SN Oper. Res. Forum (2021) 2: 13
https://doi.org/10.1007/s43069-021-00054-3

EXPOSITORY REVIEWS

Teaching Operations Research Before University: A Focus on Grades 9–12

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Received: 18 June 2020 / Accepted: 6 January 2021/ Published online: 20 Febuary 2021

Abstract
Operations Research (OR) is a branch of applied mathematics usually taught in undergraduate and graduate courses at the university level. Nevertheless, during the last years, various initiatives have been developed to introduce some topics to younger students. These initiatives usually aim to improve students' interest and motivation towards mathematics and other STEM disciplines. They also allow students to acquire new skills and abilities as well as increase their awareness of a discipline with several applications. In this review, we analyze the state of the art related to Grades 9–12. We present and classify local, national and international OR educational initiatives we were able to collect. Then, we compare their objectives and focus, topics introduced, teaching methods, instruments and software chosen, and feedback received. Finally, we discuss some of the main current international and national guidelines for mathematics education of Grades 9–12, showing how OR could be suitable to fulfil many of these.

Keywords Operations research teaching · Mathematics education · Grades 9–12

1 Introduction

Operations Research (OR) is a branch of applied mathematics usually taught in undergraduate courses of Mathematics, Computer Science and Economics, but also in some Engineering curricula. Thanks to its interdisciplinary nature, OR finds

This article is part of the Topical Collection on *The AIROYoung Experience: Operations Research for Young*

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applications in several fields such as logistics, production and facilities planning, marketing and finance.

Being so relevant and pervasive, OR can help increase pupils' motivation and interest towards STEM (i.e., *Science, Technology, Engineering and Mathematics*) disciplines. In fact, OR can show the possible connections between mathematics and the real world, which can support the students' learning process, helping them to reason and develop problem-solving and analytical skills. Moreover, a positive attitude towards mathematics could foster students to pursue a career in scientific disciplines or to continue the study at university level. For more details about motivation towards mathematics or science and technology, the reader is referred to Middleton and Spanias [1] and Potvin and Hasni [2], respectively.

We observe that, for some OR topics, there are no particular prerequisites that students would learn in the following years of their education (i.e., at university level). For instance, formulating a real problem as a mathematical model would ask them to know the concepts of variable, constraint and objective function. Moreover, applying the graphical method to solve a linear programming problem involving two variables would require solving equations or inequalities and being familiar with analytic geometry. To perform the steps of the Simplex algorithm, only some algebraic concepts would be needed (e.g., matrices). Therefore, some of the mathematical skills required to approach OR are already part of a standard mathematical background of pupils. In particular, we consider Grades 9–12 students, referring to the US/International Grades scale, corresponding to 14–17-year-old students (see, e.g., [3]).

Thus, we wondered whether there exist educational initiatives to introduce OR before university level. The main purpose of our work is to collect, classify and compare these experiences, identifying how they have been designed and developed, what goals have been pursued, what results have been obtained so far and what difficulties have been encountered.

This review paper is structured as follows. In Section 2, we illustrate how we have collected educational initiatives related to teaching OR in Grades 9–12. We describe and classify the initiatives in Section 3, comparing them on several aspects. Then, in Section 4 we discuss some of the main current international and national guidelines for mathematics education, to highlight explicit and implicit references to OR. Finally, in Section 5 we draw our conclusions and talk about future developments.

1.1 Motivation

During our PhD studies, we had many opportunities to try our hand at teaching OR, as teaching assistants and collaborators in various university courses. This increased our interest towards the subject and its related teaching methods. In parallel, we also had several experiences with younger students of different ages.

Every year, the University of Verona organizes *Kidsuniversity* [4], i.e., a whole week of activities and seminars for pupils and teachers of primary and secondary schools in the province of Verona (Italy). Each activity takes place in a university class, arranged by one or more academics and carried out as a real lecture. Since

2018, a lecture based on problem-solving games and exercises has been proposed, presenting simplifications of famous OR problems (e.g., Knapsack) and concrete applications of mathematics. It has also allowed us to introduce the *Olympiads in Problem-Solving* [5] to both students and teachers.

From the Kidsuniversity experience, collaborations with some local primary schools were born, to organize similar workshops addressed to 7–10-year-old students (i.e., Grades 2–5). We played with sorting algorithms, graphs, scheduling and planning. The feedback was positive and pupils were enthusiastic to learn mathematics in unconventional and funnier ways. Similar workshops have been offered into the free sessions of *Math 4 Kids* [6], addressed to little users of Biblioteca Fausto Sabeo, a public library in the province of Brescia (Italy).

Regarding Grades 9–12, each fall every institute can participate in the local stage of *International Olympiads in Informatics* [7], and so does a technical school in Verona (Italy). To get prepared, students can attend a series of training workshops, where exercise lectures involve dynamic programming, graph theory and modelling.

By designing, developing and collaborating in all these initiatives, we have become aware of how different branches of applied mathematics and computer science can be introduced in schools, even to very young students. Therefore, we were curious to discover which initiatives had been undertaken to foster OR knowledge among pupils.

2 Data Collection

The idea of investigating the state of the art about teaching OR to young students came right after reading the research papers written by De Leone et al. [8] and Schettino [9]. They give an overview of the main OR educational initiatives in the USA and in Europe, in particular in Italy, until 2012. We have started to search for other initiatives, both previous and successive to the ones described by De Leone et al. [8] and Schettino [9]. We have considered the following categories of initiatives:

- *promotion activities*, done by societies and communities to make OR more known, through public events, conferences, lectures and social media accounts;
- *national* and *international projects*, in which schools or single teachers can participate;
- *competitions*, i.e., local, national and international challenges;
- *training courses for teachers* and *workshops for students*, lasting one or more days and organized not by schools themselves but by other associations;
- *didactic units or lectures*, tested in one or more classrooms, typically organized by schools themselves.

Because of the interdisciplinary nature of OR, sometimes an initiative can be related to it even without explicitly mentioning the term ”Operations Research” (or ”Operational Research”). To consider an initiative as *inherent to OR*, we have searched for some keywords in its description (e.g., *modelling*, *mathematical/linear/integer programming*, *graph theory*, *optimization*, *algorithms*, *problem-solving*).

As mentioned in Section 1, we have filtered only the initiatives targeting students and/or teachers of Grades 9–12. According to the US/International Grades scale, these correspond to 14–17-year-old students (see, e.g., [3]).

The research has been conducted exploiting different search engines (e.g., *Google* and *Google Scholar*) and consulting repositories and social networks for scientists and researchers (e.g., *Scopus* and *ResearchGate*). We have used the aforementioned keywords, crossed with less specific terms, to find scientific works related to OR and concerned with the target Grades. We have checked the programs of the recent conferences of *Associazione Italiana di Ricerca Operativa* (AIRO) [10–12] and *European Operational Research Societies* (EURO) [13], where there were sessions related to the teaching of OR. Moreover, we have also examined some repositories, like *Optimization Online* [14], and available issues of well-known scientific journals, such as *INFORMS Transactions on Education* [15], *International Transactions in Operational Research* [16], *Decision Sciences Journal of Innovative Education* [17], *OR/MS Today* [18], *ZDM Mathematics Education* [19] and *Mathematics Teacher: Learning and Teaching PK-12* [20]. Furthermore, in October 2019, we proposed a survey to Italian Grades 9–12 teachers. We wanted to investigate whether OR was known and/or taught to students in anyways. *MaddMaths!*, an Italian website dedicated to mathematics education and didactics, helped us promote the survey [21]. We also designed a second survey addressed to worldwide academic personnel, published online in March 2020 and forwarded through several channels (e.g., the EURO newsletter and other OR mailing lists). This had the purpose to discover other existing activities and techniques. For both surveys, results are still being analyzed. Anyway, they made us aware of other initiatives or specific journals addressed to target students, like *KOMAL* [22], an Hungarian monthly periodical for high school students, with exercises and problems in mathematics and physics. However, being more general, we have decided not to include KOMAL in the list presented in Section 3.

Once identified the OR educational initiatives for our target students, in some cases we proceeded to contact the organizers or the participants by email. In particular, we did this when there was no research paper, official documentation or website describing the initiative. The information found, with a little difficulty, was not always sufficient to have a clear picture of the aims of the initiative, or to know and analyze the feedback received.

Before moving to the core part of our review, we would like to observe that the list of the OR educational initiatives we collected is certainly *not exhaustive*. There may be plenty of initiatives we are not aware of, that we did not find through our research. Thanks to the first survey we developed, we have been able to know several initiatives done in Italy. However, we have tried to include as many other countries as possible.

