

# Deforestation rate in the long-run: the case of Brazil

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## Abstract

In this article we study the long-run average rate of forest conversion in Brazil. Deforestation results from the following trade-off: on the one hand, the uncertain value of benefits associated with forest conservation (biodiversity, carbon sequestration and other ecosystem services), on the other hand, the economic profits associated with land development (agriculture, ranching, etc.). We adopt the model by Bulte *et al.* (2002) as theoretical frame for studying land conversion and then derive, following Di Corato *et al.* (2013), the associated long-run average rate of forest conversion. We then identify the parameters to be used in our model. The object of our simulation is Brazil and 27 states. Our aim is to compute under several scenarios the time required to develop the remaining forested land in these states. We provide potential future scenarios, in terms of forest coverage, for the next 20, 100 and 200 years. Our results suggest that the uncertainty characterizing forest benefits plays a relevant role in deterring deforestation. We find that these benefits, if growing at a sufficiently high rate, may significantly slow down the conversion process. In contrast, a higher volatility accelerates the process of deforestation. We indicate the Brazilian states where forests are expected to be saturated earlier. In this respect, we find that forestland currently available may be expected to be fully converted within a 200-year horizon.

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# 1 Introduction

For many years, Brazil has been the single country with the highest clearing areas of tropical forest in the world (Börner and Wunder, 2008). The forces that drive the deforestation have been extensively studied and include cattle ranching, agriculture, poorly defined property rights, road reconstruction, population, rainfall and trade (Faria and Almeida, 2016; Andrade de Sá et al., 2013; Kirby *et al.*, 2006).

Among them, the dynamic agribusiness sector and international markets for timber and agricultural commodities, since the enactment of free trade agreements in the 1990s (Brandão *et al.*, 2006; Faria and Almeida, 2016), have led "an aggressive expansion of the agricultural frontier in the Amazon region" (Börner and Wunder, 2008, p. 197).

Conversion of land from forest to agriculture has two opposite effects: on the one hand, it may lead to irreversible reduction of the environmental services, such as biodiversity conservation, carbon sequestration<sup>1</sup>, watershed control and tourism benefits. On the other hand, conservation implies opportunity costs in terms of foregone profits from economic activities<sup>2</sup> (i.e., agriculture, commercial forestry, etc.). The relationship/struggle between these two effects triggers land-conversion in both the short and long run. Nevertheless "little is known about the future of environmental benefits of forest conservation" [...] and "about the future demand for the natural amenities" (Bulte *et al.*, 2002; p. 150). In this line, we study deforestation when forest conservation benefits are uncertain and we model it by using a geometrical Brownian motion. Several contributions are close to ours (Leroux *et al.*, 2009; Schatzki, 2003; Isik and Yang, 2004; Engel *et al.*, 2015) but we refer to two papers in particular. Our base model is Bulte *et al.* (2002) where the authors determine the socially optimal forest stock to be held in Costa Rica by trading off profit from agriculture and the value of environmental services/benefits attached to forest conservation. Their analysis highlights the value of the option to postpone the irreversible development of natural habitat under uncertainty about conservation benefits. We then use Di Corato *et al.* (2013) in which the authors study land conversion under competition on the market for agricultural products when voluntary and mandatory measures are combined by the Government to induce habitat conservation. They show that land conversion can be delayed by paying landholders for the provision of environmental services and by limiting the individual extent of developable land. However, it is found that the presence of ceilings on aggregate conversion may lead to runs which rapidly exhaust the targeted amount of land. They study the impact of uncertainty on the optimal conversion policy and discuss conversion dynamics under different policy scenarios on the basis of the relative long-run expected rate of deforestation. We use a procedure provided in Di Corato *et al.* (2013) for determining the long-run average rate of forest conversion in Brazil. Having identified this rate, we study the time needed for saturation of the available land in the 27 Brazilian states.

Summing up, the novelties of our paper with respect to Bulte *et al.* (2002) are: a long-run analysis and the study of a different country (Brazil) and its states. In doing this we used Instituto Brasileiro de Geografia e Estatística (IBGE)<sup>3</sup> and World Bank data and we computed the surface of available forestland in 2010 as the total minus the protected and the indigenous lands for each state in Brazil. As a second step we estimated the average demand function in Brazil following the estimate by Bulte *et al.* (2002) for Costa Rica. After that, for each state we calculated how many years are required in order to totally clear the available forestland and/or the percentage (for

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<sup>1</sup>Land-use change and agriculture account for approximately one third of global greenhouse gas emissions (see, among others, Smith *et al.*, 2007; FAO, 2011; Cacho *et al.*, 2014).

<sup>2</sup>Recent empirical studies have found evidence that opportunity costs of forested land vary widely over time and space (Lu and Liu, 2013; Wheeler *et al.*, 2013).

<sup>3</sup>Data are available at <http://www.ibge.gov.br/>.

each state) of available land not deforested after 20, 100 and 200 years. We therefore try to see whether deforestation is sustainable in the long-run. Finally we study the impact of uncertainty on the timing of deforestation. Our results show that the uncertainty is the main variable for deforestation and can accelerate its process. In addition, we show that a sufficiently robust growth of forest benefits can slow down the process. However, the impact of growth turns out to be less relevant if compared to the impact of volatility. It seems clear that the saturation process can be more or less slow depending on the size of the land already developed and its total size.

It has therefore been observed that some Brazilian states will be saturated earlier although, in general, it seems that deforestation is not inevitable in the short run (20 years) and it might partially be a problem within 100 years. Deforestation, if undertaken at the rates on the basis of our data, could become non-reversible for the majority of the states, after about 200 years.

In the first part of our paper (sections 2-4) we present our theoretical frame. In section 3 we determine the optimal conversion threshold while in section 4 we derive the associated long-run average rate of forest conversion. In the second part (sections 5-6) we present some descriptive statistics concerning 27 Brazilian states and discuss the parameter' values chosen in order to illustrate, under different scenarios, the potential changes in long-run average rate of forest conversion. In section 6 we calculate the saturation timing associated with each considered state. In the last section we comment our results.

## 2 The model

We adopt the model<sup>4</sup> by Bulte *et al.* (2002) examining land conversion decisions by a social planner trading off benefits<sup>5</sup> from Environmental Services (hereafter, ES) associated with forest conservation and social surplus from agricultural activities.<sup>6</sup> Consider a country where at each time period  $t \geq 0$  the total available land,  $L$ , is allocated between cultivated land  $A(t)$  and forestland  $F(t)$  as follows:

$$L = A(t) + F(t), \text{ with } A(0) = A_0 > 0 \quad (1)$$

where  $A_0$  denotes the cultivated land at the current time which for convenience we indicate by zero.

