s,2 \text{ stock}} = 0.03 \text{ €/unit-year}$	Physical component of holding costs of the j-th supplier for the raw materials
$\rho_{s,j}$	$\rho_{s,1} = 0.05 \text{ €/€·year}$ $\rho_{s,2} = 0.07 \text{ €/€·year}$	Interest rate of the financial component of holding costs of the j-th supplier for the raw materials
$\varphi_{s,j}$	$\varphi_{s,1} = 0.6$ $\varphi_{s,2} = 0.4$	Fraction of manufacturer's demand that the j-th supplier covers

The parameters defined for describing how the price of seasoned cheese varies for buyers concerning the seasoning time have been obtained from the analysis of actual values from the CCIAA Parma source for the year 2020 (Table 2).

Table 2. Comparison between real and estimated prices for the considered seasoned cheese.

Aging time [months]	Real price of the seasoned Parmigiano Reggiano [€/kg] <sup>18</sup>	Estimated value of price of the seasoned cheese in our model [€/kg]
12	8.54	8.59
18	9.94	9.90
24	10.73	10.95
30	11.72	11.84

Figure 5 shows the goodness of the proposed price formulation as a function of its maturity. Figure 6 presents the final demand as a function of the prices and, consequently, the maturing period.

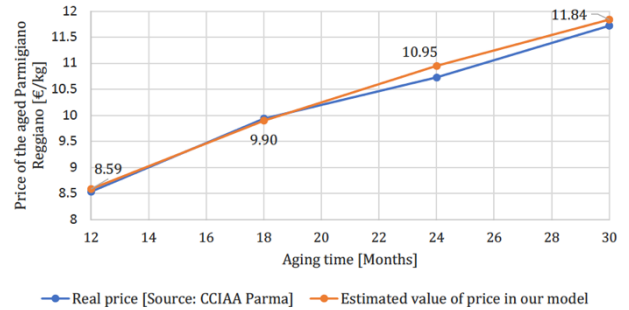


Figure 5. Price trends over different aging periods of the Parmigiano Reggiano

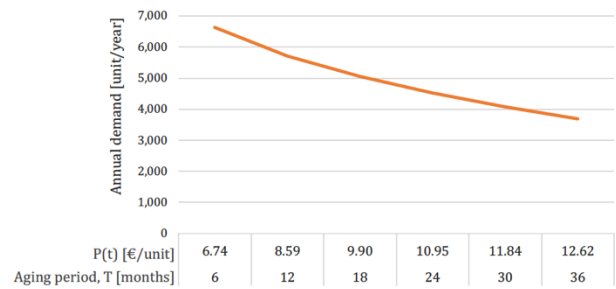


Figure 6. Annual demand as a function of the sales price for the seasoning period of the cheese.

In the single-manufacturer scenario, the optimal maturation period found is 32 months. The production batch size is consequently 10571 units, resulting in a maximum annual profit of 16,698 [€/year] for the seasoned cheese producer. The results of the average profit of the single manufacturer varying the maturing/aging period of cheese are shown in Figure 7.

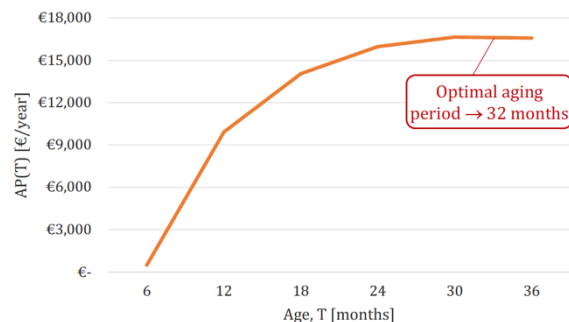


Figure 7. Average profit and optimal solution of the single-manufacturer scenario.