3 OR Educational Initiatives

In this section, we present and classify the OR educational initiatives discovered through the data collection described in Section 2. Then, we make a comparison based on the following aspects: focus and objectives, topics, teaching methods, instruments and software, and feedback.

3.1 Classification

We have defined the following categories of initiatives: promotion activities, national and international projects, competitions, training courses for teachers and workshops for students, and didactic units or lectures tested in classrooms. Actually, an initiative can belong to more than one class (e.g., when it includes both workshops for students and training courses for teachers).

Table 1 lists the collected OR educational initiatives by chronological order. When an initiative does not have a title, we use the surnames of the authors who described it. The last column indicates whether an initiative is still ongoing or not.

Among organizations and communities active in the promotion of OR, one of the oldest is *The OR Society* [23], founded in 1948, with headquarters in Birmingham (UK). Its online platform supports professional operations researchers across industries and academia. It offers an extensive training program in OR and analytics, organizes national and international conferences and raises the profile of OR through schools outreach. A particular section of the website is dedicated to the *OR in Education* (ORiE) project. ORiE involves teachers, lecturers and educators, with the aim to make every student know about OR. Many materials, such as mathematics modules, booklets and a short film about OR, are available in the *Resources Centre* [24], where teachers can also contribute by uploading their own works. The OR Society provides free workshops for teachers to use in their classrooms and holds courses like *Continuous Professional Development* outlining how to exploit the resources. Upon request, Evelyn Hardy (Education Officer at The OR Society) matches schools with volunteers who can visit and deliver a workshop or a careers talk, or attend a careers fair. Volunteers also attend science fairs and other enrichment events to interact with students and teachers [25]. The OR Society is also very active on social networks, like Twitter.

Another international society widely known is the *Institute for Operations Research and the Management Sciences* (INFORMS) [26]. Since 1995, it has been connecting OR practitioners from nearly 90 countries. Its purposes include to promote advances in OR and to increase public awareness and understanding. It provides access to several kinds of resources suitable for all types of organizations and fields of application, education too. In particular, there are various resources (e.g., promoted projects and educational videotapes) addressed to teachers, to support the introduction of OR to pupils [27]. Even if the material is not up-to-date, still it is accessible and exploitable. Also INFORMS has several social media accounts. Throughout the years, INFORMS has supported the development of education programs also in schools and universities which did not include OR or statistics courses. For instance, teaching effectiveness colloquia have been organized, in conjunction with academic conferences outside the USA. Prof. James J. Cochran (University of Alabama) describes these colloquia as a series of 60-minute workshops given by professors in the USA and by colleagues of the host organization [28]. The main goal is to encourage Grades 9–12 teachers to introduce OR, statistics and analytics to their students. In the past, these colloquia were organized in Uruguay, South Africa, Colombia, India, Argentina, Fiji, Tanzania, Nepal, Kenya, Cameroon, Cuba, Moldova, Croatia, Mongolia [28, 29] and recently in Caribbean

Table 1 Classification of OR educational initiatives addressed to Grades 9–12

OR initiative	Country	Year	Promotion	National project	International project	Competition	T. courses for teachers	Workshops for students	Didactic units	Still active
The OR Society [23]	UK	1948	✓	✓	✓		✓	✓		✓
IOI [7]	World	1989		✓	✓	✓				✓
INFORMS [26]	World	1995	✓	✓	✓		✓	✓		✓
HSOR [32]	USA	1996		✓			✓			
Kaiser [58]	Germany	2000					✓	✓		
Schuster [68]	Germany	2000						✓	✓	
McDuffie [67]	USA	2001							✓	
Cechlárová [50]	Slovakia	2005				✓		✓		
AIRO Challenges [53]	Italy	2008		✓		✓				
InGenio Community [41]	Chile	2008	✓	✓			✓	✓	✓	✓
MINDSET [35]	USA	2008		✓			✓		✓	
Logimat [56]	Italy	2008					✓			
Ottimizziamo! [61]	Italy	2009					✓	✓		✓
OPS [5]	Italy	2011		✓		✓				✓
TORCH [76]	Canada	2011		✓		✓				✓
Schettino and Bonetto [70]	Italy	2013							✓	✓
AMP21 [37]	USA	2015					✓	✓		✓
Lonati et al. [71]	Italy	2017							✓	
Colaço et al. [72]	Portugal	2018							✓	
BLOSSOMS [31]	USA	2018	✓	✓	✓					✓
Taranto et al. [62]	Italy	2018					✓	✓		
RabbitMath [43]	Canada	2019		✓			✓	✓	✓	✓
Rangel [63]	Brazil	2018						✓		✓

[30]. In the case of Mongolia, the coverage of OR, statistics and analytics was insufficient to warrant a colloquium. Thus, a different strategy was adopted. First, in 2016, a series of meetings in Mongolia was arranged, including a two-day workshop for students and teachers (see Sonomtseren et al. [29] for details). Then, in 2017, seventeen Mongolian instructors and government statisticians attended a two-week workshop hosted at the University of Alabama. The participants had the opportunity to learn several topics (e.g., teaching OR with cases and active learning, probability for statistics and OR, mathematical programming with Excel Solver and simulation). Once they came back to Mongolia, they worked with other colleagues to create the first ”*Handbook for Statistics Teachers*” [28].

Founded in 2008 by Richard Larson (MIT) and Elizabeth Murray (MIT Learning International Network Consortium), *MIT BLOSSOMS* [31] aims to enrich the students' learning experience through an online platform. Indeed, ”BLOSSOMS” stands for *Blended Learning Open Source Science or Math Studies*. The platform offers a video library with more than 150 mathematics and science lessons, that teachers and students can download or watch in streaming, in classroom or at home. Moreover, each module is usually accompanied by a teacher guide, some resource sheets and student worksheets and further documentation. A video lasts about 50 minutes and is designed for viewing in brief segments, each lasting less than 5 minutes. After each segment, the teacher can guide the students through an active learning exercise, building from the video segment. BLOSSOMS lessons connect seemingly abstract or dry concepts to the real world, giving students the chance to play and think. BLOSSOMS has provided training to more than 1500 Grades 9–12 teachers in the USA and partnering countries.

One of the longest national projects, developed to insinuate OR into Grades 9–12 mathematics classrooms, has been *High School Operations Research Outreach* (HSOR) [32, 33], since 1996. The initiative, run by Profs. Kenneth R. Chelst and Thomas G. Edwards (Wayne State University), was sponsored by the Industrial & Manufacturing Engineering Department of Wayne State University, INFORMS and the National Security Agency. HSOR consists of a series of 13 teacher instructional modules, each one provided with one or two student activity worksheets and additional teacher resource material, case studies, objectives, project ideas and possible extensions. It also offers a reference book (i.e., *Does this line ever move? Everyday applications of OR*, by Chelst and Edwards [34]). All HSOR resources are still available online. HSOR was the base of the *Project MINDSET* (i.e., Mathematics INstruction using Decision Science and Engineering). MINDSET was a 3 million National Science Foundation project, organized by three universities of North Carolina and realized by Chelst et al [35]. The main goals were to develop and enhance mathematics skills by using concepts and examples of OR and Industrial Engineering. They designed a course articulated in two semesters, dedicated to deterministic and probabilistic modelling, respectively. The key for the success of MINDSET was supporting and training teachers to be more a ”guide on the side” than a ”sage on a stage”, as Chelst et al. reported [36]. In 2007 and 2008, Chelst and his team wrote two book drafts and trained more than 30 teachers, receiving feedback from them about the drafts. In 2009 and 2010, more than 50 teachers started using the teaching material. When the two textbooks were published in 2012, MINDSET reached about 1500 students. From MINDSET, Chelst and Edwards derived another initiative called *Applied Math Practices* (AMP21) [37], nowadays still active. Schools can request 1-day workshops for students and professional development workshops of three days for up to 30 local teachers. In the following comparisons, we also report some comments by Thad Wilhelm (operations research teacher at Ernest W. Seaholm High School, Birmingham, Michigan) [39] which has developed his courses adopting Chelst and Edwards' textbooks. According to Chelst [40], Wilhelm is one of the most experienced teachers of high school OR.

Directed by Evelyn Nahuelhual, *InGenio Community* [41] is the outreach program of the Complex Engineering Systems Institute (ISCI), a research center in Chile. Among the research lines of ISCI, there are ”Operations Management and Analytics”, ”Data Science” and ”Transportation Engineering”. The main goal of InGenio Community is to create educational resources, helping Grades 9–12 students to approach engineering, mathematics and computing. It organizes direct initiatives addressed to students, such as workshops, courses, conferences and interactive exhibitions (with the collaboration of some Engineering students). Still, the main axis of InGenio Community are the teachers, for which there are training courses and seminars. These support teachers in the implementation in the classrooms and thus in reaching their students. Since 2008, more than 600 teachers have been trained, who have managed to implement what they have learned in more than 16,000 students. In addition to this, more than 10,000 young people have participated in direct workshops guided by undergraduate engineering students and more than 18,000 adolescents have participated in lectures with ISCI researchers. Satisfaction surveys are carried out to students and teachers who attend seminars, conferences or lectures [42]. Moreover, InGenio Community is also very active in social networks, disseminating graphics and video.