Let  $g(t)$  denote the annual flow of forest benefits provided by each hectare of forestland at time period  $t$ . Assume that

i) forest benefits are uncertain and evolves according to the following diffusion:

$$dg(t)/g(t) = \alpha dt + \sigma dZ(t), \text{ with } B(0) = B_0 \quad (2)$$

where  $\alpha$  and  $\sigma$  are known and certain drift and volatility parameters, respectively, and  $dz(t)$  is the increment of a standard Wiener process;<sup>7</sup>

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<sup>4</sup>See also Di Corato *et al.* 2013 for a model examining land conversion decisions in i) a centralized economy populated by a multitude of homogenous landholders, in the presence of ii) a Payments for Environmental Services (hereafter, PES) scheme compensating landholders for conservation and iii) limits set by the Government to the development of land.

<sup>5</sup>These may include biodiversity conservation, carbon sequestration, watershed control, provision of scenic beauty for recreational activities and ecotourism, timber and non-timber forest products. See e.g. Conrad (1997) and Reed (1993).

<sup>6</sup>Note that for the sake of simplicity we assume that the only use, once forestland has been converted, is agriculture. This may, however, be easily adapted to allow for other uses such as ranching, commercial forestry, etc..

<sup>7</sup>The assumed Brownian motion for the evolution over time of the value associated with forest benefits is quite standard in the literature, see for instance Conrad (1997), Reed (1993), Bulte *et al.* (2002), Leroux *et al.* (2009), and Di Corato *et al.* (2013).

- ii) total benefits associated with forest conservation,  $M(t)$ , are linearly related to the forest surface, i.e.:

$$M(g(t), F(t)) = g(t)F(t) = g(t)(L - A(t)); \quad (3)$$

- iii) at each  $t$ , forestland may be irreversibly cleared and used as an input for agriculture. Forest conversion entails a sunk cost,  $c$ , per hectare which includes the cost for clearing and settling land for agriculture;<sup>8</sup>

- iv) returns from agriculture are illustrated by the following constant elasticity demand function:

$$P(A(t)) = \delta A(t)^{-\gamma} \quad (4)$$

where the parameter  $\delta > 0$  illustrates different states of the demand and  $1/\gamma > 0$  is the demand elasticity.

Hence, at a generic time period  $t$  given a generic land allocation  $(A(t), F(t))$ , the periodical flow of social benefits accruing from agriculture and forest conservation is:

$$W(A(t), g(t)) = N(A(t)) + (\bar{L} - A(t))g(t) \quad (5)$$

where

$$N(A(t)) = \int_0^{A(t)} P(a) da = \delta \frac{A(t)^{1-\gamma}}{1-\gamma} \quad (6)$$

is the total surplus associated with agriculture.

## 2.1 Forest stock and timing of land development

The social planner sets the optimal conversion policy by maximizing the expected present value of social benefits associated with agriculture and forest conservation. Then, at time zero, the problem to be solved is the following:<sup>9</sup>

$$V(A_0, g_0) = \max_{A(t)} E_0 \left[ \int_0^\infty e^{-rt} (W(A(t), g(t)) - c dA(t)) dt \right] \quad (7)$$

where  $r$  is the constant discount rate.<sup>10</sup> The increase in cultivated land ( $dA(t)$ ) will in turn imply a drop in revenues from agriculture along the demand function  $P(A(t))$  which will restore the conditions for conserving land. The new cultivated land surface,  $A(t) + dA(t)$ , will then remain stable until the value of  $g(t)$ , reaches a level low enough to trigger further land development. In particular, solving the problem in (7), we can show that<sup>11</sup>:

**Proposition 1** *New forestland is converted every time current forest benefits reach the critical threshold:*

$$g^*(A_0) = [\beta/(\beta - 1)](r - \alpha)[(\hat{A}/A_0)^\gamma - 1]c \quad (8)$$

where  $\beta < 0$  is the negative root of the characteristic equations  $\Gamma(\beta) = (1/2)\sigma^2\beta(\beta - 1) + \alpha\beta - r = 0$ ,  $\hat{A} = (\delta/rc)^{1/\gamma}$  is the maximum extent for which conversion makes economic sense.

<sup>8</sup>We assume, without loss of generality, that the conversion cost is linear in the cleared surface. Note that  $c$  may also be negative when, for instance, benefits from logging are higher than the conversion cost.

<sup>9</sup>The expectation in Problem (7) is taken with respect to the joint distribution of  $A$  and  $g$  and it is conditional on the information available at time zero.

<sup>10</sup>Introducing risk aversion would not impact on the quality of our results. In order to allow for it, it would suffice to develop the analysis under a risk-neutral probability measure for  $g(t)$  (see e.g. Cox and Ross, 1976).

<sup>11</sup>For the derivation of this result, see the Appendix.



Equation (8) provides a standard result in the real option literature. The so-called option multiple,  $[\beta/\beta - 1] < 1$ , adjusts the standard Net Present Value rule, i.e.  $g^{NPV}(A_0) = (r - \alpha)[(\hat{A}/A_0)^\gamma - 1]c$ , in order to account for the presence of uncertainty and irreversibility (Dixit and Pindyck, 1994). The effect of trend,  $\alpha$ , and volatility,  $\sigma$ , on the threshold  $g^*(A_0)$  is negative or null (see Table 1). If benefits from forest conservation are characterized by a higher growth rate and/or volatility, the threshold value for land conversion decreases. This in turn implies, in expected terms, a delayed land conversion. The result is standard in the literature and it is explained by the presence of option value associated with the decision to be taken. An increase in the interest rate,  $r$ , should induce an earlier exercise of the option to convert land. This effect is however more than balanced by the impact that a higher  $r$  has, via  $\hat{A}$ , on the opportunity (marginal) cost of conversion. Note in fact that  $c - \delta A_0^{-\gamma}/r$  is increasing in  $r$ . Summing up, as  $g^*(A_0)$  does not increase in  $r$ , a delayed land conversion is associated with a higher discount rate.

	$\delta$	$c$	$r$	$\gamma$	$\alpha$	$\sigma^2$
$\hat{A}$	$> 0$	$< 0$	$< 0$	$< 0$	$-$	$-$
$g^*(A_0)$	$\geq 0$	$\leq 0$	$\leq 0$	$\leq 0$	$\leq 0$	$\leq 0$

**Table 1:** Derivatives of  $\hat{A}$  and  $g^*(A_0)$  with respect to the relevant parameters

Let's now comment on the effect on the threshold passing through the term  $\hat{A}$ .  $\hat{A}$  represents the last parcel for which conversion makes economic sense and results from the comparison between marginal agricultural benefits (driven by the demand function (4)) and marginal cost of land conversion. Note in fact that it solves the equation  $\delta \hat{A}^{-\gamma}/r = c$ . In Table 1, we present some comparative statics illustrating the effect that changes in  $\delta$ ,  $\gamma$ ,  $r$  and  $c$  have on  $\hat{A}$ . Concerning the effect of demand parameters, we notice that  $\hat{A}$  is increasing in the demand for agricultural goods, i.e. higher  $\delta$ , and/or in the demand rigidity, i.e. lower  $\gamma$ . This makes sense considering that as higher profits are associated with agriculture, it is profitable to convert a larger land surface. Similarly, as converting land becomes cheaper, i.e. lower  $c$ , a larger land surface is allocated to agricultural activities. Lastly, as the discount rate  $r$  decreases, the higher, ceteris paribus, the marginal benefit associated with the conversion of land, thus, again, the higher the surface to be converted to agriculture. Finally, commenting again on  $g^*(A_0)$ , we notice that whenever in response to changes in  $\delta$ ,  $\gamma$ ,  $r$  and  $c$  more profitable conditions are associated with land conversion, the threshold is higher and as a consequence land conversion is, in expected terms, anticipated.