The total cost of the cheese producer amounts to 30,987.5 [€/year], according to the maturing period of 32 months, with cost components as follows: acquisition costs are the most substantial for 76.7%, setup costs are almost completely irrelevant (0.6%) while holding costs impact on total costs for

22.7% even if a discount was applied on the capital cost thanks the warehouse financing practice. Cheese maturation of 32 months leads to an increase in the selling price of the aged product to buyers, equal to 12.15 [€/unit], which consequently corresponds to an annual revenue of 47,685 [€/year]. The maximum annual profit for the single-manufacturer model, considering all the aspects described above, is therefore 16,697 [€/year]. The optimal cheese maturing period for the supplier-manufacturer chain obtained in the model is 23 months (Figure 8). The inclusion of suppliers in the model caused an increase in demand for the manufacturer in the optimal solution but, due to the decrease of the optimal maturing period and the deterioration, this resulted in a reduction of the production batch size (equal to 8781 units) and a reduction of the selling price. Despite this, the annual profit of the considered chain is 30,574 [€/year].

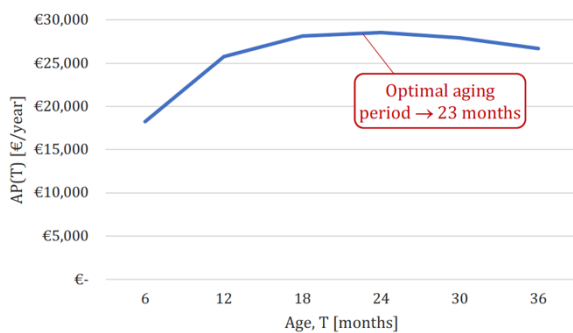


Figure 8. Average profit and optimal solution of the “multi-supplier single manufacturer” scenario.

By considering other actors, the manufacturer's profit becomes slightly worse (- 6%) because it moves from its local optimum to the global supply chain optimum. However, this allows the improvement of the economic performance of the supply chain (i.e., 30,480 €/year, + 7% of the annual profit). There is an increase in purchasing costs (82% in this context) but a decrease in holding costs due to the shorter maturation period obtained (17%); setup costs are still completely irrelevant (less than 1% of total costs). For the suppliers, production costs account for almost all their costs (92%), while setup and holding costs represent 4% and 1% of the total costs respectively. Storage conditions from regulations lead to minimized degradation costs, thus affecting only 3% of suppliers' total costs. The cheese maturation of 23 months generates a selling price of the matured product to the buyers of 10.75 [€/unit], which consequently leads to an annual revenue of 49,717 [€/year] for the manufacturer.

## 6. CONCLUSIONS

This study proposes and discusses a particular category of products defined as the class of “aging items”, associated with a specific financing strategy, i.e., warehouse financing. This peculiar, but significant, class of products presents a salient feature, which is the increase in their value during the maturing/aging process. This process takes place in the manufacturer's warehouse of finished goods, the so-called 'aging warehouse': the stock within this warehouse acts as a guarantee for the implementation of WF practice. This practice

allows the manufacturer to obtain a loan from the bank with lower interest rates since the goods are offered as collateral for the loan. The two central aspects of the presented work are directly related: as the aging period increases, the selling price of aged products increases, which directly affects the discount applied. This leads to a substantial reduction in the financial component of the holding costs for the loan applicant. The analysis of the seasoned cheese case studies shows that the implementation of this strategy leads to relevant savings. The managerial problem is the identification of the optimal maturation/aging period: to find a solution to this problem, a model has been proposed based on the maximization of the average profit. This profit maximization problem was initially introduced for a decentralized scenario with a single actor perspective (e.g., single cheese producer), and then the focus shifted to a centralized scenario with the manufacturer and multi-suppliers with a raw material that degrades over time. The results clearly show that the optimal situation varies according to the type of supply chain considered. Possible research lines for future developments are considering a limited storage space of warehouses, defective production and storage, a multi-item inventory model, different inventory policies, and including multi-retailers in the supply chain. Finally, the practice of warehouse financing is considered, but in further developments, other interesting financial instruments could be introduced: this also applies to raw materials on the supplier side (e.g., advance payment, etc.).

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