RabbitMath [43] is an ongoing initiative by Peter Taylor (Queen's University) and Chris Suurtamm (University of Ottawa). They observed that most first-year students at Queen's University lack mathematical thinking skills. Supported also by postdocs, undergraduate and graduate students, they developed RabbitMath with the goal of providing enrichment activities for high schools, working with motivated students in a collaborative approach and discovery, rather than individual study. The central theme is mathematical modelling. At the moment, Grades 9, 10 and 12 curricula are under construction, whereas the Grade 11 curriculum is up and already running in some Ontario high schools. The Grade 11 curriculum consists of 14 ”one-week” projects with activities challenging for both students and teachers. Up to now, only one project over these 14 is related to OR (more specifically, to combinatorial optimization).

Problem Posing & Solving (PP&S) is an Italian project launched in 2012 by the Ministry of Education, University and Research (MIUR) [44–46]. Originally addressed only to Grades 9–12 curricula, from 2017, it has been extended also to Grades 6–8. The main objective of PP&S is to define education processes which combined logic, mathematics and informatics. In order to formulate, quantify, simulate and analyze problems of various complexity, the use of new methodologies and technologies is promoted. In fact, PP&S involves the use of *Moodle* [47], an e-learning platform, integrated with the *Maple Suite* [48], an Advanced Computing

Environment, and a web conference system. The two key strengths of the project are the creation of best practices communities (both for teachers of different schools and for students of the classrooms enrolled) and the training courses available for teachers. PP&S also aims to build a strong relationship with the industry world, proposing exercises related to companies. Despite having this aspect in common with OR, we have decided to not include PP&S in the list of OR educational initiatives. The project seems to be more focused on producing innovative teaching modules to strengthen computer science culture. Moreover, it does not mention any topics in particular. Thus, a comparison would not be justified.

Another way to promote OR to pupils is through competitions. Although there are no international competitions exclusively on OR topics, the *International Olympiads in Informatics* (IOI) [7] includes some of them, such as graph theory, combinatorial optimization and dynamic programming. Sponsored and initiated in 1989 by UNESCO, IOI is one of the five international science Olympiads addressed to secondary school students (i.e., Grades 9–12). In 2019, 83 countries participated and sent a team of four students each. Students cannot participate individually but have to be selected through a national competition, usually preceded by local and regional phases.

In 2005, Prof. Katarína Cechlaárová (Pavol Jozef Šafárik University) designed a workshop for *STROM* [49], i.e., a Slovakian competition in mathematics problem-solving for primary and secondary schools. Cechlarova [50] defined several variants of increasing difficulty of a transportation problem. Then, she fully described the solution approaches proposed by different groups of students.

Analogous to STROM [49], the *Olympiads in Problem-Solving* (OPS) [5] are an Italian competition for primary, middle and secondary schools (Grades 3–9), promoted by MIUR since 2011. Arranged in local and regional phases, OPS can be regarded as preparatory to IOI [7]. Students can participate in groups or individually, according to their Grade. The main purpose of OPS is to encourage the development of computational thinking, foster creativity, stimulate problem-solving skills and enhance computer science culture, integrating coding and programming. Other similar international competitions, not included in the comparison, are *Kangourou sans Frontiéres* [51] and *Bebras* [52], dedicated to promote more mathematics and informatics, respectively.

From 2008 to 2011, in Italy there were also the *AIRO Challenges* [9, 53], organized by Prof. Giovanni Righini (University of Milan). The competition was structured in a local phase followed by a national phase. In the former, students could participate individually, solving a series of exercises in an iterative way. They were free to use all kinds of instruments and solvers they preferred; the only constraint was reporting everything [9]. They sent their resolutions by email to a local spokesperson. In the national phase, which used to last only one day, the finalists challenged themselves on other problems, connecting online to a platform offered by the University of Milan. Based on classical problems of OR, the AIRO Challenges represented a good alternative to the traditional competitions of mathematics and informatics, reaching many teachers and students from several Italian regions. They were initially recognized by MIUR as a national initiative to promote excellence. However, they were not funded and in the years the criteria to keep this title changed. Moreover, such initiatives do not contribute to the advance of an academic career in Italy. When Righini's term as AIRO councilor ended, the competitions were suspended [54].

The Canadian version of the AIRO Challenges [53] is *The Operations Research Challenge* (TORCH), an annual competition organized by OR researchers and undergraduate students at three Canadian universities (i.e., the Concordia University in Montréal, the University of Toronto and the University of Waterloo). TORCH is a one-day free contest where teams of 3 to 4 students solve 5 to 10 problems related to different application areas of OR. The main goals of TORCH are to interest high school students in OR and to encourage them to consider studying OR at the university level. Students are not required to know OR to participate in the contest; they can practice doing the exercises of previous editions, available on the website. Beyond sponsors, TORCH success is "*heavily dependent on the time and effort of many student volunteers, university staff and professors*" [55]. Indeed, TORCH was founded in 2011 by Daria Terekhov, Maliheh Aramon Bajestani, Tony T. Tran, T. K. Feng and Christian Muise, which at that time were graduate students at the Toronto Intelligent Decision Engineering Laboratory (TIDEL), coordinated by Prof. J. Christopher Beck (University of Toronto).

Training courses for teachers are one of the ways to reach a much higher number of students, guiding teachers on OR topics and indicating which kinds of exercises to propose in the classrooms. Sforza et al. [56] presented *Logimat 1* (2008/2009) and *Logimat 2* (2009/2010 and 2010/2011), three editions of a series of training courses addressed to Italian mathematics teacher of middle and secondary schools (i.e., Grades 6–12). Logimat and Logimat2 were part of an agreement between MIUR and the Campania Region. Each series was composed of 17 seminars or lectures (about 100 hours in total). These were based on problem-solving and decision-making concepts. The main theme was using mathematical modelling to solve problems arising from real applications (e.g., traffic, transportation, environment, energy, etc.). Thus, the subjects taught included modelling and OR, but also data collection and processing, statistics, simulation, informatics, logic, games and history of mathematics. Moreover, some lectures were more focused on teaching and on the relationship between teachers and students [57].

Sometimes courses for teachers may also imply direct experience with students. It is exactly the case of the German project "*Mathematical Modelling in School*", described by Prof. Gabriele Kaiser (University of Hamburg) [58, 59]. Established in 2000 within the framework of the initiative "*Mathematics at the Interface between School and University*" and active until 2013, the project was financed by the Volkswagen Foundation and conducted at the University of Hamburg by the Department of Mathematics and the Department of Education. The activity was addressed to both prospective teachers, enrolled in the university, and Grades 10–12 students from the Gymnasium Tonndorf, Gymnasium Grootmoor, Albert-Schweitzer-Gymnasium in Hamburg and Gymnasium Harksheide in Schleswig-Holstein. In particular, students were divided in groups and supervised by prospective teachers for two semesters. Groups worked on authentic open problems (e.g., "*Pricing of Air Berlin*", "*Pricing of an internet café*" and "*Risk management*") proposed by applied mathematicians working in industry. At the same time, a university course was carried out for prospective teachers, to discuss students' solution attempts, issues and experiences.

Usually training courses for teachers and workshops for students run in parallel, like in *Ottimizziamo!* [60, 61]. Organized by Profs. Giovanni Righini and Alberto Ceselli (University of Milan), Ottimizziamo! offers to students a two-day workshop, which can be also regarded as a young apprenticeship program. Students are introduced to OR and its origin to understand the relevance of mathematical models. The activity is performed in a computer laboratory of the University of Milan, where each student works on a computer. Exercises from real problems are tackled with linear and nonlinear programming using spreadsheets and solvers. Instead, addressed to teachers, Ottimizziamo! includes more than 45 hours of training courses about modelling, linear, integer and nonlinear programming, how to use solvers, designing algorithms, preparing and evaluating exercises and, last but not least, developing didactic units and resources.