## 2.2 The long-run average rate of forest conversion

Starting from the short-run optimal conversion policy described by Eq. (8), we are able to derive the optimal land conversion dynamics in the long-run. This is done by determining the expected long-run growth rate of forest conversion associated with  $g^*(A_0)$ . In this respect, we follow the procedure proposed by Di Corato *et al.* (2013).<sup>12</sup> Specifically, using Eq. (8), let's define the regulated process:

$$\omega(t) = g(t)/[(\hat{A}/A_0)^\gamma - 1], \text{ for } \omega(t) > \bar{\omega} = [\beta/(\beta - 1)](r - \alpha)c \quad (9)$$

where  $\bar{\omega}$  is a lower reflecting barrier (see Harrison 1985, Chapter 2). The process (9) illustrates the long-run land conversion in response to fluctuations in the value of benefits from forest conservation

<sup>12</sup>For the derivation of this result, see our Appendix or Di Corato *et al.* (2013).

$g(t)$ . As  $\omega(t)$  moves, driven by a reduction in  $g(t)$ , downward toward  $\bar{\omega}$ , the profitability of land conversion increases. Then, in technical parlance, in order to prevent  $\omega(t)$  from crossing  $\bar{\omega}$ , a reflection,  $dA(t) > 0$ , occurs, i.e. additional land is converted to agricultural activities. Newly converted land, by determining a drop along the demand for agricultural commodities,  $P(A(t))$ , drives  $\omega(t)$  away from the barrier  $\bar{\omega}$  restoring conditions for keeping the new land allocation just reached  $(A(t) + dA(t), L - dA(t))$ . The process will stop only when the amount of land developed reaches the amount  $\hat{A}$  where, as explained above, further land conversion is not profitable. In the Appendix we show that

**Proposition 2** *For any initial land allocation  $A_0 \leq \hat{A}$  the expected long-run growth rate of forest conversion is given by*

$$\frac{1}{dt}E[d\ln A] \simeq \begin{cases} [(1/2)\sigma^2 - \alpha] \frac{1-(A_0/\hat{A})^\gamma}{\gamma} & \text{for } (1/2)\sigma^2 > \alpha \\ 0 & \text{for } (1/2)\sigma^2 \leq \alpha \end{cases} \quad (10)$$

**Proof.** See Appendix. ■

Commenting on Eq. (10) it is worth highlighting that in order to have a positive long-run growth rate of forest conversion, the trend in the change over time of the value associated with forest benefits must be sufficiently low, i.e.  $\alpha < (1/2)\sigma^2$ . Otherwise, i.e. if  $\alpha \geq (1/2)\sigma^2$ , the rate is null since the trend is strong enough to keep  $\omega$  away from the barrier  $\bar{\omega}$  or, in other words, forest conservation is expected to pay better than agriculture. Note that the condition  $(1/2)\sigma^2 > \alpha$  is always met for  $\sigma > 0$  and  $\alpha \leq 0$ . Studying the impact of each parameter we notice that the rate of forest conversion is decreasing in  $\alpha$  and increasing in the volatility<sup>13</sup> associated with forest benefits,  $\sigma$ . Furthermore, the rate is, not surprisingly, increasing in the demand elasticity,  $1/\gamma$ . Lastly, the rate of land conversion responds negatively to changes in the term  $(A_0/\hat{A})^\gamma$ . As  $\hat{A}$  is the maximum extent for which conversion is profitable, the ratio  $A_0/\hat{A} \leq 1$  is a measure of the profitability associated with additional land conversion when the converted surface is equal to  $A_0$ . Note that, consistently, the higher  $A_0/\hat{A}$ , the lower the rate of forest conversion. This result is easily explained by noting that, as land is converted, the levels of  $g$  needed in order to trigger land conversion become gradually lower (see Eq. 8). Hence, as the probability of hitting the threshold  $g^*(A_0)$  decreases, the rate of forest conversion converges to zero.

### 3 The Brazilian Case

In this section we first present some figures relative to the destination of land in Brazil in 2010. We then briefly discuss the choices made for i) the characterization of the scenarios to be studied in Section 4 and ii) the calibration of our numerical analyses. We then apply the model described in the previous section in order to calculate the long-run average deforestation rate in Brazil. To do this, first of all, we calculate and define the parameters of equation (10).

#### 3.1 Forestland in Brazil

We provide figures relative to land use in Brazil distinguishing among its 27 states. In figure 1, we provide for each state the available land surface in 2010 as a percentage of the total land of each state. The available land is defined as the difference between the total land surface minus the sum of protected and indigenous lands and land previously converted (see figures 2 and 3,

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<sup>13</sup>This makes sense considering that as volatility increases, due to the increased positive skewness of the distribution of  $\omega$ , the probability of reaching the barrier  $\bar{\omega}$  is higher. See Di Corato et al. (2013) for further details.

respectively). The total Brazilian land previously converted ( $A_0$ ) is 273 421 000 hectares in 2010, while the total available land,  $L$ , is 604 618 605 hectares and the data are taken from the World Bank. Figure 4 provides in percentage the land with permanent and temporary crops in the same year and uses the IBGE Brazilian database. Regarding the definition of protected areas, we used three references. The first one is the Isa and Imazon report by Verissimo *et al.*, (2011) in which the authors define the protected and indigenous land in 2010 of the following states: Acre, Amapá, Amazonas, Maranhão, Mato Grosso, Pará, Rhodonea, Roraima and Tocantis. They use data from the official IBGE Brazilian database in 2010.<sup>14</sup> The second source is Börner *et al.* (2010). They show the percentage of the "indigenous lands" (about 22% of the total) and the "strictly protected areas" (about 7%). The total is 29% and refers to the states of Acre, Amapá, Amazonas, Pará, Rhodonea, Roraima and only partially the states of Maranhão, Mato Grosso and Tocantis. The last source is the World Bank database<sup>15</sup> that provides a percentage of 26.28% of protected areas in Brazil. Since it is roughly in line with the World Bank database, we keep the percentage of 29% as a reference for our calibration. This figure is also used in order to calculate the residual protected areas within the states for which Verissimo *et al.* (2011) do not provide any data.

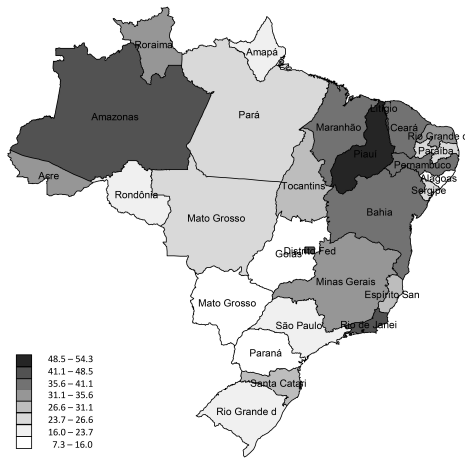


Figure 1: Available land in 2010, percentage

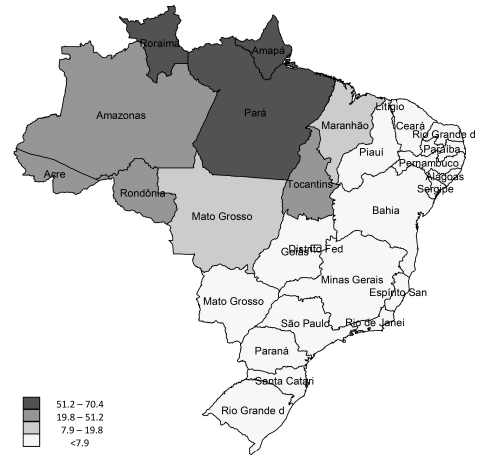


Figure 2: Protected areas in 2010, percentage

<sup>14</sup>See <http://www.ibge.gov.br/> for further details.

<sup>15</sup><http://data.worldbank.org/country/brazil>.

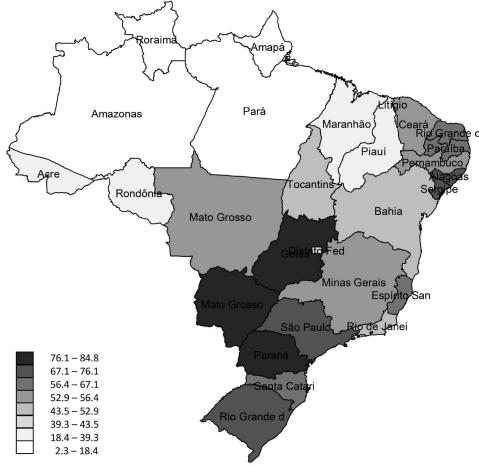


Figure 3: Used land in 2010, percentage

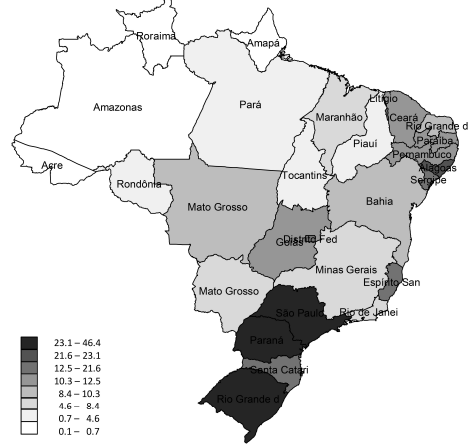


Figure 4: Land for crops in 2010, percentage

By looking at Figures [1-4], it is not surprising that only a limited amount of land is still forested in the South of Brazil, while most of the protected areas are located in the Northwest of Brazil and focus on Para, Roraima, Amazonas Acre, Tocantis and Rondonia. The low use of the land in this area means that large areas are available, but constrained by the protected and indigenous land. Our expectation is that as some areas saturate the available land, the deforestation rates of the other states still covered by forests will increase.

### 3.2 Agricultural commodities: demand parameters

In line with Börner *et al.*, (2010), we assume that agricultural expansion mirrors forest loss. In order to estimate the parameters  $\delta$  and  $\gamma$  in Eq. (3) we use IBGE data for permanent and temporary crops cultivated in Brazil. We use a 1994-2000 panel for 60 different crops and for each year regress their prices with respect to the agricultural land allocated to each specific crop for a set of  $i$  types of crops where  $i = 1...60$ . We find that:

$$\ln(P_{it}) = 12.16 - 0.727 \ln(A_{it}) \quad (11)$$

(0.44)      (0.057)

where the subscripts  $i$  and  $t$  stand for crop and year considered. Standard errors are provided in parentheses while the adjusted  $R^2$  is equal to 0.33. Using the estimated figures in Eq. (11) yields  $\gamma \simeq 0.727$  and  $\delta = \exp(12.16) = 190786$ .

### 3.3 Conversion costs

We set the forest conversion cost  $c$  equal to 0. By doing this we are implicitly assuming that actual conversion costs are covered by benefits from logging or that conversion costs are not significant. This can be quite realistic in the context of "slash and burn" agriculture (Leroux et al. 2009).<sup>16</sup> Further, setting  $c = 0$  the limit amount of land for which conversion is profitable, i.e.  $\hat{A}$ , tends to infinity. This in turn implies that the expected long-run growth rate of forest conversion is equal to

$$\frac{1}{dt} E[d \ln A] \simeq \begin{cases} [(1/2)\sigma^2 - \alpha]/\gamma & \text{for } (1/2)\sigma^2 > \alpha \\ 0 & \text{for } (1/2)\sigma^2 \leq \alpha \end{cases} \quad (10.1)$$

<sup>16</sup>Note that also Bulte et al. (2002) set the conversion cost equal to 0 in their numerical analysis.

Hence, as  $(A_0/\hat{A})^\gamma < 1$ , by our assumption we are potentially providing an overestimation of this rate. In our analysis this would imply a faster and more aggressive conversion of the forest stock considered.

### 3.4 Trend and volatility of forest benefits

As is evident from equation (10.1), the trend ( $\alpha$ ) and volatility ( $\sigma$ ) in the change over time of the forest benefits, are relevant parameters for calculation of the long-run average rate of forest conversion. Nevertheless, in economic literature there is no consensus on these values, especially for the volatility. Then, to encompass most of the values suggested by other authors, we choose  $\alpha$  within the range  $\alpha = \{0.00, 0.025, 0.05\}$  and  $\sigma = \{0.15, 0.175, 0.2, 0.225, 0.25\}$ .<sup>17</sup>

Using these values, Table 2 below reports the combinations of parameters that satisfy the constraint  $\frac{1}{2}\sigma^2 - \alpha > 0$  which guarantees a positive rate of deforestation.

	$\alpha = 0.000$	$\alpha = 0.025$	$\alpha = 0.05$
$\sigma = 0.150$	0.011		
$\sigma = 0.175$	<b>0.015</b>		
$\sigma = 0.200$	0.020		
$\sigma = 0.225$	0.025	<b>0.0003</b>	
$\sigma = 0.250$	<b>0.031</b>	<b>0.006</b>	

**Table 2:** Trend strength and volatility

Note that irrespective of the volatility level the rate of conversion is null for  $\alpha = 0.05$ . In contrast, when  $\alpha = 0.025$ , the rate is positive only for  $\sigma = \{0.225, 0.25\}$  and null otherwise. Finally, the rate is potentially higher when considering the combinations where  $\alpha = 0$ . Hence, among all these combinations, for the sake of realism, we simulate our model considering only the cases in bold. In our opinion these figures are sufficiently general to include both cases with low trends and low volatility. Bearing this in mind, Table 3 summarizes the parameters' values that will be used in our numerical analyses.