At AIRO-ODS 2018, Dr. Eugenia Taranto (University of Catania) presented an activity supported by the research project "*Mathematical models in teaching and learning mathematics*" [62]. The goal was to integrate teaching with strategies and assessment tools to solve problems of everyday life, eventually to develop problem-solving skills. These can help reduce the gap between the students' education provided by the school system and the demand for competencies in the workplace. In fact, problem-solving skills are linked to seeking solutions instead of memorizing procedures, to exploring patterns instead of just using formulas, and to formulating conjectures, rather than just doing exercises. The activity, carried out as a young apprenticeship program (about 40 hours in total), involved some researchers of OR and mathematics education and about 50 Grade 12 students of different schools. The first four meetings were dedicated to OR theory, whereas, in the last three, students were divided into groups. They had to model, discuss and solve some proposed problems, using Microsoft Excel and its Excel Solver add-in. The initiative also included a training course for teachers.

At EURO 2019, Prof. Socorro Rangel (Universidade Estadual Paulista) described an activity called "*Traveling around 20 Brazilian cities*". This is part of the major project "*Matemtica Discreta: Modelando a Realidade*" (i.e., "*Discrete Mathematics: Modeling Reality*"), which includes the "*Coloring the map of Brazil*" and "*Producing tables and chairs*" workshops [63]. The project is funded by the Center for Research in Mathematical Sciences Applied to Industry (CEMEAI) and participates to the "*Embaixadores da Matemtica*" (i.e., "*Mathematics Ambassadors*") program [64]. The main purpose is to introduce concepts of graph theory and mathematical optimization (not included in the Brazilian national mathematics syllabus) to students at pre-university level, as well to the general public. Beyond encouraging young people to pursue a career in the STEM area, the project may contribute to the formation of citizens able to understand the complexities of the current world [65]. Addressed to Brazilian secondary schools, the activity was implemented during four workshops, held from September 2016 to October 2018. Each workshop had between 30 and 42 students, and at least one teacher. After a brief presentation about the theory behind the TSP, students were provided with the "*Game of Hamilton*", i.e., a wooden board with the projection of a dodecahedron in the plane (see [66] for more details). After a playing part, a discussion about the problem complexity and the number of possible solutions was proposed, also relating the problem to real-life applications.

Finally, we present some OR-based didactic units developed in the last twenty years. In 2001, Prof. Amy Roth McDuffie (Washington State University) described how to introduce graph theory in classrooms starting from an airline application [67]. Graphs, in fact, offer the opportunity to analyze such problem situations as networks and circuits. The activity is described by four different sheets, each one requiring one to two class periods. First, students get familiar with the context and analyze an airline map of flights to cities in the USA. Then, some parts are dedicated to building the foundation for communicating in the language of graph theory. Given a set of cities, students have to decide which routes are important to preserve, considering real aspects like population, demand for services, weather conditions and so on. Students are encouraged to discover the relationship between the number of edges and the sum of the degrees of the vertices in a graph. In the last part, a whole new problem is proposed to discuss.

From 2000 to 2002, Prof. Andreas Schuster (University of Würzburg) developed five projects to introduce combinatorial optimization to Grades 9–12 students of some German high schools [68]. In particular, two projects were implemented in a Grade 9 classroom, whereas the remaining three were proposed as workshops at the University of Würzburg. The problems (i.e., Traveling Salesman Problem, Minimum Spanning Tree and Shortest Path Problem) were proposed in a very open manner, leaving students to cooperate autonomously in groups, simulating working in a software development company. Teamwork and discussion through a plenary assembly alternated.

At EURO 2013, Alberta Schettino and Maria Celeste Bonetto, two secondary school teachers at the Technical Institute ”G. Galilei” in Imperia (Italy), presented a path for two Grade 10 classrooms. The Italian guidelines for technical schools explicitly mention OR in the economic and technological curricula [69]. The course of *Mathematic Complements for Logistics and (Maritime) Transport*, part of the *Logistics and (Maritime) Transport* curriculum, comprehends complex numbers, differential equations, spherical trigonometry, statistics and OR. In particular, each classroom spent 5 hours studying characteristics problems of OR (e.g., inventory management, production and allocation problems) and criteria for problems under uncertainty. Moreover, Schettino and Bonetto [70] described *What Is OR* (WIOR), an extra-curricular project to introduce OR and reinforce listening skills in English. In the six lessons of WIOR, about 20 students watched a film produced by The OR Society [23], studied classical OR problems and explored applications in transportation, manufacturing, sport, government, supply chain and education and careers areas.

At AIRO-ODS 2017, Dr. Violetta Lonati (University of Milan) described the importance of developing greedy strategies, considered as a natural way to cope with optimization problems [71]. The didactic unit proposed to Grades 10–12 students was first developed and fine tuned with bachelor computer science students. It was composed of two main phases. In the former, students had to solve a problem about giving change using the minimum number of available coins and bills. They were organized first in pairs and then in groups; each group read its algorithm and the others executed the instructions. The first goal was to define an abstract description of a greedy procedure. In the latter, the same approach was used to solve a scheduling problem, discussing analogies and differences with the former. Students were provided with an ad-hoc software tool supporting their tests and strategies.

In 2018, Susana Colaço (Escola Superior de Educação and Instituto Politécnico de Santarém) showed how to adapt three optimization problems for creating mathematics tasks for classrooms from Grade 1 to Grade 12 [72]. In particular, the first (”*Optimizing flow information problems*”) and the third (”*Diet problem*”) ones were addressed to middle and high school students (Grades 6–12). Colaço et al. [72] designed tasks to enhance mathematical reasoning and problem-solving, to foster the use of mathematical representations and the exploration of their connections. Moreover, they evaluated the role of communication when the teacher facilitates discussions of mathematical ideas and asks purposeful questions.

3.2 Objectives and Focus

Based on the descriptions in the previous subsection, we can affirm that all initiatives aim to increase students' interest and motivation towards OR and mathematics: the activities help students to recognize when and where they are going to ”*use this stuff*” [34]. But what can we say about the initiatives main focus, in terms of *mathematical modelling* and *algorithmics*?

Mathematical modelling is the process through which a problem, usually described with words, is translated into mathematical terms (i.e., variables, constraints and, if required, an objective function). Once a problem is formulated within a model, the whole body of knowledge for that model can be applied, in order to solve it. Using available tools and software to compute a solution, one can concentrate more on correctly understanding the problem, abstracting it to obtain a more accurate representation. However, a model remains a simplification, thus it may not include all aspects of all the possible instances. Sometimes a problem may be too hard to be optimally solved by a solver. In such a case, to find a solution we can use, for instance, a greedy algorithm. An algorithm is a finite sequence of instructions or steps designed to solve the problem. Its properties can be studied mathematically in order to define, for instance, its efficiency, correctness and completeness. Exact methods, heuristics and approximation algorithms: they are all relevant to solve OR problems. Developing algorithms requires analytical skills, computational thinking and creativity. Both models and algorithms are indispensable when studying OR, but which concept is the most proposed in the collected initiatives for Grades 9–12 students? We can try to answer by looking at Table 2.

According to Evelyn Hardy, the resources provided by The OR Society [23] are focused on a mix of both modelling and algorithms. The main aim is to engage the students' interest in OR approaches to problem-solving, to build their awareness of OR (as a potential career, or as a future user of OR skills) and to encourage them to continue with numerate subjects in their education [25]. These purposes are shared by other communities. BLOSSOMS [31] tries to connect abstract concepts to the real world, engaging students in observation, experiment and discussion. Most of the videos available in the BLOSSOMS platform are about modelling, but a few present some algorithms (e.g., the Simplex method) and some are dedicated to programming. Also InGenio Community [41] is mostly focused on mathematical modelling, but still integrating algorithms [42]. HSOR [32, 33], as well as its related projects MINDSET [35] and AMP21 [37], provides materials which engage students in exploring, conjecturing and reasoning about problems. Wilhelm [39] would like students to see and appreciate how relevant and applicable mathematics is in the real world.

Competitions like IOI [7] and OPS [5] are more focused on strategies and methods to solve the proposed exercises rather than modelling. However, it may depend on the kind of training students receive, in order to be prepared for the challenges. Cechlrov [50] showed linear programming to the students who were training for STROM [49]. AIRO Challenges [53] and TORCH [76] are both inspired by real-world applications or based on fantasy settings. Only the former requires explicitly the formulation of models. The latter is more centered on students' problem-solving skills and creative thinking to find optimal solutions.