Parameter	Value
$(1/2)\sigma^2 - \alpha$	0.0003; 0.006; 0.015; 0.03
$\delta$	190786
$\gamma$	0.727
$c$	$\simeq 0$
$A_0$	273 421 000 ha
$L$	604 618 605 ha

**Table 3:** Parameter values

<sup>17</sup>Bulte *et al.* (2002) use  $\alpha = \{0.00, 0.025, 0.05\}$  and  $\sigma = \{0.00, 0.125\}$ . Engel *et al.* (2015) use a time series indexed to the returns of transferable permits in the European market with  $\alpha = 0.00$  and  $\sigma = \{0.01, 0.025\}$ ; Brauneis *et al.* (2012) use a carbon price standard deviation of  $\sigma = 27\%$  and a price process of CO<sub>2</sub> emission allowances with an expected growth rate of 6.99% taken from different databases and test the sensitivity of their model by letting  $\alpha$  vary in a range similar to Bulte *et al.* (2002), i.e.,  $\alpha \in [0; 0.14]$ , but with higher volatility, i.e.,  $\sigma \in [0.15; 0.45]$ .

## 4 Deforestation rate and saturation timing

In this part in table 4 we calculate the saturation timing in the long run, that is the number of years in order to totally clear the available land. This was done taking into account the equation (10.1) according to the parameters of table 2 and in particular according to the following expected growth rate of  $g(t) : \alpha - \frac{1}{2}\sigma^2 = 0.0003; 0.006; 0.015; 0.03$ . In tables 5-7 we show the percentage of land still available after 20, 100 and 200 years, respectively. The results for Brazil are shown in the first line of tables 4-7, while the other lines show the results for the 27 Brazilian states. For each state we started with their  $A_{0j}$ , where 0 is the year 2010, while  $j$  is name of the state.

For conservation, not surprisingly,  $\alpha - \frac{1}{2}\sigma^2 = 0.0003$  is the best case while higher  $\alpha - \frac{1}{2}\sigma^2$  imply an increase in deforestation rates and a reduction in the number of years to totally clear the available land. We remember that 0.0003 is the result of a low uncertainty (0.025) and low trend (0.000) or a higher level of variance (0.225) and drift (0.025). Therefore if the future scenario is characterized by these two pairs of values, deforestation might not be a serious problem. As said above, if we compare column three and five of table 3 we study the effect of the trend of the benefit process, while the comparison of column four and five shows the uncertainty effect.

The States that maintain forests longest are the following; Amazonas, Parà, Bahia, Minas Gerais and Mato Grosso. Not all the states are in the North of Brazil. The worst are Paraíba, Rio Grande do Norte, Alagoas, Sergipe, Espirito Santo and Rio de Janeiro. Almost all are in the Eastern part of Brazil, where the size of each state is smaller than the other states and where the percentage of available land is low.

	$\alpha - \frac{1}{2}\sigma^2$			
	<b>0.0003</b>	<b>0.006</b>	<b>0.015</b>	<b>0.03</b>
<b>Brazil</b>	45672	942	466	2293
<b>Rondônia</b>	23625	487	241	1186
<b>Acre</b>	885	492	244	1199
<b>Amazonas</b>	12199	6791	3363	16536
<b>Roraima</b>	1290	718	356	1749
<b>Pará</b>	5602	3118	1544	7593
<b>Amapá</b>	566	315	156	767
<b>Tocantis</b>	1253	697	345	1698
<b>Maranhão</b>	2299	1280	634	3116
<b>Piauí</b>	2302	1281	635	3120
<b>Ceará</b>	971	541	268	1316
<b>Rio Grande do Norte</b>	282	157	78	382
<b>Paraíba</b>	238	132	66	322
<b>Pernambuco</b>	607	338	167	823
<b>Alagoas</b>	75	42	21	102
<b>Sergipe</b>	90	50	25	122
<b>Bahia</b>	3777	2103	1041	5120
<b>Minas Gerais</b>	3525	1962	972	4778
<b>Espírito Santo</b>	237	132	65	321
<b>Rio de Janeiro</b>	332	185	92	450
<b>São Paulo</b>	994	553	274	1347
<b>Paraná</b>	498	277	137	676
<b>Santa Catarina</b>	464	258	128	629
<b>Rio Grande do Sul</b>	946	526	261	1282
<b>Mato Grosso do Sul</b>	439	244	121	595
<b>Mato Grosso</b>	4008	2231	1105	5433
<b>Goiás</b>	872	486	240	1182
<b>Distrito Federal</b>	47	26	13	64

Table 4. Deforestation timing in Brazil and its states.

Another way of looking at the same issue is by determining how much land, with respect to the total available land will not be cleared after a certain period. This should illustrate how inevitable the problem is and in which states it is more important. We show our results for the 4 net benefits and for the following periods: 20 years (in table 5), 100 years (table 6) and 200 years (table 7 and figures 5-8). Let us start from the 4 cases after 200 years in figures [5-8].

First of all, let us start with figure 5. In all the cases, after 200 years more than 84% of the available land remains in all the states. This means that deforestation is not inevitable in the medium-run and also that the effect of uncertainty and the trend of the natural amenities is too weak. Figure 8, which intuitively should have the strongest deforestation effect, is characterized by a reduction in available land but the majority still have more than 67% of available land. Figure 6

is more interesting with some states, especially on the East coast, totally exhausted after 200 years. This is the case in the following states: Rio Grande do Norte, Paraíba, Alagoas, Sergipe, Espírito Santo, Rio de Janeiro and Distrito Federal. Also Mato Grosso will have a critical situation with 4% of available land. Other states will show a strong reduction in forestland. They are Amapá, Paraná, Pernambuco, Santa Catarina, with a percentage of available land lower than 50% or around 50% for the states of Acre, Rondonia, Rio Grande do Sul and Goiás. Figure 7 with a expected growth rate of  $g(t)$  equal to 0.015 is the worst case because it is characterized by many states with no available land in the long run. These are the following: Rio Grande do Norte, Paraíba, Pernambuco, Alagoas, Sergipe, Espírito Santo, Rio de Janeiro, São Paulo, Acre, Rondônia, Amapá, Ceará, Paraná, Rio Grande do Sul, Mato Grosso, Goiás, Distrito Federal and Santa Catarina, that is 18 states out of 27, amounting to 67%. Roraima and Tocantins will have around 10% of available land and for Pará and Amazonas there will be a percentage over 80%.