Table 2 Main focus of OR educational initiatives

OR initiative	Country	Year	Modelling	Algorithms
The OR Society [23]	UK	1948	✓	✓
IOI [7]	World	1989		✓
INFORMS [26]	World	1995	✓	
HSOR [32]	USA	1996	✓	
Kaiser [58]	Germany	2000	✓	
Schuster [68]	Germany	2000	✓	✓
McDuffie [67]	USA	2001	✓	✓
Cechlrov [50]	Slovakia	2005	✓	✓
AIRO Challenges [53]	Italy	2008	✓	
InGenio Community [41]	Chile	2008	✓	✓
MINDSET [35]	USA	2008	✓	
Logimat [56]	Italy	2008	✓	
Ottimizziamo! [61]	Italy	2009	✓	✓
OPS [5]	Italy	2011		✓
TORCH [76]	Canada	2011	✓	✓
Schettino and Bonetto [70]	Italy	2013	✓	
AMP21 [37]	USA	2015	✓	
Lonati et al. [71]	Italy	2017		✓
Colaço et al. [72]	Portugal	2018	✓	✓
BLOSSOMS [31]	USA	2018	✓	✓
Taranto et al. [62]	Italy	2018	✓	
RabbitMath [43]	Canada	2019	✓	
Rangel [63]	Brazil	2019		✓

During training courses and workshops, the focus is more on modelling and how to translate a problem into mathematical language (e.g., Kaiser [58]), leaving computational tasks to computers (e.g., Ottimizziamo! [60] and Taranto et al. [62]).

Regarding initiatives in classrooms, Schettino and Bonetto [70] are more focused on modelling applications, identifying variables and constraints. Instead, McDuffie [67], Schuster [68] and Colaço et al. [72]) examine both modelling and investigation of algorithms. Lonati et al. [71] and Rangel [63] propose the development of algorithms, in particular greedy and heuristics, respectively. In this way, students are encouraged to think, elaborate and compare different approaches, analyzing their limits and searching for properties regarding optimality, correctness and completeness. Cechlrov [50] affirmed that optimization problems are not typical for school mathematics, but if correctly posed they can stimulate students' active approach to problem-solving, invention, creativity and informal search for appropriate methods from their pool of knowledge.

Therefore, it seems that most of the initiatives are based on both modelling and algorithmics, which are intertwined together during the activities. Usually, mathematical modelling is preparatory to design a correct algorithm, because it allows to get a mathematical formulation of the problem. Only in competitions and games the focus is mainly put on algorithms, probably because it is more important to find, hopefully quickly, the solutions to the puzzles.

3.3 Topics

We summarize in Table 3 the topics presented in the collected initiatives, with the following abbreviations: *Mathematical Modelling* (MM); *Linear Programming and Integer Linear Programming* (LP/ILP); *Graph Theory* (GT); *Combinatorial Optimization* (CO); *Dynamic Programming* (DP); others less common.

Platforms like The OR Society [23] and INFORMS [26] have the purpose to introduce OR to teachers and students, thus most of their material is about mathematical modelling and linear programming. Being the base for more advanced topics, these are introduced in almost all initiatives, in particular in training courses (e.g., Logimat [56], Ottimizziamo! [61]), workshops (e.g., Cechlrov [50], Ottimizziamo! [61], Taranto et al. [62]), and didactic units in classrooms (e.g., Schettino and Bonetto [70], Colaço et al. [72]). Linear programming allows for further extensions like integer linear programming and sensitivity analysis (e.g., Schettino and Bonetto [70], AMP21 [37]). Graph theory concepts can be introduced easily presenting real-world applications (e.g., McDuffie [67]) and then used to deal with shortest paths and minimum spanning trees (e.g., Schuster [68]). Some techniques of combinatorial optimization are also taught (e.g., Rangel [63], Schuster [68]), because they offer plenty of materials for developing heuristics strategies. Some initiatives may also intrigue students presenting nonlinear programming or bilevel programming (e.g., Ottimizziamo! [61]) or project management techniques like PERT and CPA (e.g., Schettino and Bonetto [70]).

Being centered not only on mathematics but also on other STEM disciplines, BLOSSOMS [31] offers general contents, not all related to OR. Still, we can find

Table 3 Main topics of OR educational initiatives

OR initiative	Country	Year	MM	LP/ILP	GT	CO	DP	Others
The OR Society [23]	UK	1948	✓	✓	✓	✓		-
IOI [7]	World	1989			✓	✓	✓	Computational Complexity
INFORMS [26]	World	1995	✓	✓				Probability; Statistics; Simulation
HSOR [32]	USA	1996	✓	✓	✓	✓		See Table 4
Kaiser [58]	Germany	2000	✓					Nonlinearity; Probability
Schuster [68]	Germany	2000	✓		✓	✓		-
McDuffie [67]	USA	2001	✓		✓			-
Cechlrov [50]	Slovakia	2005	✓	✓				-
AIRO Challenges [53]	Italy	2008	✓	✓	✓	✓		Nonlinearity; Probability
InGenio Community [41]	Chile	2008	✓	✓	✓	✓		MCDM
MINDSET [35]	USA	2008	✓	✓	✓	✓		-
Logimat [56]	Italy	2008	✓					Statistics; Informatics; History of OR
Ottimizziamo! [61]	Italy	2009	✓	✓				Nonlinearity; History of OR
OPS [5]	Italy	2011			✓	✓	✓	-
TORCH [76]	Canada	2011	✓		✓	✓	✓	Probability
Schettino and Bonetto [70]	Italy	2013	✓	✓	✓			Project Management; Stochastic Programming
AMP21 [37]	USA	2015	✓	✓	✓	✓		MCDM; Probability; Queuing Theory
Lonati et al. [71]	Italy	2017	✓					Greedy algorithms
Colaço et al. [72]	Portugal	2018	✓	✓		✓		-
BLOSSOMS [31]	USA	2018	✓	✓	✓	✓		MCDM
Taranto et al. [62]	Italy	2018	✓	✓	✓			-
RabbitMath [43]	Canada	2019	✓			✓		-
Rangel [63]	Brazil	2019	✓		✓	✓		-

some video lectures that address traditional topics such as linear programming and graph theory (e.g., ”*Optimizing Your Diet: What Linear Programming Can Tell You!*”, ”*Taking Walks, Delivering Mail: An Introduction to Graph Theory*”), but also more advanced subjects (e.g., ”*Choosing a College Roommate: How Multi-Criteria Decision Modeling (MCDM) Can Help*).

HSOR [32] (and its related projects MINDSET [35] and AMP21 [37]) provides modules which cover several different OR topics, some of them not so common among other initiatives (e.g., Queuing Theory, Simulation, Multi-attribute Utility Theory; see Table 4 for more details). Wilhelm [39] teaches two OR classes: one semester on deterministic modelling and one semester on probabilistic modelling. In the former, he presents topics like MCDM, linear and integer programming, sensitivity analysis; in the latter, randomness and variability, conditional probability, decision trees and distributions.

Regarding competitions, the syllabus of the 2019 edition of IOI [109] includes the following subjects: ”*Graphs and trees*”, ”*Algorithms and problem solving*”, ”*Basic algorithmic analysis*” (only the basics of computational complexity), ”*Algorithmic strategies*” (greedy algorithms, dynamic programming and heuristics), ”*Algorithms*” (for shortest-path, minimum spanning tree and maximum bipartite matching problems). OPS [5], as described in the guide for the 2019/2020 edition [108], often proposes exercises about planning and scheduling, combinatorial optimization problems like *Knapsack* and graphs. The AIRO Challenges [53] consisted of exercises about transportation, production, assignment, diet, logistics and knapsack problems. Students could use mathematical modelling, linear and integer programming but also nonlinear programming to tackle them.

Table 4 HSOR modules

HSOR module	Topic
Decision Tree: Making a Decision with Impact	Decision Analysis
Arm-and-a-Leg: Ticket Counter	Queuing Theory
Gamz, Inc.: Targets the Markets	Marketing Science
Torn Shirts Inc.: Telephone Orders	Simulation
Frankfurter High: Hot Dog Sales	Simulation
Speedy Delivery: Service Woes	Routing in Network
Outel Semiconductor: Recruiting Circuit	TSP
Miguel Chooses a College: Tradeoffs	Multi-attribute Utility Theory
Latisha Develops an Investment Plan: Decision Variables	Integer Programming
Jurassic Oil: Fuel Blending	Linear and Non-Linear Programming
Cutting Times: Trimming Losses	Linear Programming
High Step Shoes: Product Mix	Linear Programming
Pizza Pi: Work Force	Linear Programming

3.4 Teaching Methods

According to Drake [106], any pedagogical approach should facilitate the students' development of certain desired knowledge, insight or skills. Nowadays there are several options, alternative to the traditional lecture method, that can help in reaching the learning objectives set. Table 5 illustrates the teaching methods mentioned in the collected initiatives, tested or suggested to work with students.

Active learning is a teaching method where students are not passively receiving concepts and notions, but they personally act and participate, conquering competences and knowledge by themselves. Keeping students' attention is a difficult task: during a traditional lecture they can get bored and distracted, nullifying the lecture itself. Working in pairs or groups is an example of a technique which supports active learning (see Felder and Brent [77] for further information). Ottimizziamo! [60, 61] alternates guided discussions with mathematical modelling

Table 5 Teaching methods tested or suggested in OR educational initiatives

OR initiative	Country	Year	Teamwork	Cooperative learning	Project-based learning	Cases	Games	Blended learning
The OR Society [23]	UK	1948	✓		✓	✓	✓	
IOI [7]	World	1989	✓	✓				
INFORMS [26]	World	1995			✓	✓	✓	
HSOR [32]	USA	1996	✓			✓		
Kaiser [58]	Germany	2000	✓	✓	✓	✓		
Schuster [68]	Germany	2000	✓	✓	✓			
McDuffie [67]	USA	2001	✓	✓		✓		
Cechlrov [50]	Slovakia	2005	✓	✓				
AIRO Challenges [53]	Italy	2008				✓		
InGenio Community [41]	Chile	2008				✓	✓	
MINDSET [35]	USA	2008	✓	✓		✓		
Logimat [56]	Italy	2008				✓		
Ottimizziamo! [61]	Italy	2009		✓		✓		
OPS [5]	Italy	2011	✓	✓				
TORCH [76]	Canada	2011	✓	✓		✓		
Schettino and Bonetto [70]	Italy	2013	✓	✓		✓		✓
AMP21 [37]	USA	2015				✓		
Lonati et al. [71]	Italy	2017	✓	✓				
Colaço et al. [72]	Portugal	2018	✓	✓				
BLOSSOMS [31]	USA	2018		✓				✓
Taranto et al. [62]	Italy	2018	✓	✓				
RabbitMath [43]	Canada	2019	✓		✓			
Rangel [63]	Brazil	2019	✓	✓			✓	

and resolution. Each student can use a personal computer, thus can actively work in solving the given exercises. The importance of the active role of students is also shared by Chelst and Edwards in all their projects (i.e., HSOR [32, 33], MINDSET [35] and AMP21 [37]). Wilhelm's courses [39] involve a lot of whole-class discussions during the modelling process, but there is also time for students to practice formulating and solving problems either individually or in small groups.

Cases from several applications are used to improve students' motivation and foster connections between mathematics and the real world. We note that *case studies* and *teaching cases* are two teaching methods often confused. In fact, the former presents a situation or a problem taken or inspired by real life, whereas for the latter this is not a strong requirement. Usually, a case study already contains information about how the problem has been addressed, whilst in teaching case no solution is provided and it is up to students to find one (or more). Thus, the former may only lead to analysis and discussion, whereas the latter helps students develop problem-solving skills [78]. Since in the literature the two terms are used interchangeably, we have decided to indicate both with the word *cases*. Using cases, students enjoy the challenge of working on an open-ended problem with several potentially correct answers, also having the opportunity to justify their answers. Indeed, cases have been used during workshops, like the INFORMS colloquia [28] or the courses offered by InGenio Community [41, 42]. Inquiry-based learning, cases and games are the preferred teaching methods adopted in InGenio Community activities. Among the educational resources provided, we can find "*Fixture of Chilean soccer*", "*Design of a newspaper*", "*Multicriteria Analysis*", "*Traveling salesman problem*" and "*Minimal road*". Cases intend for students to formalize a real problem, with a methodology that allows them to go step by step [42].

Also the initiatives organized by The OR Society [23] are often based on *games and puzzles*. Examples of such activities are "Play with Lego", "Play a board game" and "Cooking Bolognese". Students are interested in new and unfamiliar topics or methods, and the discussion is also encouraged. The available resources are designed to be interactive, encouraging groups of students to compete with each other, or building models with Lego, or leading to class discussions [25]. Rangel [63] exploited a wooden version of the Icosian Game to challenge students to find cheaper and cheaper Hamiltonian circuits.

To increase students' motivation, *project-based learning* can be an effective and comprehensive strategy to apply. Blumenfeld et al. [73] affirmed that students are engaged in investigation to create "real artifacts": they discuss ideas about a problem to be solved; they ask questions, acquire concepts and plan their actions; they collect and analyze data, experiment, use trial-and-error process; they tell about their results. Students can really perceive they are solving authentic problems, while growing collaborative skills and gaining a deep and critical understanding of the problem. However, structuring a project-based activity could be a difficult task: knowledge required from students have to be evaluated very carefully, as much as being able to switch in their point of view and not regarding the problem as experts; furthermore, students' different skills and level of competences must be taken into account. Schuster [68] implements learning by performing projects in the context of combinatorial optimization problems, simulating an industrial environment. Each semester, students and prospective teachers participating in the activity described by Kaiser [58] were assigned with a project that involved analysis and modelling; they had to present their results at the end of each period. RabbitMath [43] uses project-based activities to implement the following three main concepts: paying attention to the stage of first apprehension (i.e., how students feel when they are learning, called *romance* by Whitehead [79]); providing students experiences, according to Dewey's meaning [80]; and considering them as *sculptors*, following Papert's constructionism theory [81].

To train for competitions such as IOI [7], students can attend onsite or online camps and participate in simulation contests. Often, this kind of training implements the principles of *cooperative learning*, where the teacher's role is to be a coach. A coach, or also called mentor, supports a student to find innovative ways for solving exercises. At the same time, this may open up to the teacher new aspects and ways to approach competitive tasks. Thus, both of them contribute to the development of the creative potential of the other [82, 83]. This methodology, together with group assignments, is usually exploited also in training for other competitions (e.g., OPS [5], STROM [50]).

McDuffie [67] recommends to give students time to think about each problem on their own, either individually (also as a homework assignment) or within groups of three to four people. When students *work in groups*, they can share competences and ideas. Once the classroom is gathered together, groups share their findings and strategies, comparing approaches and solutions, thus developing problem-solving and collaborative skills, as observed in McDuffie [67] and Taranto et al. [62]. One of the most common strategies reported by Colaço et al. [72] is trial and error. A group can be considered as a place to belong to, where to find support and motivation. Indeed, Lonati et al. [71] underlines that the group promotes cooperation and the activation of latent cognitive potentials. This allows students to carry out the main parts of the activity by themselves, except for some brief explanations provided by teachers. Lonati et al. [71] proposed their own teaching method, denoted by *algomotricity*, which consists of active workshops exploiting kinesthetic learning activities. These have the aim of informally exposing students to a specific informatics topic, followed by an abstract learning phase devoted to let students build their mental models of the topic under investigation, and a final computer-based phase to close the loop with their previous acquaintance with applications. Schettino and Bonetto [70] divided students into groups too, with uniform levels of competence but internally heterogeneous. Cooperation and interaction were enhanced through a Facebook group. In their WIOR project, they applied blended learning too.

Thus, we can observe that most of the collected initiatives are based on *constructionism*, a student-centered learning theory defined by Papert [81] and based on Piaget's epistemological constructivism theory. Students are encouraged to work with creative experimentation, making tangible objects through authentic and real learning opportunities, arising from the real world. They exploit previous knowledge in order to construct and acquire more notions and concepts, working in collaborative environments and discussing among each other. Papert strongly recommended the use of tools, media and technology in classrooms to teach mathematics to children. Teachers assume the role of guides, but more in the sense of facilitators, rather than giving lectures in the traditional way. Constructivism is opposite to *behaviorism*, the other common approach to teach mathematics [85, 86]. Knowledge is transmitted to students through teacher-centred lessons or with direct instructions. Students are thus conditioned and motivated to do or to get something by external causes. Knowing is considered as an organized accumulation of associations and components of skills, acquired through the learning process. Instead, constructivists view learning as an active process centered on students, which are constantly reviewing their prior knowledge, revising and adapting it to new information. Anyway, implementing one approach does not have to necessarily exclude the other. In fact, both behaviorism and constructionism may be used alternatively to maximize students' engagement and achievement [86]. Other teaching methods adopted to teach OR are *interactive tools* (e.g., Nurre and Weir [87]) and *flipped classroom* (e.g., Asef-Vaziri [38]). The interested reader is referred to Cochran [74] for further information about OR pedagogy.

3.5 Instruments and Software

This part describes the tools exploited or recommended in the collected initiatives. In general, the final decision is left to teachers.

The OR Society [23], INFORMS [26] and BLOSSOMS [31] offer repositories of problems, teachers' manuals and video libraries. They do not recommend any particular instruments instead of others, because they usually try to use tools already available in schools. For instance, for workshops The OR Society [23] utilizes Microsoft PowerPoint and downloadable PDF files [25].