#### Percentage of available lands after 200 years

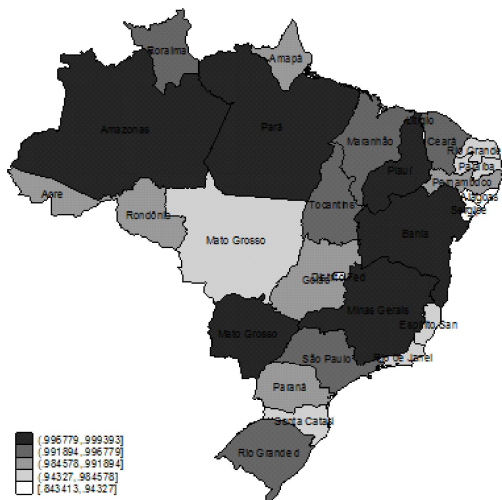


Figure 5:  $\alpha - \frac{1}{2}\sigma^2 = 0.0003$

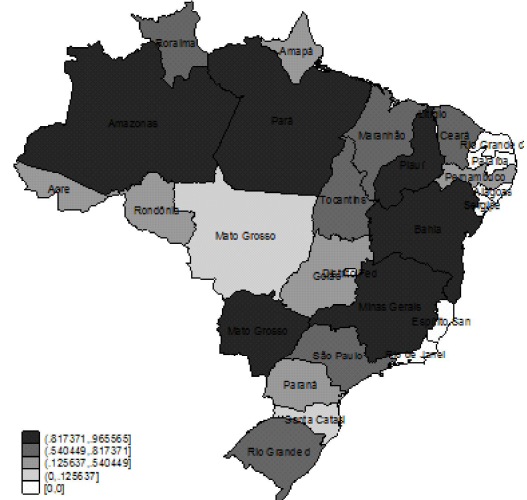


Figure 6:  $\alpha - \frac{1}{2}\sigma^2 = 0.007$

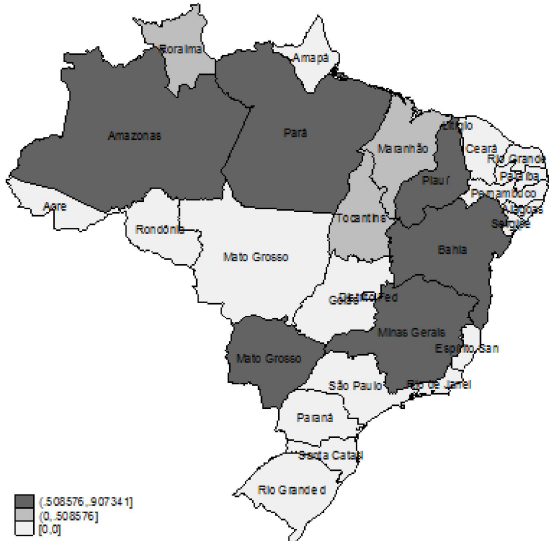


Figure 7:  $\alpha - \frac{1}{2}\sigma^2 = 0.015$

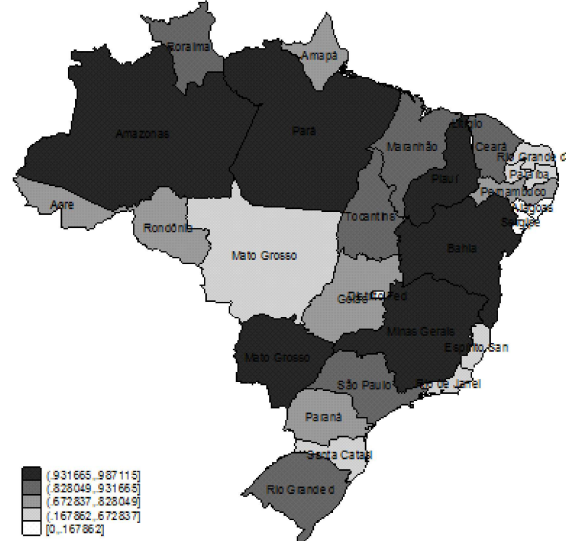


Figure 8:  $\alpha - \frac{1}{2}\sigma^2 = 0.03$



In the following tables 5-7 we show the percentage of available land after 20, 100 and 200 years respectively. After only 20 years, only three states (Alagoas, Sergipe and Distrito Federal) will have a percentage of available land lower than 69%. The majority of the other states will have percentages over than 90% due to a non-inevitable problem in the short run, regardless volatility and drift.

The problem will arise after roughly 200 years with a high level of volatility or a moderate level of uncertainty and benefits drift. As can be observed in table 7, in column "0.015", the majority of the states will have exhausted their land. A high percentage will remain in Amazonas, Pará, Bahia, Mato Grosso do Sul and Minas Gerais.

	$\alpha - \frac{1}{2}\sigma^2$			
20 years	<b>0.0003</b>	<b>0.006</b>	<b>0.015</b>	<b>0.03</b>
<b>Brazil</b>	99.96	97.88	95.71	99.13
<b>Rio Grande do Norte</b>	99.74	87.27	73.95	94.77
<b>Paraíba</b>	99.69	84.90	69.10	93.80
<b>Pernambuco</b>	99.88	94.08	87.89	97.57
<b>Alagoas</b>	99.01	52.07	1.93	80.32
<b>Sergipe</b>	99.18	60.17	18.50	83.64
<b>Minas Gerais</b>	99.98	98.98	97.91	99.58
<b>Espírito Santo</b>	99.69	84.82	68.93	93.76
<b>Rio de Janeiro</b>	99.78	89.19	77.87	95.56
<b>São Paulo</b>	99.93	96.39	92.60	98.52
<b>Roraima</b>	99.94	97.22	94.30	98.86
<b>Amazonas</b>	99.99	99.71	99.40	99.88
<b>Pará</b>	99.99	99.36	98.69	99.74
<b>Acre</b>	99.92	95.94	91.69	98.33
<b>Rondônia</b>	99.92	95.89	91.60	98.31
<b>Amapá</b>	99.87	93.65	87.02	97.39
<b>Tocantins</b>	99.94	97.13	94.13	98.82
<b>Maranhão</b>	99.97	98.44	96.80	99.36
<b>Piauí</b>	99.97	98.44	96.81	99.36
<b>Ceará</b>	99.92	96.30	92.43	98.48
<b>Paraná</b>	99.85	92.79	85.25	97.04
<b>Rio Grande do Sul</b>	99.92	96.20	92.23	98.44
<b>Mato Grosso</b>	99.98	99.10	98.17	99.63
<b>Mato Grosso do Sul</b>	99.83	91.81	83.25	96.64
<b>Goiás</b>	99.92	95.88	91.57	98.31
<b>Distrito Federal</b>	98.43	24.07	0	68.82
<b>Bahia</b>	99.98	99.05	98.05	99.61
<b>Santa Catarina</b>	99.84	92.25	84.15	96.82