In 2004, some HSOR modules listed in Table 4 have been adapted to compose a book titled "*Does this line ever move? Everyday Applications of Operations Research*" [34]. The book was aligned with the *National Council of Teachers of Mathematics Standards* (see Subsection 4.2). Two semester books were also developed during MINDSET [35] and published in 2012. Then, they were improved and published again in 2015 with the titles "*Mathematical Modeling with Algebra: Using Authentic Problem Contexts*" and "*Mathematical Modeling with Probability: Using Authentic Problem Contexts*", respectively. These are part of the "*When will I ever use this stuff?*" series, suggested in AMP21 [37]. A third book concludes the series, i.e., "*From Percentages to Algebra: Using Authentic Problem Contexts*", available in two different versions for teachers and students.

Designing her activity in the classrooms, McDuffie [67] suggested the use of calculators, almanacs and an airline flight map and system timetable, in order to make the experience more concrete. Schuster [68] exploited computers to do computations. Schettino and Bonetto [70] relied on HSOR book [34] and The OR Society resources [24]. They exploited the free version of Wolfram Alpha [88] and an interactive whiteboard. For problems related with geometry, Colaço et al. [72] suggest GeoGebra [89].

Another tool exploited during didactic units and also workshops is Excel Solver (e.g., Ottimizziamo! [61], Taranto et al. [62]). A good alternative is the free and accessible Google Drive Spreadsheet, as suggested by InGenio Community [41, 42]. Both teachers and students are usually familiar with Microsoft Office and Google environments, thus the effort in learning new features of these is probably little. More rare is the use of modelling languages like AMPL [107] (e.g., Ottimizziamo! [57]) or MATLAB [90] (e.g., Kaiser [58]). Sometimes ad-hoc software is developed and utilized, as done by Lonati et al. [71] and by Cechlrov [50] with CASSIM (i.e., *Computer ASsisted SImplex Method*).

For competitions such as IOI [7], OPS [5] and TORCH [76], texts and exercises of previous editions are available, but in general the tools to utilize are not indicated. Usually computers are not allowed, but sometimes they are. In fact, the AIRO Challenges [53] had to be tackled using Excel Solver.

We observe that solvers often exploited in university courses, such as IBM ILOG CPLEX Optimization Studio [91] or Gurobi Optimizer [92], are not used. We assume this is because they are not as accessible as the tools mentioned above in terms of cost and, mostly, in terms of knowledge from teachers. Moreover, the choice of the tools also depends on the learning objectives to reach. If the goal is to introduce OR concepts, Excel Solver or similar software may be sufficient and simpler.

3.6 Feedback

Students have been very enthusiastic participating in OR-based initiatives. Sometimes they have been much more actively engaged in their learning than in other mathematics courses (e.g., Wilhelm [39]). Even if they never worked with similar problems, they were able to get to an adequate interpretation and a profitable strategy (e.g., Colaço et al. [72], Kaiser [58]). Sometimes they did not want to know the correct answer but they even asked to delay the explanation, in order to have the possibility to think further about the problem (e.g., Lonati et al. [71]). Results showed that complex modelling examples are not reserved for highly talented and high performing students but can be carried out by average students in ordinary schools (e.g., Kaiser [58]). In fact, students did not have many difficulties understanding OR concepts. They appreciated the chance to solve challenging problems related to real life, starting to think about how to optimize other practical and concrete situations. Students' interest increased also during competitions (e.g., TORCH [55]) and didactic units (e.g., Lonati et al. [71], Schettino and Bonetto [70], Schuster [68]). Moreover, students enjoyed learning new software tools like Excel Solver (e.g., Ottimizziamo! [60], Taranto et al. [62]). About InGenio Community [41], Nahuelhual reported that in 2018 and 2019 students evaluated the workshops with an average score of 6.5 (on a scale of 1 to 7, where 1 is very bad and 7 is very good). 80% of the participating students said they agree or totally agree with the statement: "*The workshop allowed me to see mathematics as something useful*" [42]. Rangel [63] said that most of the participants evaluated the activity about Brazilian cities very positively. Most students expressed their wonder, perceived the importance of using mathematics and appreciated the development of different strategies to solve a problem. However, there was also a sign of disinterest from some others.

Teachers enjoyed that students were more interested in mathematics, in particular in some branches not usually part of standard curricula (e.g., AIRO Challenges [54]). In fact, they were satisfied by the educational value of a model-oriented approach that integrates thinking and modelling exercises with software and tools. On the other hand, teachers have underlined the difficulty of integrating the approach in the traditional school curriculum, where time can be very scarce and some students have gaps in basic mathematical concepts (e.g., Ottimizziamo! [60]). Moreover, teachers may have never attended any OR courses during their studies, as underlined by Taranto et al. [62]. This is also why they have considered training courses as a way to go deeper into the subject and to support the teaching of mathematics and science (e.g., Logimat [56, 57]). Furthermore, prospective teachers stated that practical training should generally be more integrated into university study (e.g., Kaiser [58]).

InGenio Community [41] supports teachers in several ways. During the training, there is a permanent monitoring system, implemented through a Moodle virtual platform, email, telephone, personal meetings. Teachers have to plan how they are going to implement games and cases in their classrooms and, at the end, they must submit a report about achieved objectives and opinions. In 2018 and 2019, 97% and 100% of teachers, respectively, reported having met all their set goals of motivating students and testing new methodology. Teachers attending The OR Society [23] online courses were excited to use the available resources and interested in the field of OR. Throughout the years, they have been giving constructive feedback on possible ways to improve the workshops [25]. About HSOR [32], MINDSET [35] and AMP21 [37] resources, Chelst and Edwards did not collect proper information about the usage of the published books [40].

4 OR in Guidelines for Mathematics Education

This section illustrates some of the main international and national guidelines for mathematics education. In particular, we look for explicit or implicit references to OR, in order to show how studying OR could help to achieve the objectives and levels of mathematics education set by international authoritative organizations and several countries.

4.1 International Guidelines

The Program for International Students Assessments (PISA)

Every three years since 2000, the *Organisation for Economic Co-operation and Development* (OECD) has been promoting the international survey PISA, targeting 15-year-old students all over the world [93]. This survey aims to assess whether students are able to apply to real-life situations what they have learned in schools. Thus, the test is not focused on verifying the gained knowledge but rather the acquired skills. PISA is of great importance to the participating countries because it provides a clear picture of students' competences and skills. Thus, it can highlight any critical issues in students' curricula. Results in hand, each national government can decide whether to undertake system reforms or strengthen certain aspects of the school system. Each test is mainly about a specific learning area among reading, mathematics and science. The main subject of PISA 2003 and PISA 2012 was mathematics, with a particular focus on *problem-solving*. Mathematics will also be the central theme for PISA 2021, in which 86 different nations will participate. In view of this next survey, PISA has drawn up a document [94] to explain how the assessments will be based on the important concept of *mathematical literacy*. This is the ability to learn and rework mathematical concepts, in order to formulate and solve problems in various real-world contexts, as well as to learn the main notions of computational thinking. The document specifies which skills a mathematician-literate should have, starting with *mathematical reasoning*. Through this process, students should be able to draw assured and not questionable conclusions from acquired mathematical notions. These conclusions need to be valid when exploited in real contexts. Mathematical reasoning should be based, among other things, on the capacity for abstraction, i.e., in knowing how to build mathematical models that represent real scientific phenomena. Only thanks to these, if correctly formulated through mathematical language, it will be possible to study and analyze the phenomena represented, using suitable algorithms and software.

Thus, it is straightforward how studying and learning such an applicative discipline as OR could easily foster this skill. In particular, the central part of the document discusses three keywords: *formulate*, *employ* and *interpret*. PISA associates to the first keyword all the activities related to: the identification of the *mathematical aspects* of a problem situated in a real-life context, highlighting *input and output variables*; the identification of *constraints* gathered from the problem context; the translation of a problem into *mathematical representation*, using appropriate variables and symbols and knowing how to connect them to the context-specific language. The activities connected to the second keyword are instead the following: the project and implementation of *strategies* for finding mathematical solution to the problem; the use of *technology and mathematical tools* (e.g. algorithms or graphics) to help find *exact or approximate solution*. Finally, the last keyword is related to the following activities: the *interpretation* of information presented in diagrams; the *evaluation* and interpretation of a mathematical result in terms of its *real world context*; the evaluation of the *reasonableness* of found solutions and how to apply them.

United Nations Educational, Scientific and Cultural Organization (UNESCO)

UNESCO is the United Nations agency created to promote peace among countries through education, science, culture, communication and information. Thus, it holds all scientific disciplines in high regard. In particular, UNESCO currently organizes and promotes some educational activities about mathematics, especially addressed to young students to make them appreciate the importance of the discipline in society and in everyday life [95, 96]. Moreover, in 2015, on the occasion of the 70th session of the United Nations General Assembly, 17 Suitable Development goals were set. The fourth one is *Quality education*, which considers the *strengthening of* *technology and mathematics education* as necessary, not only to improve the quality of education. In fact, it would help to develop and innovate values and attitudes to allow citizens to lead a healthy, satisfying life, being aware of the challenges to be faced [97].

European Union

In May 2018, the European Parliament and the Council of the European Union drafted a document for the member states to identify and define the key competences necessary for employability, personal fulfilment and health, active and responsible citizenship and social inclusion [75]. The eight competences are the following: literacy competence; multilingual competence; competence in mathematics and science and technology and engineering; digital competence; personal, social and learning to learn competence; citizenship competence; entrepreneurship competence; cultural awareness and expression competence. Four of these can be connected to OR topics, methods and techniques. The mathematical competence is described as the *ability to develop and apply mathematical thinking to solve a range of problems in everyday situations*. It involves the ability and willingness to use mathematical modes of thought and presentations, through the use of *formulas*, *models*, constructs, graphs and charts. The part related to science, technology and engineering involves being able to utilize and handle *technological tools and machines*, as well as *scientific data*, to achieve a goal or to reach an *evidence-based decision* or conclusion. The digital competence is about understanding how digital technologies can support communication, creativity and innovation. It comprises skills such as the ability to use, access, filter, evaluate, create, program and share digital content. In the part about personal, social and learning to learn competences, one can find reference to *problem-solving*, collaboration, assertiveness and integrity, *identification and setting of goals*. Last but not least, a *sound understanding of written information* and *knowledge of vocabulary and grammar* is key in the literacy competence, to identify, understand, express, create and interpret concepts, feelings, facts and opinions in both oral and written forms. Correctly modelling and solving a problem without a proper understanding of its features and requisites is not possible.

4.2 National Guidelines

United States of America

Begun in 2009, the *Common Core State Standards Initiative* [98] is a USA educational initiative sponsored by the National Governors Association and Council of Chief State School Officers. It is about the standard levels of knowledge and skills to reach in arts and mathematics in Grades K-12. Thanks to a great effort by teachers, schools administrators and experts of higher education, business and industry, these standards aim to uniform local school districts in more than 45 US states (up to March 2019, 7 states repealed and 36 formally adopted them). In particular, regarding mathematics, *modelling* and *problem-solving* are highlighted as two of the main standards to be achieved [99]. The standards describe the former through the abilities to identify variables representing a real problem and to formulate the model that describes the relationships and operations among these variables. Moreover, students should be able to interpret the results obtained from the resolution of a model, by applying them to the starting context, to validate the model built and improve it. Regarding the latter, the standard specifies how problem-solving is not a particular topic. Instead, it must involve the entire course of study of mathematics. In particular, students must be encouraged to abstract strategies and knowledge applied in different contexts to apply them in new ones, thus fostering curiosity and the ability to solve problems even in unfamiliar contexts.

Australia

The Australian education system considers mathematics as a central tool for developing *critical thinking* and *problem-solving skills* [100]. In particular, students of Grades 10 and 11 must be able to ”*develop and apply mathematical models in routine and non-routine problems in a variety of contexts*”, in order to achieve the highest grade in mathematics [101]. The NSW Government is engaging in a multi-year project to disseminate the importance and various applications of mathematics to both students and their parents. It trusts that, thanks to this discipline, students will be able to understand and manage the real needs of the labour market.

China

Mathematics in Chinese education is regarded as the science of thinking. Developing *mathematical thinking* in students is one of the main goals of Chinese teachers. The concept can be traced back to *mathematical reasoning*, recently introduced by PISA (see Subsection 4.1). *Learning by doing* and problem-solving teaching techniques play a central role in Chinese education. In particular, students should be able to induce new knowledge individually from a reworking of previous knowledge, to tackle real-life problems and authentic situations [102]. This trend is also clearly evident in the latest school reforms conducted by the government. In fact, in 2000, one of the main objectives of the *Basic Education Curriculum Reform Outline* was to promote *constructivist learning* to improve students' skills in acquiring knowledge, problem-solving and cooperative learning. Moreover, in 2014, the concept has been confirmed and strengthened with a reform where *mathematical abstraction* and *mathematical modelling* are included as two of the six skills needed to develop and increase the mathematical thinking competence [103].

Sweden

Problem-solving and *modelling* as well as the development of simple *algorithms* and their implementation are objectives of Swedish mathematics education from the first years of compulsory school (i.e., Grades 2–8). In particular, problem-solving is considered an important and consistent macro-topic of the discipline of mathematics in the curriculum for Grades 9–11. It is organized into different sub-objectives which include, among others: knowing how to elaborate and choose strategies for problem-solving in *everyday situations* and in various contexts; building mathematical formulation of models representing real situations; developing, testing and improving algorithms that exploit mathematical models created to solve problems [104].

Italy

In 2010, MIUR published three documents regarding education in high schools [84], technical schools [69] and trade schools [105], i.e., the three main categories of Italian secondary schools. All of these discuss mathematics education introducing the following four macro-topics: algebra and arithmetic; geometry; functions and relations; data and predictions. OR is explicitly mentioned in the economic and technological curricula of technical schools, in Grade 12 and in Grades 10–11, respectively. In high schools curricula, instead, OR is just cited in the margin in the applied science curriculum. Anyway, several implicit references to OR can be found in all curricula. In fact, *modelling* and *problem-solving* are suggested to deal with complex situations and *taking decisions* according to several variables. Practical aspects of mathematics, related to economic, social and technological environments, are discussed in technical and trade schools, whereas high schools are more centered on the cultural value of mathematics, as a product of the human thought developed in time. All schools are recommended to opt for *informatics tools*: high schools curricula focus more on understanding concepts, designing algorithms and resolution strategies; technical and trade schools are more addressed to executive objectives (e.g., *use*, *operate*, *compute*, *execute*).

5 Conclusions and Future Developments

In this work, we have collected, presented and compared a list of OR educational initiatives addressed to Grades 9–12 students. We have identified various types of initiatives: promotion activities, national and international projects, training courses for teachers, workshops for students and didactic units in classrooms. Overall, we managed to classify 23 different initiatives, most of which were international, while the others came from 9 different countries. If the reader is aware of other initiatives not included, we would be very glad to know them.

All the initiatives aim to increase students' interest and motivation towards mathematics and other scientific disciplines. Either these initiatives are entirely OR-based or anyway they can be related to OR. More than half of the initiatives are still active. Most of them are workshops for students or didactic units. We have noticed that many of the projects were aimed not only at students but also at teachers, arranging training courses or seminars in order to train them and reach a much higher number of students all over the different countries. A preliminary classification was made to distinguish the initiatives according to two macro-topics: modelling and algorithmics. We have been able to ascertain that almost all initiatives deal mainly with the former, or with both. The most common topics presented are linear and integer programming, graph theory and combinatorial optimization. Regarding teaching methods, most of the collected initiatives are based on constructionism: they usually implement active learning through cases and games inspired by real-world situations. Collaborative skills have been enhanced adopting cooperative learning and teamwork. Algorithms and resolution strategies have been developed in classrooms or also exploiting computer science laboratories. Usually, computations have been left to solvers or other software. A few textbooks have been published to guide teachers in implementing OR-based didactic units and activities. Feedback from both students and teachers were very positive. However, some teachers underlined that including OR in standard mathematics curricula could be difficult, because time is very scarce or sometimes teachers themselves may have never studied OR during their education.

Finally, we have examined some guidelines on mathematics education defined by international organizations (i.e., PISA, UNESCO and the European Union) and countries of Europe and worldwide. From the documentation consulted, it is clear that one of the main objectives in mathematics education is fostering students to understand how to apply mathematics in order to model, analyze and solve real-world problems. To reach this goal, problem-solving and modelling skills, as well as the ability to design and build algorithms, should be enhanced to play a central role throughout the entire schooling. All of these skills could be acquired by studying OR.

Making treasure of all the information collected and analyzed in this work, we are going to design and develop an OR-based didactic unit to be tested in some Italian secondary schools. The main goal is to propose an innovative way of integrating mathematics and computer science through OR. The project will be mainly based on active learning and constructionism, studying examples and cases closely connected with students' everyday life or with the Italian reality, balancing modelling and algorithmics.

Acknowledgements We thank the two anonymous referees for their precious advice that allowed us to better define and explain our work. We also thank Kenneth R. Chelst (Wayne State University), Evelyn Hardy (The OR Society), Evelyn Nahuelhaul (ISCI), Giovanni Righini (University of Milan), Socorro Rangel (Universidade Estadual Paulista) and Thad Wilhelm (Ernest W. Seaholm High School) for helping us retrieve useful information. Last but not least, we would like to thank also Daniela Ambrosino (University of Genova), Pietro Di Martino (University of Pisa) and Roberto Natalini (Italian National Research Council) for supporting us in this project, especially in the development of the surveys.

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