Table 5. Available lands in percentage after 20 years

	$\alpha - \frac{1}{2}\sigma^2$			
100 years	<b>0.0003</b>	<b>0.006</b>	<b>0.015</b>	<b>0.03</b>
<b>Brazil</b>	99.78	89.38	78.56	95.64
<b>Rio Grande do Norte</b>	98.69	31.52	0	73.49
<b>Paraíba</b>	98.44	18.77	0	68.56
<b>Pernambuco</b>	99.39	68.17	30.06	87.68
<b>Alagoas</b>	95.06	0	0	0.22
<b>Sergipe</b>	95.89	0	0	17.08
<b>Minas Gerais</b>	99.89	94.52	87.96	97.88
<b>Espírito Santo</b>	98.43	18.33	0	68.39
<b>Rio de Janeiro</b>	98.89	41.84	0	77.49
<b>São Paulo</b>	99.63	80.56	57.28	92.48
<b>Roraima</b>	99.71	85.02	67.09	94.20
<b>Amazonas</b>	99.97	98.42	96.52	99.39
<b>Pará</b>	99.93	96.55	92.42	98.66
<b>Acre</b>	99.58	78.16	52.00	91.55
<b>Rondônia</b>	99.58	77.92	51.47	91.45
<b>Amapá</b>	99.35	65.87	25.01	86.79
<b>Tocantis</b>	99.70	84.57	66.10	94.03
<b>Maranhão</b>	99.84	91.59	81.53	96.75
<b>Piauí</b>	99.84	91.60	81.55	96.75
<b>Ceará</b>	99.62	80.10	56.28	92.30
<b>Paraná</b>	99.26	61.23	14.80	84.99
<b>Rio Grande do Sul</b>	99.61	79.56	55.10	92.09
<b>Mato Grosso</b>	99.91	95.18	89.41	98.13
<b>Mato Grosso do Sul</b>	99.16	55.98	3.27	82.96
<b>Goiás</b>	99.58	77.85	51.32	91.43
<b>Distrito Federal</b>	92.17	0	0	0
<b>Bahia</b>	99.90	94.88	88.76	98.02
<b>Santa Catarina</b>	99.20	58.33	8.44	83.87

Table 6. Available lands in percentage after 100 years

	$\alpha - \frac{1}{2}\sigma^2$			
200 years	<b>0.0003</b>	<b>0.006</b>	<b>0.015</b>	<b>0.03</b>
<b>Brazil</b>	99.56	78.76	57.12	91.28
<b>Rio Grande do Norte</b>	97.37	0	0	44.31
<b>Paraíba</b>	96.89	0	0	33.94
<b>Pernambuco</b>	98.78	30.83	0	74.12
<b>Alagoas</b>	90.12	0	0	0
<b>Sergipe</b>	91.79	0	0	0
<b>Minas Gerais</b>	99.79	88.09	67.95	95.54
<b>Espírito Santo</b>	96.87	0	0	33.59
<b>Rio de Janeiro</b>	97.77	0	0	52.70
<b>São Paulo</b>	99.25	57.75	0	84.19
<b>Roraima</b>	99.43	67.45	12.43	87.82
<b>Amazonas</b>	99.94	96.56	90.74	98.71
<b>Pará</b>	99.87	92.50	79.83	97.19
<b>Acre</b>	99.16	52.53	0	82.24
<b>Rondônia</b>	99.15	52.01	0	82.04
<b>Amapá</b>	98.69	25.83	0	72.25
<b>Tocantins</b>	99.41	66.47	9.80	87.46
<b>Maranhão</b>	99.68	81.73	50.85	93.16
<b>Piauí</b>	99.68	81.75	50.91	93.17
<b>Ceará</b>	99.24	56.76	0	83.82
<b>Paraná</b>	98.51	15.74	0	68.47
<b>Rio Grande do Sul</b>	99.22	55.59	0	83.38
<b>Mato Grosso</b>	99.82	89.52	71.81	96.08
<b>Mato Grosso do Sul</b>	98.31	4.32	0	64.20
<b>Goiás</b>	99.15	51.86	0	81.99
<b>Distrito Federal</b>	84.35	0	0	0
<b>Bahia</b>	99.80	88.88	70.09	95.84
<b>Santa Catarina</b>	98.40	9.44	0	66.11

Table 7. Available lands in percentage after 200 years

## 5 Conclusions

In this article we studied the long-run average rate of forest conversion in Brazil, trying to understand i) how the process can evolve and ii) what the main variables are accelerating the process. We studied the rate of deforestation in a context of uncertainty, identifying drivers of deforestation as the demand and supply of agricultural products. These drivers that push deforestation move antithetical to the forces that lead to conservation, such as the value of benefits related to biodiversity, tourism, carbon sequestration and watershed control. On the one hand, deforestation implies a reduction of environmental services, on the other hand it implies an increase in agricultural profits. It is the struggle of these two opposite values that finally drives the net effect of deforestation. In our theoretical frame which builds on Bulte *et al.* (2002) and Di Corato *et al.* (2013), we determine the long-term deforestation rate. Secondly we calculate the parameters for Brazil and its 27 states, and use these parameters to define the time required to clear all the available land in the Brazilian states. Moreover we study the situation of the available land after 20, 100 and 200 years. The

results demonstrate that uncertainty appears to be a crucial variable for deforestation and accelerates its process. In addition to this, the biodiversity benefits trend can slow down the process but turns out to be less effective compared to the volatility. It seems clear that the saturation process can be more or less slow depending on the size of the land already developed and its total size. It is therefore observed that some Brazilian states are saturated earlier than others although, in general, it seems that the total exhaustion of the forest stock cannot occur in the short run (20 years), and could start representing an issue, but only partially, within a 100-year horizon. Finally, deforestation, if it continued at the rates calculated with the data in our possession, could become a problem for the majority of the states after about 200 years.

## A Appendix

### A.1 Proof of Proposition 1

In this section, we study the optimal conversion policy. The value associated with the current land allocation,  $(A(t), F(t))$ , is given by:

$$V(A(t), g(t)) = \max_{A(s)} E_t \left[ \int_t^\infty e^{-rs} (W(A(s), g(s)) - cdA) ds \right] \quad (\text{A.1.1})$$

s.t.  $dA(s) \geq 0$  with  $A(s) \leq \hat{A} \leq L$ , and (2) for all  $s$ ,

where  $r$  is the constant risk-free interest rate.

Dropping the time index for notational convenience and using standard arguments, we can express Eq. (A.1.1) as follows<sup>18</sup>

$$V(A, g) = \{W(A, g)dt + \frac{E_0[V(A, g + dg)]}{1 + rdt}\} \quad (\text{A.1.2})$$

By applying Ito's Lemma to expand  $dV(A, g)$ , we obtain:

$$\Gamma V(A, g) = -W(A, g) \quad (\text{A.1.3})$$

where  $\Gamma$  is the differential operator:  $\Gamma = -r + \alpha p \frac{\partial}{\partial g} + \frac{1}{2} \sigma^2 g^2 \frac{\partial^2}{\partial g^2}$ .

Differentiating (A.1.3) with respect to  $A$ , we have:

$$\Gamma v(A, g) = -w(A, g) \quad (\text{A.1.4})$$

where  $v(A, g) = \partial V(A, g) / \partial A$  and  $w(A, g) = \partial W(A, g) / \partial A$ .

The solution of Eq. (A.1.4) takes the following functional form:

$$v(A, g) = m(A, g) + K_1(A)g^{\beta_1} + K_2(A)g^{\beta_2} \quad (\text{A.1.5})$$

where  $\beta_1 > 1$ ,  $\beta_2 < 0$  are the roots of the characteristic equation  $\Gamma(\beta) = \frac{1}{2} \sigma^2 \beta(\beta - 1) + \alpha \beta - r = 0$ ,  $K_1(A)$ ,  $K_2(A)$  are two constants to be determined and  $m(A, g)$  is the particular solution of the nonhomogeneous equation. Note that given the current surface,  $A$ , allocated to agriculture,  $m(A, g)$  represents the expected net present value from converting an additional unit of land. That is:<sup>19</sup>

$$m(A, g) = E_0 \left[ \int_0^\infty e^{-rt} w(A, g) dt \right] = E_0 \left[ \int_0^\infty e^{-rt} (\delta A^{-\gamma} - g) dt \right] = \delta \frac{A^{-\gamma}}{r} - \frac{g}{r - \alpha}$$

The boundary conditions for (A.1.5) are

$$v(A, g^*(A)) = c, \quad v_g(A, g^*(A)) = 0 \quad (\text{A.1.5a-A.1.5b})$$

$$K_1(A) = 0, \quad K_2(\hat{A}) = 0 \quad (\text{A.1.5c-A.1.5d})$$

Substituting (A.1.5) into the system [A.1.5a-A.1.5b] yields

$$\begin{aligned} K_2(A)g^{*\beta_2} + \delta \frac{A^{-\gamma}}{r} - \frac{g^*}{r - \alpha} &= c \\ K_2(A)\beta_2 g^{*\beta_2 - 1} - \frac{1}{r - \alpha} &= 0 \end{aligned}$$

<sup>18</sup>We drop the time index for notational convenience.

<sup>19</sup>For the calculation of this expected present value, see Dixit and Pindyck (1994, pp. 315-316).

Solving for  $g^*(A)$  and  $K_2(A)$  we obtain:

$$g^*(A) = \frac{\beta_2}{\beta_2 - 1}(r - \alpha) \left[ \left( \frac{\hat{A}}{A} \right)^\gamma - 1 \right] c \quad (\text{A.1.6a})$$

$$v(A, g) = m(A, g) + \frac{g^*(A)}{\beta_2(r - \alpha)} \left( \frac{g}{g^*(A)} \right)^{\beta_2} \quad (\text{A.1.6b})$$

where  $\hat{A} = (\frac{\delta}{rc})^{1/\gamma}$  is the last unit of land for which conversion is worthwhile, i.e.,  $\delta \hat{A}^{-\gamma}/r = c$ .

## A.2 Proof of Proposition 2

In order to determine the long-run rate of deforestation we use the procedure proposed by Di Corato *et al.* (2014). Again, dropping the time index for notational convenience and using Eq. (8), we define

$$\omega = \frac{g}{(\hat{A}/A)^\gamma - 1}, \quad \text{for } \omega > \bar{\omega} = \frac{\beta}{\beta - 1}(r - \alpha)c \quad (\text{A.2.1})$$

where  $\{\omega\}$  is a regulated process in the sense of Harrison (1985, chp. 2) with  $\bar{\omega}$  as lower reflecting barrier.

Taking logarithm on both sides of (A.2.1) we obtain

$$\ln \omega = \ln g - \ln \left[ e^{\gamma(\ln \hat{A} - \ln A)} - 1 \right] \quad (\text{A.2.2})$$

Using a first-order approximation on the RHS around the point,  $\widetilde{\ln A}$ , we have

$$\ln \omega \simeq x_0 + x_1 \ln A + \ln g \quad (\text{A.2.3})$$

where

$$\begin{aligned} x_0 &= -\left\{ \ln \left[ e^{\gamma(\ln \hat{A} - \widetilde{\ln A})} - 1 \right] + \frac{\gamma}{1 - e^{-\gamma(\ln \hat{A} - \widetilde{\ln A})}} \widetilde{\ln A} \right\} \\ x_1 &= \frac{\gamma}{1 - e^{-\gamma(\ln \hat{A} - \widetilde{\ln A})}} \end{aligned}$$

By a straightforward application of Ito's lemma,  $\ln \omega$ , evolves according to the same Brownian motion that drives  $\ln g$ , that is

$$d \ln g = \left( \alpha - \frac{1}{2} \sigma^2 \right) dt + \sigma dZ$$

Following Dixit (1993, p. 61) the long-run density function for  $\ln \omega$  fluctuating between an upper reflecting barrier,  $u \rightarrow \infty$ , and a lower reflecting barrier,  $\ln \bar{\omega}$ , is given by the following truncated exponential distribution:

$$\begin{aligned} f(\ln \omega) &= \begin{cases} 0 & \alpha \geq \frac{1}{2} \sigma^2, \\ -(2 \frac{\alpha}{\sigma^2} - 1) e^{(2 \frac{\alpha}{\sigma^2} - 1)(\ln \omega - \ln \bar{\omega})} & \alpha < \frac{1}{2} \sigma^2. \end{cases} \\ \text{for } \ln \bar{\omega} &< \ln \omega < \infty \end{aligned}$$

Note that every time  $\ln \omega$  reaches  $\ln \bar{\omega}$  ( $g$  is low enough) then  $A$  increases to prevent  $\ln \omega$  from passing the barrier (reflection), i.e.,  $d \ln \omega = 0$ . Hence, using (A.2.4), it follows that

$$d \ln A \simeq -d \ln g / x_1 \quad (\text{A.2.4})$$

Taking the expected value on both sides, we obtain

$$\frac{E\{d \ln A\}}{dt} = [(1/2)\sigma^2 - \alpha] \frac{1 - e^{-\gamma(\ln \hat{A} - \widetilde{\ln \hat{A}})}}{\gamma} \quad (\text{A.2.5})$$

Note that by the monotonicity property of the logarithm,  $\tilde{A}$  must exist such that  $\ln \tilde{A} = \widetilde{\ln \hat{A}}$ . This implies that the long-run average rate of deforestation can be written as follows:

$$\frac{E\{d \ln A\}}{dt} = [(1/2)\sigma^2 - \alpha] \frac{1 - (\tilde{A}/\hat{A})^\gamma}{\gamma} \quad (\text{A.2.6})$$

Eq. (10) can be then obtained by setting  $\tilde{A} = A_0$ .

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